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MULTI-ATTRIBUTE SUBJECTIVE EVALUATIONS
OF MANUAL TRACKING TASKS
VS. OBJECTIVE PERFORMANCE OF THE HUMAN OPERATOR

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ABSTRACT

Cooper-Harper ratings have been common measures for evaluating the response of aircrafts from the pilots' point of view. It has been noted, however, that in general the rendering of such subjective evaluations for the handling qualities of engineering systems can neither account for the multi-dimensional nature of tracking and/or monitoring tasks nor is it invariant to operator-centered variables, including the operator's perception of the task.

This paper develops a computational method to deal with these problems by defining matrix -rather than one-dimensional- ratings based on multi-dimensional scaling techniques and multi-variate analysis (no utility functions involved). The method consists of two distinct analytical steps: 1) to determine the mathematical space of subjective judgements of a certain individual (or group of evaluators) for a given set of tasks and experimental conditionings, and 2) to relate this space with respect to both the task variables and the objective performance criteria used. In this space there exist vectors along which the one-dimensional C-H normative scale (in McDonnell's sense of the term) or any multi-attribute utility function can be validly assessed.

Results from this method have been obtained for a variety of second-order trackings with smoothed noise-driven inputs, and they clearly show how: 1) many of the internally perceived task variables form a non-orthogonal set, and 2) the structure of the subjective space varies among groups of individuals according to the degree of familiarity they have with such tasks.

DEFINITION OF THE PROBLEM

In manual tracking tasks one deals with a man/machine system according to Fig.1. For a given human operator and extra-systemic conditions, the task is a blend of the requested mission, the display configurational mode and the dynamics of the machine or vehicle with its associated controlled elements.

$$task(\underline{x}, D, C) \tag{1}$$

While the vehicle dynamics and type of controls can be easily characterized by certain parameters (like gain, damping ratio, etc), there are not even known and widely applicable parameters of statistical nature that are capable of characterizing feedback displays. scalings, luminosity, and measures of legibility and information contents have been the focus of various people's efforts. For reasons of this basic lack of understanding, as well as for simplicity in the design of control/display configurations, it is customary to fix all pre-mentioned parameters to a certain level. On the other hand one requires maximum flexibility in what the input might be, so that the mission can be both time-varying and performance-dependent

$$\underline{X}(t, \underline{Y})$$

For a vehicle, a possible state description might be $(\underline{x} \ \dot{x} \ \underline{x})$, where $\underline{x}$ the position and $E(\dot{x}, \dot{e}, V)$ a measure of energy expenditure, environmental impact, effort allocation, and vehicle economics and kinematics. An associated constraint on permitted accelerations might be of the form $\ddot{x} < f(\underline{x}, t; task)$; and so on, ... the model proceeding to its natural or unnatural conclusion.

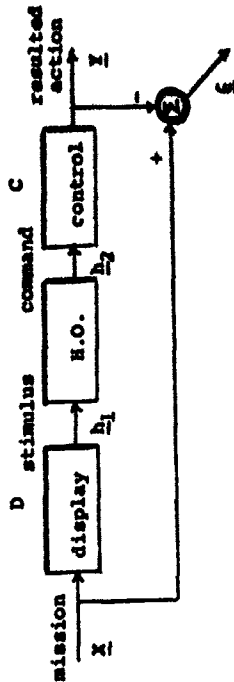In order to stabilize the performance of the system one feeds $\underline{x}$ back to the input

$$\frac{Y}{X} = \frac{1}{1 + G_h G_c} \qquad \text{or} \qquad Y = (1 + CHD)^{-1} CHD \underline{x}$$

so that essentially from the operators point of view the mission becomes a function of the error and its past history

$$h_1(\underline{\dot{x}}, D, f(\dot{x}), \dot{e}_i)$$

while his command actions are still governed [1] by

$$h_2(h_1, \Phi, \psi, \eta, G_m).$$

procedural η
environmental $\dot{e}$
operator-centered $\dot{e}$
physiological Φ
psychological ψ

Figure 1

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THE EXPERIMENTAL PARAMETERS

According to definition (1) each task is characterisable from three entities. For single one-dimensional compensatory tracking tasks displayed on a CRT in standard proportional fashion, these reduce to two: the mission and the controls. Therefore, there are two kinds of parameters needed to specify a task: standard control parameters and parameters capable of describing the essential elements of the mission.

