THE OPTIMAL CONTROL FREQUENCY RESPONSE PROBLEM IN MANUAL CONTROL

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SUMMARY

An optimal control frequency response problem is defined within the context of the optimal pilot model. The problem is designed to specify pilot model control frequencies reflective of important aircraft system properties, such as control feel system dynamics, airframe dynamics, and gust environment, as well as man-machine properties, such as task and attention allocation. This is accomplished by determining a bounded set of control frequencies which minimize the total control cost. The bounds are given by zero and the neuromuscular control frequency response for each control actuator. This approach is fully adaptive, i.e., does not depend upon user entered estimates. An algorithm is developed to solve this optimal control frequency r[illegible]nse problem. The algorithm is then applied to an attitude hold [illegible] for a bare airframe fighter aircraft case with interesting dynamic properties.

INTRODUCTION

Application of the optimal pilot model to complex aircraft systems and real world tasks has identified deficiencies in the control frequency response specification [28]. Existing methods rely on user supplied estimates, either directly with respect to the control filter cutoff frequencies or time constants, or indirectly with respect to control amplitude and rate penalties [3-20,30]. Reference [28] provides some insight into the dependency of control frequency response upon aircraft system dynamics for a particular application. But, in general, there exists limited mathematical guidelines for control frequency response specification.

This paper presents a method for optimal pilot model control frequency response specification which is suitable for complex aircraft systems and tasks. The method is predictive and reflects important manned aircraft system properties, such as control feel system dynamics, airframe dynamics, gust environment, task, and attention allocation.

SYMBOLS

A_a	Augmented open loop dynamics matrix ($n_s \times n_s$)
A_p	Augmented open loop dynamics matrix containing control filter ($n_s \times n_s$)
$\bar{A}$	Augmented closed loop dynamics matrix ($n_s \times n_s$)
B_a	Augmented control distribution matrix ($n_s \times n_c$)
C_a	Augmented measurement distribution matrix ($n_m \times n_s$)
E	Expected value
E_a	Augmented disturbance distribution matrix ($n_s \times n_d$)
E_f	Filtering error matrix ($n_s \times n_s$)
E_p	Prediction error matrix ($n_s \times n_s$)
F_a	Augmented feedback matrix ($n_c \times n_s$)
J	Control cost
L	Effective feedback matrix to unaugmented state system ($n_c \times n_x$)
n_c	Number of controls
n_d	Number of disturbances
n_m	Number of measurements
n_s	Number of states in augmented system
n_x	Number of states in unaugmented system
P_a	Riccati control gain matrix for augmented system ($n_s \times n_s$)

Q	Measurement penalty matrix ($n_m \times n_m$)
R	Control rate penalty matrix ($n_c \times n_c$)
$\underline{u}$	Pilot's control input, a vector of dimension n_c
V_m	Autocovariance of motor noise, a vector of dimension n_c
V_y	Autocovariance of measurement noise, a vector of dimension n_m
$\underline{w}$	A disturbance vector of Gaussian white noise, a vector of dimension n_d
x_a	State of the augmented system, a vector of dimension n_s
X_a	State covariance matrix of the augmented system ($n_s \times n_s$)
$\underline{y}$	A vector of measurements available to the pilot, of dimension n_m
Δ	Step size
Γ	Transformation matrix
Σ	Riccati filter covariance matrix
τ	Pure time delay
ω_c	Control cutoff frequency
ω_N	Neuromuscular cutoff frequency
Φ_c	Control filter matrix

Subscripts

a	Augmented
p	Perceived

Superscripts

T	Transpose
*	Optimal
^	Estimated parameter

OPTIMAL PILOT MODEL

The optimal pilot model concept, developed by Kleinman, Baron, and Levison [3-20], has demonstrated success in modeling complex, time varying control tasks. The optimal pilot model is a mathematical construct designed to synthesize pilot control performance and behavior. The model is based on the assumption that the human operator will control a dynamic, stochastic system optimally subject to his inherent limitations. These limitations are considered to be

1. A time delay, representing cognitive, visual central processing, and neuromotor delays.

2. "Remnant" signals, divided into an observation noise to represent signal degradation due to work load, scanning effects, and signal thresholds, and a motor noise to represent random errors in executing the intended control.

3. A "neuromuscular lag" to represent neuromuscular dynamics.

The control commands are synthesized by a continuous linear equalization network which contains a full state optimal filter (Kalman filter), a full state optimal predictor, and a full state optimal feedback control law. The control law is derived for an augmented state system which results from introducing the neuromuscular lag by means of a control rate penalty. The structure of the model results from a suboptimal solution to a control problem involving a time delay and observation noise. The model is shown in Figure 1.

