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PREDICTION OF PILOT-AIRCRAFT STABILITY  
BOUNDARIES AND PERFORMANCE CONTOURS

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ABSTRACT

Control-theoretic pilot models can provide important new insights regarding the stability and performance characteristics of the pilot-aircraft system. Optimal-control pilot models can be formed for a wide range of flight conditions, suggesting that the human pilot can maintain stability if he adapts his control strategy to the aircraft's changing dynamics. Of particular concern is the effect of sub-optimal pilot adaptation as an aircraft transitions from low to high angle-of-attack during rapid maneuvering, as the changes in aircraft stability and control response can be extreme. This paper examines the effects of optimal and sub-optimal effort during a typical "high-g" maneuver, and it introduces the concept of minimum-control effort (MCE) adaptation. Limited experimental results tend to support the MCE adaptation concept.

INTRODUCTION

Since Tustin first likened the command and response patterns of anti-aircraft gunners to rudimentary control systems (Ref. 1), the intriguing notion that control-theoretic mathematical models can characterize the human operator has been carried to a high state of development. Frequency-domain models have proven capable of capturing fundamental aspects of the human operator's behavior in a straightforward and logical fashion (as in Refs. 2 and 3), while time-domain models have demonstrated that a well-motivated subject can, in fact, behave like an optimal control system in various single- and multi-axis tracking tasks (Refs. 4 to 6). Nevertheless, a number of perplexing problems remain in the study of what might be called the pilot's "discretionary control behavior," i.e., given that the subject is physically and emotionally capable of performing a task in an optimal manner, why might that subject choose to do otherwise?

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The matter is of particular concern when the subject is a skilled pilot and the task is controlling a maneuvering aircraft, for the success of the mission and the pilot's own safety are strong motivating factors. During rapid maneuvering, the aircraft's dynamic characteristics can change markedly in a matter of seconds, and the pilot may be called upon to make changes in his control strategy just to maintain stability, much less carry out his mission. More often than not, the pilot who performs such maneuvers has mastered the necessary procedural adaptation and executes it with precision. On rare occasions, even the skilled pilot may get into trouble, adapting his control strategy to suit poorly chosen criteria, or perhaps not adapting at all. In high-performance aircraft, this apparent lapse can cause a pilot-induced "departure," i.e., a loss-of-control incident which, if not corrected immediately, can lead to a spin and possible loss of the aircraft. The problems for study are not only how to model the pilot's discretionary behavior in departure-prone maneuvering tasks, but how to relate such a model to the more frequent, near-optimal behavior of the skilled pilot.

The approach taken in this paper is to define a sequence of optimal-control pilot models which correspond to the aircraft's changing dynamics as it performs a nominal maneuver and to examine the effects of pilot-aircraft model mismatch on closed-loop stability and statistical tracking error. The maneuver -- a "wind-up turn" -- begins at low angle of attack ( $\alpha_0$ ) and proceeds to a high  $\alpha_0$ . As the maneuver progresses, there is a dramatic variation in the optimal piloting strategy, including, in some cases, a change in sign of the pilot's stabilizing commands to the aircraft. From the outset, it is clear that sufficient mismatch could lead to closed-loop instability, but the rationale for large mismatch remains to be determined.

A hypothesis for mismatch is found in the minimum-control-effort model of pilot adaptation, which simply stated, suggests that the pilot selects not the optimal strategy but the one which minimizes the variance of stick and pedal motions (in the mismatched case). With the minimum-control-effort (MCE) pilot model, closed-loop stability can be maintained, but the margin for error is reduced, by comparison to the optimal strategy. Where the pilot has a choice of control outputs, (e.g., stick alone, pedal alone, or combination of the two), the MCE pilot model also predicts the point during the maneuver at which the pilot may choose to transition from one command mode to another to maintain stability with minimum effort.