Fig 2 shows the task space compiled out of the well-accepted parameters for second order plants

$$G_c = \frac{K}{(s/\omega_n)^2 + 2(7/\omega_n)s + 1}$$

In contrast to the natural frequencies and the damping ratios, the gains are not prespecifiable; they are rather derived as a consolidation between input strength and display dimension.

$$\left[\int\left[\int_0^T g_c(t)u(t)^2 dt\right]dVdh\right] = T^{rv}(Z)L/\xi_s^2(T;\theta)$$ (3)

which says that the RMS output due to an RMS input be the same specified level for all tasks (where T, a certain duration of the unobstacted signal and L the height of the screen). Of course, if the LHS is an unbounded quantity, a meaningful selection of T is of critical importance.

Choosing parameters to represent the mission is a more difficult task, even for one-dimensional trackings. It involves two aspects: specifying a nominal path and superimposing a certain disturbance. Because many of the recent experiments involve non-stochastic rather than smooth slow-varying deterministic inputs, this study was compelled towards representing the input signal in terms of a statistical description which has also the property of becoming an exact representation for the case the inputs are non-stochastic.

Starting from a Taylor-like expansion for the property f associated with the Nth value of the running process $x(t)$

$$f(x_{r,N}) = Nf(r) - \sum_{k=1}^{N-1} f(x_{r,k}) + N \sum_{h=1}^{\infty} \frac{d^h f(r)}{dr^h} \frac{S_h(N;r)}{h!}$$

one can express the Fourier series of a continuous process $n(t)$ in terms of the statistics S_h and a reference time t_r, by

$$\eta(t) = \frac{1}{2} \sum_{k=-\infty}^{\infty} \left\{ \eta(t_r) + \sum_{h=1}^{\infty} \frac{c_{nh}(k)}{h!} S_h(k,t_r,\theta) \right\} e^{i\omega_k(t-t_r)}$$ (6)

where $\omega_k = \frac{k\pi}{r}$, and ζ_{hh} a recursive coefficient with $\zeta_{k,o} = \eta(k)$ and

$$c_{k,h} = 1 - i\omega_k \frac{c_{k,h-1}}{\zeta_{k,k-1}}$$

Given a set of N disjoint tasks, the operator can rate these manual tracking tasks in an internal frame of reference of his by comparing the "innermost experience" he encountered during their execution. The hypothesis done is that this comparison is not exclusively based on his objective performance, but is a verbalization of his inner state of mind, physique, mood, or whatever one might be inclined to call it, hopefully relevant to the execution of the tasks [2].

These comparisons have to be done in the most elemental of fashions, ie. by pairs, so that one can investigate the most minute details of preference, confusion, inconsistency, etc. characterizing the subjective evaluations rendered by the operator. Otherwise, a direct comparison of all N tasks will only reveal judgements on a one-dimensional basis [3]. How can one get into the internal structure of the operator's evaluation mechanism might be a metaphysical question. If it were not for gathering and interpreting these comparisons in a very engineering and mathematical sense respectively.

Comparison information is gathered in the form of a certain numerical measure rendered by the operator on a physical variable on which humans are known to exhibit a fair aptitude for precise estimates (for ex. length, frequency; not weight or time etc). In general, this judgement-to-numbers mapping performed by the operator . subject to the sequence of task pairs already presented and to the sensory and motor mechanisms employed, as dictated from the physical analogy used. Focusing on the psychophysical, rather than on the physiological aspects of this mapping, if the numerical value for the similarity between the tasks of the nth pair is $d_{n,jh}$, then

$$d_{n,jh} = d_{i,jh}\left(n, \zeta_{s,n-1}\right)$$ (2)

Comparison information is interpreted as a measure of dissimilarity between the tasks of a given pair, each task being represented by a point in an abstract "subjective" space, the distance of the point from the rest being prescribed by the corresponding d_{jk}'s. The closer two points the more similar the tasks corresponding to these points are. The multi-dimensional nature of the manual tracking task suggests that in order to fit all those points in their appropriate position within the space one has to resort to a space of enormous dimensionality. If there are N different tasks compared to each other, then in general $N-1$ would be a sufficient space to accommodate without any discrepancies all points of interest subject to their mutual constraints, as dictated by their distances.

The particular form of the metric used to denote distances in this space, of the coordinate representation used for labelling the points in it, and of the reduction used to alleviate unnecessary dimensions, is an altogether different matter.(See discussion under "The Subjective Space").