The mathematical algorithm of the optimal pilot model is deve c from the following control problem:

Given the quadratic cost functional of the form

$$J = 1/2 \int_0^\infty E \{\underline{y}^T(t)\, Q\, \underline{y}(t) + \underline{u}_a^T(t) R \underline{u}_a(t) \qquad (1)$$

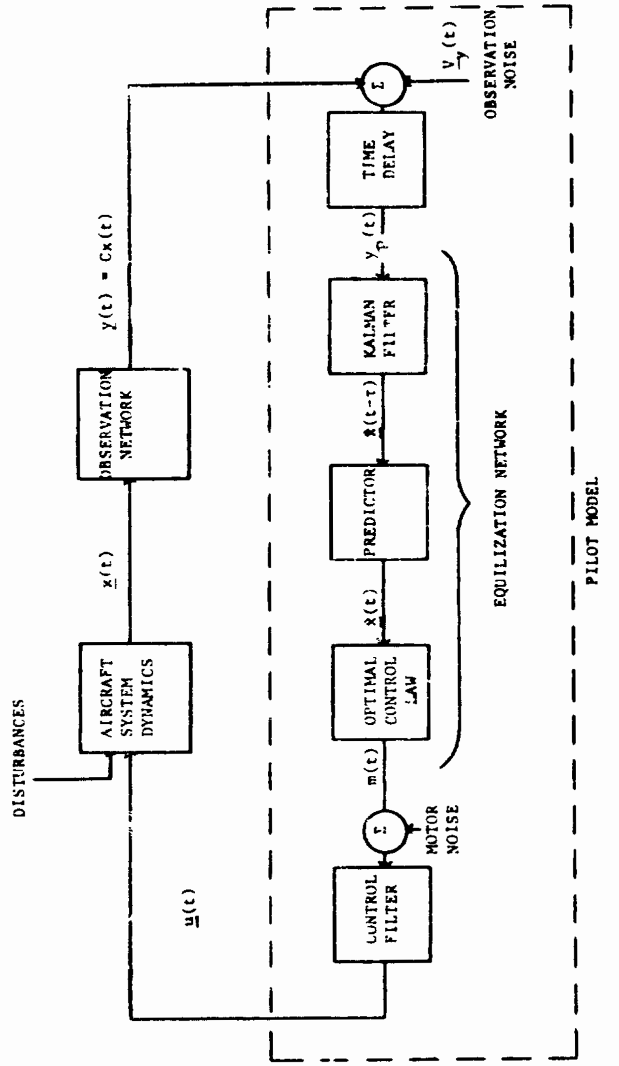

Figure 1 Structure of Optimal Pilot Model

Subject to the constraints

$$\underline{\dot{x}}_a(t) = A_a \underline{x}_a(t) + B_a \underline{u}_a(t) + E_a \underline{w}_a(t) \quad (2)$$

$$\underline{y}(t) = C_a \underline{x}_a(t - \tau) + \underline{V}_y(t - \tau), \quad (3)$$

determine the non-anticipative feedback control $\underline{u}^*_a(t)$ which minimizes the cost functional.

CONTROL FREQUENCY RESPONSE IN THE OPTIMAL PILOT MODEL

The pilot control frequency response is regulated in the optimal pilot model by a first order filter matrix which processes the commanded control signals such that

$$\underline{\dot{u}} = - \Omega_c L \underline{x} - \Omega_c \underline{u} \quad (4)$$

The filter matrix, Ω_c, is derived from the augmented control Riccati solution

$$A_a^T P_a + P_a A_a + C_a^T Q C_a - P_a B_a R^{-1} B_a^T P_a = 0 \quad (5)$$

The closed loop dynamics matrix '- then given by

$$\bar{A} = A_a - B_a F_a \quad (6)$$

where the augmented feedback matrix F_a is given by

$$F_a = R^{-1} B_a^T P_a \quad (7)$$

The augmented feedback matrix F_a can be partitioned such that

$$F_a = [\Omega_c L \vdots \Omega_c] \quad (8)$$

where the filter matrix Ω_c is given by

$$\Omega_c = R^{-1} P_f \tag{9}$$

and

$$P_a = \left[\begin{array}{c|c} P & P_e^T \\ \hline P_e & P_f \end{array}\right] \tag{10}$$

The optimal control $\underline{u}_a^*$ is then given by

$$\underline{u}_a^* = -F_a \underline{x}_a$$

which yields equation (4)

$$\underline{u}_a^* = \dot{\underline{u}} = -\Omega_c L \underline{x} - \Omega_c \underline{u}$$

The diagonal elements ω_{c_i}, $i=1, \ldots, n_c$ of Ω_c represent the first order cut-off frequencies of the control inputs u_i, $i=1, \ldots, n_c$. This can be shown by rearranging equation (4) to isolate the ith control input:

$$u_i = \frac{\omega_{c_i}}{s + \omega_{c_i}} \left[\frac{1}{\omega_{c_i}} \left(- \sum_{j=1}^{n_x} (\Omega_c L)_{ij} x_j - \sum_{\substack{j=1 \\ j \neq i}}^{n_c} \Omega_{c_{ij}} u_j \right) \right] \tag{11}$$

The cut-off frequencies are constrained such that

$$0 \le \omega_{c_i} \le \omega_{N_i}, \; i=1, \ldots, n_c \tag{12}$$

where $\omega_{c_i} \ge 0$ by definition and ω_{N_i} is the pilot's neuromuscular frequency limit for the ith control input.

