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STUDY OF THE USE OF A NONLINEAR, RATE LIMITED, FILTER ON

PILOT CONTROL SIGNALS

by

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SUMMARY

Analysis of pilot response while performing in a closed loop control situation has shown that there is a large remnant in the pilot's control output that does not add to the goodness of the control, but does add unwanted motion to the system response. The use of a filter on the pilot's control output could improve the performance of the pilot-aircraft system. What is needed is a filter with a sharp high frequency cut-off, no resonance peak, and a minimum of lag at low frequencies. The present investigation studies the usefulness of a nonlinear, rate limited, filter in performing the needed function. The nonlinear filter is compared with a linear, first order filter, and no filter. An analytical study using pilot models and a simulation study using experienced test pilots was performed.

The results showed that the nonlinear filter does promote quick, steady maneuvering. It is shown that the nonlinear filter attenuates the high frequency remnant and adds less phase lag to the low frequency signal than does the linear filter. It is also shown that the rate limit in the nonlinear filter can be set to be too restrictive, causing an unstable pilot-aircraft system response.

INTRODUCTION

Analysis of pilot response while performing closed loop control of dynamic systems has shown that the pilot's response is composed of a signal that is linearly related to the input signal and a random noise with a band pass equal to the band pass of the linear signal. The study which led to

these conclusions is presented in reference 1, where the vehicle being controlled was an acceleration response type of plant, $\frac{K}{s^2}$, and the pilot band pass was around 10 radians per second. Since the pilot remnant does not contribute to the goodness of the system response, any means of reducing its effect would be beneficial. In fly-by-wire control systems, it is possible to use a low pass filter on the pilot's control signal which, ideally, would eliminate the high frequency remnant signal while having no effect on the low frequency, linear part of the control signal. What is needed is a filter with a very sharp cut-off, but with no resonance peak, and with very little phase shift below the cut off frequency. The purpose of the present investigation is to examine the usefulness of a nonlinear, rate limited, filter in providing this needed function. The nonlinear filter was compared with a no filter condition, and with a linear, first order filter.

Reference 2 is a study that is similar to the present study in many ways. In reference 2, flight tests were conducted with an elevator control booster which contained a variable rate limit. It was found that the rate limit could be restricted to 7 degrees per second with no detrimental effects on the controllability of the system. It should be noted that in reference 2 the control rate limit is not included in any stability augmentation loop closure, and the present study does not suggest that the filter be included in any stability augmentation loop. Reference 3 shows that including a rate limit in a stability augmentation loop can destroy the effectiveness of the stability augmentation.

SYMBOLS

Values are given in SI Units. The measurements and calculations were made in U.S. Customary Units.

F_x x axis force, N
h altitude, m

I_y	moment of inertia, Kg-m ²	ξ_a	pilot-model, aircraft system short period mode damping ratio
K	general gain	ω_δ	pilot-model, aircraft system control mode frequency, rad/sec
K_θ	pilot model pitch loop static gain	ξ_δ	pilot-model, aircraft system control mode damping ratio
K_h	pilot model altitude loop static gain	ω_h	pilot-model, aircraft system altitude mode frequency, rad/sec
K_n	remnant static gain	ξ_h	pilot-model, aircraft system altitude mode damping ratio
L_a	$-\frac{1}{mV} \frac{\partial F_z}{\partial \alpha}$ per sec	γ	flight path angle, rad
M_y	y axis moment, N-m	ϕ	frequency response phase angle, deg
H_a	$\frac{1}{I_y} \frac{\partial M_y}{\partial \alpha}$ per sec	Subscript	
M_q	$\frac{1}{I_y} \frac{\partial M_y}{\partial q}$ per sec ²	c	command
M_{δ_e}	$\frac{1}{I_y} \frac{\partial M_y}{\partial \delta}$ per sec ²	e	error
m	mass, Kg		
q	pitching velocity, rad/sec		
s	Laplace variable, per sec		
V_{x0}	velocity, m/sec		
α	angle of attack, rad		
δ_e	elevator deflection, rad		
θ	pitch angle, rad		
ω_{sp}	short period natural frequency, rad/sec		
ξ_{sp}	short period damping ratio		
λ	pilot-model, aircraft system pitch mode root, rad/sec		
ω_a	pilot-model, aircraft system short period mode frequency, rad/sec		