This implies that even if one knew the exact time-history of the statistics one would not in principle reproduce exactly the input signal, since for infinitely differentiable $n(t)$'s one would truncate the series. However, the power of the above result is that, if one could start specifying some of the properties on frequency rather than just on amplitude, one could conceivably acquire information about the signal in a faster rate. For example, there have been in the past pseudo-random trackable signals generated only by combining a few sinusoids with random mixing of amplitudes. No series on amplitude statistics involved, just a randomly selected $n(t_r)$ which is different for each ω_k.

Returning to our general result of (4), the question arises: which of the ω_k's are most essential, beyond specifying certain of the S_k's? In other words, what is the minimum prescription for defining a frequency in a signal which has no apparent primary mode of oscillation? It is suggested in this paper that such a primitive notion of frequency is provided by the average rate the signal crosses its mean. The RMS_ω of fig 2 signifies such a frequency.

It can be noted that this primitive frequency is a function of time both explicitly and implicitly:

$$RMS_\omega(t, E, \{t\})$$

Explicitly, because the rate of crossings depends from the incoming signal flow; implicitly, because the mean of the signal itself changes. For a purely random sequence one would of course expect the mean to be constant, and the incoming signal to have an equal chance to go over or under the mean, in which case the frequency remains on the average constant.
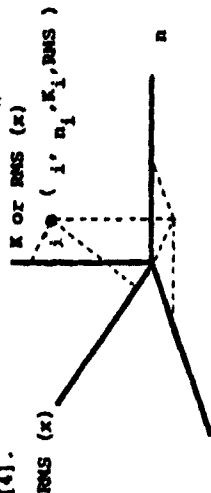
Summarising the above, one could say that all non-procedural experimental parameters are incorporated in the notion of task variables which, in the case of one-dimensional tracking on fixed display format and proportional imaging, are:

$$x = [S_h(t;n(t_r)) \text{ with } k-1, -]$$
$$D = [K(L) \text{ of } G_c \text{ as given by (3)}]$$
$$c = \{a_k \text{'s of } G_c = a_m s^{-} ... + a_0 t^{-} ... + a_n s^n\}$$

or combinations thereof. For any practical reasons $k-1$, -2 and $m = n-2$ for PID plants. An interesting application where x is filled to keep more of the statistics S_k characterising x is treated in [4].

The Task Space
Figure 2

After having defined the nature of the non-procedural experimental parameters, comes the time to talk about the procedural parameters of the experiment. Definition (2) explains the object of the experimental sessions: the rendering of subjective judgements about the similarity of the tasks of given pairs in a numerically quantifiable form. When is the sequence of given pairs more inconsequential to both the objective performance and the subjective evaluations of the operator? A standard sequence of randomly mixed pairs with each pair appearing at least two times seems to be a satisfactory answer.

However, this approach inevitably leads to long sessions where fatigue and boring play a very important role, and induces the tendency for insufficient in-between-task time of de-adaptation and for ridiculously low task durations. For example a 2-hour session with three values for each task variable of fig.2 would result in a task duration of ... .55 secs! Besides, is such an environment of abstract simulation and of intense need for continuous re-adaptation, the subjective sense of time for the human operator is severely offset [5].

A heuristic selection from the set of possible combinations of task variable values seems a much sounder method than an arbitrary splitting up of the experimental session. The selection can be achieved via the idea of derangements and partitions [6]. Of course, this way we get far less d_{ij}'s than necessary to fix our R^{M-1} space, so we need to extract d_{ij} from the human operator other kinds of comparison information as well. For example, a measure of combined difficulty, or any other characteristic f, in addition to similarities [4]. This type of experimental procedure would constitute a "fractional replication factorial design" according to [7].

THE SUBJECTIVE SPACE

Now, it is time to go back to the concluding remark made on the definition of the problem. How is the numerical information, gathered from the operator comparing manual tracking tasks in pairs, mathematically interpreted as a function $f(x_1(i),...,x_{k-1}(i); x_1(j)...x_{k-1}(j))$ of certain "hypothesised" $k-1$ coordinates $x_k(j)$ in the abstract space of subjective evaluations or "subjective space"? The essence of multi-dimensional scaling techniques lies in interpreting dissimilarity as a distance d_{ij}, and distance as a Minkowski metric [8]:

$$d_{ij} = \left[\sum_{k=1}^{M-1} |x_k(i) - x_k(j)|^p \right]^{1/p}$$

For Euclidean spaces $p=2$ and in order for the measured d_{ij}'s to be true physical distances one requires that d_{ij} is never greater than $d_{ik} + d_{kj}$. However there is no guarantee that this will be the case. The standard transformation used to ensure that this will hold is given by [9]:

$$\delta_{ij} = d_{ij} + \text{OFFSET}$$
$$\text{where OFFSET} = \max_{\{i,j,k\}} (d_{ij} - d_{ik} - d_{kj} > 0)$$

