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-- LASER DRIVEN LIGHT SAILS --

AN EXAMINATION OF THE POSSIBILITIES FOR INTERSTELLAR PROBES AND OTHER MISSIONS

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"Light," Potter said firmly.

"Light sail!" Rod shouted in sudden realization. "Good thinking." The whole bridge crew turned to look at the Captain. "Renner! Did you say the intruder is moving faster than it ought to be?"

"Yes, sir," Renner answered from his station across the bridge. "If it was launched from a habitable world circling the Mote."

"Could it have used a battery of laser cannon?"

"Sure, why not?" Renner wheeled over. "In fact, you could launch with a small battery, then add more cannon as the vehicle got farther and farther away. You get a terrific advantage that way. If one on the cannon breaks down you've got it right there in your system to repair it."

"Like leaving your motor home," Potter cried, "and you still able to use it."

"Well, there are efficiency problems. Depending on how tight the beam can be held," Renner answered. "Pity you couldn't use it for braking, too. Have you any reason to believe-"

Rod left them telling the Sailing Master about the variations in the Mote. For himself, he didn't particularly care. His problem was, what would the intruder do now?

It was twenty hours to rendezvous when Renner came to Blaine's post and asked to use the Captain's screens. The man apparently could not talk without a view screen connected to a computer. He would be mute with only his voice.

"Captain, look," he said, and threw a plot of the local stellar region on the screen. "The intruder came from here. Whoever launched it fired a laser cannon, or a set of laser cannon-probably a whole mess of them on asteroids, with mirrors to focus them-for about forty-five years, so the intruder would have a beam to travel on. The beam and the intruder both came straight in from the Mote."

From, "The Mote in God's Eye"
by Larry Niven and Jerry Pournelle

I. INTRODUCTION

Although the concept of interstellar travel has tantalized the imagination of many a scientist and science fiction afficionado, it has not fared very well when subjected to quantititive scrutiny. A few serious ideas have been proposed which may lie within both the realm of known physics and the range of possible technology. There is no denying, however, that the task of sending a probe even to the nearest stars will be supremely difficult.

To establish a historical context, it is interesting to consider two previous ideas for interstellar probes which have been proposed as marginally feasible within the next half-century or so. In 1962, Spencer and Jaffe[1] of JPL pointed out that the same advantages which accrue from using multiple stages for chemical rockets can be usefully exploited in rockets propelled by thermonuclear fusion. Even though the mass ratio (and, presumably, the cost!) would be very large, they reached the important conclusion that a five stage fusion rocket could send a one-way probe to Alpha Centauri in less than ten years. Numerous other interesting possibilities are raised by the Spencer and Jaffe paper, which should have attracted wider attention than it did.

Later, in 1968, Freeman Dyson[2] examined the use of nuclear explosions for propelling enormous starships similar in concept to the Orion ships studied in some detail (c. $10,000,000) by the Department of Defense in the early 1960's. He found that 3×10^5 one-megaton nuclear devices exploded at the focus of an enormous paraboloid could propel a craft weighing some 10^5 tons at an optium mean mission velocity of 10,000 Km/sec. Thus many nearby stars could be reached by large crews of explorers on voyages lasting a few centuries. It has been argued that such an expedition could follow naturally from the space colony concepts now being considered. Dyson pointed out the important result that good energy efficiency ($\xi > 50\%$) is obtainable with a single-stage ship and a reasonably small mass ratio ($R < 4$) provided that the mission velocity V is significantly less than the maximal exhaust velocity U (say $V < 3U/4$). This conclusion is universally applicable; and, of course, it implies the need for the maximum possible exhaust velocity. Ideally the most efficient possible exhaust product would thus be photons, for which $U = c$, where c is the speed of light.

Although photon propelled "rockets" have been discussed for some years, their exploitation probably would depend upon the efficient harnessing of matter/anti-matter annihilation. It seems safe to assume that this accomplishment lies far in the future. Another idea for photon propulsion has lately evolved, however, which may not be quite so fantastic. This is the concept of launching comparatively light-weight light sail probes to nearly relativistic velocities using enormous lasers in cislunar space. This would permit most of the capital hardware to remain near to the earth, where it could be frequently reused or perhaps employed for other projects, thus relieving many of the cost constraints.

To our best knowledge, the proposal that high energy lasers (hereafter sometimes called HELS) might be used to propel light sails was originated in the science fiction novel, "The Mote in God's Eye"[3]. Upon reading that story, the son of one of the authors of the present paper requested verification that such a propulsion scheme is possible.* This query led to the work reported here.

It has been only three years since Gerald O'Niell[4] proposed the construction of enormous colonies in space. Now this project is widely regarded as being not only feasible but probable on a time scale of a few decades. The present authors and their colleagues at W. J. Schafer Associates have shown that lasers may play a key role in the construction of such space colonies because of the economies which can result from using lasers to propel rockets using hydrogen as a reaction mass. In fact, it can be shown that

* Richard Rather -- private communication

Gigawatt (10^9 watt) HELS can deliver millions of tons of material from the earth's surface to high orbit more efficiently than any other means now being contemplated. Hence one "dedicated Grand Coulee Dam" could power the laser system which could build the space colony.

In a future where laser propulsion and huge space colonies become commonplace, it will not be unthinkable to step up to projects such as the laser propelled light sail. It is even possible that laser-light sails could provide a key link in colonizing the solar system because of greatly reduced travel times and payload limitations. Beyond the solar system, the need for interstellar probes will be guided by the findings of astronomy and the S.E.T.I. (Search for Extraterrestrial Intelligence) programs over the next two or three decades. We shall outline in this paper some possibilities for such missions. It will be shown that the laser light sail concept is probably feasible for one-way fly-by probe missions, provided that certain key technological advances can be achieved.

II. RULES OF THE GAME

To debate the possible efficacy of lasers for light sail propulsion we must probe well beyond the bounds of present technology. It is possible, however, to make some predictions of those technological facets of the conceptual design that will be most tractable and those that will not. Accordingly, in this section we will attempt to set some reasonable boundaries for the proposed system before beginning our evaluation.

A. The Sail

An obvious point of departure is the characterization of the sail itself. A strong upper limit on the capabilities of the sail is set by the need for it to survive the very intense photon flux which must impinge upon its shiny surface continuously for a year or more. Regardless of what material is chosen for the sail, it must be able to transfer all absorbed energy from the back of the sail (the side being irradiated) to the front side, where it can be radiated away to space. Hence the thermal diffusion time must be short, and the thermal loading must not exceed the damage threshold of the material. To minimize absorption, the back side will be coated with materials which are as reflective as possible at the wavelength of the laser, hopefully approaching 99% reflectivity. The front side will be black to provide maximum radiative cooling. The heat loading problem may be described as follows:

The time, Δt, required for heat absorbed at the back side of the sail to reach the front side is governed by the thermal diffusion constant, K, of the material. For a sail of thickness Δx, the relationship is

$$(\Delta x)^2 = 4K\Delta t$$

or

$$\Delta t = \frac{(\Delta x)^2}{4K} \qquad (1)$$

Materials of thickness 0.1 mil (i.e. 2.54μm) and having typical thermal diffusion constants $K = 10^{-2}$ cm^2/second give values of $\Delta t \lesssim 5 \times 10^{-6}$ seconds. This means that the heat is transferred almost instantaneously to the front of the sail. Hence the key requirement is for efficient radiation at the front side.

The energy absorbed per unit area is related to the incident irradience by

$$E_{absorbed} = \alpha E_{incident} ,$$

where, for 99% reflectivity, $\alpha = 0.01$.

In equilibrium,

$$E_{radiated} = E_{absorbed}$$

and thus, assuming black body radiation,

$$E_{radiated} = \sigma T^4 .$$

$$\therefore E_{incident} = 100 \sigma T^4 \qquad (2)$$

where $\sigma = 5.67 \times 10^{-8}$ $J/m^2/^{0}K^4$/second.