An iterative technique has been developed by the author [31] by which the control solution can be regulated so that the cutoff frequencies ω_{c_i}, $i=1, \ldots, n_c$, attain any desired set of values, subject to the constraints of equation (12). A technique is then required by which these cutoff frequencies be specified.

CONTROL FREQUENCY RESPONSE REQUIREMENTS

The importance of proper specification of the cutoff frequencies ω_{c_i}, $i=1, \ldots, n_c$, must be emphasized. The accuracy of the specified cutoff frequencies can greatly impact the fidelity of the optimal pilot model. Flight and simulation data [1,2] indicate real world pilot control frequency behavior not amenable to a priori estimation by existing modeling methods. User supplied estimates can thus be grossly inaccurate. In addition, these estimates impact other elements of the optimal pilot model, including the covariance propagation and optimal attention allocation algorithms. Thus the entire spectrum of performance predictions generated by the optimal pilot model depend upon the control frequency response specification.

The utility of the optimal pilot model depends, for many applications, on its predictive and adaptive capabilities. The capability to predict pilot control frequency response reflective of important manned aircraft system properties, and consistent with the other model performance predictions, would thus greatly enhance the utility of the model.

OPTIMAL CONTROL FREQUENCY RESPONSE

It is hypothesized that the human operator will adapt his control frequency response to control a dynamic, stochastic system optimally subject to his inherent limitations. These limitations include "neuromuscular lags" to represent neuromuscular dynamics. This hypothesis is a simple extension of the formulation of the optimal pilot model [3-20] as well as recent developments such as optimal attention allocation [30,32].

It is furthermore hypothesized that the control frequency response minimizes the same quadratic cost function from which the optimal pilot model is developed:

$$J = 1/2 \int_0^\infty E\{ y^T(t) Q\, \underline{y}(t) + \underline{u}_a^T(t)\, R\, \underline{u}_a(t) \}\, dt \quad (1)$$

Thus it is required to determine the cutoff frequencies ω_{c_i}, i=1, ..., n_c, which minimize the cost functional. The cutoff frequencies are bounded such that

$$0 \le \omega_{c_i} \le \omega_{N_i}, \quad i=1, \ldots, n_c \quad (12)$$

where $\omega_{c_i} \ge 0$ by definition and ω_{N_i} is the human operator's neuromuscular frequency limit for the ith control input.

OPTIMAL CONTROL FREQUENCY RESPONSE ALGORITHM

Quadratic Cost Functional [33]

The cost functional J can be rewritten in the steady state as

$$J = E\{\underline{y}^T(t)\, Q\, \underline{y}(t) + \underline{u}_a^T(t)\, R\, \underline{u}_a(t)\} \quad (13)$$

or

$$J = E\{\underline{y}^T(t)\, Q\, \underline{y}(t) + \underline{\dot{u}}^T(t)\, R\, \underline{\dot{u}}(t)\} \quad (14)$$

This may be further rewritten as

$$J = tr[\underline{y}^T(t)\, Q\underline{y}(t) + R\, E\{\underline{\dot{u}}^T(t)\, \underline{\dot{u}}(t)\}] \quad (15)$$

The control ate, $\underline{\dot{u}}(t)$, may be approximated by the pilot's own estimate of $\underline{\dot{u}}(t)$. N e that the actual $\underline{\dot{u}}(t)$ is modeled to contain white motor noises. Thus

$$\begin{aligned} \underline{\dot{u}}(t) \doteq \hat{\underline{\dot{u}}}(t) &= -\Omega_c\, L\, \underline{\hat{x}}(t) - \Omega_c\, \underline{u}(t) \\ &= -\Omega_c L\, \underline{\hat{x}}(t) - \Omega_c\, \underline{\hat{u}}(t) - \Omega_c\, \underline{u}_e(t) \\ &= -F\, \underline{\hat{x}}_a(t) - \Omega_c\, \underline{u}_e(t) \end{aligned} \quad (16)$$

where $\underline{u}_e(t)$ is the error $(\underline{u}(t) - \underline{\hat{u}}(t))$. Since $\underline{\hat{x}}_a(t)$ and $\underline{u}_e(t)$ are uncorrelated for linear optimal estimation,

$$E\{\underline{\dot{u}}(t)\, \underline{\dot{u}}^T(t)\} = F\, \hat{X}_a F^T + [0 \vdots \Omega_c]\, (E_f + E_p) \begin{bmatrix} 0 \\ \hline \Omega_c^T \end{bmatrix} \quad (17)$$

where E_f, E_p and $\hat{X}_a$ are the filtering error, prediction error and augmented state estimate covariance matrices, respectively. The cost function J can therefore be given by

$$J = tr[\underline{y}^T(t) Q \underline{y}(t) + R\{F\, \hat{X}_a F^T + [0 \vdots \Omega_c](E_f + E_p) \begin{bmatrix} 0 \\ \hline \Omega_c^T \end{bmatrix} \}] \quad (18)$$