Since we know N dissimilarity measurements δ_{ij}, and at the same time have [N tasks • (N-1) coordinates] = 2 N^2 unknowns, there are N^2 degrees of freedom. [4] specifies an other N^2 constraints by taking into account symmetric measurements f_{ij} on an extra compari-son measure. [10] adds constraints that fix the origin at the cent-roid of all N points, and point the axes at certain orientations. The advantage over [4] and [10] of the method presented below is that one can obtain directly a closed form solution when setting the upper triangular table of fig. 3 to zero: the first point is at the origin, the second is on one of the axes, the third on any of the planes that incorporate the previously considered axis, and so on.

$$x_k(i) = \frac{\delta_{ki}^2 - \delta_{k,i+1}^2 + x_k^2(k+1) - \sum_{r=1}^{k-1}\left\{[x_r(k+1)-x_r(i)]^2 - [x_r(k)-x_r(i)]^2\right\}}{2\,x_k(k+1)} \qquad (5)$$

Equation (5) is the coordinate representation derived for labeling the point i, and is based on a Euclidean metric. It comp-utes the coordinate values in terms of the coordinates $x_r(k)$ of previously computed points, and of previously computed r coordina-tes $x_r(i)$ of the point in question.
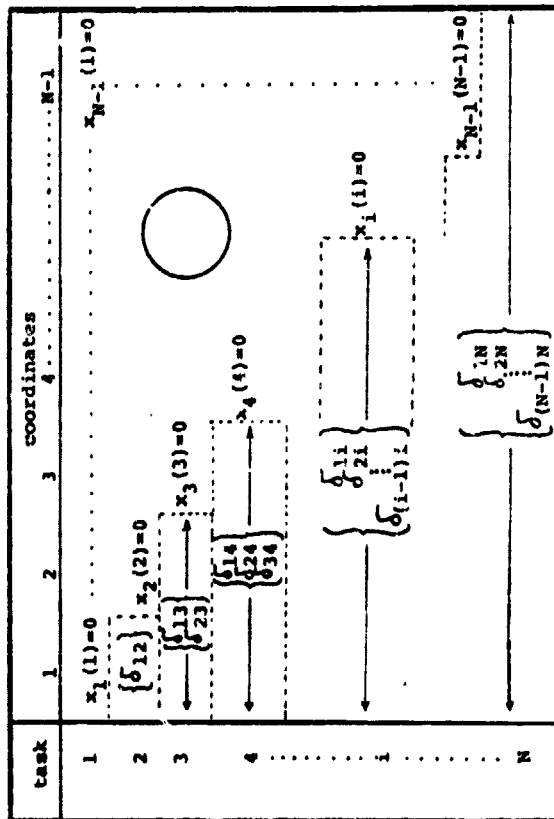
Since the dimensionality of the subjective space far exceeds the total number of task variables, one would like to arrive at a reduced space representation of the essential character portrayed by the "exact space" with much fewer dimensions. [11] lists a number of methods that do one of three things: either trim the exact space from low RMS coordinates and compensate the remaining of it with a best fit rearrangement, or partition the space in regions of high concentration and derive a lower order space corresponding to only the distances between the centroids of these clusters, or directly apply a statistical hyperplane fit (of prespecified dimensionality) to the exact space. The advantage of the method presented below is that the procedure outlined is exact and non-iterative in nature.

First one reorders the coordinates in order of least import-ance $[x's] \to [x's]$. Looking at figure 3 and accepting that the sig-nificance of coordinate k is the probability that a point has a non-zero kth coordinate times the average relative importance of coor-dinate k for all points i that have values on k:

$$\text{SIGNIFICANCE OF COORD } k = \frac{N-k}{(N-1)\,(N-1)/2\,N}\cdot\frac{\sum_{i=k+1}^{N} x_k(i)}{\sum_{i=k+1}^{N} E2(i-1)} \propto \sqrt{\sum_{i=k+1}^{N}\frac{(i-1)\,X_k^2(i)}{N-k}}$$