Hence it can be seen that a sail which can withstand temperatures of only $373^{0}K$ ($100^{0}C$) can handle incident fluxes no larger than 10^5 watts/m^2, while a sail that can sustain temperatures of $750^{0}K$ ($477^{0}C$) can tolerate 1.5×10^6 watts/m^2.

The weight of the sail material is, of course, another critical factor. The sail must be as light as possible consistent with the thermal constraints and the mechanical loading caused by the payload. Although materials such as tantalum, nickel, and gold have excellent thermal properties and are extremely malleable and ductile, they are quite dense. In order to keep the weight reasonable, these materials would have to be formed into foils much less than 1 micron thick. Such foils are extremely fragile. -- By comparison, 0.1 mil (i.e. 2.5 micron) thick aluminized Kapton, a polymer material now under consideration for solar propelled light sails, is some ten to twenty times lighter per unit volume, and very resistant to tearing. On the other hand, Kapton has a much lower melting point.

A neat resolution for this dilemma may be found in very thin composite materials made from graphite fibers. Such materials are both lightweight and very strong. In addition, they have a melting point which is quite high. Curiously, graphite also has very high thermal conductivity and readily accepts metal coating. Hence it seems to meet all of the necessary criteria.

As a working hypothesis we will assume the use of a 1 μm thick graphite composite which weighs 1000 kg/km^2. From laser vulnerability studies we can estimate the maximum permissible heat load (i.e. absorbed heat flux) to be $\sim 10^4$ $watts/m^2$.* This sets a strong upper bound on the ultimate performance achievable by the light sail because it essentially establishes the maximum thrust per square meter that can be delivered by the laser.

B. Interstellar Drag Forces.

Another property of the sail itself which must be considered is its resistance to the interstellar gas particles which it encounters as it proceeds along its path. This may seem strange because we are accustomed to thinking of interstellar space as a nearly perfect vacuum. Indeed, the interstellar gas contains, on the average, only about ten neutral hydrogen atoms per cubic centimeter and, perhaps, two neutral helium atoms. All other atomic and molecular species have concentrations

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* This corresponds to incident irradiance $I = 10^6$ w/m^2

which are ignorably small. At velocities of interest for interstellar probing, however, the volume of space swept out by the sail each second is so enormous that a significant back pressure might develop.

The pressure on the sail is the negative change of momentum suffered by all atoms incident upon unit area in unit time. The atoms bury themselves in the sail because it is moving much faster than the thermal velocity of the particles in the gas. Hence, the total change of momentum, Δp, is given by

$$\Delta p = N m_H v_s \quad ,$$

where m_H is the mass of a hydrogen atom, v_s is the velocity of the sail and N is the number of atoms encountered by the sail per second:

$$N = A_s \times \rho \times v_s$$

A is the area of the sail, and ρ is the gas density.

Therefore:

$$\Delta p = A_s \rho m_H v_s^2 .$$

substituting numbers into this equation, one finds for a velocity of 0.2c the surprisingly large back pressure of 60 Newtons per square kilometer of sail area, assuming that the atoms are stopped by the sail.

If the foregoing pressure actually obtained, one might think that it would be difficult to achieve the needed velocity by photon pressure. This is not the case, however, because it will be seen below that the photon pressure is actually quite large enough to dominate the drag from the gas. Surprisingly, the situation is made even better by the interesting fact that the sail would become transparent to hydrogen atoms at the speeds of interest:

The rest energy of hydrogen atoms is slightly less than 10^9 electron volts (931MeV). The kinetic energy, T, of a particle at any velocity is given by

$$T = m(1/(1-\beta^2)^{1/2} - 1) ,$$

where m is the rest mass and $\beta = v/c$. Referring to Figure 1, we see that even at a "modest" speed of $5 \times 10^{-3}c$ (i.e. 1500km/sec) the particle energy is already up to to 10 KeV.

Data from collision cross-section measurements for laboratory ion sources is available for many materials. The stopping range for hydrogen atoms penetrating lightweight composites is very long. In the energy range below 10KeV there is undoubtedly a significant dE/dx loss in graphite. At energies above 100KeV, however, the particles would pass through the material virtually unimpeded. This corresponds to a velocity of 0.015 c, or 4500 km/sec. The resistance force would probably never exceed 10^{-3} Newtons/km^2 of sail area at any velocity, and thus is ignorable in in our calculations.

C. The Laser Devices

The properties of the proposed hardware for delivering energy to the sail are sufficiently horrendous at first glance to be dangerous for the weak-hearted! Laser powers in the hundreds of Gigawatts must be contemplated. (Present technology is pushing toward the hundreds of Kilowatts regime). Nevertheless, when we consider the history of technological progress in the past forty years, it is entirely reasonable to assume that powers at least in the hundred Megawatt regime will be available by the year 2000. We must then consider the feasibility of using tens or hundreds of such lasers in parallel to achieve the required power level. It will be seen that this "ganging" concept fits well with the need for an extended, systhesized projection aperture to be discussed below. Figure 2 shows a conceptual design for a solar powered space laser proposed by one of the authors (JDGR) for another application. This so-called "STAG" device exemplifies one of the elements of the proposed projector: A "light bucket" approximately 250 meters in diameter delivers solar energy to a hypothetical closed-cycle visible light laser which produces a continuous coherent output power of ∿ 50 megawatts. The laser beam is expanded and transferred to a multi-faceted projection aperture 50 meters in diameter. The facets are driven by piezo-electric transducers to achieve full wavefront control in the manner of a phased microwave array. (Small controlled phasefront devices of this sort already exist (c.f. Ref. 5). Diffraction limited optical appertures 30 meters in diameter have already been studied by NASA (Ref. 6) and have been found to be feasible.)

The STAG device described above would comprise one element of a large distributed array of projectors. If a total power of 100 GW were required to power the light sail, 2000 of these devices would have to be phase-locked together by laser reference beams. The nearly filled synthesized aperture thus resulting would be a co-orbiting array some 6 kilometers in diameter. Such an array could project a low-loss nearly diffraction-limited beam to a 1.3 km diameter (i.e. $1 km^2$) light-sail at a range of 2×10^{10} kilometers. The equation for the transfer of nearly diffraction-limited gaussian beam (truncated at $1/e^2$ points) from a projector aperture of effective diameter D_p to a light sail of diameter D_s is

$$D_p D_s \doteq \frac{8 \lambda R}{\pi},$$

where R is the range. This becomes

$$D_p D_s \doteq 1.3 \times 10^{-9} R$$

for a wavelength of 0.5 μm. Although the flux on the sail decreases inversely with the square of the distance beyond the Fresnel zone, appreciable power will be transferred to ranges exceeding 10^{11} Km. (See text below and Figure 5).

D. Beam Pointing. Assuming that the ability to generate and project sufficient photon flux to power a light-sail can be achieved, there still remains the problem of keeping the light on the sail at vast ranges. Clearly, a pointing accuracy of $\sim 10^{-12}$ radians r.m.s. is required at a range of 10^{11} km. This might be impossible to achieve if it were not for the fortuitous circumstance that the target star would loom large and bright in the field of view of the projection optics. It should be possible to sense an image of the star at the focal point of the projection system and to maintain a precision bore-sight on the star's center because of the excellent photon statistics available. Then, the light sail itself could take care of the fine pointing by simply riding the beam. In fact, the sail might be shaped so as to be self-centering and dynamically stable.

The combined interactions of laser power, projector size, sail size and pointing accuracy are shown in the carpet plot" of Figure 3.

