

N79-17481

PARAMETER ESTIMATION IN A HUMAN OPERATOR DESCRIBING FUNCTION MODEL FOR A TWO-DIMENSIONAL TRACKING TASK.

Antonie van Lunteren

Man-Machine Systems Group
 Laboratory for Measurement and Control
 Department of Mechanical Engineering
 Delft University of Technology
 Mekelweg 2
 Delft
 Netherlands



0 Summary

A previously described parameter estimation program has been applied to a number of control tasks, each involving a human operator model consisting of more than one describing function. One of these experiments is treated in more detail. It consisted of a two-dimensional tracking task with identical controlled elements. The dynamics of these controlled elements have been chosen as $K, K/s$, and K/s^2 . The tracking errors were presented on one display as two vertically moving horizontal lines. Each loop had its own manipulator. The two forcing functions were mutually independent and consisted each of 9 sine waves. A human operator model was chosen consisting of 4 describing functions, thus taking into account possible linear crosscouplings. From the Fourier coefficients of the relevant signals the model parameters were estimated after alignment, averaging over a number of runs and decoupling. The results show that for the elements in the main loops the crossover model applies. A weak linear cross-coupling existed with the same dynamics as the elements in the main loops but with a negative sign.

1 Introduction

In an earlier paper [1] a parameter estimation method has been treated which gives consistent estimates in closed loop systems. This is an essential requirement for identification of human operator models in tracking experiments. The method has been based on the application of forcing functions consisting of a number of sinusoids. It has been shown that, after application of a decoupling procedure, it can also be applied for human operator models with more than one input and more than one output. The paper just mentioned does not give applications of the method. Since then, however, the method has been applied in a number of experiments.

Because much knowledge is already available on human operator behavior in single loop compensatory tracking tasks, applications have been chosen involving more complex human operator describing function models. These applications were:

- a multiple loop control task.
- a multiloop control task,

- a control task with preview. They have been described in a recent Progress Report of the Man-Machine Systems Group [2]. This paper deals with one of the three topics just-mentioned viz the multiple loop control task.

2 Multiple loop control tasks

A multiple loop control task consists of controlling a number of mutually independent systems. Fig. 1 gives the block diagram for the case of a multiple loop task with two control loops. The most general representation of a human operator describing function model for this type of task is given in Fig. 2. In the ideal case the describing functions 21 and 12 would be zero. However, as a result of task interference these describing functions may have a nonzero value.

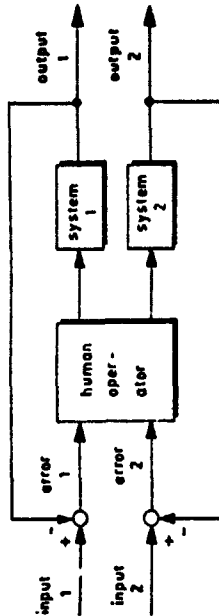


Figure 1: A multiple loop control task with two control loops.

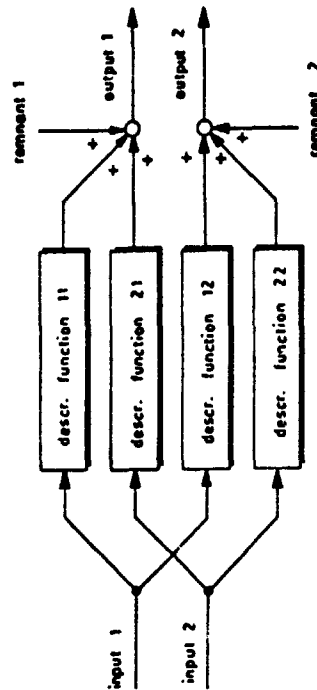


Figure 2: Human operator model with two inputs and two outputs.

96

Since 1960 a number of attempts to investigate multiple loop systems have been made by Chernikoff et al [3,4,5] using two predictable forcing functions, each consisting of a single sinusoid. As a measure for the human operator performance only the error scores were determined. Investigations resulting in human operator describing functions in a multiple loop control task date from about 1965 and have been published by Iodosiev et al [6] and by Levison [7,8]. Several experimental parameters were varied, like system dynamics (K, K/s, K/s², equal or different systems) control-display configurations (one two-dimensional display and control; two one dimensional displays and controls, both signals foveally visible or not) and forcing function bandwidth. In all studies no a priori assumption was made that linear cross-coupling, due to task interference might exist within the human operator model. The absence of such an assumption greatly facilitates the data processing procedure.