Experimental Procedure

The three filter configurations, no filter, nonlinear filter, and linear filter, were examined with pilot models in an analytical study and with real pilots in a fixed base simulator. Three different tasks were executed in each case: (1) performing a step pitch attitude change, (2) performing a step altitude change, and (3) following a sinusoidal altitude command. These tasks were performed with three different aircraft configurations which represented a medium speed condition of approximately Mach number 0.6 at an altitude of 25,000 feet, a high speed condition of approximately Mach number 1.0, and a low speed, low altitude condition.

The pilot model used in the analytical study was

$$\frac{\delta_e}{\theta_a} = \frac{K_g}{(1 + .2s)^2}$$

for pitch control. No lead term has been included in the pilot model because the intention of this study is to combine the pilot model with aircraft that

The pilot model was combined with a simplified, two degree-of-freedom representation of the aircraft

$$\ddot{\alpha} - \dot{\alpha} = -L_{\alpha}\alpha$$

$$\ddot{\theta} = M_{\dot{q}}\dot{q} + M_{\alpha}\alpha + M_{\delta_e}\delta_e$$

and the relationship for altitude

$$\dot{h} = V_{x_0}(\theta - \alpha)$$

The coefficients for the three aircraft configurations are given in Table I, together with the aircraft response characteristics. Also given are the pilot model gains, K_{θ} and K_h , and the pilot-model aircraft system characteristics.

The nonlinear filter equations are:

$$\dot{\delta}_e = 100 \delta_c - 100 \delta_e \quad |\dot{\delta}_e| < \text{limit value}$$

$$\dot{\delta}_e = \text{limit value} \quad |\dot{\delta}_e| > \text{limit value}$$

An analog diagram for the nonlinear filter is shown in the sketch. The linear filter had a 5 radians per second break point, and was represented in a straight forward manner.

In the simulation tests, three experienced test pilots performed the tasks. The simulator cockpit used by the pilots was equipped with a televised, out-the-window display of the horizon and a target airplane. The included angle of the display was 20 degrees vertically and 35 degrees horizontally. The control stick was a force stick with an unlimited, linear

are considered to have satisfactory handling qualities, and it has been shown that no lead is required to represent a pilot's response with these aircraft. The lag time constant of 0.2 second has been shown to be proper value for the pilot model when controlling a aircraft with at least tolerable handling qualities. The gain K_{θ} was adjusted to provide a pilot-model aircraft system response with the real root larger in magnitude than -0.4 radian per second and a damping ratio of the oscillatory mode of motion greater than 0.1. These selections for the lag time constant and gain provide typical pilot-aircraft system characteristics. The selected pilot model was used without any further adjustment with each filter configuration to provide a clear indication of the effect of the filter on the system response. It was, of course, necessary to adjust the gain, K_{θ} , for each aircraft configuration to provide the desired system response, but the pilot model coefficients were kept constant for each filter configuration. For altitude control the pilot model consisted of an outer loop added to the pitch control loop as shown in figure 1, with a constant gain, K_h , on the outer loop control block. For the altitude control cases the gains K_h and K_{θ} were adjusted to provide a system response with the lower frequency greater than about 1 radian per second and the lowest damping ratio greater than 0.1. Again, these system characteristics are assumed to be typical for altitude control by a real pilot. With the high speed aircraft configuration, a small amount of lead was added to the pitch control loop pilot model in the altitude control system. Also, for all three aircraft configurations, a limit was placed on the pitch command (the output of the K_h block) in the altitude control systems. To complete the pilot models, a random white noise signal was filtered with a second order filter $\frac{K_n}{(1 + .2s)^2}$ and added to the output of the pilot model to represent the remnant of the real pilot. The amplitude of this remnant signal was adjusted so that the variance of the remnant was between 40 to 50 percent of the total control signal. All of these items have been shown to be reasonable for the representation of pilot response.

output and no hysteresis. The force stick was mounted on a rubber block base which gave it a small amount of rotational movement. There was no restriction in the rate of movement of the control stick. When the nonlinear filter was added to the system, only the input to the aircraft was rate limited.

