

PLANS FOR  
WIND ENERGY SYSTEM SIMULATION

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ABSTRACT

Two new analysis tools, one a digital computer code and the other a special purpose hybrid computer, are introduced. The digital computer program, the Root Perturbation Method or RPM, is a new implementation of the classic Floquet procedure which circumvents numerical problems associated with the extraction of Floquet roots. The hybrid computer, the Wind Energy System Time-domain simulator (WEST), yields real-time loads and deformation information essential to design and system stability investigations.

INTRODUCTION

In the realm of wind energy system simulation, the MOSTAB-HFW and WINDLASS digital computer programs have attained highly developed states for loads and deformation predictions. Except for empirical refinement of some data, and other minor adjustments, the ability to predict loads and deformations has been fully developed. The next task in the simulation problem is to determine the stability margins of the complex (coupled) aeroelastic-structural system present in the windmill designs, including the effects of periodic coefficients. Two methods for the solution of this problem are possible.

A rigorous stability analysis using digital techniques can be performed. This analysis would yield characteristic roots which reveal the system stability characteristics directly. The classic Floquet procedure is essential in this regard since it treats linear operators that are periodic functions of time.

An alternative method ties in real-time simulation of the system which would provide assessment of the system stability in the time domain, and an interactive ability to test different control algorithms. This paper introduces new concepts on these classic methods, both of which are planned for development in CY 1978.

## PROPOSED DIGITAL STABILITY ANALYSIS PROGRAM

### Overview of Floquet Method

In the past, digital verification of the stability of periodic systems has been treated using Floquet procedures. However, the Floquet procedures suffer from two very important drawbacks.

Firstly, in determining the stability of a system whose eigenvalues are, say,  $\lambda_1, \lambda_2 \dots \lambda_n$ , the eigenvalues computed by the Floquet procedure are  $\gamma_1, \gamma_2 \dots \gamma_n$ , where

$$\gamma_j = e^{T\lambda_j} \quad j = 1, 2 \dots n \quad (1)$$

and  $T$  is the period of the physical system. Equation (1) says that the Floquet eigenvalues are exponentials of physical dynamic modes represented by  $T\lambda_j$ . While the values of  $\lambda_j$  will be spaced in magnitude to reflect the natural frequencies of the system, the  $\gamma_j$ 's will be spread out due to the exponentiation. This results in considerable significant figure accuracy requirements in order to achieve even moderate accuracy in the calculations of  $\lambda$ . This effectively limits the number of dynamic modes that can be computed using practical computer implementation of the Floquet procedures.

The second difficulty with Floquet emerges when one notes that if

$$\lambda_j = a_j + i b_j \quad (2)$$

is a solution to Equation (1), so is

$$\lambda_j = a_j + i (b_j + n2\pi) \quad (3)$$

Hence, the imaginary part cannot be uniquely determined, but only within multiples of  $2\pi$ .

### The Root Perturbation Method

A new method for estimating the characteristic roots of linear systems with periodic coefficients has been proposed. The Root Perturbation Method (RPM) is an alternative implementation of the Floquet procedure, but circumvents the drawbacks of Floquet. The RPM begins by assuming a mean value of the constant coefficient matrix and determining the eigenvalues of that system. For example, if the system is modelled

$$\dot{x} - M(t) x = N(t) u \quad (4)$$

then the eigenvalues and eigenvectors  $\mu$  and  $x$  are defined so that

$$M_0 X = X U \quad (5)$$

where  $M_0$  is the mean value of  $M(t)$  and  $U$  is the diagonal matrix made of the  $\mu$ 's. Defining the perturbation quantity

$$\bar{M}(t) \triangleq M(t) - M_0$$

and the vector

$$Z(\lambda) \triangleq e^{-u\lambda} X^{-1} \bar{M}(\lambda) X e^{u\lambda} \quad (6)$$

which is then used in the Neumann Series

$$H \triangleq \int_{\tau}^{\tau+t} Z(\lambda) d\lambda + \int_{\tau}^{\tau+t} Z(\lambda) \int_{\tau}^{\lambda} Z(\eta) d\eta d\lambda + \int_{\tau}^{\tau+t} Z(\lambda) \int_{\tau}^{\lambda} Z(\eta) \int_{\tau}^{\eta} Z(\beta) d\beta d\eta d\lambda + \dots \quad (7)$$

Some straightforward algebra will show that the sought characteristic roots,  $\lambda_j$ , are given by

$$\lambda_j = \mu_j + \frac{1}{T} \ln (h_{jj} + 1) \quad j = 1, 2 \dots n \quad (8)$$

where  $h_{jj}$  are the diagonal elements of H defined in Equation (7).

The above equation is a perturbation result which should be much less susceptible to numerical difficulties. Also, since the log term is small,  $\lambda_j$  will be close to  $\mu_j$  such that the  $2\pi$  multiple associated with  $\lambda_j$  is easily discerned.

#### PROPOSED HYBRID SYSTEM FOR STABILITY AND CONTROL ANALYSIS

As mentioned earlier, another viable method to examine stability, loads and deformations of a wind turbine system is to construct a hybrid simulation model. Special purpose hybrid computers are especially attractive here since they can simulate real-time aeroelastic phenomena. At this time, construction of such a simulator is planned for CY 1978. Dubbed WEST (Wind Energy System Time-domain simulator), the same aeroelastic math models incorporated in MOSTAB-HFW (not stability derivative models!) will be solved simultaneously with the tower, pod and drive train models, to create real-time loads and deformation information for the entire system. WEST will be equipped with external potentiometers and a patch panel which allow the user to easily reconfigure the control system algorithms. The unit will be supported by a Digital Support System (DSS) that aids the programmer in modelling any number of physical system geometry changes. WEST will produce accurate time domain results (time histories) of the operation of a given wind turbine plant. A wind spectral synthesizer will also be incorporated in WEST, which will enable calculation of loads and motions in a statistical wind environment.

#### SUMMARY

Two useful new tools have been outlined and are currently planned for next year; these will provide methods of determining the stability, loads and motions of an entire wind energy system. The Root Perturbation Method, a post-processor for the WINDLASS system, implements the classic Floquet method in a new way which is less susceptible to numerical difficulties. The WEST special purpose hybrid computer uses the best features of both digital and analog technology to model a wind energy system in real time. Both of these new tools promise to be major contributors to the future study and evolution of wind turbine systems.