In the present study the possible occurrence of linear cross-couplings due to task interference was taken into account, so a model structure according to Fig. 2 was assumed. To eliminate effects due to scanning behavior as well as possible it was decided to display both tracking errors foveally on one CRT. A display-control configuration with a good compatibility was desired in order to prevent effects resulting from suboptimal task conditions. Two possibilities exist: Application of one joystick which can be moved in left-right direction for one control loop and in a forward-backward direction for the other. The tracking errors can be indicated by one lightspot which can move horizontally and vertically (integrated configuration). The second possibility is to use two joysticks which can be moved in a forward-backward direction with the left hand and the right hand respectively. The corresponding tracking errors can be displayed as two vertically moving dots on the left hand and right hand part of the CRT (separated configuration). In the integrated configuration differences in performance may be found in the horizontal and in the vertical direction. In the separated configuration left-right differences may occur resulting from left- or righthandedness of the subject. According to Levison and Elkind [8] task interference may easily occur in an integrated configuration with a heterogeneous control task (each loop a different controlled element). Although the present study was limited to homogeneous control tasks (both loops the same controlled element) the separated control configuration was chosen.

3 System identification

In the introduction it has already been mentioned that the method applied has been treated in an earlier paper [1]. Therefore only the main points will be summarized here. For more details see also [9]. When applying estimation procedures it is always important to have a measure for the reliability of the results. Therefore a method has been developed to compute bias and standard deviation for estimates in closed loop systems based on a finite observation time. An extensive treatment is given in an internal report [10] (in Dutch). Here only the resulting expressions are given.

3.1 Identification method

Consider an unknown system with transfer function $H_1(v)$ (v =frequency in Hz) in a closed loop disturbed by noise. The configuration is given in Fig. 3. Suppose that the external forcing function $u(t)$ can be chosen as a

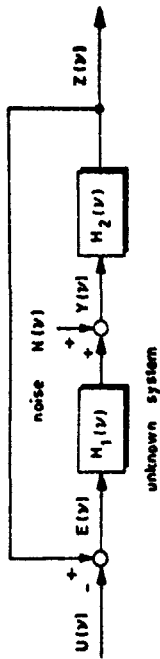


Figure 3: Unknown system in a closed loop disturbed by noise.

sum of sinusoids and that the error signal $e(t)$ and the disturbed output $y(t)$ of the unknown system can be measured. If the transfer function $H_1(v)$ can be characterized by the relation

$$H_1(v) = \frac{B(v)}{A(v)} e^{-j2\pi v\tau} \quad (1)$$

$$\text{with } B(v) = b_0 + b_1(j2\pi v) + \dots + b_M(j2\pi v)^M \quad (2)$$

$$A(v) = a_0 + a_1(j2\pi v) + \dots + a_L(j2\pi v)^L \quad (3)$$

then the following relation exists:

$$A(v)Y(v) - B(v)e^{-j2\pi v\tau}E(v) = A(v)N(v) \quad (4)$$

The term $A(v)N(v)$ is called the system residue $V(v)$. The signals $Y(v)$ and $E(v)$ each can be split up in two parts resulting from the forcing function $U(v)$ and the noise $N(v)$ respectively, thus:

$$E(v) = E_u(v) + E_n(v) \quad (5)$$

$$Y(v) = Y_u(v) + Y_n(v) \quad (6)$$

For the undisturbed signals $E_u(v)$ and $Y_u(v)$ the following relation holds:

$$A(v_i)Y_u(v_i) - B(v_i)e^{-j2\pi v_i\tau}E_u(v_i) = 0 \quad (7)$$

where v_i are the frequencies for which the forcing function $U(v)$ contains a sinusoidal component ($i=1, \dots, n$).

From the measured signals $e(t)$ and $y(t)$ estimates $\hat{E}_u(v_i)$ and $\hat{Y}_u(v_i)$ can be made for $E_u(v_i)$ and $Y_u(v_i)$ by means of Fourier analysis. If the system is modelled by a transfer function

$$\hat{H}_1(v) = \frac{\hat{B}(v)}{\hat{A}(v)} e^{-j2\pi v\tau} \quad (8)$$

then for this model the following equation holds:

$$\hat{A}(v_i)\hat{Y}_u(v_i) - \hat{B}(v_i)e^{-j2\pi v_i\tau}\hat{E}_u(v_i) = \hat{V}(v_i), \quad i=1, \dots, n \quad (9)$$

In this equation the quantity $\hat{V}(v_i)$ is called the model residue at frequency v_i . The model parameters are estimated by minimizing the following cost function:

$$J = \sum_{i=1}^N \lambda_i |\hat{v}(v_i)|^2 \quad (10)$$

The quantities λ_i are weighting factors. For a given value τ of the delay time τ to be estimated, minimization of the cost function J to the unknown parameters \hat{a}_j ($j=0, 1, \dots, L$) and \hat{b}_k ($k=0, 1, \dots, M$) leads to a set of linear equations in these unknown parameters. Without loss of generality one of these parameters, for instance a_0 , can be chosen as equal to 1. Substitution of these solutions in the original equation and differentiation with respect to the model parameter τ leads to one non-linear equation from which the parameter τ can be solved, after which also the other parameters can be calculated.