Secondly, one would like to define transformations on x's such that R linear combinations of them ξ are equal to zero. This then would mean that all information contained in x_{N-1} will be contained after the transformations are found in x_{N-k}. Suppose $f_k(\vec{x}_k)$ the transformation rule on the kth coordinate and T the matrix that maps the basis $\{\xi_{N-1}\}$ of x^{N-1} to a new basis $\{\xi_{N-1}\}$ in a space x^{N-1} which contains the sought after x_{N-k}. Then our complete description of the mapped transformation is:

$$\begin{pmatrix} \xi_1, \xi_1' \\ \xi_k, \xi_k \\ \vdots \\ \xi_{N-1} \\ 0 \end{pmatrix} = T \begin{pmatrix} f_1(\vec{x}_1)\,\xi_1 \\ f_k(\vec{x}_k)\,\xi_k \\ \vdots \\ f_{N-1}(x_{N-1})\,\xi_{N-1} \end{pmatrix}$$

← order of greater importance

Thirdly, one would like, if possible to have an exact descri-ption on the new coordinates $\xi_1, \xi_k, \ldots, \xi_{N-1}$ in terms only of the transformations f of the most important coordinates, in other words of $f_i(\vec{x}_i)$ rather than of $f_{i+1}(\vec{x}_{i+1})$. This implies that T should be lower triangular-like in form. Fig 4 indicates how such a T can be formed. Each row k represents a unit vector $\vec{e}_k$ which should be orthogonal to all others, ie. $\vec{e}_k \cdot \vec{e}_k = \delta_{kk}$.

$$\tau_{ii} = -\left(-\lambda^2 + \sum_{k=1}^{i}\tau_{ik}^2\right)/\tau_{i-1,i}$$

$$\tau_{i,i+1} = \left[1-\lambda^2 - \sum_{k=1}^{i}|\tau_{kk}|^2\right]^{1/2}$$

task	1	2	3	4	.....	N-1
1	$x_1(1)=0$				.....	$x_{N-1}(1)=0$
2	$\{\delta_{12}\}$; $x_2(2)=0$					
3	$\{\delta_{13},\delta_{23}\}$	$x_3(3)=0$				
4	$\{\delta_{14},\delta_{24},\delta_{34}\}$		$x_4(4)=0$			
i	$\{\delta_{1i},\delta_{2i},\ldots,\delta_{(i-1)i}\}$				$x_i(1)=0$	
N	$\{\delta_{1N},\delta_{2N},\ldots,\delta_{(N-1)N}\}$					$x_{N-1}(N-1)=0$

coordinates

Figure 3

and

$$\left\{ \varepsilon^2 \left[\alpha^2 \sum_{k=2}^{N-1} \overline{\tau_{Nk}^2} \right] + \beta^2 \overline{\tau}_{(N-1)(N-1)} = 0 \right\}$$

$$\left\{ \varepsilon^2 \left[\alpha^2 \sum_{k=2}^{N-1} |\tau_{Nk}|^2 \right] + \beta^2 = 1 \right\}$$
determine ε and β.

Fourthly, one would like to have meaningful transformations f's. Polynomial stretching-shrinking functions $\hat{w}(\tilde{\chi}_{ki}) = \sum_{\ell=0}^{Q} \alpha_{ki} \tilde{\chi}_k^{\ell} + \chi_{k}$ that preserve the order of the points in the untransformed space are used. Suppose Q_{ki} the order of the projection of the ith point on the kth coordinate, then

$$O_{ki} \cdots \tilde{\chi}_{k}^{Q+1}(i) = \sum_{\ell=0}^{Q+1} \alpha_{k\ell} \tilde{\chi}_k^{\ell}(i)$$

are $N(N-1)$ equations for the $(N-1)(Q+1)$ unknown coefficients $a_{k\ell}$. The rest RN equations come from requiring that $\zeta_k = \sum_{\sigma=1} \tau_{k\sigma} \tau_{r\sigma} f_\sigma(\tilde{\chi}_\sigma) = 0$ for all new coordinates beyond the desired reduction; in other words

$$-\sum_{\sigma=1}^{N-1} \tau_{k\sigma} \tilde{\chi}_\sigma(i) = \sum_{\sigma=1} \tau_{k\sigma} \alpha_{\sigma\ell} \tilde{\chi}_\sigma^{\ell}(i) \quad \text{for } k = N-R \text{ to } N-1.$$