E. Velocity.

With all of the foregoing assumptions, it will be seen below that it is still difficult to launch a massive probe at speeds sufficient to reach the nearer stars in less than a human lifetime. An upper limit to credability is reached at a velocity of about 0.2c, which would permit reaching Proxima and α Centauri (A & B) in 22 years. To send a 10,000 Kilogram probe on this mission would require a total energy of 1.8×10^{19} Joules, which equates, in nuclear parlance, to 45 Megatons of energy. This corresponds to a transfer of 570GW-years of laser power.

The difficulties of reaching 0.2c are so great that it is an idle pursuit to investigate reaching higher velocities by the light sail method. At 0.2c, relativistic corrections to the velocity and mass amount to only ∿ 2%. We shall, consequently, ignore relativity in our calculations.

Another reason for ignoring higher velocities is that, without some means for slowing the probe as it approaches the target star, the encounter time becomes too short. At 0.2c a probe would traverse our solar system in about 50 hours. The most important data on the characteristics of the terrestrial planets would have to be acquired in approximately 2 hours, which is difficult but not impossible.

F. Probe Mass

To stay within the bounds of reason, the total mass of the payload plus light sail should not exceed 10,000 Kg. The weight must clearly be kept to a minimum. Assuming continuing progress in micro-miniaturization of electronic and computer components, it does not seem unreasonable that probes weighing a few thousand Kilograms could return much useful data. It is particularly interesting to consider using the light-sail itself as a huge transmitting antenna to greatly reduce the needed transmitter power to return the data to the earth.

G. Radiation Shielding

Referring again to Section IIB. and Figure 1, we see that the scientific payload of a probe moving at the velocities of interest will be subjected to a constant flux of very energetic particles. At 0.2c, 6 x 10^{14} atoms/m^2/second having energies of about 10^7eV will impinge upon the sail (and, presumably, upon the payload). This could be a serious problem for microelectronics intended to operate flawlessly for some 30 years! It appears, however, that this problem could be surmounted in a straightforward way:

It has already been shown that most of the neutral atoms encountered in instellar space will pass through the light sail with small dE/dx energy losses. Experience with laboratory ion sources shows that the principle result will be simple ionization of the atoms which penetrate the sail. The recombination time for the resulting charged nuclei and electrons will be quite long because of the long mean free paths and low collision frequencies. Hence, the payload will encounter essentially only protons and electrons, provided that it remains in a position such that all particles pass through the sail before reaching it. It is easy, then, to employ a superconducting coil in the probe to produce a magnetic field to shunt the charged particles past the payload. A clever design might even use the current of charged particles to generate power to regenerate the magnetic field or trickle-charge batteries.

H. Wavelength

In empty space the diameter of the focal spot, D_s, is related to the projector diameter, D_p, by

$$D_s = 2.44\frac{\lambda R}{D_p},$$

where R is the range to the sail. But the irradience, I, (i.e. irradient flux density impinging on the sail) is related to the power transmitted, P_t, by

$$I = \frac{4P_t}{\pi D_s^2},$$

whence, substituting, we get

$$I = \frac{4P_t D_p^2}{6\pi\lambda^2 R^2}$$

Hence the irradience is inversely proportional to λ^2. We shall see later how the irradience is the critical factor determining acceleration. Thus, there is an obvious need for the shortest possible wavelength.

Presently the highest energy lasers available operate at wavelengths of 3.7, 5.0 and 10.6 μm in the infrared. There are numerous major research efforts to develop shorter wavelength HELs, however. It is reasonable to assume that these efforts will succeed within the next decade or so. Hence, we feel that we can safely anticipate the availability of lasers yielding tens to hundreds of megawatts at 0.5 μm (i.e. visible) by the early 21st century. While shorter wavelength operation will undoubtedly be achieved eventually, the advantages for light sail propulsion may be insignificant. The reason for this is that the photons would become so energetic that they would penetrate the shiny metallic sail coating, producing surface damage and much greater heating of the sail material. We shall therefore assume a wavelength of 0.5 μm for our remaining deliberations.

III. ANALYSIS

Any mission involves the following parameters:

X = total distance to be traversed.

X_a = distance traversed during acceleration (i.e. required range of laser).

P_L = total power delivered to sail by laser.

t_a = time elapsed during acceleration (i.e. required laser on-time)

t = total trip time assuming no deceleration at destination.

V_a = velocity at termination of acceleration.

Then, it is easy to see that (ignoring relativistic effects):

$$X = \tfrac{1}{2} a t_a^2 + V_a (t - t_a) \qquad (1)$$

Or, substituting $V_a = at_a$,

$$X = \tfrac{1}{2} a t_a^2 + a t_a (t - t_a)$$

$$X = a t_a t - \tfrac{1}{2} a t_a^2 \qquad (2)$$

Now, since $X_a = \tfrac{1}{2} a t_a^2$, we have

$$X = at \sqrt{\frac{2X_a}{a}} - X_a$$

So, the key equation for the trip may be written

$$X = t\sqrt{2aX_a} - X_a \qquad (3)$$

Also, from equation 2 , we have

$$\frac{(at_a)^2}{2a} - (at_a)t + X = 0$$

Hence, from the quadratic formula,

$$at_a = \frac{t - \sqrt{t^2 - \frac{4X}{2a}}}{\frac{1}{a}}$$

Thus the maximum velocity is related to the acceleration by:

$$V_a = at - \sqrt{(at)^2 - 2ax}. \qquad (4)$$

A further useful permutation produces the form:

$$t_a^2 = \frac{4\ (V_a t - X) X_a}{V_a^2} \quad .$$

Now, given a light sail system capable of achieving a specified acceleration, we can choose a star at distance X, specify an acceptable trip time, t, and calculate the needed velocity. We can then back-calculate through equations 2 and 3 to obtain the needed laser range X_a, and irradience time, t_a. Alternately, we can decide on a maximum range and then calculate the other parameters.

The achievable acceleration is directly related to the deliverable laser power, P_L, by the radiation pressure:

$$\text{Radiation Force} = ma = \frac{2 \cdot P_L}{c}$$

Or:

$$a = \frac{2\ P_L}{mc} \qquad (5)$$

To achieve more acceleration, there are only two choices:

1. Increase the laser power.
2. Decrease the mass of the system.

Since the sail will always be operated at the maximum possible irradiance, which we have assumed to be 10^6 w/m^2, any increase in laser power will necessitate an increase in sail area. It might seem that the large sail might outweigh other disadvantages. From equation 5, the radiation force per unit area (i.e. pressure) is given by

$$F_a = \frac{2I}{c} ,$$

where I is the irradiance intensity. This gives

$$F_a = 6700 \text{ Newtons/km}^2$$

for $10^6 w/m^2$ (again calling attention to the ignorability of the 60 Newton/km^2 back pressure). But there is an asymptotic effect, because the mass of the sail increases as the square of its diameter. It turns out that, for a fixed intensity, increasing the sail area cannot possibly help the situation by more than a factor of ∿5, but this will be bought at a large price because of the greatly increased laser power required. This can be seen clearly by rewriting equation 5 in terms of the separate payload mass, m_p, and sail mass, m_s, as follows:

$$a = \frac{2 \quad P_L}{(m_p + m_s) c} \qquad (6)$$

Then we can state P_L in terms of the irradiance, I, and the sail area, A:

$$P_L = IA \qquad (7)$$

Further, we can relate the total sail mass, m_s, to the mass per unit area, m_s' by

$$m_s = m_s' A . \qquad (8)$$

So now, upon substituting equations 7 and 8 into 6, we get

$$a_{max} = \frac{2IA}{(m_p + m_s' A)c} \qquad (9)$$

If we allow the sail area to become very large, equation 9 approaches the limit

$$a = \frac{2I}{m_s' c} , \qquad (10)$$

Thus, the functional behavior is as shown in figure 4. If we adopt as a compromise a sail having total mass equal to the instrument probe mass, we find that

$$a = \tfrac{1}{2} a_{max} .$$

This provides a crude optimization for the apportionment of payload and sail mass.