For the estimator of the parameters \hat{a}_j and \hat{b}_k the following properties can be derived [9]:

- The estimator is consistent.
- The estimator will be a minimum variance estimator if the weighting factors are chosen as $\lambda_i = S_{vv}(v_i)$, where $S_{vv}(v_i)$ is the auto spectral density of the system residue $v(t)$ at the frequency v_i . According to (4) this quantity can also be given by

$$\lambda_i = S_{vv}(v_i) = A(v_i)^2 S_{nn}(v_i) \quad (11)$$

In practice both $A(v_i)$ and $S_{nn}(v_i)$ are unknown. A possible procedure is to make a first estimate with weighting factors $\lambda_i=1$, based on these estimates, to obtain an estimate for the weighting factors. These factors are used in a new estimate. If necessary this procedure can be repeated.

3.2 Reliability of the estimates

Consistency of an estimator means that for an infinite observation time the estimate will become equal to the quantity to be estimated with probability 1. For a finite observation time the estimator will have a certain variance and may even have a bias, i.e. a consistent estimator may only be asymptotically unbiased. It has been shown elsewhere [10] that in the case of Fig. 3 a system in the loop can only be identified by an asymptotically unbiased estimator. For a finite observation time, for a normally distributed remnant, and for a forcing function $u(t)$ described by:

$$u(t) = \sum_{i=1}^N a_i \cos 2\pi v_i t + b_i \sin 2\pi v_i t \quad (12)$$

the bias in an estimate $\hat{H}_1(v_i)$ of $H_1(v_i)$ is equal to:

$$\text{Bias}[\hat{H}_1(v_i)] = E\{\hat{H}_1(v_i) - H_1(v_i)\} = -(H_1(v_i) + \frac{1}{H_2(v_i)}) e^{-x_i} \quad (13)$$

where

$$x_i = \frac{|E\{u_i\}|^2}{E\{n_i\}} = \frac{\int_0^{kT} e_u(t) e^{-j2\pi v_i t} dt}{\int_0^{kT} e_n(t) e^{-j2\pi v_i t} dt} \quad (14)$$

and where k is an integer and T is the basic period of the periodic test signal $u(t)$. Thus the quantity x_i can be considered as the signal to noise ratio of the signal $e(t)$ at the frequency v_i . This ratio increases with increasing value of the integer k belonging to the observation time kT . For the variance of the estimator $\hat{H}_1(v_i)$ the following expression has been derived:

$$\text{Var}\{\hat{H}_1(v_i)\} = E\{|\hat{H}_1(v_i) - E\{\hat{H}_1(v_i)\}|^2\} = |H_1(v_i) + \frac{1}{H_2(v_i)}|^2 \sum_{i=1}^N e^{-2x_i} e^{-x_i} E_1(x_i) \quad (15)$$

where the function $E_1(x)$ is defined as:

$$E_1(x) = \int_0^{\infty} \frac{e^{-v}}{v} dv \quad (16)$$

The functions $E_1(x)$ and $x e^{-x} E_1(x)$ are given in tables, for instance in Abramowitz and Stegun [11].

If the input $u(t)$ is stochastic the expressions for bias and variance are almost identical. Only the quantity x_i must be replaced by $x_n(v)$, where

$$x_n(v) = \frac{n(\Gamma_{ue}(v))^2}{\Gamma_{ue}(v)} \quad (17)$$

In this expression $\Gamma_{ue}(v)$ is the coherence between the signals $u(t)$ and $e(t)$ at the frequency v ; n is the number of elementary frequency bands with width $\Delta v = 1/T$, over which an averaging is executed in the estimation procedure for the spectral densities $S_{yy}(v)$ and $S_{ue}(v)$ [12]. These estimates are used to obtain the transfer function estimate:

$$\hat{H}_1(v) = \frac{S_{yy}(v)}{S_{ue}(v)} \quad (18)$$

A study of the expressions for bias and variance shows that the bias decreases much more rapidly with increasing observation time, so that in many practical cases the bias can be neglected.