Gradient Expressions

It is apparent from equation (18) and equations (4) through (10) that the cost functional J has a very complex and indirect relationship to the control cutoff frequencies ω_{c_i}, i=1, ..., n_c. Since it is unlikely that a closed-form solution for the optimal control cutoff frequencies can be found, the optimization process will be performed via a gradient algorithm similar to that developed by Kleinman [32]. Expressions will be developed for the gradient vector

$$g_{\omega_c} = \frac{\partial J}{\partial \omega_c}$$

that will be used in the subsequent optimization algorithm. Since the number of controls is usually small, this gradient vector can be evaluated numerically without incurring excessive computational cost.

This is accomplished by modifying the gradient g_{ω_c} whenever

$$g_{\omega_{c_i}} > 0 \quad \text{and} \quad \delta > \omega_{c_i} > 0 \qquad (23)$$

or

$$g_{\omega_{c_i}} < 0 \quad \text{and} \quad (\omega_{N_i} - \delta) < \omega_{c_i} < \omega_{N_i} \qquad (24)$$

where δ is some small frequency. Since moving in a direction opposite to the gradient would result in either a negative or a physically unattainable control cutoff frequency, the only feasible direction is given by $g_{\omega_{c_i}} = 0$. Thus ω_{c_i} remains fixed for the next iteration. The resulting gradient vector is given by $g^P_{\omega_c}$, the projected gradient vector.

Control Frequency Response Optimization

The optimization scheme to minimize the quadratic cost functional J is developed as follows. A small change in the control cutoff frequency vector along the projected gradient vector such that

$$\omega_c(k+1) = \omega_c(k) + \Delta\omega_c(k) \qquad (25)$$

will still satisfy the constraints and will cause a small change in J(k),

$$J(k+1) = J(k) + (g^P_{\omega_c})^T \Delta\omega_c(k) \qquad (26)$$

If $\Delta\omega_c(k)$ is selected as

$$\Delta\omega_c(k) = -\frac{\epsilon\, J(k)\, g^P_{\omega_c}}{\| g^P_{\omega_c} \|^2}, \quad \epsilon < 1 \qquad (27)$$

then

$$J(k+1) = (1-\epsilon)J(k) \qquad (28)$$

Thus, each successive iteration will result in a lower cost of 100ε% if ε is sufficiently small.

A step size of the form

$$\epsilon = \beta\, \epsilon_{MAX}, \quad \beta \le 1 \qquad (29)$$

will reduce the step size when near the optimum and still satisfy the constraints if ε is sufficiently small so that

$$0 < \omega_{c_i}(k+1) < \omega_{N_i}, \; i=1, \ldots, n_c \qquad (12a)$$

This can be assured by defining ϵ_{MAX} to be the greatest value less than unity for which

$$0 \le (\omega_{c_i}(k) - \frac{\epsilon_{MAX} J(k)}{\| g_{\omega_c} \|^2} g_{\omega_c}) < \omega_{N_i} \qquad (30)$$

for all i, i=1, ..., n_c. If the step size causes an increase in J, a smaller step size can be taken by reducing β. Convergence occurs when J(k+1) is sufficiently close to J(k). Note that $\omega_c(k+1)$ will continue to satisfy the constraints imposed on $\omega_c(k)$. It is only required that the user supplied initial estimate satisfy the constraints.

Computation Requirements

The optimal control frequency response gradient algorithm exercises major blocks of the basic optimal pilot model. Several considerations allow for efficient execution of this algorithm. The algorithm requires n_c (usually only one or two) attaches to a control frequency response regulation algorithm [31] to determine the transformation matrix Γ.

A frequency step size of .5 radians/second is recommended to identify appropriate control rate penalty matrix step sizes for the subsequent determination of the gradient $\frac{\partial J}{\partial r}$. This gradient requires n_c control solutions with the specified control rate penalty matrix steps, as well as n_c performance and total cost predictions. It has been found suitable for these computations to maintain constant attention allocation, thus significantly reducing the computation time which would be required by an optimal attention allocation algorithm.

The optimal control frequency response optimization algorithm forms an outer loop about the basic optimal pilot model. A complete pilot model solution, including attention allocation optimization, is required for each step. However, convergence is usually achieved in only one to five steps.

The computations required by both algorithms can be performed by a single subroutine. A flow chart of the computations required is presented in Figure 2.