For a unique determination of the coefficients one has to put $Q = N + R + 1$ and specify arbitrarily a total of $2(N-1) - R$ of them. After all a's have been determined, one arrives at the reduced
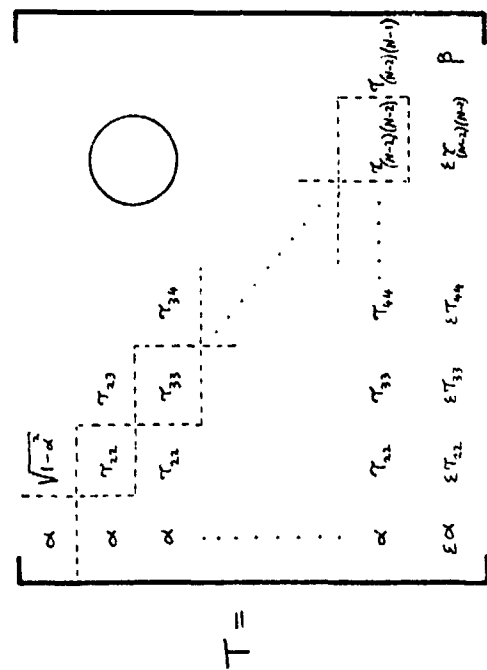

Figure 4

subjective space
$$\zeta_K = \left\{ \left| \sum_{\sigma=1}^{k+1} \left| \tau_{K\sigma} \cdot \left(\sum_{\ell=0}^{a} \alpha_{\sigma\ell} \tilde{\chi}_\sigma^{\ell} + \tilde{\chi}_\sigma^{Q+1} \right) \right|^2 \right| \right\}^{1/2}$$
(6)

which is an exact representation in terms of the original subjective space. Information from the least significant of the coordinates $\tilde{\chi}$ is not lost, because that information is carried implicitly in the determination of the coefficients.

PROPERTY VECTORS

Having arrived at a reduced space for subjective evaluations, with a dimensionality $N - 1 - R = M$ comparable to the hypothesized independent task variables, one would like to know on which directions in this space a certain property P of the tasks experiences maximum change.

$$[\nabla P(\zeta_1, \zeta_2 \cdots \zeta_M)]_{max} \text{ implies } \sum_{k=1}^{M} \frac{\partial P}{\partial \zeta_k} \frac{\partial^2 P}{\partial \zeta_j \partial \zeta_k} \zeta_k = 0 \quad \forall j = 2, M$$

where ∇ denotes the gradient in the reduced subjective space. There are three issues involved here: which are the properties P of interest; how does one arrive at their functional form in the reduced space formulation; and which are the best linear functions $f_m(\zeta_1), f_{m-1}(\zeta_1), \cdots \zeta_n(\zeta_1)$ that represent the vector along which such a change occurs.

There are only three kinds of task properties one could think of:

(a) the task variables (see discussion under "The Experimental Parameters")

(b) various functions of subjective nature. The normative Cooper-Harper scale [3], multi-attribute utility functions [11],etc.

(c) various measures of the operator's performance. These are observed and computed statistics characterizing the objective performance of the human operator on any given task, and they comprise the O b j e c t i v e s p a c e (figure 5).
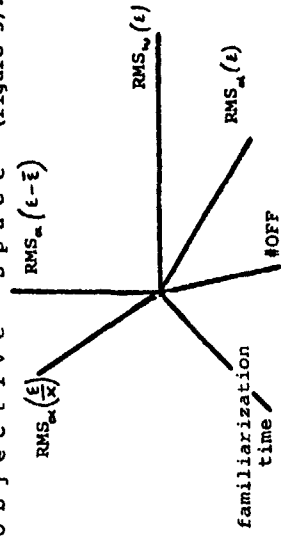
The Objective Space
Figure 5

There are two classes of interesting statistics $S_k(t;\tau)$ that characterize the objective performance, statistics on $\epsilon(t)$ and statistics on $\epsilon(t)/x(t)$. Suppose $\phi(\epsilon(t),x(t))$, then in order for one to know any

$$\frac{1}{t}\int_0^t \phi^\kappa(\tau)d\tau = E_1^\kappa(t) + \sum_{k=1}^K \frac{\kappa!/k!}{(\kappa-|k-1)!} E_{1,k}^{\kappa-k}(t)\, S_k(t;E_1(t))$$

one also needs $E_1(t)$. Figure 5 portrays the case $k=2$, along with the RMS frequency at which the error becomes + or -.

In addition to the measures that are, one way or another, statistical functions of the error and the input h_1 (in our case x), there are other performance measures which are display-dependent or non-accumulative in character. An example of the first kind is the number #OFF of times the display output gets off the limits of the CRT screen. An example of the second kind is the time required for the human operator to remain within a certain percentage of failure from exceeding the standard deviation achieved by task's end.