3.3 Complex configurations

For systems with more than one input and more than one output a number of separate transfer functions has to be estimated. In such a case (see for instance Fig. 2) more than one external forcing function has to be applied and a decoupling procedure has to be executed on the available inputs and outputs. This procedure has already been described earlier [1,13]. It involves a number of subtractions and divisions of Fourier coefficients of the measured inputs and outputs. As a consequence thereof it is no longer possible to get reliable estimates for bias and variance as in the simple case of Fig. 3.

4 Experiments and data processing

The control configuration is given in the block diagram of Fig. 4. The

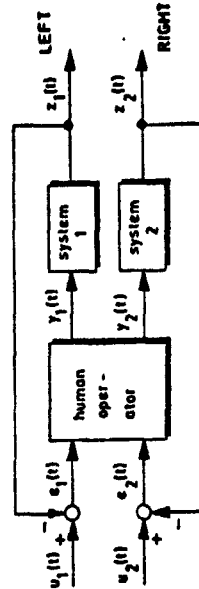


Figure 4: The control configuration for the multiple loop task.

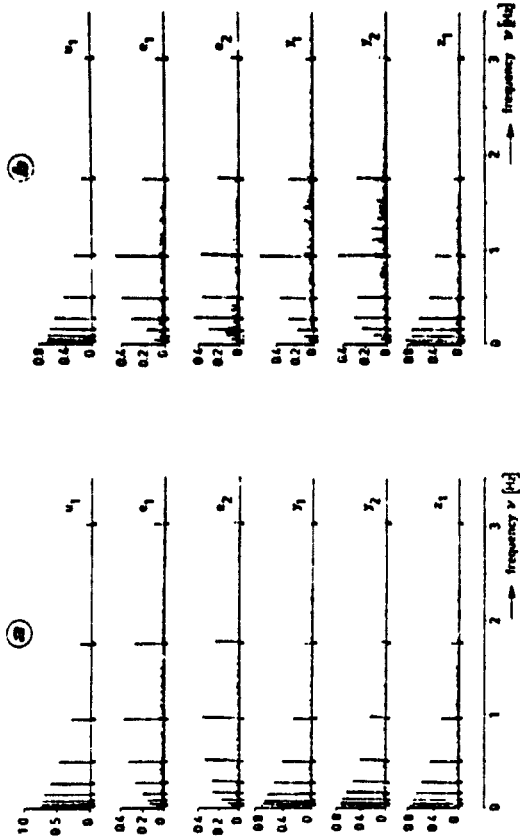


Figure 5: Examples of amplitude spectra in cm display for three different controlled elements

a) $H(v) = 1$
 b) $H(v) = \frac{4}{j2\pi v}$
 c) $H(v) = \frac{(j2\pi v)^2}{8}$

Indices 1 refer to the left hand task, 2 to the right hand task.

The table shows that the standard deviation σ_e of the error signal $e(t)$ was always about 0.6 cm. In fact, using information from the training period, the inputs $u_1(t)$ and $u_2(t)$ were chosen in such a way that σ_e was more or less the same during all experiments. The relative noise content M_1 of the input signals $u_1(t)$ and $u_2(t)$ was always very low (< 0.01). This is an indication that the signal/noise ratio of the recorder was sufficiently low, and that the Fourier analysis was indeed based on an observation time equal to an integer number of periods for all input frequencies. As might be expected the relative tracking error increases with increasing system order. It looks as if in a single task the left hand performed slightly better than the right hand although all three subjects were righthanded. Of the same order are the differences which give an indication for a performance deterioration in the double task compared with the single task.

Fig. 5 shows examples of the amplitude spectra for the three different systems in a double task. The figures on the relative noise content in Table 3 already indicate an increase of human operator remnant with increasing system order. The spectra, moreover, illustrate how the remnant is distributed along the frequency axis.