APPLICATION TO AN ATTITUDE HOLD TASK

The optimal control frequency response scheme is applied to an attitude hold task for a bare airframe fighter aircraft case. An attitude hold task for a bare airframe provides a simple example which can be easily duplicated for further research in this area. The bare airframe fighter aircraft case selected has unstable longitudinal dynamics and more conventional lateral dynamics, which will be useful for the illustration of basic properties of the optimal control frequency response scheme.

Aircraft System

The fighter aircraft case involves straight and level flight at an altitude of 3,048 meters (10,000 feet) at an airspeed of 262 meters/second (862 feet/second). The airframe dynamics for this case are modeled by standard, linearized, primed, longitudinal and lateral body axis equations of motion [31]. The stability derivatives and other parameters pertinent to this case are presented in Table 1. For simplicity, as well as to accentuate the aircraft dynamics, no models are included for the stability augmentation system or control feel system. The airframe is disturbed by turbulence with gust intensities of 5 feet/second. MIL SPEC 8785B turbulence is provided, as modeled by Heath [24].

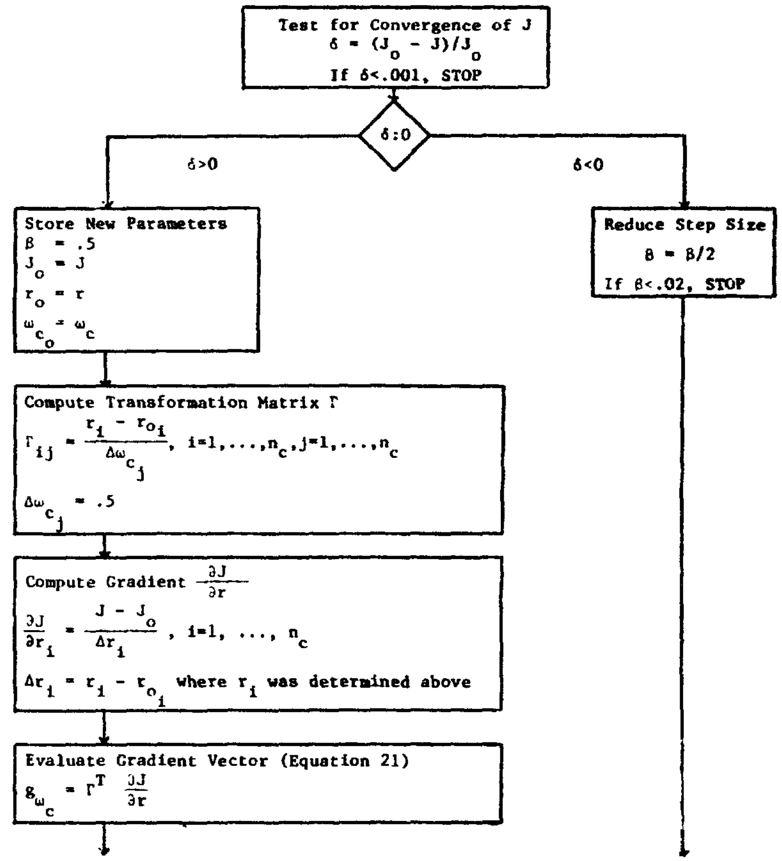

FIGURE 2. OPTIMAL CONTROL FREQUENCY RESPONSE OPTIMIZATION ALGORITHM FLOW CHART (1 of 2)

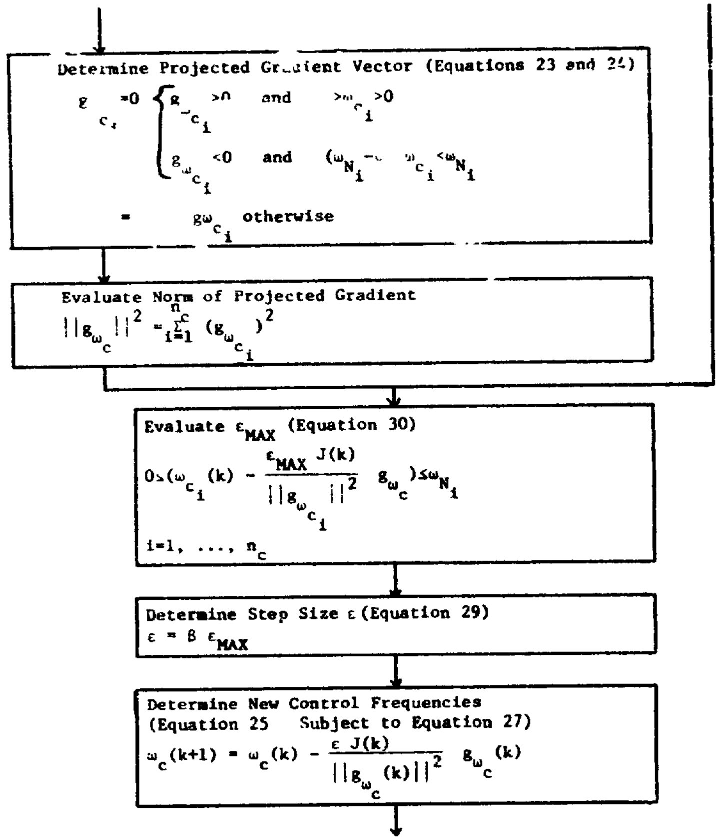

FIGURE 2. OPTIMAL CONTROL FREQUENCY RESPONSE OPTIMIZATION ALGORITHM FLOW CHART (2 of 2)