The MCE pilot model has yet to be validated by exhaustive experimentation, but there is remarkable agreement between

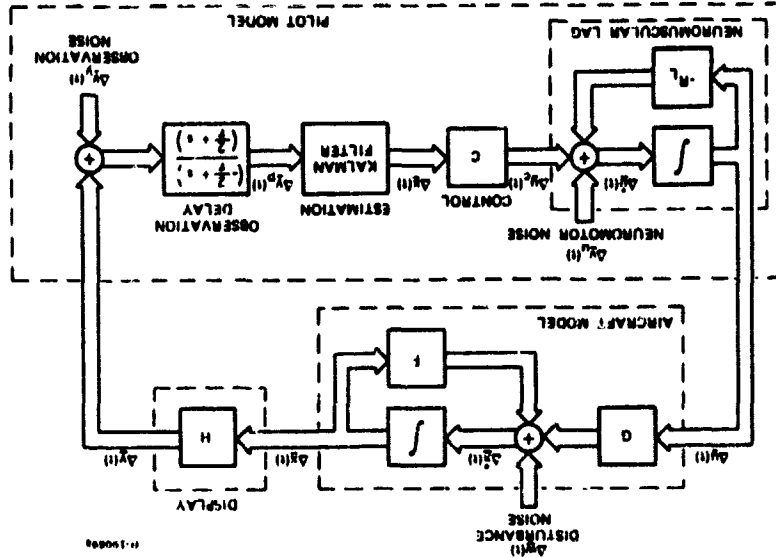
the model's predictions and piloted, ground-based simulation results, one of which is shown below.

#### PILOT AND AIRCRAFT MODELS

A block diagram of the pilot and aircraft models is shown in Fig. 1, and it is seen to be similar in structure to the systems of earlier studies (Refs. 5 and 6). The aircraft is modeled as a linear, time-invariant system with state vector  $\Delta x$ , control vector  $\Delta u$ , and output vector  $\Delta y$ . The pilot observes the output, adding noise,  $\Delta v$ , in the process and introducing a perceptual delay  $\tau$ . The pilot model estimates the aircraft's states (as well as the added states due to the observation delay and neuromuscular lag), from the delayed outputs,  $\Delta y$ , and forms a feedback control strategy based upon  $\Delta x$ . Neuromotor noise,  $\Delta w$ , is added to the pilot's internal command,  $\Delta u$ , and these are subjected to a neuromuscular lag prior to commanding the aircraft model via  $\Delta u$ . The primary parameters of the pilot model are: the Kalman filter gain matrix,  $K$ ; the linear-optimal regulator matrix,  $C$ ; and the neuromuscular lag matrix,  $R$ . Further details of the model structure can be found in Refs. 7 to 9, which illustrate the distinguishing features of the model: use here (including use of the Pade approximation, uncoupled multiple pilot commands with differing neuromuscular time constants, and use of a "contraction mapping" sequence in finding the gains of the pilot model).

Attention is directed to the effects that the pilot model has on a high-performance aircraft which is in a "wind-up turn" maneuver, described by the first four columns of Table 1. The pilot model is formulated to control only the lateral-directional modes of the aircraft using observations of Euler angles and angular rates. The pilot model can control the aircraft with lateral stick motions (which control differential stabilators and spoilers), foot pedal motions (which command rudders), or both. Beginning in straight-and-level flight, the aircraft is rolled into a turn that eventually achieves a steady-state body-axis pitch rate of 7.5 deg/sec. As velocity decreases, the angle of attack,  $\alpha$ , must increase to maintain the earth-relative turn rate (hence, the name "wind-up turn"), and the dynamic characteristics of the aircraft change markedly during a short period of time. In particular, the aircraft's lateral control surfaces (used primarily for roll control at low  $\alpha$ ) develop an adverse yaw characteristic for a beyond 12 deg. their control "power" is reduced, and there is a region of Dutch roll instability for a beyond 18 deg.