For the experiments with two control loops the mathematical description of the man-machine system is illustrated in the block diagram of Fig. 6. The human operator model transfer functions were identified as follows: From the averaged and decoupled fouriercoefficients the raw data points for the Bode plots of the four transfer functions $H_{11}(v)$, $H_{12}(v)$, $H_{21}(v)$ and $H_{22}(v)$ were determined. These data points are given by the crosses in Fig. 7. Based on these data points models were chosen for the four transfer functions for each of the three experimental conditions. By means of the parameter estimation program the parameters in these models were estimated from the fouriercoefficients of the decoupled signals. Using these parameter values the model transfer functions were obtained and also drawn in Fig. 7 (solid lines). The same procedure was applied to the single loop measurements, be it without the decoupling operation. It is interesting to note that linear cross couplings were clearly present in the cases that two proportional systems or two integrators had to be controlled. All cross couplings had a negative sign. In the case of the two double integrators there was a cross coupling from the right hand display (e_2) to the left hand control (y_1), again with a negative sign. From the left hand display to the right hand control no significant linear coupling could be found. Attempts to fit a model in the latter case yielded modelling errors in the order of 90%, while in all other cases these values were in the order of 10% or lower. The raw data points and the Bode plots of the human operator models for the single loop experiments are given in Fig. 8.

In the averaging procedure for the fouriercoefficients not only the measurements but also the standard deviations were determined. In this way estimates for the signal to noise ratio for a number of frequencies were available. Using these values it was possible to estimate bias and standard deviation according to Eqs. 13 and 15 from section 3.2. The resulting one sigma limits are also indicated in Fig. 8.

From the results given in Table 4 the cross-over frequencies and phase margins were determined for the single loop tasks and for the comparable main loops in the double loop tasks. The results are given in Table 5.

ORIGINAL PAGE IS
 OF POOR QUALITY

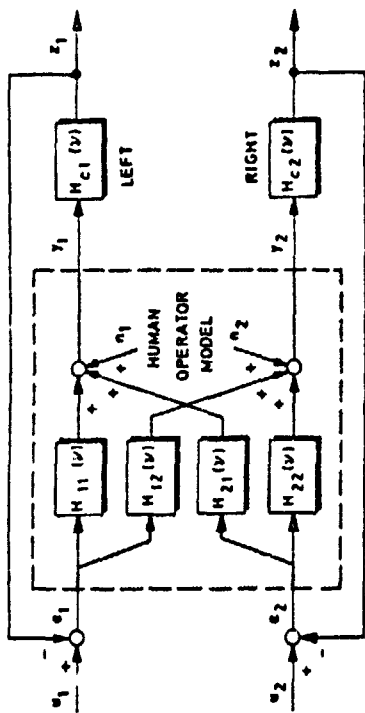
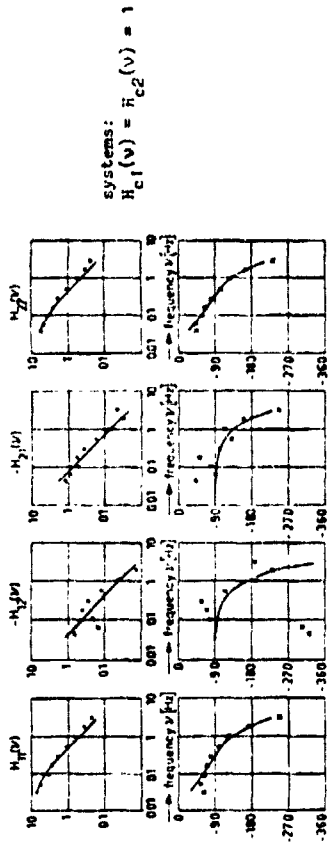


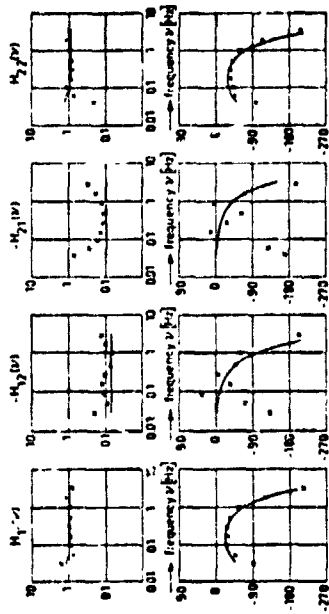
Figure 6: Block diagram of the human operator model in a two dimensional multiple loop system.

Table 4: Human operator describing functions.