PARAMETER	LONGITUDINAL AXIS SYSTEM	LATERAL AXIS SYSTEM
Unprimed Stability Derivatives	X_u = -.01578	Y_v = -.2932
	X_w = .02144	$Y_{\delta a}$ = 14.15
	$X_{\delta e}$ = 20.6	$Y_{\delta r}$ = 50.72
	M_u = -.001169	L_β = -70.95
	M_w = .005766	L_p = -2.551
	M_q = -.6528	L_r = -.06964
	$M_{\delta e}$ = -28.2	$L_{\delta a}$ = -62.92
	$M_{\dot{w}}$ = -.0003094	$L_{\delta r}$ = 17.01
	Z_u = -.06086	N_β = 11.51
	Z_w = -1.692	N_p = -.01704
	$Z_{\delta e}$ = -157.3	N_r = -.2266
		$N_{\delta a}$ = -2.261
		$N_{\delta r}$ = -9.057
Moments		I_{xx} = 9017.5 I_{zz} = 58104. I_{xz} = 144.93
Wing Span	30.	30.
Total Velocity	862.0	862.0
Flight Path Angle	0.	0.
Angle of Attack	1.5285	1.5285
Altitude	10000.	10000.
Units	Feet, Seconds, Radians	Feet, Seconds, Radians

TABLE 1. FIGHTER AIRCRAFT STABILITY DERIVATIVES AND OTHER PARAMETERS

Controls include a side stick for pitch and roll and pedals for yaw. The neuromuscular frequency response limits are estimated to be 10, 10, and 5 radians/second for the pitch stick, roll stick, and pedals, respectively [25,26]. The multi-dimensional control for the lateral axis system will be useful to demonstrate the utility of the optimal control frequency response algorithms.

Measured quantities include pitch angle, vertical velocity, roll angle, and yaw angle. It is assumed that the rates of these quantities are simultaneously available. The perceptual thresholds for the measured quantities are presented in Table 2.

Task

The task is defined to be precision attitude hold in turbulence. Control action is required as soon as a deviation in measured quantities is noticed, and deviations must be maintained within specified limits. A total of 80% attention is allocated to this task.

Immediate control action implies that the indifference thresholds are the same as the perceptual thresholds, as presented in Table 2. The maximum allowable deviations are also presented in this table. The optimal pilot model state penalties are derived by the conventional rule

$$Q_{ii} = \left(\frac{1}{MAX|y_i(t)|}\right)^2 \qquad (31)$$

and are presented in Table 2.

Model Predictions

The optimal control frequency response scheme is implemented on the AFFDL optimal pilot model computer program FGDPILOT [31], which is designed for complex aircraft system and simulation analysis. The program solves the longitudinal and lateral axis systems separately, which results in a significant cost reduction. However, this presents a problem for specification of attention allocation between the two axis systems. For the purpose of this paper, the naive assumption will be made of equal attention allocation, i.e., 40% total attention per axis system. This problem is addressed by the related conference paper "An Approach to the Multi-Axis Problem in Manual Control" by this author [34].

MEASURED QUANTITY	UNITS	PERCEPTUAL THRESHOLD	INDIFFERENCE THRESHOLD	MAXIMUM ALLOWABLE DEVIATION	STATE PENALTY
Pitch Angle	deg.	.8	.8	10.	.01
Pitch Rate	deg./sec.	1.6	1.6	32.	.001
Vertical Velocity	ft./sec.	.27	.27	10.	.01
Vertical Acceleration	ft./sec.^{2}	.54	.54	32.	.001
Roll Angle	deg.	.9	.9	10.	.01
Roll Rate	deg./sec.	1.8	1.8	32.	.001
Yaw Angle	deg.	.9	.9	10.	.01
Yaw Rate	deg./sec.	1.8	1.8	32.	.001

TABLE 2. MEASURED QUANTITIES AND RELATED PARAMETERS

The computer program is completely automated, and requires essentially only those parameters presented in this section. The program contains nominal values for all the basic optimal pilot model parameters. Program predictions include optimal control frequency response, optimal attention allocation [32], and control and aircraft system performance. Predictions for this case are presented in Figures 3 and 4 for the longitudinal and lateral axis systems, respectively.