Figure 1: Block Diagram of the Pilot-Aircraft Model



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Table 1  
Eigenvalues of the Wind-Up Turn with Pilot  
Using Lateral Stick Alone for Control

MANUEVER COND'ION	PILOT LATERAL STICK/SPIRAL		DUTCH ROLL		ROLL		YAW		
	$\sigma_1$ deg	$\sigma_2$ deg/sec	$\zeta_1$ rad/sec	$\zeta_2$ rad/sec	$\zeta_3$ sec	$\tau_3$ sec	$\tau_1$ sec	$\tau_2$ sec	
244	1.02	1.0	0.0	7.38	0.740	2.40	0.468	0.892	6.9
244	8.72	4.3	7.5	7.00	0.679	1.62	0.614	0.781	1.72
213	11.1	3.8	7.5	6.69	0.655	1.11	0.683	0.642	1.84
183	15.4	3.3	7.5	6.48	0.635	0.296	0.775	0.535	1.19
152	19.8	2.9	7.5	6.11	0.642	0.486	0.861	0.521	2.39
137	24.6	2.5	7.5	6.09	0.614	0.265	0.722	0.532	0.855

Using lateral stick motions alone, the optimal pilot model develops adapted control strategies which stabilize the entire maneuver very well, as shown by the remaining six columns of Table 1. The real roots associated with the pilot's lateral arm motions and the aircraft's spiral mode coalesce to form a damped, oscillatory mode; the aircraft's Dutch roll mode remains well-damped, although its natural frequency decreases sharply in the region of adverse yaw; the roll mode time constant decreases by 40 percent during the maneuver, and the mode which results from feeding back yaw angle observations quickens with increasing  $\alpha$ .

A close look at the adapted control gains (Table 2) reveals that substantial variations in strategy are required to maintain optimal control during the maneuver. Not only must the gain increase in magnitude to account for decreasing control power, but several gains change sign -- in fact, the yaw rate gain ( $\delta\delta/\delta r$ ) changes sign three times. The first change occurs as a result of the increased pitch rate, which changes the nature of the Dutch roll mode. The second change occurs at the onset of adverse yaw, while the third can be attributed to the unstable open-loop Dutch roll mode. Although it is well within an actual pilot's psychomotor ability to behave as an optimal controller

at each flight condition, the demands for smooth adaptation during the course of the maneuver are excessive. It is likely, therefore, that the pilot may choose to adapt in sub-optimal fashion when using the stick alone for control.

Table 2  
Control Gains of the Adapted Pilot Using  
Lateral Stick Alone

AIRCRAFT ANGLE OF ATTACK, deg	SIDE VELOCITY $\delta\delta/\delta v$ , deg/ftps	YAW RATE, $\delta\delta/\delta r$ deg/deg/ sec	ROLL RATE, $\delta\delta/\delta p$ deg/deg/ sec	ROLL ANGLE, $\delta\delta/\delta\theta$ deg/deg	YAW ANGLE, $\delta\delta/\delta\psi$ deg/deg
1.02	+0.522	-0.123	-4.93	-0.345	-1.68
8.72	-0.509	-0.619	+1.74	-0.582	-1.23
11.1	-0.544	-1.02	+1.43	-0.827	-1.62
15.4	-0.433	-3.52	-2.98	-3.40	-3.72
19.8	-0.417	-3.26	+18.8	+2.89	+3.74
24.6	-1.50	-3.66	+18.07	+1.80	+4.12

As shown in Table 3, if the pilot chooses to use foot pedals as well as lateral stick motions for control, the need to make large changes in strategy to maintain optimal control is greatly reduced. The only sign change in lateral stick gains occurs near the onset of adverse yaw for side velocity feedback, while roll and yaw angle feedback to foot pedals change sign when positive pitch rate is established. The range of adapted gain magnitudes still is large, but the procedural changes required of the pilot are less than in the previous case.

Should the pilot choose to stabilize the maneuver using foot pedals alone, his strategy is not significantly different from the foot pedal strategy required for dual controls. A comparison of Tables 3 and 4 shows that the foot pedal gains have the same signs at all but the first flight condition, and the gain magnitudes are very close as well. Thus, it is clear that the degree of adaptation required for foot pedal control is relatively low, although there is substantial change in gain magnitudes to account for changing control power.