The simulator was controlled by a full set of nonlinear aircraft equations of motion presented in Appendix A. The pilot performed the attitude control tasks with reference to the horizon, and the altitude control tasks with reference to the target airplane. While the pilots were performing these longitudinal tasks, they also had to regulate the lateral-directional response of the aircraft as an additional task. The target aircraft was driven so that it remained at a constant 183 meters in front of the test aircraft. The target flew either straight and level or with a sinusoidal variation in altitude. The simulator equations of motion were solved with a digital computer that operated with a sample rate of 32 samples per second. In order to properly represent the high frequency response of the nonlinear filter when it was operating on its linear region, it was necessary to use special computational techniques.

RESULTS

Comparison of Filters - To illustrate the differences in the operation of the nonlinear and linear filters, the frequency response of the two devices can be compared. The data for the nonlinear filter is actually describing function data, and was obtained with an analog representation. Sinusoidal input signals with a number of difference frequencies were applied to the nonlinear filter, and the time history of the output recorded. The frequency response phase angle data was obtained by measuring the time difference in the zero crossings of the input and output, and using the formula

$$\phi = \frac{\Delta t_{0.360}}{2\pi}$$

The amplitude ratio was obtained from the ratio of the peak values of the input and output. These data are shown in figure 2a, where it is compared with the frequency response of the linear filter. It can be seen that the phase lag for the nonlinear filter is less than for the linear filter at frequencies below 3 radians per second. This small phase lag at low frequencies for the nonlinear filter is the result of the 100 radians per second break point used in the linear part of the nonlinear filter. It can also be seen that the reduction in amplitude ratio with increasing frequency is much steeper for the nonlinear filter than for the linear filter. The results of these two factors is that the nonlinear filter would have less effect on low frequency signals, and would attenuate high frequency signals better than would the linear filter.

Where the response of the linear filter would be invariant with the amplitude of the input, the nonlinear filter response is affected by the amplitude of the input. This is illustrated in figure 2b, where the describing function data of the nonlinear filter for three different input amplitudes is presented. The figure shows that the larger the amplitude of the input, the more phase lag is created at any given frequency. This situation indicates a potential stability problem with large inputs for a system incorporating the nonlinear filter.

To indicate the effect on stability of the nonlinear filter for a typical pilot-model, aircraft system, figure 3 is presented. The pilot model was simplified in this case by leaving out the remnant term. Figure 3a shows the response of a typical system with the nonlinear filter included, but with the rate limit set so high that it does not come into effect. The commanded pitch angle change is 5 degrees in this case. Figure 3b shows the response of the same system with the nonlinear filter rate limit set so that it does come into operation. It can be seen that while the nonlinear filter does noticeably change the control moment time history as compared to figure 3a, there is no noticeable effect on the pitch angle time history. Figure 3c shows the response of the same system used in figure 3b to a 10 degree pitch angle change command. In this last case the initial overshoot in pitch angle is noticeably larger in proportion to the steady state value of pitch angle than in the case with the 5 degree pitch angle change.

This change in the stability of the response of the system with an increase in the size of the input command illustrates a possible disadvantage of the nonlinear filter.

In this report, control action will be presented as the normalized control moment, $M_c \delta_c$, instead of using control deflection. The purpose for using this particular method of data presentation is to generalize the results rather than leaving the results as the function of a particular control effectiveness value.

Pilot Model Analysis - To test the usefulness of the nonlinear filter, a study using pilot models for both attitude and altitude control was undertaken to compare the nonlinear filter with both no filter and with a linear, first order filter. The comparison was made with each of three different aircraft configurations. The nonlinear filter rate limits were established initially in these tests by noting the maximum rate required in the 5 degree attitude change maneuver, and setting the rate limit at one-half of this maximum value. Further restrictions in the rate limit were then tried.