Having defined above what is meant by property in the context of psychophysical engineering, one can address the second question: how does one arrive at their functional form. Given, for example, C-H ratings corresponding to the N tasks, one has N equations to solve for a total of N suitably selected a's,

$$P(i) = \sum_{\{K\}} a(K, K_1 \ldots K_M)\, \xi_1^{K_1} \xi_2^{K_2} \ldots \xi_M^{K_M}$$

The sequence in which one would employ the a's should be governed by the rationale that lower order Taylor-expansion-like terms are preferred to higher order ones. This procedure is known as multi-variate synthesis.
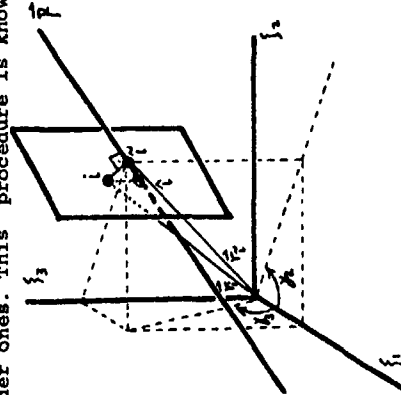
Property Fitting
Figure 6

Now suppose $\tilde{r}$ is the projection of the point i, corresponding to the ith task in the reduced subjective space, on the sought after vector $\tilde{P} = \xi_1' + \sum_{k=1}^M \ldots$ on which $\ldots$ minimizes the cost functional $J = \sum_{i,k}[P(i)-\tilde{P}(i)]$. By combining the relation $(\tilde{r}_i-\tilde{r}_i')\tilde{P}=0$ (which renders $\tilde{r}_i'$ the projection of i) with the idea of neglecting higher order terms in the expansion

$$P(i) = P(i) + \sum_{j=1}^M \frac{\partial P}{\partial \xi_j}\Big|_i \big(\xi_j(i)-\xi_{ji}(i)\big) + \sum_{k=2}^K \frac{1}{k!}\left[\sum_{j=1}^M (\xi_j-\xi_{ji})\frac{1}{\partial \xi_j}\right]^k P$$

one arrives at expressions for $\partial J^2/\partial \gamma_k = \partial J^2/\partial \delta_k = 0$ that can be solved iteratively to find the γ's and δ's which best specify the direction (the γ's) and the origin (the δ's) of $\tilde{P}$.

The advantage of this method over the methods mentioned in [11] is that one fixes completely the property vector in the reduced subjective evaluation space, while the rest assume that in all regions of the space the property direction keeps always parallel to itself. Also, as one can inspect from figure 6, the best estimates $\tilde{r}$'s for the points are exact projections on the fitted vector rather than mere images. $\tilde{r}$'s on planes parallel to the $\{\tilde{M}$ axis. If $\tilde{P}_1, \tilde{P}_2, \ldots \tilde{P}_L$ the property vectors corresponding to the task variables, then one can derive the matrix $\prod_{L \subseteq M}$ that defines the mapping from the subjective space to the task vectors, and call $\prod$ the "matrix rating" associated with this subjective space.

DISCUSSION

It is this last technique that allows us to find out if the mapping of the task space in the subjective space maintains the orthogonality between the task vectors or not. If, for example, we arrive at an experimentally deduced law of universal validity about certain task variables not comprising an orthogonal set in the subjective space, then it might mean only three things:

(a). In our abstract modelling of the task we might have misconceived or altogether skipped certain interdependencies amongst the so-called task variables.

(b). In our internal model of the task we might have misperceived -or inadequately learned to appreciate-, the independence of the task variables.

(c). Mathematical misformulations and psychophysical misjustifications concerning developments in the theory of subjective spaces.

I would like to suggest that all three factors come into play:

(A). Not accounting for the higher order statistics in the representation of the input disturbance, in the new parameters introduced by the digitization of the control structures in the simulation environment, etc.

(B). The human operator might subconsciously introduce artificial dependencies among the various independent parameters in order to facilitate data reduction and information storage mechanisms that might govern his/her internal response and decision processing.