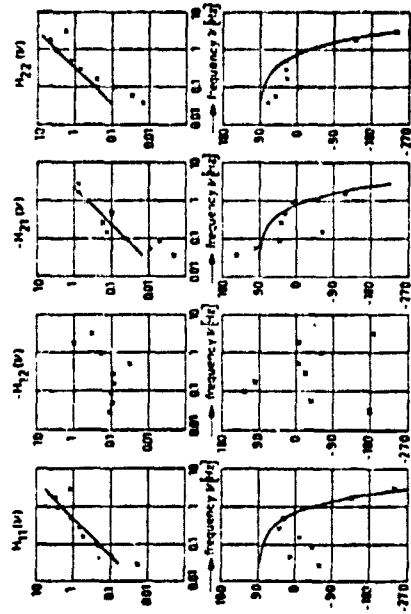
$H_c(s)$	hand	single system	
1	LL	$\frac{10}{1+3s} e^{-0.11(s\tau)}$	$\frac{1}{1+2.5(s\tau)} e^{-0.12(s\tau)}$
	LR	$-\frac{0.2}{3s\tau} e^{-0.22(s\tau)}$	$-\frac{0.2}{3s\tau} e^{-0.22(s\tau)}$
	RL	$-\frac{0.5}{3s\tau} e^{-0.13(s\tau)}$	$-\frac{0.5}{3s\tau} e^{-0.13(s\tau)}$
	RR	$\frac{6}{1+1.8s} e^{-0.10(s\tau)}$	$\frac{6}{1+2(s\tau)} e^{-0.15(s\tau)}$
$\frac{1}{3s\tau}$	LL	$\frac{0.25(s\tau+1)}{3s\tau} e^{-0.15(s\tau)}$	$\frac{0.25(s\tau+1)}{3s\tau} e^{-0.18(s\tau)}$
	LR	$-0.09 e^{-0.26(s\tau)}$	$-0.09 e^{-0.26(s\tau)}$
	RL	$-0.10 e^{-0.11(s\tau)}$	$-0.10 e^{-0.11(s\tau)}$
	RR	$\frac{0.25(s\tau+1)}{3s\tau} e^{-0.17(s\tau)}$	$\frac{0.25(s\tau+1)}{3s\tau} e^{-0.20(s\tau)}$
$\frac{0}{(3s\tau)^2}$	LL	$0.1(s\tau) e^{-0.27(s\tau)}$	$0.1(s\tau) e^{-0.31(s\tau)}$
	LR	0	0
	RL	$-0.36(s\tau) e^{-0.27(s\tau)}$	$-0.36(s\tau) e^{-0.27(s\tau)}$
	RR	$0.1(s\tau) e^{-0.27(s\tau)}$	$0.1(s\tau) e^{-0.30(s\tau)}$



systems:
 $H_{c1}(s) = H_{c2}(s) = 1$



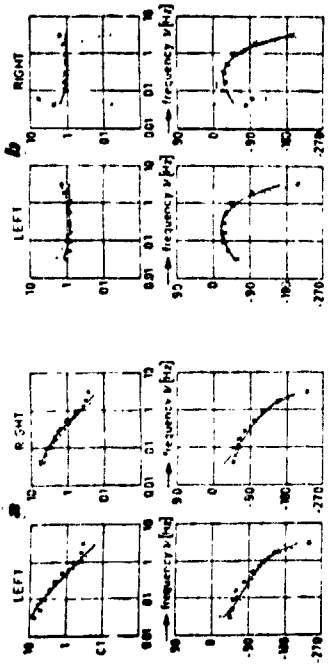
systems:
 $H_{c1}(s) = H_{c2}(s) = \frac{1}{3s\tau}$



systems:
 $H_{c1}(s) = H_{c2}(s) = \frac{0}{(3s\tau)^2}$

1=left hand
 2=right hand

Figure 7: Raw data points (crosses) and estimated model transfer functions (solid lines) of the averaged control behavior of the three subjects with each of the three configurations.



system to be controlled

a) $H_C(v) = 1$

b) $H_C(v) = \frac{k}{j2\pi v}$

c) $H_C(v) = \frac{8}{(j2\pi v)^2}$

Figure 8: Raw data (crosses) and human operator model transfer function estimates (solid lines) with estimated one sigma limits (dashed lines) for the single loop experiments.

Table 5: Cross-over frequencies and phase margins.

$H_C(v)$	hand	single system			double system		
		ω_c [Hz]	ϕ_c [deg]	ω_c [Hz]	ϕ_c [deg]	ω_c [Hz]	ϕ_c [deg]
1	L	0.25	75	0.51	3.2	79	
	R	0.51	81	0.17	9.0	79	
$\frac{k}{j2\pi v}$	L	0.57	50	0.61	4.0	46	
	R	0.70	44	0.58	3.0	43	
$\frac{8}{(j2\pi v)^2}$	L	0.51	41	0.35	2.4	43	
	R	0.51	40	0.51	3.2	35	

From Fig. 8 and also from Table 5, it can be seen that in the single loop task no difference occurs between left and right handed tasks. The same is true for the double loop task, with an exception for the locking left to right cross coupling in the control of the double integrator. A comparison between single loop control and double loop control shows that in the double loop experiment the transfer functions $H_1(v)$ and $H_2(v)$ show a slight increase in delay time in relation to the transfer functions in the single loop experiments. The cross-over frequencies and phase margins from Table 5 do not indicate that there are left-right differences of single loop-double loop differences. The performance measure ω_c/ζ in Table 3 does not differ significantly between left and right or between single loop and double loop task in controlling a single or double integrator. Only in the case of controlling a proportional system the error score for a left-handed single loop task is lower than for the right handed task or for the double loop task. This may be related to the higher value of the standard deviation σ of the forcing function $u(t)$ for that case.