#### STABILITY BOUNDARIES

While the adapted pilot model maintains a high level of stability throughout the wind-up turn, it is apparent that adap-

Table 3  
Control Gains of the Adapted Pilot Model Using  
Lateral Stick and Foot Pedals

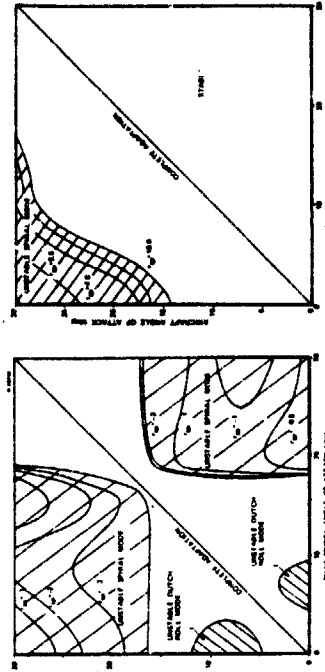
CONTROL	AIRCRAFT ANGLE OF ATTACK, deg	SIDE VELOCITY $\frac{\partial \delta}{\partial v}$ , deg/fps	YAW RATE $\frac{\partial \delta}{\partial r}$ , deg/deg/ sec	ROLL RATE $\frac{\partial \phi}{\partial p}$ , deg/deg/ sec	ROLL ANGLE $\frac{\partial \phi}{\partial \psi}$ , deg/deg	YAW ANGLE $\frac{\partial \delta}{\partial \psi}$ , deg/deg
Lateral Stick	1.02	-0.645	-0.367	-0.386	-0.764	-2.08
	8.72	-0.0299	-0.026	-0.559	-0.853	-2.19
	11.1	+0.0302	-0.42	-0.947	-1.22	-3.29
	15.4	+0.548	-2.84	-2.84	-2.98	-8.78
	19.8	+0.884	-4.34	-3.31	-3.00	-9.16
Pedals	1.02	+0.0342	-0.995	+0.0572	+0.109	+0.101
	8.72	+0.0137	-1.074	+0.0087	-0.0901	-0.397
	11.1	+0.0474	-1.48	+0.0451	-0.169	-0.721
	15.4	+0.154	-2.39	+0.073	-0.369	-1.57
	19.8	+0.339	-3.90	+0.259	-0.625	-2.82
24.6	+0.454	-5.42	+0.119	-1.04	-4.66	

Table 4  
Control Gains of the Adapted Pilot Model Using  
Foot Pedals Alone

AIRCRAFT ANGLE OF ATTACK, deg	SIDE VELOCITY $\frac{\partial \delta}{\partial v}$ , deg/fps	YAW RATE, $\frac{\partial \delta}{\partial r}$ , deg/deg/ sec	ROLL RATE, $\frac{\partial \phi}{\partial p}$ , deg/deg/ sec	ROLL ANGLE, $\frac{\partial \phi}{\partial \psi}$ , deg/deg	YAW ANGLE, $\frac{\partial \delta}{\partial \psi}$ , deg/deg
1.02	+0.0276	-1.049	-0.0281	-0.1618	-0.339
8.72	+0.0574	-1.097	+0.0327	-0.181	-0.324
11.1	+0.127	-1.449	+0.0526	-0.218	-0.440
15.4	+0.276	-2.01	+0.126	-0.269	-0.625
19.8	+0.503	-2.96	+0.294	-0.403	-0.987
24.6	+0.772	-4.33	+0.373	-0.729	-1.74

tation is itself a difficult task. The pilot may choose to adopt response patterns which are more consistent and, therefore, sub-optimal with respect to the criteria used to generate the pilot model. Furthermore, even if the pilot chooses to adapt optimally in a dynamic maneuver, there is likely to be a lag between the aircraft's actual flight condition and the pilot's adaptation point. Consequently, it is instructive to examine cases in which the aircraft's dynamics and the control strategy adopted by the pilot are mismatched. In the examples which follow, it is assumed that the pilot formulates an optimal control strategy for an assumed angle of attack,  $\alpha_p$ , which may or may not be the same as the aircraft's angle of attack,  $\alpha_A$ , during the maneuver.