The first aircraft configuration to be discussed represents a medium speed flight configuration of a fighter type aircraft. The aircraft speed was 214 meters per second, and the aircraft short period response characteristics were $\omega_{sp}^2 = 20 \text{ radians}^2 \text{ per second}^2$, $2\zeta\omega_{sp} = 5 \text{ radians per second}$ ($\omega_{sp} = 4.48 \text{ radians per second}$, $\zeta_{sp} = .56$). The results obtained when the filters were inserted in a system containing this aircraft and the typical pilot model, and a step change in pitch angle is performed are shown on figure 4. The figure shows a reduction in the pitching motion activity that occurs in this maneuver with each of the two filters included in the system as compared to that which occurs with no filter. The nonlinear filter brings about a greater reduction in pitching activity than does the linear filter. Each of the filters reduces the effect of the pilot remnant, but the linear filter also reduces the damping of the oscillatory mode of motion of the system. This reduction in system damping is illustrated more clearly in figure 5, where the pilot remnant has been removed from the pilot model.

The same type of result was obtained when a step change in altitude was computed using the multi-loop pilot model. These results are shown on

figure 6 and again a decrease in system damping can be seen to occur when the linear filter is added to the pilot-model, aircraft system, and slightly less pitching motion activity occurs with the nonlinear filter as compared with the linear filter.

When the high speed aircraft configuration ($V = 305 \text{ meters per second}$, $\omega_{sp}^2 = 100 \text{ per second}^2$, $2\zeta\omega_{sp} = 3 \text{ per second}$) was considered, the reduction in pitching activity that occurred with the nonlinear filter as compared to the no filter configuration or the linear filter was very evident. The results are shown on figure 7, where the response to a step change in altitude is shown. The same result was obtained with the step change in pitch angle computation. It should be noted that a small amount of pilot lead (a lead time constant of 0.2 second) was used in the computation of the altitude change shown in figure 7. This amount of lead is an addition that a pilot is very likely to try in his control response in an attempt to improve the system response.

With the low speed aircraft configurations ($V = 122 \text{ meters per second}$, $\omega_{sp}^2 = 5 \text{ per second}^2$, $2\zeta\omega_{sp} = 5 \text{ per second}$), the results as regards the step pitch angle and step altitude change were the same as for the first two configurations. One test in which the filters had a pronounced effect with the low speed aircraft configuration was in following a sinusoidal altitude command. A typical computer run is shown in figure 8. The command in this case was a sine wave with a period of 30 seconds and an amplitude of 120 meters. A summary of the results obtained from these computations are presented in Table II. Presented are the root-mean-square values for altitude error for all three aircraft configurations with all three filters. These root-mean-square error values have been normalized to the no filter case so as to show at a glance the effect of the nonlinear filter and the linear filter. The results show that with the low speed aircraft configuration both the nonlinear filter and the linear filter provide considerable improvement in the sinusoidal command following ability of the pilot-model, aircraft system. Less improvement was provided by the two filters with the high speed aircraft, and with the medium speed aircraft the filters reduced the accuracy of the sinusoidal altitude following.

Simulation Tests - Three experienced test pilots served as subjects in the simulation tests. Each pilot tested each filter configuration with each of the three aircraft configurations. In this investigation the rate limit in the nonlinear filter was set by combining the pilot model with the 5 degree-of-freedom, nonlinear aircraft representation, noting the maximum control moment rate that was required in a 3 degree pitch angle change maneuver, and setting the rate limit at one-half of this maximum value. While this method of setting the rate limit proved to be very useful in determining the value to use initially, preliminary tests showed that the rate limit was restricted a little bit more. The task of following the target plane which was moving vertically in a sine wave with a period of 30

seconds and an amplitude of 120 meters proved to be the most sensitive tests of required control moment rate, and so this task was used in this preliminary investigation. Sample tests with the high speed aircraft are shown in figure 9. With the initial $M_0 \delta_e$ limit value of 45 per second³, the pilot was able to perform the maneuver with no difficulty. When the limit value was reduced to 25 per second³, the pilot experienced some difficulty at first, as is indicated by the one cycle of a divergent oscillation that can be seen in the figure. However, the pilot made the necessary adjustment and regained control. It was concluded from this test that the limit value of 25 per second³ was close to the greatest amount of restriction that could be used, and this is the value that was used in the remainder of the investigation with the high speed aircraft configuration. Similar tests were made with the other two aircraft configurations, and resulted in values of $M_0 \delta_e$ of 8 per second³ for the medium speed configuration and 6 per second³ for the low speed configuration being selected for use in the remainder of the investigation.