6 Discussion.

Up till now only a few investigators have been involved in identifying human operator transfer functions in multiple loop control tasks. Consistent estimation procedures with an unpredictable input and with human operator transfer functions involving a delay time were applied by Levison and Elkind [7, 8]. In [7] Levison describes a number of experiments with one control and one display. The variables of the two loops corresponded to the x- and y-direction of control and display. In the homogeneous control situation (identical systems; double integrator; equal forcing function bandwidths: 1.5, 2.5 and 3.5 rad/sec) only small differences were found between x and y and between single axis and double axis control. In [8] experiments have been described with separate displays and controls (left and right). This study was aimed at investigating the influence of display separation on human operator performance (peripheral vision, scanning). Experiments which can be compared with the present study were those with a display separation of 0.8° and controlled elements consisting of a single integrator. The human operator transfer function was modelled with four parameters: a static gain, one pole, one zero and a time delay. The main differences with reference to a one axis (foveal) control task were a slight decrease in gain (3dB) and an increase in the time delay from 0.12 to 0.15 sec.

In the present study a noticeable decrease in gain could not be found, but there was an increase in time delay of 0.03 sec. In this case a model with three parameters was chosen. A four parameter model could not be identified reliably because of the more complex overall model which included two cross-coupling terms. Even in the three parameter transfer function models the reliability will probably be lower than in the estimates of Levison and Elkind. Taking into account these differences it can be stated that the results for the transfer functions $H_1(v)$ and $H_2(v)$ do not contradict those of Levison and Elkind.

It is interesting to note that linear cross-couplings have indeed been found. These cross-couplings may have a visual origin; the moving left hand spot may influence the perception of the distance to the zero reference for the right hand spot and vice versa. The negative sign then would mean that the bias in the observation is such that the subject perceives the reference as shifted in the direction of the dot from the other control task. If, on the other hand, the cross-coupling has its origin in the motor system, it means that the hands tend to move in an opposite direction. A third possibility is that it results from central processes. Even a combination of these factors may be responsible.

ORIGINAL PAGE IS
OF POOR QUALITY

Why there was no cross-coupling from left hand display to right hand control in the case of controlling the double interceptors remains an intriguing question.

7. Conclusions and Future Work

The study of human operator control behavior is primarily a problem of identification of a non-stationary system with a noise source in a closed loop. Therefore conclusions from this study will partly refer to the system identification problem and partly to the human operator control behavior in the task considered.

With respect to the system identification problem the following statements can be made:

- The identification method consisting of a parameter estimation procedure based on Fourier coefficients is found to be useful for this type of problem.
- The deconvolution procedure enables the identification of *cross-couplings*, extra consisting of more than one describing function (cross-couplings, extra feedback element, feedforward element).
- For identification of complex models with more than one input it is necessary to introduce more than one test signal in order to identify the transfer functions from the input to the output(s) of the model to be identified. These test signals must be statistically independent.
- Because in the tasks observed the human control behavior can be considered *quasi-steady state* (i.e. *steady state* (fatigue, boredom) it is necessary to change the parameters of the system to obtain a reasonable signal to noise ratio in tasks involving complex human operator models.
- Averaging should be applied on the Fourier coefficients, i.e. *longer observation time* (i.e. *longer observation time*), in order to obtain a reliable choice for the structure of the describing function, and to decrease possible biases resulting from the finite observation time.
- The method applied, like most of the methods based on the use of periodic forcing functions and estimation of Fourier coefficients, yields consistent estimates of the parameters. This means that for an infinite observation time bias and standard deviation of the estimator will be zero. The estimator is only *quasi-steady state* in closed loop systems.
- For a *finite observation time* bias and standard deviation of a describing function can be computed as a function of the injected noise in the system to be identified and of the dynamics of the systems in the closed loop.
- With *longer observation time*, i.e. with increasing signal to noise ratio of the Fourier coefficient estimates the bias approaches asymptotically to zero in comparison with the standard deviation.
- From the viewpoint of *steady state* (i.e. *steady state*) a *forcing function* consisting of a sum of sinusoids should be chosen in such a way that the sinusoids should be located at those frequencies where a good *transfer function* exists to the *measured* elements to be estimated.
- The methods of the forcing function components should be chosen such that the *transfer function* in the rest of the transfer function to be identified is about equal for all components.