Figure 2 illustrates the boundaries between pilot-aircraft stability and instability for independent variations of  $\alpha_A$  and  $\alpha_p$  (during the wind-up turn). When the pilot model output is lateral stick command alone (Fig. 2a), mismatch introduces regions of Dutch roll and spiral mode instability which are approximately symmetric about the line of perfect adaptation. There is a "stability neck" in the vicinity of  $\alpha_A = 18$  deg, where  $\alpha_p$  must be very close to  $\alpha_A$  to maintain stability. Not only is adaptation in this region potentially difficult (as indicated by Table 2), it is crucial that it be nearly optimal to prevent loss of control. If the pilot model uses foot pedals as well as lateral stick (Fig. 2b), stability margins are substantially increased, and there are no unstable regions for  $\alpha_p$  greater than  $\alpha_A$ . If the pilot model uses foot pedals alone for control, the entire region is stable; in other words, a mismatch of as much as 30 deg between  $\alpha_A$  and  $\alpha_p$  causes no instability if the pilot model does not use lateral stick motions for control.



a) Lateral Stick Alone  
b) Lateral Stick and Foot Pedals  
Figure 2: Effects of Pilot Model Adaptation on  
Maneuvering Flight Stability

The "separation theorem" of stochastic, linear-optimal control (Ref. 10) is only partially applicable to the optimal-control pilot model. The stability results presented here depend on the pilot's control strategy, but not on his estimation law; therefore, control results are separated from estimation results, but the converse is not true. The pilot model estimator gains (K) and eigenvalues depend on the control strategy as a consequence of signal-dependent neuromotor and observation noise, but the pilot model control gains (C) and "closed-loop" aircraft eigenvalues do not depend on K. Thus, the stability boundaries of Fig. 2 apply as long as the pilot model is able to make a stable estimate of the aircraft's state. "Vertigo" is an example of a circumstance in which the pilot estimator does not converge. In the present case, the pilot model experiences estimator divergence with dual control at high  $\alpha_A$  (Fig. 2b) because the signal-dependent neuromotor noise is free to grow without bound in the estimator solution (Ref. 9); however, if the neuromotor noise level is bounded (standard deviation of 1 in for stick motion and 0.7 in for pedal motion), a stable estimator solution is obtained for all  $\alpha_p$ .

#### PERFORMANCE CONTOURS

In addition to predicting stability boundaries for varying flight conditions, the pilot-aircraft model (Fig. 1) can be used to predict the statistics of tracking errors and control usage within the stable regions. (Outside the stable regions, these statistics grow without bound). The method applied here is covariance analysis (Ref. 11), in which the aircraft is assumed to be driven by a gaussian disturbance (turbulence with an rms level of 1.52 m/sec (5 fps) and a correlation time of 0.3 sec), and the pilot-aircraft model is used to compute the rms values of state and control perturbations which result\*. Since the covariances of system variables are used to define the pilot model estimator gains, the same equations can be used to evaluate the statistical performance of the pilot. In order to use these equations, the pilot model estimator is assumed to be adapted at each flight condition (defined by  $\alpha_A$ ); hence, when  $\alpha_p \neq \alpha_A$ , only the pilot model

\*The state covariance matrix, X, is defined as the expected value of the products of the states, i.e.,  $X = E(\Delta X \Delta X^T)$ , and the rms values of the states are the square roots of the diagonal elements of X. The control covariance matrix, U, and the associated rms values are similarly defined. The covariance propagation equations are detailed in Ref. 9.

control gains are mismatched. The results which follow demonstrate the effects of sub-optimal control strategy on piloting performance; the effects of sub-optimal estimation remain to be determined.

Figure 3 presents contours of equal rms values under the assumption that the pilot uses lateral stick motions alone for control. These contours would scale up or down as the turbulence level changed, so absolute values are less important than relative values in evaluating the figure.