The predicted control frequency responses for this case are presented in Table 3. The maximum physically attainable frequency response is predicted for the longitudinal axis system, as would be expected for an unstable system. Intermediate values are predicted for the roll stick and pedal frequency response. These values would have been difficult to predict a priori. However, these predictions are based on equal attention allocation to the longitudinal and lateral axis systems. It is unlikely that equal attention allocation is appropriate for this case. Since the control frequency response predictions are dependent upon attention allocation, the predictions are in error. Thus the multi-axis attention allocation problem must be addressed.

CONTROLLER	NEUROMUSCULAR CUTOFF FREQUENCY	OPTIMAL CONTROL CUTOFF FREQUENCY
Pitch Stick	10 radians/second	10 radians/second
Roll Stick	10 radians/second	7.6 radians/second
Pedals	5 radians/second	4.4 radians/second

TABLE 3. PREDICTED CONTROL FREQUENCY RESPONSE

The multi-axis problem is addressed by reference [34], in which corrected predictions are made for this case. The multi-axis solution indicates a large shift in attention allocation to the unstable longitudinal axis system. The predicted attention allocation is .697 and .103 for the longitudinal and lateral axis systems, respectively. The corrected control frequency responses are presented in Table 4. The maximum physically attainable frequency response is again predicted for the unstable longitudinal axis system. But, a substantial reduction in control frequency response is predicted for the lateral axis system. This reduction is most consistent with the reduction in attention allocation to this axis system. Note that the pedal cutoff frequency of .2 radians/second corresponds to a control cycle time of about 30 seconds. These values could not be predicted by previous methods.

CASE DESCRIPTION

LONGITUDINAL DYNAMICS, M=.8
PRECISION ATTITUDE HOLD TASK
LONGITUDINAL DYNAMICS

ALTITUDE = 1.000000E+04 FEET
NOMINAL AIRSPEED = 8.616933E+02 FEET/SECOND
NOMINAL ANGLE OF ATTACK = 1.528500E+00 DEGREES
TURBULENCE CONDITIONS
SIG U GUST = 5.000000E+00 FEET/SECOND
SIG V GUST = 5.000000E+00 FEET/SECOND
SIG W GUST = 5.000000E+00 FEET/SECOND

AIRFRAME EIGENVALUES

(-3.6681283)+J(0.)
(1.0855377)+J(0.)
(-.22298678E-01)+J(.13527864)
(-.22298678E-01)+J(-.13527864)

PILOT MODEL PARAMETERS

TOTAL ATTENTION ALLOCATION TO PRIMARY FLIGHT TASK (ATTN) = .40000E+00
PURE TIME DELAY (TAU) = .20000E+00 SECONDS
MOTOR NOISE TO SIGNAL RATIO (PM) = 3.000000E-03
MEASUREMENT NOISE TO SIGNAL RATIO (PY) = 1.000000E-02
TOTAL COST (COST) - 2.505697E+00
SCANNING COST (SCOST) = 1.790228E+00

FIGURE 3. OPTIMAL PILOT MODEL PERFORMANCE PREDICTIONS FOR LONGITUDINAL F-16 BARE AIRFRAME ATTITUDE HOLD TASK (1 of 2)

CONTROLS

CONTROLLER	NEUROMUSCULAR FREQUENCY	OPTIMAL CONTROL FREQUENCY	CONTROL RATE PENALTY (RINV)	CONTROL FORCE STD DEVIATION	CONTROL SURFACE STD DEVIATION
PITCH STICK	10.000000	10.314665	9.8533666	.31868657	.31868657

OBSERVED QUANTITIES

OBSERVED QUANTITY	UNITS	TASK VECTOR	PERCEPTUAL THRESHOLD	INDIFFERENCE THRESHOLD	OPTIMAL ATTENTION ALLOCATION	STANDARD DEVIATION
PITCH ANGLE	DEG	.10000000E-01	.80000000	.80000000	.30604225E-01	4.7593264
PITCH RATE	DEG/SEC	.10000000E-02	1.6000000	1.6000000	.30604225E-01	1.9888165
VERTICAL VELOCITY	FT/SEC	.10000000E-01	.27000000	.27000000	.36939577	10.666126
VERTICAL ACCEL	FT/SEC**2	.10000000E-02	.54000000	.54000000	.36939577	26.346301

FIGURE 3. OPTIMAL PILOT MODEL PERFORMANCE PREDICTIONS FOR LONGITUDINAL F-16 BARE AIRFRAME ATTITUDE HOLD TASK (2 of 2)

CASE DESCRIPTION

LATERAL DYNAMICS, BARE AIRFRAME, M=.8
PRECISION CONTROL WITH ROLL STICK AND PEDALS
LATERAL DYNAMICS

ALTITUDE = 1.000000E+04 FEET
NOMINAL AIRSPEED = 8.616933E+02 FEET/SECOND
NOMINAL ANGLE OF ATTACK = 1.528500E+00 DEGREES
TURBULENCE CONDITIONS
    SIG U GUST = 5.000000E+00 FEET/SECOND
    SIG V GUST = 5.000000E+00 FEET/SECOND
    SIG W GUST = 5.000000E+00 FEET/SECOND