The results obtained with the pilots in the simulator closely parallel those obtained with the pilot model in the analytical study. Time history records obtained with pilot P for the step change in pitch angle are shown in figure 10. These results are typical for all the subjects. The pilots performed these tests in a systematic manner by first performing a very slow maneuver (using a low gain) which they were sure would be well

damped. They increased the maneuver rate in the next try, and then made a final maneuver which was done as rapidly as they felt they would ever do the maneuver. It is this final maneuver that should compare with the pilot model results. It can be seen in figure 10 that there is a definite

reduction in system damping for the rapid maneuver with the linear filter included in the system as compared to the response with either the no filter or nonlinear filter, and that the pitching activity is the least with the nonlinear filter. Figure 11 shows the step altitude change maneuver, and again, the pitching activity is slightly less with the nonlinear filter than with either the no filter or the linear filter configurations.

With the high speed aircraft configuration, the pitching activity is clearly the smallest with the nonlinear filter in both the attitude change and the altitude change tasks. These results are shown in figures 12 and 13, where pilot P was the subject. These figures show that not only does the nonlinear filter reduce the random noise generated by the pilot, but also it does not effect the linear portion of the pilot's response. The result is that with the nonlinear filter in the system, the final steady state condition of the maneuver is arrived at quickly and with only a small oscillation about this steady state value. This type of response character would be a great value doing maneuvers that must be done rapidly with great accuracy.

With the low speed aircraft configuration, the most pronounced effect in the simulation tests was, as it was in the pilot model analysis, in the task of following a sinusoidal altitude command. A set of typical time histories is shown in figure 14. There was a great deal of learning involved in this task. The performance measure used was the difference in the altitude of the target airplane and the controlled airplane, but the pilot had a tendency to want to only keep the gun sight pipper on the target. As they learned that a good score would result from staying at the same altitude as the target, and, at the same time, learned to use a small amount of lead to accomplish this task, the scores improved by a large amount. To show the final result, the last three scores of the one subject who performed a complete set of tests are given in Table II. It can be seen that no improvement in root-mean-square values of the altitude error was provided by either the nonlinear filter or the linear filter with the medium speed aircraft

configuration, there was some improvement with the high speed aircraft, and there was a very noticeable improvement due to the filters with the low speed aircraft. These results closely match the results obtained in the pilot model analysis.

The pilots were asked to rank the different filter configurations as best (1), in between (2), and worst (3). This rating data is given in Table III. It can be seen that there was no agreement among the pilots in their rankings. Further, any one pilot varied in his ratings when different aircraft configurations were involved. The effect of the filter, as shown in the time histories presented previously, was small, and this is probably the reason the pilots were not able to reach an agreement in rankings in the small amount of experience they had with the different filters in the course of the present experiment. Nevertheless, it is concluded that the nonlinear filter does promote quick, nonoscillatory system response, and that it deserves further consideration.

In the present investigation, the filters were not located inside any stability augmentation loops. The intention was that the filters be inside only pilot loop closures, and, therefore, the airplane was represented as having no stability augmentation. Even if the airplane did include stability augmentation, the intention is to locate the filter outside these loops.

The nonlinear filter was also tried on the aileron control system. Tests were made both with pilot models in an analytical study and with real pilots in the simulation study. In each situation it was found that a small amount of rate restriction caused a very noticeable deterioration in the stability of the system response. For this reason the use of the nonlinear filter, as defined in this study, is recommended for use only in the elevator control system.

In the present investigation, the rate limit in the nonlinear filter was set individually for each aircraft configuration, and was a different value for each aircraft. This situation would indicate that if the nonlinear filter were to be used in an airplane which had a large flight envelope the rate limit value would have to be scheduled as a function of flight conditions to achieve the best results possible. This scheduling problem was bypassed in the present investigation.