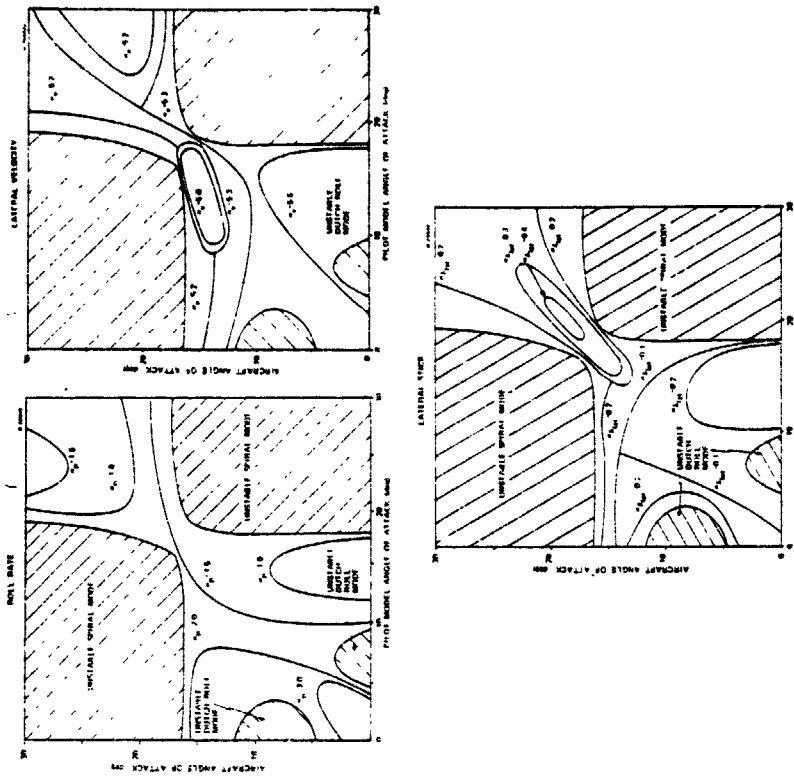


Figure 3: Performance Contours for Control with Lateral Stick Alone

The contours for roll rate rms ( $\sigma_r$ ), side velocity rms ( $\sigma_v$ ; equivalent to sideslip angle times forward velocity), and stick motion rms ( $\sigma_{\delta_{lat}}$ ) define surfaces of rms values in much the same way as a topographical map displays hills and valleys. It is apparent that the valleys in Fig. 3 do not lie along the line of perfect adaptation, nor do they overlay each other with complete regularity. There are two reasons for this. Although the control gains are meant to minimize a weighted sum of state and control covariances, this is no guarantee that the covariances of individual components will be minimized at the same time. There is a tradeoff between tracking error and control usage, so it is likely that decreases in one component will be accompanied by increases in another. The second reason is that the partial breakdown in the separation theorem leads to the possibility that alternate control strategies could yield lower values of control cost than the separately optimal control law.

Controlling to minimize  $\sigma_{\delta_{lat}}$  does have the effect of approximately minimizing  $\sigma_p$ , although increased control activity is required in the region of the stability neck (Fig. 3). Maintaining perfect adaptation for  $\alpha_A = 17$  to 20 deg requires four times the rms control motion that is used at low  $\alpha_A$ . In addition to substantial gain variation with flight condition (Table 2), perfect adaptation also leads to increased control effort, again suggesting that the pilot may choose to change his control mode or to adapt in sub-optimal fashion.

Performance contours for a pilot model using both stick and pedals demonstrate that the addition of pedal control has little effect on  $\sigma_p$  (except at low  $\alpha$ ), but it does reduce  $\sigma_v$  (as might be expected) and increase stability margins (Fig. 4). The pilot model can retain low  $\sigma_p$  and  $\sigma_v$  to high angles of attack with low control effort by fixing up in the vicinity of 10 deg (Fig. 5), a decidedly sub-optimal policy by the criteria used for pilot model computation. Sub-optimal or not, this adaptation approach could have definite appeal to the human pilot, for it provides perceptibly low tracking error and control effort with minimal adaptation; however, if the pilot wishes to fly to  $\alpha_A = 30$  deg and beyond, he must adopt a marked change in control strategy to avoid spiral mode instability and estimator divergence. Reference 9 presents additional results for pedal-alone control which demonstrate that stability and low control effort can be maintained with the stick centered for all  $\alpha_A$  considered.