- From the viewpoint of human operator control behavior study the forcing function should be such that the human operator is *not able to identify separate periodic components*, i.e. no single dominant components must occur in the human operator input.
- In general it can be stated that by taking into account the just mentioned recommendations and by making use of the information obtained in the present studies a *better choice for the forcing functions* should be possible for further investigations in these types of control task.

With respect to the results obtained in the double loop task the following remarks can be made:

- In controlling two identical loops with separate (visually observable) displays and controls no significant differences could be found between right hand and left hand performance.
- The results obtained from execution of a double task hardly differ from those obtained in a single task, only a slight increase in human operator time delay (0.03 sec) could be observed.
- A parallel model gives a good description of the human operator control behavior in this control task. It is interesting to note the existence of linear cross-coupling between the two mutually independent tasks.
- The cross-coupling terms can be described with the same dynamics as the terms in the main loops, however, they have a negative sign.
- The effect of the cross-coupling terms is small in comparison with the terms in the main loops. For practical applications they can be neglected. This means that the crossover model can be applied for each of the loops separately.
- The magnitude of the *residual* does not differ between single loop tasks and double loop tasks as is indicated by the standard deviations and the relative noise contents of the signals in the control loops. This means that *task interference* is only demonstrated in the linear cross-coupling terms.
- From the present data it is not possible to locate the origin (visual system, central system or motor system) of the task interference described by the cross-coupling terms.

8. References

- [1] Van Lunteren, A. and H.G. Stassen: Parameter estimation in linear models of the human operator in a closed loop with application of deterministic test signals, Proc. of the 9th Annual Conf. on Manual Control, MIT, (1971), pp. 289-297.
- [2] Stassen, H.G. et al.: Progress Report January 1973 until July 1976 of the Man-Machine Systems Group, Delft, Report MTHD 95, (1977), pp. 19-65.

- [3] Chernikoff, R., J.W. Ducey and F.V. Towler: Two dimensional tracking with identical and different control dynamics in each coordinate, *Journal of Experimental Psychology*, Vol. 60, No 5, (1960), pp. 318-322.
- [4] Ducey, J.W. and R. Chernikoff: The use of quickening in one coordinate of a two-dimensional tracking system, *IEEE Trans. on HFR*, Vol. HFE-1, (1960), pp. 21-24.
- [5] Chernikoff, R. and M. Le May: Effect of various display-control configurations on tracking with identical and different coordinate dynamics, *Journal of Exp. Psych.*, Vol. 66, No 1, (1963), pp. 95-99.
- [6] Todossian, E.P.; R.E. Rose and I. G. Summers: Human performance in single- and two-axis tracking systems, *Proc. 2nd Annual Conf. on Manual Control*, NASA SP-128, (1966), pp. 143-158.
- [7] Lawson, W.H.: Two-dimensional manual control systems, *Proc. 2nd Annual Conf. on Manual Control*, NASA SP-128, (1966), pp. 159-180.
- [8] Levison, W.H. and J.I. Fikind: Two-dimensional manual control systems with separate displays, *Proc. 3rd Annual Conf. on Manual Control*, NASA SP-144, (1967), pp. 29-42.
also: *IEEE Trans. on Human Factors in Electronics*, Vol. HFE-8, No 3, (1967), pp. 201-209.
- [9] Van der Bos, A.: Estimation of parameters of linear systems using periodic test signals, *Dr. Thesis*, Delfse Universitaire Pers, (1974), 136 p.
- [10] Van Lunteren, A.: Systemidentificatie en parameterschatting in open en gesloten ketens, (System identification and parameter estimation in open and closed loops), *Delft, Report N-114*, (1976), pp. 85-96.
- [11] Abramowitz, M. and I.A. Stegun: *Handbook of mathematical functions*, Dover, (1965), pp. 238-243.
- [12] Jenkins, G.M. and D.G. Watts: *Spectral Analysis and its applications*, Holden Day, (1969), pp. 363-411.
- [13] Staassen, H.G. et al.: *Progress Report January 1970 until January 1973 of the Man-Machine Systems Group*, Delft, Report WTHD 55, (1973), pp. 9-23.