During the course of the present study there were no instances found where the nonlinear filter caused a divergent oscillation to occur in an attitude control task. However, there were cases involving altitude control in which pilot induced unstable oscillations did occur. In the pilot model analytical study, it was found that a rate restriction in the nonlinear filter which did not cause an unstable oscillation in the attitude change task would cause an unstable oscillation in the altitude change task if the pitch angle command limit was not included in the pilot model. Including the pitch angle command limit would eliminate the problem. In the simulation study it was found that using a rate limit in the nonlinear filter greater than the value reported in this study would cause pilot induced unstable oscillations to occur in altitude control tasks. Figure 9b is an example of a borderline case. It is concluded that a rate restriction that is too great must be avoided. The critical tasks where trouble might arise are tasks that require rapid and accurate altitude regulation. Formation flying, short range air-to-air combat, and landing are examples of such tasks. It is felt that large altitude changes such as are required in navigation tasks, but which do not require rapid and accurate response, would not be critical.

CONCLUDING REMARKS

Analytical studies using pilot models and simulation studies using pilot subjects has led to the following conclusions on the usefulness of a nonlinear, rate limited, pilot pitch control filter.

1. The nonlinear filter will promote rapid competition of maneuvers while minimizing the oscillatory motion involved in the maneuver. Time history records obtained with pilot subjects show that the nonlinear filter allows better system response than does either a linear, first order filter, or the absence of any filter.
2. The differences between the nonlinear filter, the linear filter, or no filter were too small to be detected by the pilots in the short study of this investigation.

3. The pilot model analytical study confirmed the conclusion that the nonlinear filter will promote rapid completion of maneuvers, and indicated that the superiority of the nonlinear filter is due to the fact that it introduces less lag into the system than does the linear filter, and also attenuates the pilot's remnant better than does the linear filter.

4. The study showed that the rate limit in the nonlinear filter can be set to produce a rate restriction that is too great and will cause a system instability. The pilot model analysis was useful in establishing a safe rate limit.

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APPENDIX A

Equations of Motion for the Simulation Study

The equations of motion used for the pilot simulation experiment were:

$$A_x = 0$$

$$A_y = V_{x_0} \gamma_\beta \beta$$

$$A_z = -V_{x_0} (L_\alpha \alpha - L_0)$$

$$\dot{p} = L_p^i p + L_\beta^i \beta + L_r^i r + L_{\delta_a}^i \delta_a$$

$$\dot{q} = M_\alpha^i \alpha + M_q^i q + M_{\delta_e}^i \delta_e$$

$$\dot{r} = N_r^i r + N_\beta^i \beta + N_p^i p + N_{\delta_r}^i \delta_r$$

$$\dot{\phi} = p + q \sin \phi \tan \theta + r \cos \phi \tan \theta$$

$$\dot{\theta} = q \cos \phi - r \sin \phi$$

$$\dot{\psi} = \frac{r \cos \phi + q \sin \phi}{\cos \theta}$$

$$\xi_1 = \cos \psi \cos \theta$$

$$\eta_1 = \cos \psi \sin \theta \sin \phi - \sin \psi \cos \phi$$

$$\eta_2 = \cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi$$

$$\beta = \sin^{-1} \frac{v}{V}$$

These additional symbols are used in these equations.

a_x, a_y, a_z	body axis components of acceleration, m/sec ²
u, v, w	body axis components of velocity, m/sec
V_x, V_y, V_z	inertial axis components of velocity, m/sec
p, r	rolling and yawing velocity, rad/sec
β	sideslip angle, rad
δ_a, δ_r	aileron and rudder deflection, rad
ψ, ϕ	yaw and roll angle, rad
M_x	rolling moment, N-m
L_0	$\frac{d}{V_{x0}}$ per sec
L_i	$\left(1 - \frac{I_{xz}^2}{I_x I_z}\right)^{-1} \left(L_i + \frac{I_{xz}}{I_x} N_i\right)$ ($i = \beta, p, r, \delta_a, \delta_r$)
L_p	$\frac{1}{I_x} \frac{\partial M_x}{\partial p}$ per sec
L_r	$\frac{1}{I_x} \frac{\partial M_x}{\partial r}$ per sec