AIRFRAME EIGENVALUES
(0.)+J(0.)
(-2.5341997)+J(0)
(-.18760888E-01)+J(0)
(-.25919919)+J(3.6290849)
(-.25919919)+J(-3.6290849)

PILOT MODEL PARAMETERS

TOTAL ATTENTION ALLOCATION TO PRIMARY FLIGHT TASK (ATTN) = .400000E+00
PURE TIME DELAY (TAU) = .200000E+00 SECONDS
MOTOR NOISE TO SIGNAL RATIO (FM) = 3.000000E-03
MEASUREMENT NOISE TO SIGNAL RATIO (PY) = 1.000000E-02
TOTAL COST (COST) = 1.802367E-01

FIGURE 4. OPTIMAL PILOT MODEL PERFORMANCE PREDICTIONS FOR LATERAL F-16 BARE AIRFRAME ATTITUDE HOLD TASK (1 of 2)

CONTROLS

CONTROLLER	NEUROMUSCULAR FREQUENCY	OPTIMAL CONTROL FREQUENCY	CONTROL RATE PENALTY (RINV)	CONTROL FORCE STD DEVIATION	CONTROL SURFACE STD DEVIATION
ROLL STICK	10.000000	7.5831960	168.07812	.18555362	.18555362
PEDALS	5.0000000	4.3811237	390.77780	.37499172	.37499172

OBSERVED QUANTITIES

OBSERVED QUANTITY	UNITS	TASK VECTOR	PERCEPTUAL THRESHOLD	INDIFFERENCE THRESHOLD	OPTIMAL ATTENTION ALLOCATION	STANDARD DEVIATION
ROLL ANGLE	DEG	.10000000E-01	.90000000	.90000000	.30351583	3.1144329
ROLL RATE	DEG/SEC	.10000000E-02	1.8000000	1.8000000	.30351583	7.8006640
YAW ANGLE	DEG	.10000000E-01	.90000000	.90000000	.96484172E-01	.82409736
YAW RATE	DEG/SEC	.10000000E-02	1.8000000	1.8000000	.96484172E-01	1.6599728

FIGURE 4. OPTIMAL PILOT MODEL PERFORMANCE PREDICTIONS FOR LATERAL F-16 BARE AIRFRAME ATTITUDE HOLD TASK (2 of 2)

CONTROLLER	NEUROMUSCULAR CUTOFF FREQUENCY	MULTI-AXIS OPTIMAL CONTROL CUTOFF FREQUENCY
Pitch Stick	10.rad/sec	10.rad/sec
Roll Stick	10.rad/sec	4.2 rad/sec
Pedals	5.rad/sec	.20rad/sec

TABLE 4. PREDICTED MULTI-AXIS CONTROL FREQUENCY RESPONSE

The multi-axis solution also produced a substantial reduction in total quadratic cost, as shown in Table 5. This is further evidence of the adaptive capability of the methods developed herein and in reference [34].

AXIS SYSTEM	QUADRATIC COST	MULTI-AXIS QUADRATIC COST
Longitudinal	2.51	1.29
Lateral	.18	.35
TOTAL SYSTEM	2.69	1.64

TABLE 5. PREDICTED QUADRATIC COST

SUMMARY OF THE OPTIMAL CONTROL FREQUENCY RESPONSE PROBLEM

A method has been presented for optimal pilot model control frequency response specification which is suitable for complex aircraft systems and tasks. The method is predictive and reflects important manned aircraft system properties, such as control feel system dynamics, airframe dynamics, gust environment, task, and attention allocation. Dependency upon other optimal pilot model predictions results from the definition and iterative nature of the method.

The method has been applied to an attitude hold task for a bare airframe fighter aricraft case. This application demonstrated the method's capability to make realistic predictions for stable as well as unstable aircraft system dynamics and for scalar as well as multi-dimensional controls. The application identified a related deficiency in attention allocation specification for the multi-axis control task. This problem is addressed in the conference paper "An Approach to the Multi-Axis Problem in Manual Control" by this author.

RECOMMENDED AREAS OF INVESTIGATION

Control frequency response is considered to be bounded by neuromuscular control frequency response limits. First order approximations are used in the optimal pilot model. It would be beneficial to this area of research to experimentally identify these limits for conventional controllers. Suggested controllers are force and dynamic side sticks, center sticks with high and low roll pivot, a wheel and column combination, and pedals.

Further investigations are required to fully develop the optimal control frequency response concept. Modifications to the motor noise [35,36,37] from the existing implementation may have significant effect upon the total system cost and thus the optimal control frequency response. Control cost may require additional terms other than rms control activity [36], both for control frequency response optimization and for total attention allocation specification. And, of course verification against experimental data is required.

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