Another matter which is worthy of consideration is the efficiency with which kinetic energy is transferred to the sail by the laser. The force on the sail, considering perfectly elastic recoil encounters of photons with the sail surface, is given by

$$F = \frac{2h\nu}{c} \frac{P_L}{h\nu} , \qquad (11)$$

where $h\nu/c$ is the photon momentum and $P_L/h\nu$ is the number of photons. Thus we can write

$$\frac{d^2X}{dt^2} = \frac{2P_L}{mc}.$$

Integration gives

$$v = \frac{2P_L}{mc} t \qquad (12)$$

Now, the kinetic energy of the sail is

$$E_s = -\tfrac{1}{2} mv^2 \qquad (13)$$

Substituting equation 12 into 13, we have

$$E_s = \frac{2 P_L^2 t^2}{mc^2}$$

Factoring, we see that

$$E_s = \left(\frac{2P_L t}{mc}\right) \frac{P_L t}{c} = \frac{v}{c} \cdot Pt = \beta Pt .$$

This simply says that the kinetic energy of the sail at any time is proportional both to the velocity and to the total energy, Pt, transmitted by the laser. One way to visualize this result is to think of a photon in the instant of encounter with the sail: Because the sail is moving rapidly away, the photon is redshifted in the encounter proportional to the sail velocity; and, hence, more energy is transferred per encounter at high speeds than at low speeds.

Now we have all of the ingredients necessary for a unified treatment of the entire laser light sail problem. For completeness we shall include a parameter, N, which is called "beam quality". This term expresses imperfections of the beam as a multiple of the diffraction limited spot size. A beam quality of 2, which is typical of excellent performance in present-day HELs, indicates that the best focal spot is twice as large as the theoretical optimum.

Consider a Gaussian beam for which the intensity, I, at any point in the focal spot is related to the intensity on the center line, I_C, by

$$I = I_c e^{-(r/a)^2}.$$

The included power within a radius r is then

$$P' = \int_0^r I 2\pi r dr = \pi a^2 I_C \left[1 - e^{-(r/a)^2}\right]$$

$$= P \left[1 - e^{-(r/a)^2}\right] \qquad (14)$$

For a focussed beam having an initially uniform cross-sectional area A and beam quality N, the power collected by a sail of area A_S at range R would then be:

$$P' = P \left[1 - e^{-\frac{A_s A}{\lambda^2 R^2 N^2}}\right] \Rightarrow \begin{cases} \frac{PA}{\lambda^2 R^2 N^2} A_s, & R \to \infty \\ P, & R \to 0 \end{cases} \qquad (15)$$

Thus, this formulation yields the proper behavior in both the near and far fields of the projection aperture. For the case in question, $P = I_0 A_s$, where I_0 is the maximum allowable intensity based upon the heat rejection characteristics of the sail.

The dynamics of the problem are described by

$$F = ma = \frac{2}{c} P' = \frac{2}{c} P \left[1 - e^{-\frac{A_s A}{\lambda^2 R^2 N^2}} \right]$$

$$= (m_p + m_s' A_s) \frac{1}{2} \frac{dv^2}{dR} \qquad (16)$$

where m_s' is defined as the mass of the sail per unit area. It is assumed that the focussing is continuously adjusted to its optimum value. Equation 16 then leads to

$$\frac{\lambda NC (m_p + m_s' A_s) V^2}{2 P \sqrt{A_s A}} = \int_{\frac{A_s A}{\lambda^2 R^2 N^2}}^{\infty} \frac{1-e^{-x}}{x^{3/2}} dx$$

$$= \frac{2}{\sqrt{x}} (1-e^{-x}) + 2\sqrt{\pi}\ \mathrm{erf}\, c\sqrt{x} \Rightarrow \begin{cases} 2/\sqrt{x}, & x \to \infty \\ 2\sqrt{\pi}, & x \to 0 \end{cases} \qquad (17)$$

Here we have set $x = A_s A/\lambda^2 R^2 N^2$. Equation 17 now contains all of the important parameters. The behavior is shown in Figure 5. Several important features should be noted:

(1) Were the sail to be irradiated to infinite range, half the final energy (i.e. 71% of the final speed) would have been imparted in the near field, i.e. for $R < \sqrt{A_s A}/\lambda N$. Consequently, there is little point in irradiation beyond the near field.

Figure 7 TRIP TO ALPHA CENTAURI ($R = 10^{13}$ KM)

10^9

(2) With fixed power and aperture per laser (e.g. with STAG type devices illustrated in Figure 2), the required number of lasers is proportional to the square of the intended terminal velocity.

(3) Holding other parameters fixed, the laser power and the sail area are both proportional to the fourth power of the terminal velocity. Thus, if speed requirements can be relaxed, a substantial saving in laser complexity and cost can be realized.

The last point can be clarified as follows. Neglecting the mass of the payload (for simplicity), and starting from

$$F = ma = \frac{mdv}{dt} = \frac{m}{2}\frac{dv^2}{dR} = \frac{2}{c}\langle I\rangle A_s \quad , \qquad (18)$$

where $\langle I\rangle$ is the mean intensity on the sail, we have

$$\frac{dv^2}{dR} = \frac{4}{c}\frac{\langle I\rangle A_s}{m_s' A_s} = \frac{4}{c}\frac{\langle I\rangle}{m'_s} \qquad (19)$$

Then, $$v^2 = \frac{4}{c}\frac{\langle I\rangle}{m_s'} R \qquad (20)$$

But $$\frac{\sqrt{A_s A}}{\lambda N R} = 1$$

at the near-field limit.

Putting $A_s = \frac{P}{\langle I\rangle}$

we get $$v^2 = \frac{4}{c}\frac{\langle I\rangle}{m_s'} \frac{\sqrt{\frac{AP}{\langle I\rangle}}}{\lambda N} .$$

$$= \frac{4}{c m_s' \lambda N} \sqrt{AP\langle I \rangle}$$

Then $$AP\langle I \rangle = \left[\frac{c m_s' \lambda N}{4} v^2\right]^2$$

Therefore $$P \propto v^4 .$$

In Figure 6, all of the foregoing wisdom is drawn together to show laser and irradiation range requirements versus attainable final speed. For this example we have adopted the constants discussed in the foregoing text: $I_0 = 10^6$ watts/m^2, $m_s' = 10^3$kg/km^2, N = 2, and $\lambda = 0.5\mu$m.

A final analytical point that must be made is that it is not possible to stop an interstellar light sail probe by using natural light from the target star or interstellar gas drag to retard the sail's motion. The use of starlight is intuitively impossible because the laser accelerates the probe by imparting hundreds of solar constants of radiant flux for a year or more. This cannot be undone in an encounter with a star which lasts only a few hours at best. -- In Section II-B we have already shown that interstellar drag forces are negligible during the acceleration of the sail. That they continue to be negligible throughout the flight and do not integrate to any significant amount can easily be shown:

Given that the total drag force is Dv^2 (where D has units of Newton sec^2 meter^{-2}), the equation of motion is

$$M \frac{dv}{dt} + Dv^2 = 0.$$

Integration gives the time to decelerate from the initial velocity V_{-} to a final velocity V_o:

$$t = \frac{M}{D}\left(\frac{1}{V_0} - \frac{1}{V_a}\right)$$

Any reasonable combination of payload, sail area and initial velocity gives deceleration times on the order of a million years. A corollary is that a close approach to a star to utilize "stellar wind" drag will also be utterly futile as a braking mechanism.