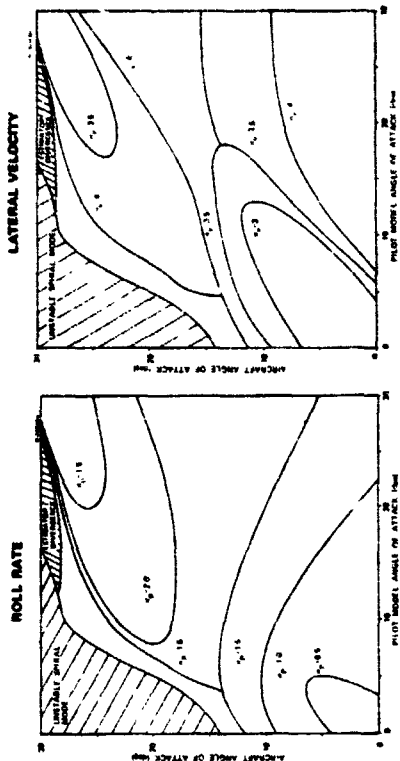


Figure 4 Performance Contours for Control with Lateral Stick and Foot Pedals

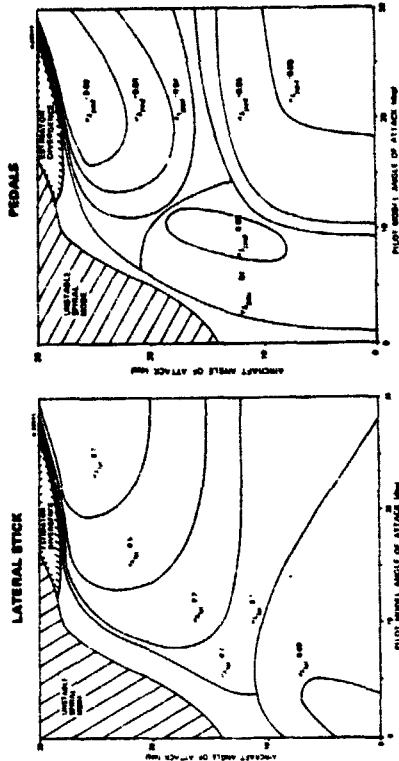


Figure 5 Control Usage Contours for Control with Lateral Stick and Foot Pedals

### MINIMUM-CONTROL-EFFORT PILOT MODEL

The combination of reduced control effort and reduced variation in control strategy suggests a hypothesis for minimum-control-effort (MCE) pilot adaptation, which also can predict at what point the pilot is likely to switch his control mode. Figure 6 illustrates two MCE adaptation patterns for the wind-up turn, with the heavy line tracing the corresponding  $\alpha_A$ -up relationship. For  $\alpha_A$  below 12 deg, there is no significant lateral control reduction associated with using the pedals as well as the stick, and the MCE model is "content" to use stick alone. The MCE model is slightly overadapted at low  $\alpha_A$  and slightly underadapted at  $\alpha_A = 12$  deg; hence, the net amount of adaptation is lower than that implied by fully optimal control.

As  $\alpha_A$  increases, the MCE strategy is headed for a stability boundary; the pilot can avoid the boundary by adopting a more nearly optimal strategy, but this requires substantially increased control effort. As alternatives, he can either blend in foot pedal command (coordinated adaptation) or use the pedals alone (stick-centered adaptation) for lateral-directional control. The advantage of the first approach is that relatively good maneuvering precision can be maintained with both controls without requiring counter-intuitive control style; however, the coordinated use of both controls is a difficult task. Judging from Table 4, the use of pedals alone may be a more easily learned task which results in modest increases in  $\sigma_p$  and  $\sigma_v$ .

Experimental results indicate that the MCE pilot model hypothesis does, in fact, describe a realistic pattern of pilot adaptation. Figure 7 is a partial time history of a wind-up turn maneuver in which a trained pilot is flying a ground-based simulation of the subject aircraft. The aerodynamic model of the aircraft is the same as that used in the linear analysis, although the nonlinear, time-varying equations of motion drive the simulator. Below 18-deg angle of attack, the pilot controls with stick alone. As  $\alpha_A$  increases beyond 10 deg, stick motions and sideslip excursions build up. At  $\alpha_A = 18$  deg, the pilot begins to use the rudder pedals actively, while his use of the stick is substantially diminished. This result tends to confirm the MCE pilot model, although further validation is warranted.