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NOMENCLATURE

c_{kn} — the sequence of the first n k-tuples presented $\{(i_1,j_1,k_1,...),(i_2,j_2,k_2,...),...,(i_n,j_n,k_n,...)\}$

(i,j) — the pair comprised of the ith and j^{th} tasks

N — the total number of tasks $\prod_{l=1}^{L} n_l$ ($L=\#$ of task variables)

$\mathcal{N}$ — all possible pairs from N tasks $N(N-1)/2$

R^m — an m-dimensional space

n_ℓ — the # of the different values on the ℓth task var.

$E_k(N)$ — the kth average of the first N values, ie. $\overline{x^k}$
$$E_k(N) = x_N^k / N + (N-1) E_k(N-1) / N$$

$S_k(N;r)$ — the statistic $\langle (x-r)^k \rangle$ of an N-member sampling x from r
$$S_k(N;r) = \sum_{k=0}^{n-2} \frac{(-)^{k+k}\, k!}{k!(k-k)!} E_k(N)$$

$\Theta_n(N)$ — the nth threshold of a regressional filter
$$\Theta_n(N) = \Theta_{n-1}(N) - \left[\frac{|S_n(N;n-1)|}{2^{n-2}} \right]^{1/n} \quad \text{with } \Theta_1(N)=E_1(N)$$

$T(\underline{e}_n,\underline{e}_n')$ — the transformation matrix that maps the coordinate system of an original R^m space to a new R^n space whose coordinate system representation basis $\underline{e}'$.

(C). Dissimilarity judgements might not be modellable by a triangular inequality, so that their mathematical interpretation as distances might not be appropriate. But even if that is not the case, some Minkowski metric might exist such that it does preserve the orthogonality of the mapped task space.

re is much utility to be found in exploring the subjective evaluation space of human operators in manual tracking tasks.

(α). Knowing a sufficient description of the reduced subjective space corresponding to such and such manual tracking tasks, one could predict the subjective evaluations and objective performance of any human operator whose psychophysical aspects are similar to those for which the particular space was constructed.

(β). One could find for which categories of people such and such task variables are not treated independent of each other.

(γ). The subjective space could become a ground for testing the objective performance indices that best co-align with the C-H scales.

Actually, this last issue is one of the most interesting. It raises the possibility of resolving the often observed incompatibility between objective performance and subjective evaluation. What one would expect from a rational processor like the human operator is to match his internal satisfaction with the task to his actual performance. There is speculation, however, about three possible antagonistic mechanisms present that could be checked via the subjective space idea:

(A). Improving one's own satisfaction with a particular task does not necessarily mean better objective performance.

(B). Increasing the disheartening aspects of a task might actually result in improved performance.

(Γ). Teaching the human operator to consider the right type of variables for his/her task -by utilizing the mapping of the task space in his subjective space- might result in a worse optimal control representation for his internal model, due to (B).

If such antagonisms between subjective evaluations and objective performance are indeed sometimes observed, one should rest easy; according to [12] these antagonisms often result from acceptable "rational" kinds of task conceptualization and internal goal-defining.

REFERENCES

[1] McRuer + Krendel "Mathematical Models of Human Pilot Behavior" NATO Agard-AG-188, Jan 1974

[2] A.Slapkara "The Cognitive Processes Underlying Human Pilot Models" 1975 (Unpublished. Can be obtained by request by writing to POB 6039,Boston,Ma 02209)

[3] J.McDonnell "Pilot Rating Techniques for the Estimation and Evaluation of Handling Qualities" AFFDL TR-68-76,Dec 68

[4] A.Slapkara "Modelling and Logic behind the Evaluation of Automated Production Centers" 1976 (Unpublished. Can be obtained by request from POB 6039,Boston,Ma 02209)

[5] J.Holubář "The Sense of Time" The State Medical Publishing House, Prague 1961

[6]   S.Even "Algorithmic Combinatorics" Ed. Collier-Macmillan;73

[7]   D.Campbell + J.Stanley "Experimental and Quasi-experimen
      Designs for Research" The American Educational Research Association, 1963

[8]   Kruskal,J.B. & F.J.Carmone "M-D_SCAL_5" Bell Telephone Laboratories,March 1969

[9]   Torgerson,W.S. "Theory and Methods of Scaling" Wiley Ed.,
      N.Y. 1960

[10] Carroll + Chang "INDSCAL analysis via an N-way Generalization
      of Eckart-Young Decomposition", Psychometrica 35,
      1970, 283-319

[11] Paul E. Green + Yoram Wind "Multi-attribute Decisions in
      Marketing" The Dryden Press (chapters on multi-dimensional scaling)

[12] D.Bobrow + A.Collins "Representation & Understanding", Academic Press, 1975 (p 131-184).