IV. CONCLUSIONS

It is now possible to summarize the conclusions of this study in terms of "practical" examples. Figures 7 and 8 show travel times versus laser parameters for various payloads. Figure 7 applies to a one-way flyby probe to α Centauri, while Figure 8 represents voyages to the planet Saturn during which the sail vehicle is accelerated for the first half of the trip and decelerated for the second half by a laser system at Saturn. In all cases, it is assumed that half of the mass of the vehicle is in the sail and the other half is in the payload, as suggested by the discussion on page 21 (c.f. ·ig.4).

The α Centauri probe mission is seen to be extremely difficult. For a 1000 kg payload to reach the star in 22 years (i.e. V_a=0.2c), The laser projector must have a power-aperture product of ∿1.6 x 10^8 GW-km^2. Thus a 1000 GW laser system would require a nearly filled projection aperture of area 1.6 x 10^5 km^2. This corresponds to a total aperture diameter of ∿ 450 km. The near-field focus would have to be maintained to a range of ∿ 3 x 10^{11} km. If the travel time could be relaxed to 60 years, 10,000 kg could be launched by a 1000 GW laser system having a diameter of "only" 142 km. Still better, a 60 year mission for a 1000 kg payload would require only a 45 km projection aperture for the 1000 GW laser system. (In all cases a beam quality of N=2 is assumed).

While the results for the interstellar probe mission can scarcely be called encouraging, the possibilities for using light sails to implement colonization of the solar system are very exciting indeed. Figure 8 shows that 100,000 kg payloads can be delivered to Saturn in 40 days by a 100 GW laser system feeding an effective aperture only 10 km in diameter. This is well within range of the capabilities of the STAG system which has been proposed by one of the present authors (J.D.G.R.) for power transmission from geosynchronons orbit to the earth and moon. In the example of Figure 8, it is assumed that a laser system has been established in orbit around Saturn's satellite Titan to decelerate incoming vehicles and accelerate return vehicles to the Earth. Such a system could be moved to Saturn by slow degrees using the dynamical principles of solar sailing. Perhaps the lasers themselves could provide the needed thrust to achieve the needed orbits.

In all of the foregoing work we have attempted to remain alert to fallacies which would relegate laser light sails to the category of impossible dreams. Although it is clear that we are conceptualizing technologies which push well beyond what is presently possible, we have not found any limitations of the *reductio ad absurdum* sort. If we are willing to accept the possibilities of very high laser power and very large, very precise projection structures, the possibilities seem realistic, particularly for efficient transportation within the solar system. A critical technological problem that muat be solved, however, is the phase-locking of large numbers of freely moving subcomponents of the synthesized projection aperture. Given sufficient time and incentive, we feel that it is safe to assume that this can be accomplished.

LIST OF REFERENCES

1. D. F. Spencer and L. D. Jaffe, "Feasibility of Interstellar Travel", JPL Technical Report #TR-32-233, (1962).

2. F. Dyson, "Interstellar Transport," Physics Today, pp. 41-45, October, 1968.

3. L. Niven and J. Pournelle, "The Mote in God's Eye", Simon and Schuster, New York, (1974).

4. Bridges, W. B.; Brunner, P.T.; Lazzara, S.P.; Nussmeier, T.A.; O'Meara, T.R.; Sanguimet, J.A.; and Brown, W.P., Jr.; "Coherent Optical Adaptive Techniques," Applied Optics, 12, 291-300 (1974).

5. Berggren, R.R. and Lenertz, G.E.; "Feasibility of a 30-Meter Space Based Laser Transmitter", NASA Technical Report #CR-134903, (1975).