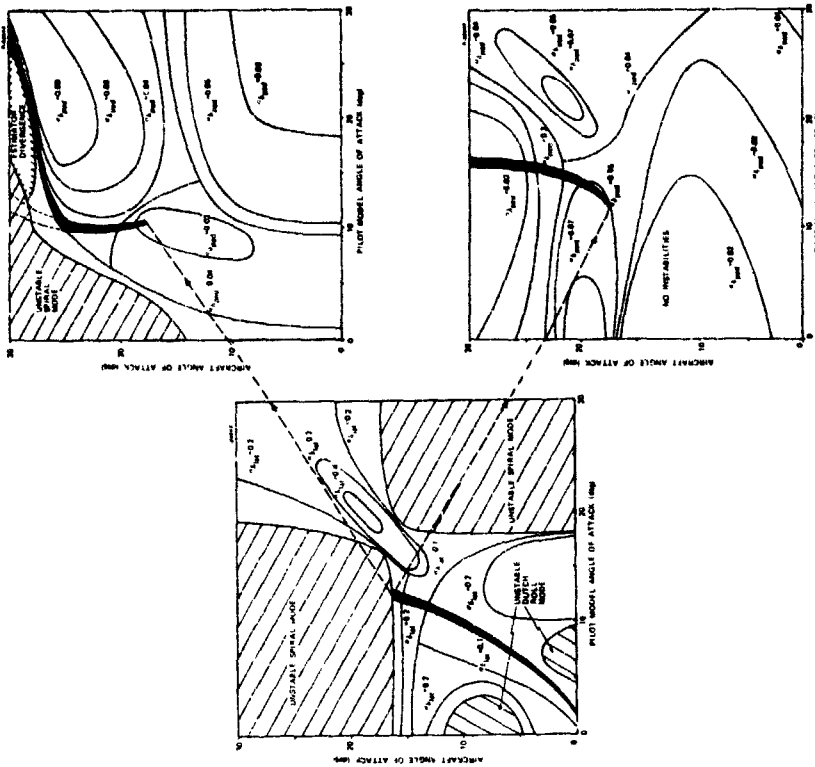


Figure 6 Prediction of Pilot Behavior with Minimum-Control-Effort (MCE) Adaptation Model

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### ACKNOWLEDGMENT

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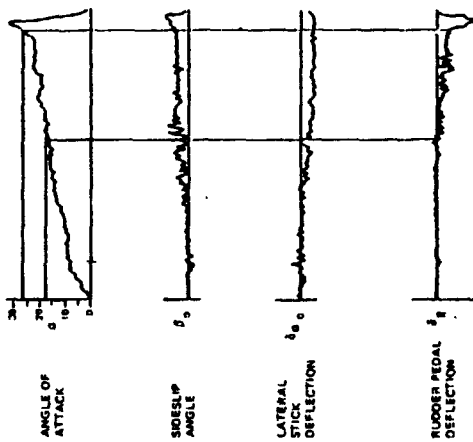


Figure 7 Results of Manned Simulation

### CONCLUSION

Whether or not a pilot experiences difficulties in maneuvering flight depends upon how he adapts his control strategy to changing flight conditions. Stability boundaries plotted as functions of the aircraft's actual  $\alpha$  and the  $\alpha$  assumed by the pilot model in forming a control strategy illustrate that the pilot's adaptation must be very nearly optimal to maintain stability in certain flight conditions. Consideration of statistical tracking error and control usage within stable boundaries leads to the concept of minimum-control-effort (MCE) adaptation in the pilot model. The MCE model provides a rationale for non-optimal adaptation which accounts for fundamental changes in the control modes selected by the pilot, such as the decision to use stick and pedals in a coordinated fashion rather than stick alone.