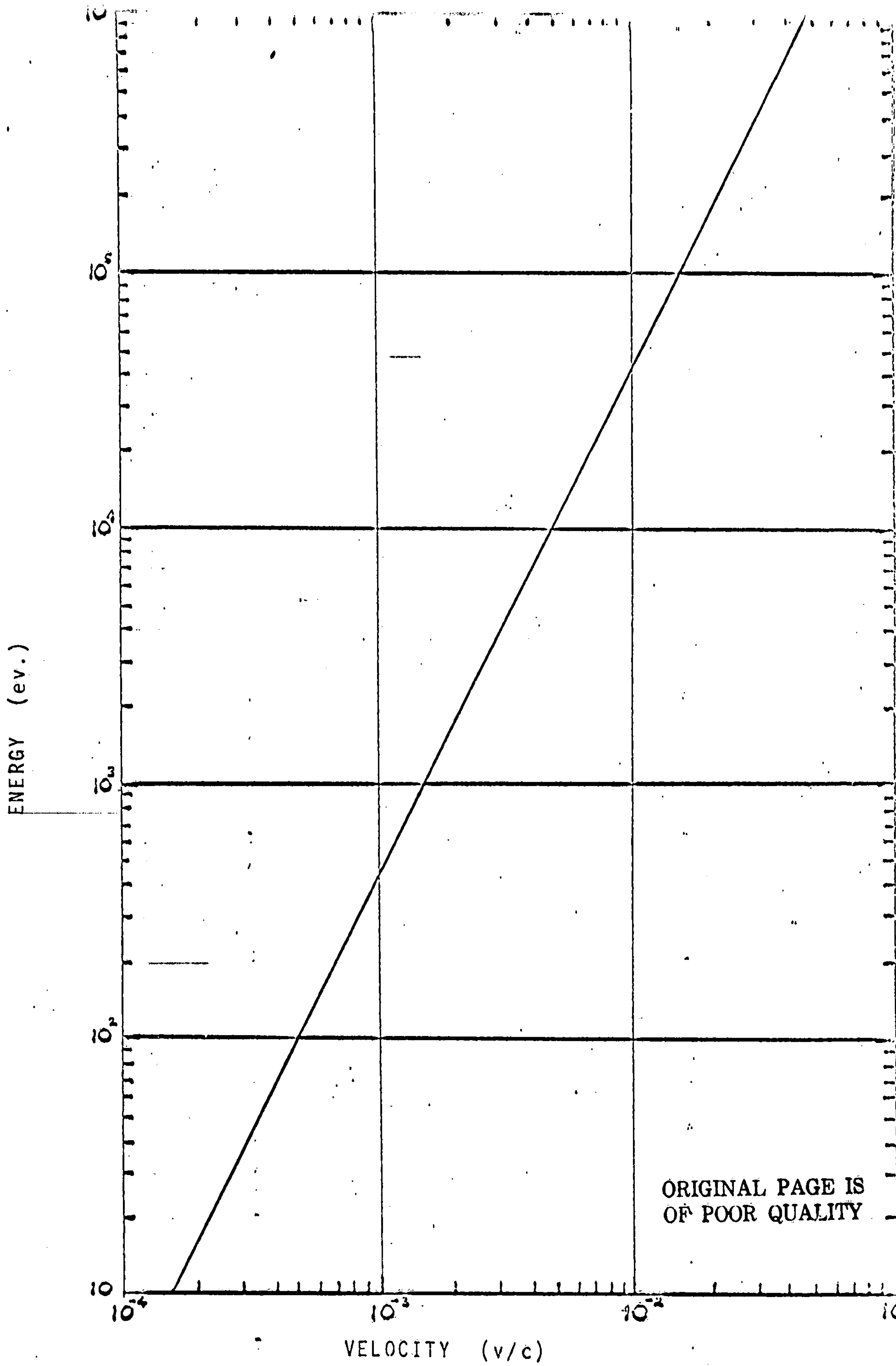


FIGURE 1. INTERSTELLAR GAS ENERGY VERSUS SAIL VELOCITY

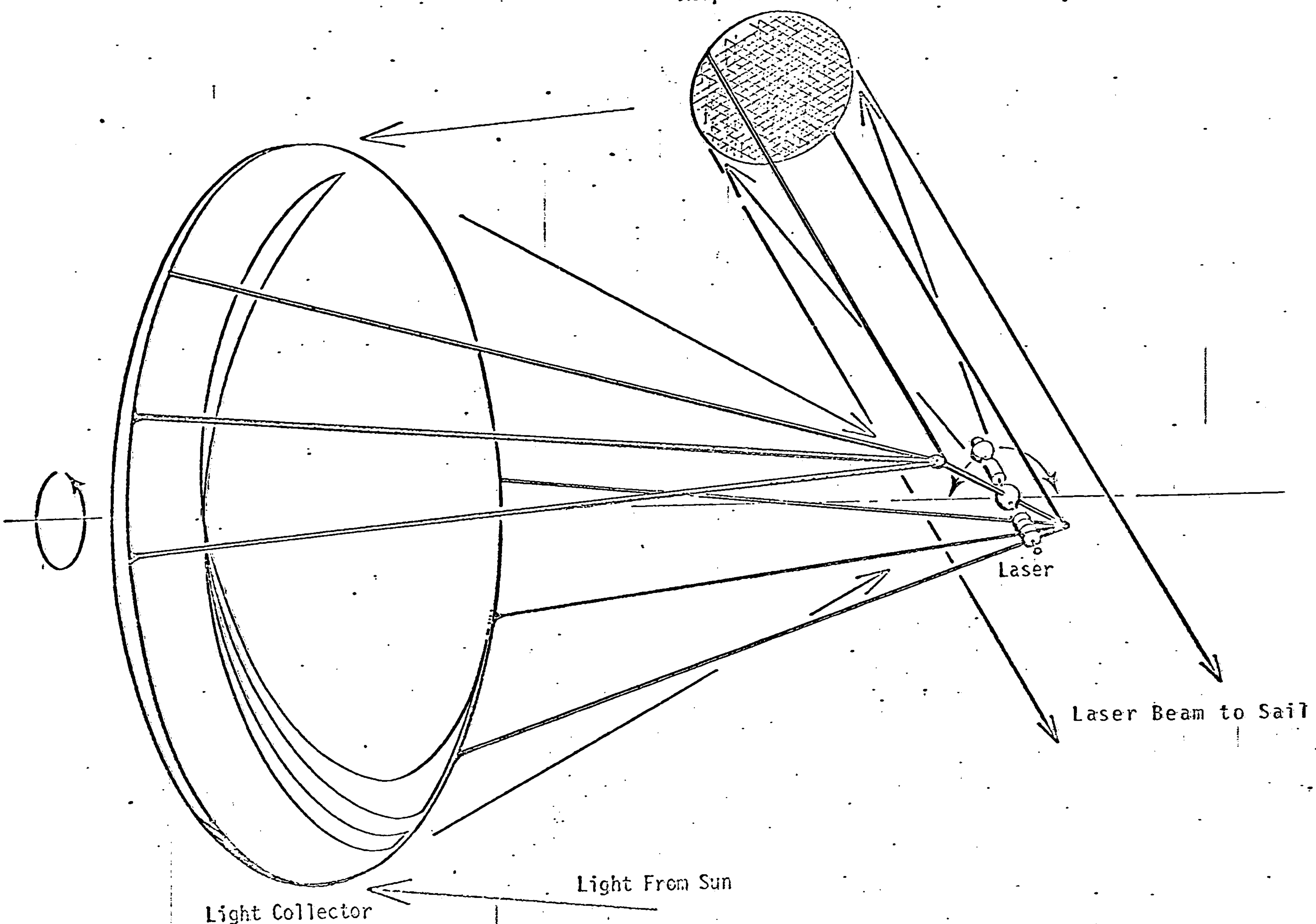
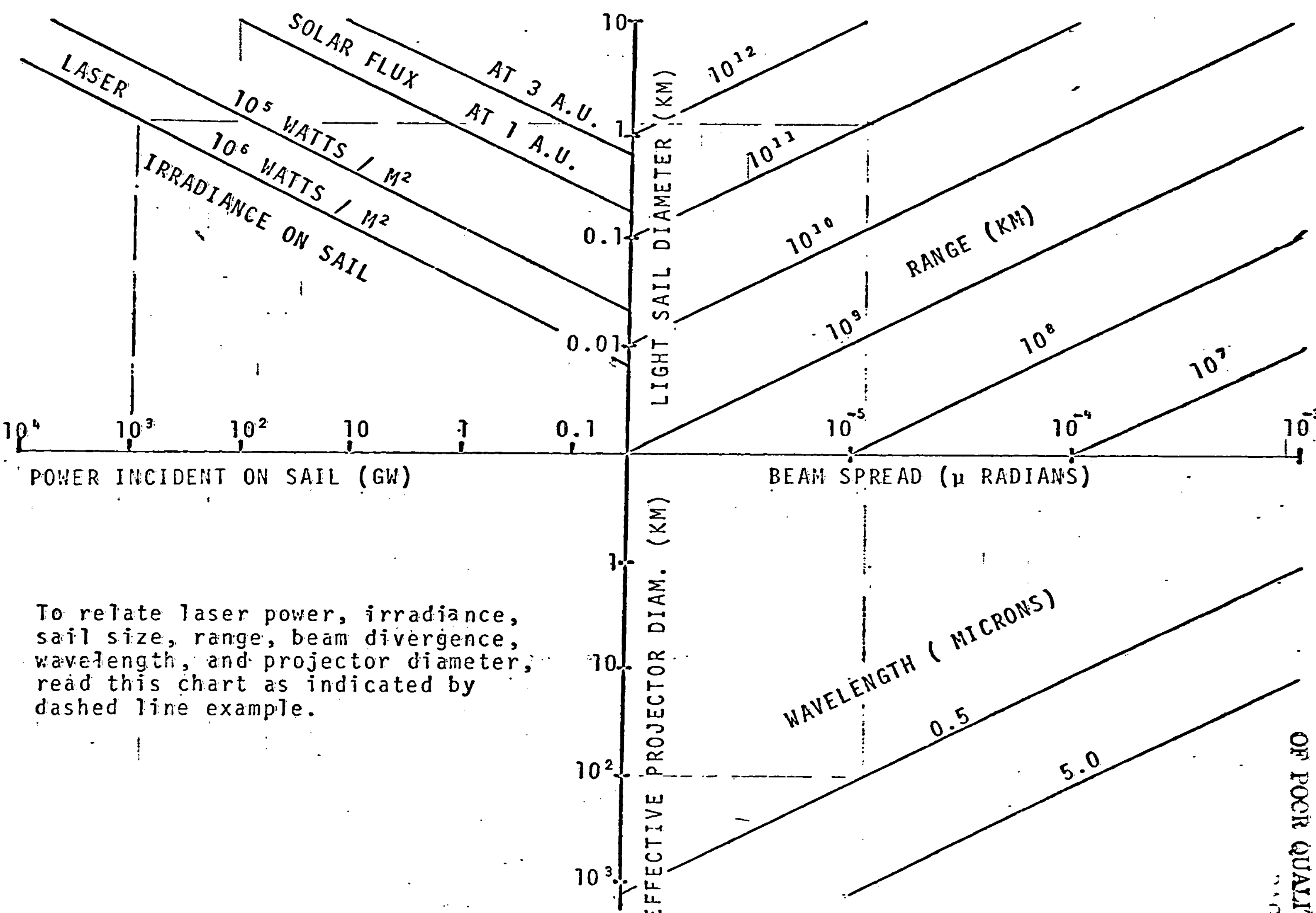


FIGURE 2. CONCEPTUAL VIEW OF A STAG DEVICE



To relate laser power, irradiance, sail size, range, beam divergence, wavelength, and projector diameter, read this chart as indicated by dashed line example.

FIGURE 3. LASER SAILING "CARPET PLOT"

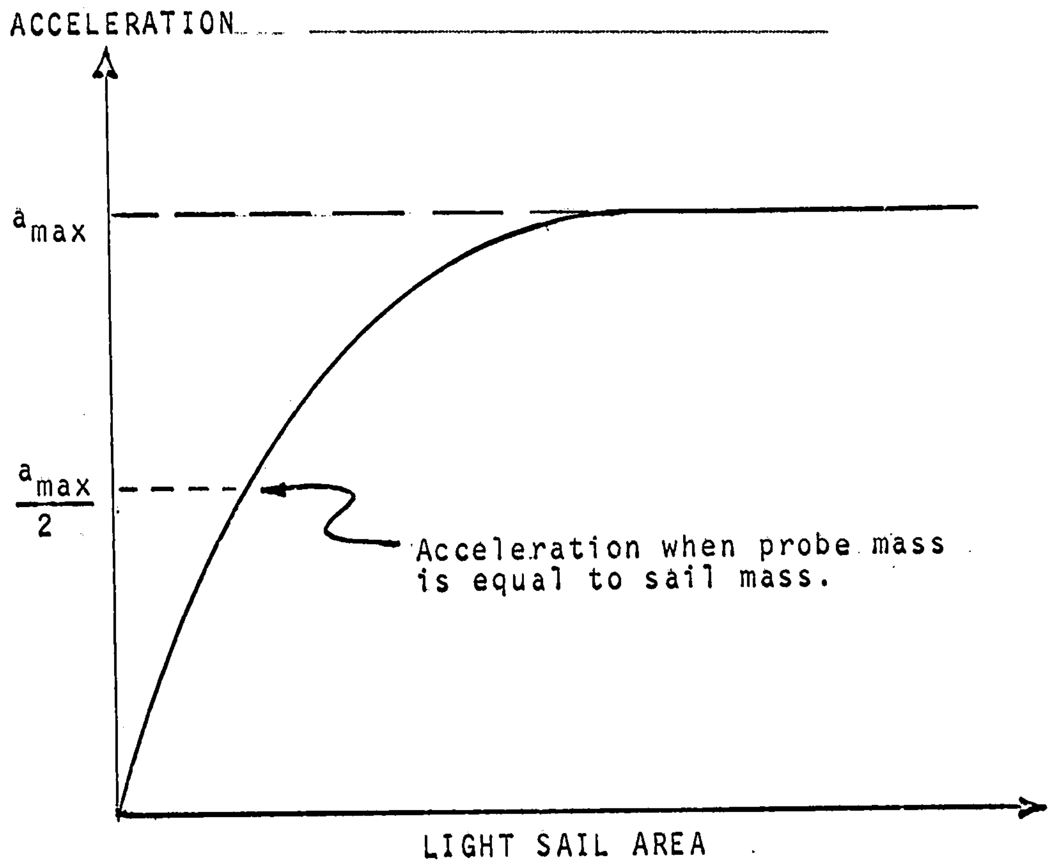


FIGURE 4. FUNCTIONAL RELATIONSHIP BETWEEN ACHIEVABLE ACCELERATION AND LIGHT SAIL AREA, ASSUMING CONSTANT IRRADIANCE.

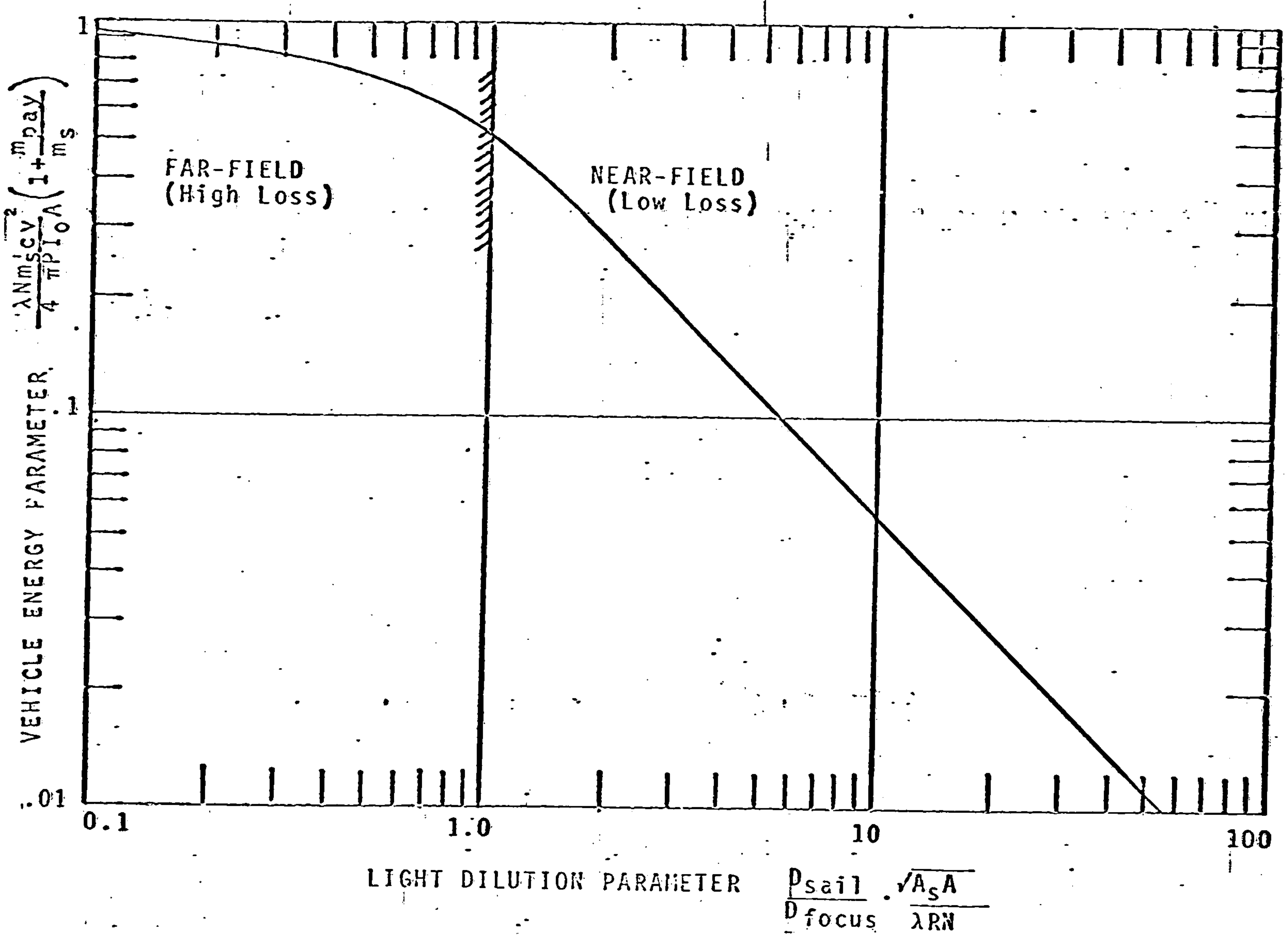


FIGURE 5 - This plot shows the efficiency of laser propulsion as a function of the concentration of power on the light sail. The Vehicle Energy Parameter is the ratio of the kinetic energy of the vehicle to the total energy transmitted by the laser. The Light Dilution Parameter indicates the amount of "spillover" of radiation at the range of the sail.

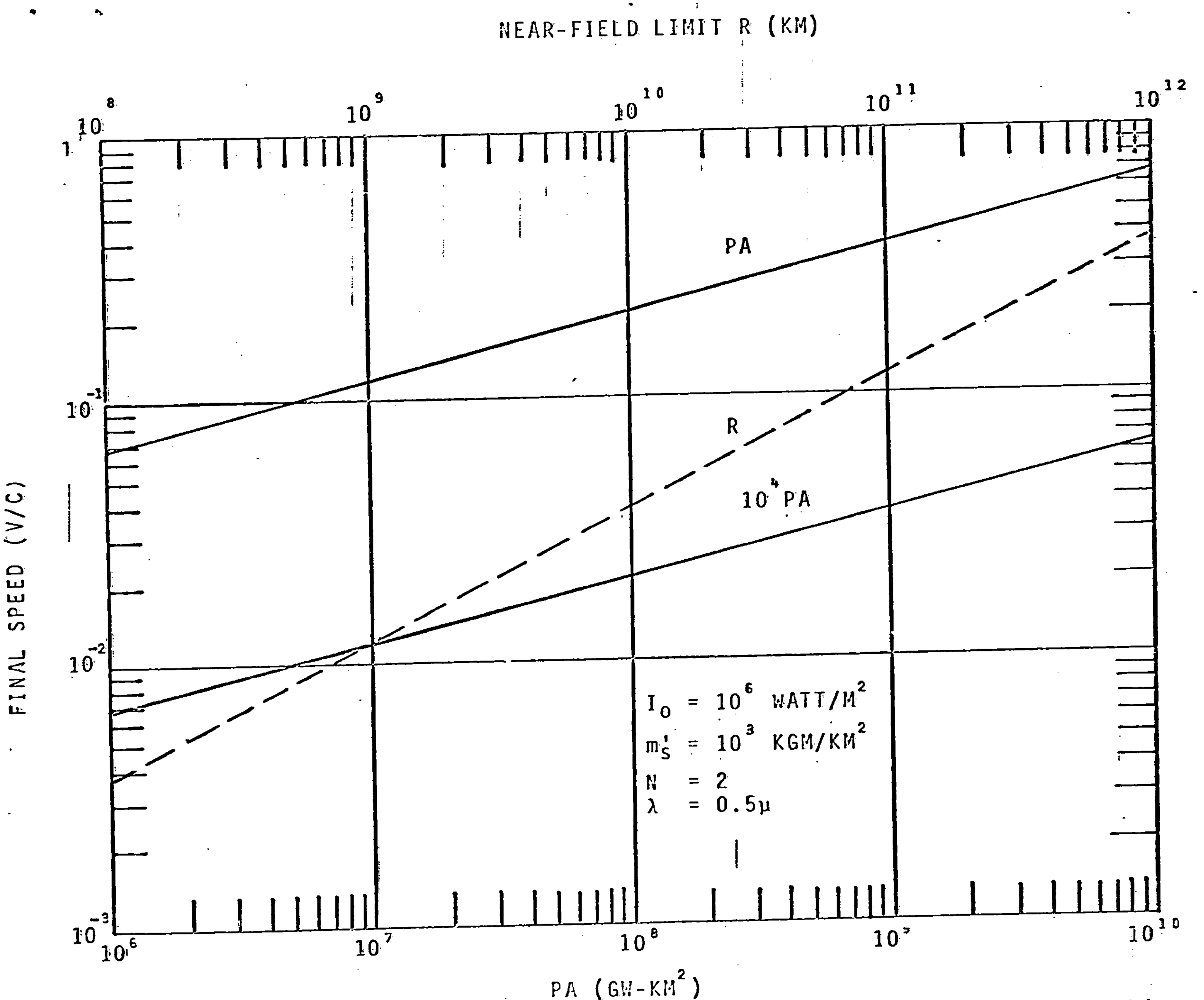


FIGURE 6 - Solid lines show required Laser Power - Projector Aperture product to achieve any desired final speed. (10^4PA line is continuation of upper PA line). In addition to the assumed constants indicated, the assumption is made that the payload mass is negligible compared to the sail mass. (Correction for finite payload mass can be applied later). -- Dashed line shows required near-field range to produce necessary acceleration to reach the desired final velocity.

Figure 7 TRIP TO ALPHA CENTAURI ($R = 10^{13}$ KM)

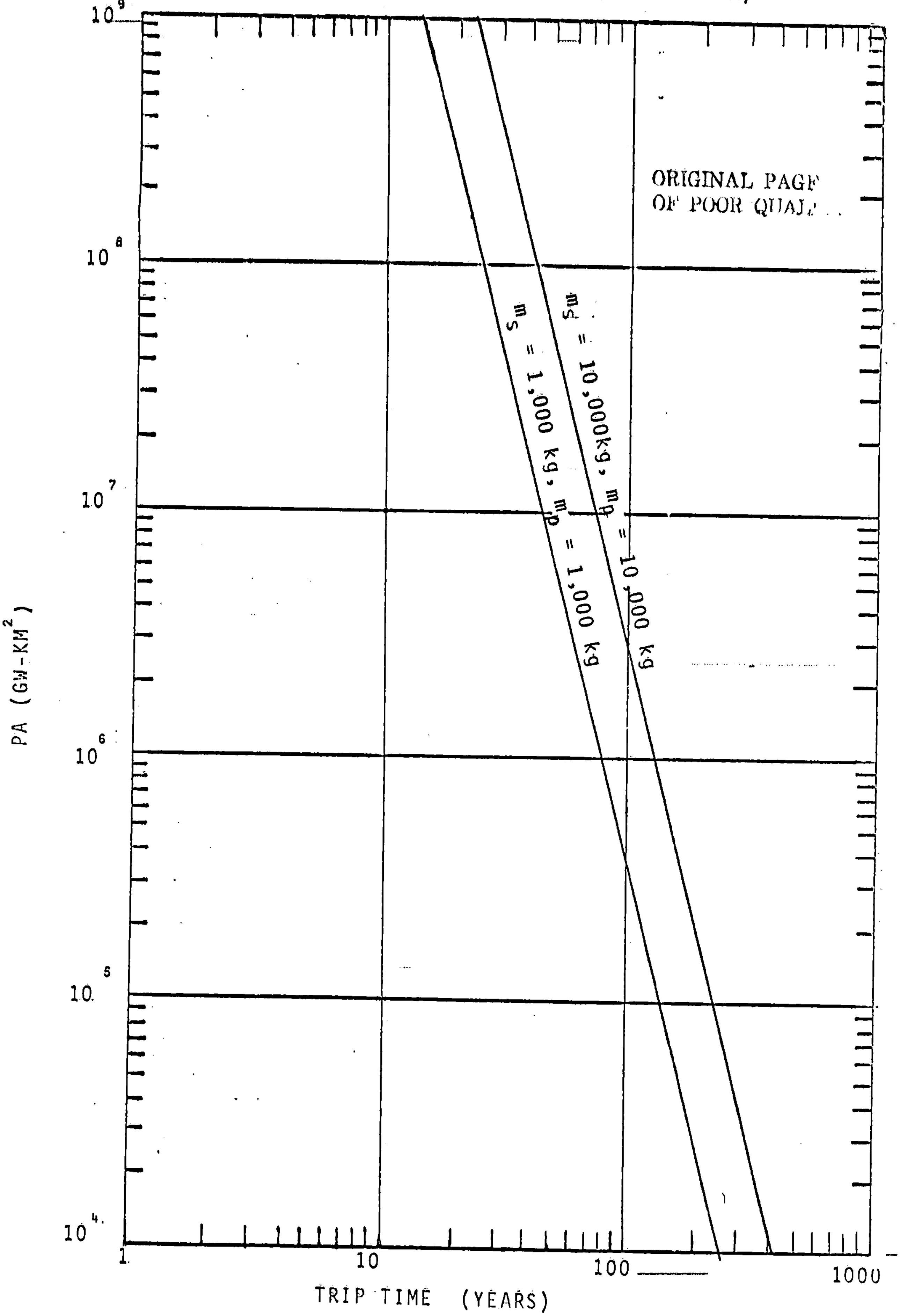


Figure 8 TRIP TO SATURN (R = 1.5 x 10^9 KM)