$$\xi_2 = \sin \psi \cos \theta$$

$$m_2 = \sin \psi \sin \theta \sin \phi + \cos \psi \cos \phi$$

$$n_2 = \sin \psi \sin \theta \cos \phi - \cos \psi \sin \phi$$

$$\xi_3 = -\sin \theta$$

$$m_3 = \cos \theta \sin \phi$$

$$n_3 = \cos \theta \cos \phi$$

$$\dot{V}_x = \xi_1 a_x + m_1 a_y + n_1 a_z$$

$$\dot{V}_y = \xi_2 a_x + m_2 a_y + n_2 a_z$$

$$\dot{V}_z = \xi_3 a_x + m_3 a_y + n_3 a_z + g$$

$$u = \xi_1 V_x + \xi_2 V_y + \xi_3 V_z$$

$$v = m_1 V_x + m_2 V_y + m_3 V_z$$

$$w = n_1 V_x + n_2 V_y + n_3 V_z$$

$$V = (V_x^2 + V_y^2 + V_z^2)^{1/2}$$

$$\alpha = \tan^{-1} \frac{w}{u}$$

L_B	$\frac{1}{I_X} \frac{\partial M_X}{\partial \beta}$ per sec ²	Y_r	$\frac{1}{mV} \frac{\partial F}{\partial r}$
L_{δ_a}	$\frac{1}{I_X} \frac{\partial M_X}{\partial \delta_a}$ per sec ²	Y_B	$\frac{1}{mV} \frac{\partial F}{\partial \beta}$ per sec
M_Z	yawing moment, N-m	I_X, I_Z	moments of inertia, kg-m ²
N_i'	$\left(1 - \frac{I_{XZ}^2}{I_X I_Z}\right)^{-1} \left(N_i + \frac{I_{XZ}}{I_Z} L_i\right)$	I_{XZ}	product of inertia, kg-m ²
N_p	$\frac{1}{I_Z} \frac{\partial M_Z}{\partial p}$ per sec	L_B'	= - 42.14
N_r	$\frac{1}{I_Z} \frac{\partial M_Z}{\partial r}$ per sec	L_p'	= - 2.74
N_B	$\frac{1}{I_Z} \frac{\partial M_Z}{\partial \beta}$ per sec ²	L_r'	= 2.058
N_{δ_a}	$\frac{1}{I_Z} \frac{\partial M_Z}{\partial \delta_a}$ per sec ²	N_B	= 5.54
F_y	side force, N	N_p'	= .0148
Y_p	$\frac{1}{mV} \frac{\partial F}{\partial p}$	N_r'	= - .278
		Y_B	= - .159
		N_{δ_r}'	= - 10.0
		L_{δ_r}'	= - 10.0

(i = B, P, r, δ_r , δ_a)

TABLE I - CONTINUED

Parameter	PILOT-MODEL, AIRCRAFT ALTITUDE CONTROL CHARACTERISTICS		
	Medium Speed Aircraft	High Speed Aircraft	Low Speed Aircraft
K_{θ}	24.	60.	15.
K_h	2.	2.	1.
ω_h	1.35	1.02	.740
ξ_h	.125	.103	.244
ω_{α}	3.85	9.33	2.67
ξ_{α}	.156	.224	.133
ω_{δ}	7.55	6.47	7.29
ξ_{δ}	.89	.82	.95

TABLE I
AIRCRAFT STABILITY DERIVATIVES AND
OPEN AND CLOSED LOOP CHARACTERISTICS

Parameter	AIRCRAFT CONFIGURATION		
	Medium Speed Aircraft	High Speed Aircraft	Low Speed Aircraft
V_{x_0}	214.	305.	122.
L_{α}	1.3	1.3	0.6
L_0	.0461	.0322	.0805
M_{α}	- 15.2	- 97.8	- 2.36
M_q	- 3.70	- 1.70	- 4.40
M_{δ}	- 10.0	- 10.0	- 10.0
ω^2_{sp}	20.	100.	5.
$2\omega^2_{sp}$	5.	3.	5.
ω_{sp}	4.48	10.	2.24
ξ_{sp}	.55	.15	1.11
PILOT-MODEL, AIRCRAFT SYSTEM ALTITUDE CONTROL CHARACTERISTICS			
K_{θ}	24.	60.	15.
λ	- .890	- .567	- .526
ω_{α}	3.92	9.30	2.75
ξ_{α}	.10	.11	.10
ω_{δ}	7.55	6.25	7.25
ξ_{δ}	.89	.83	.96

TABLE II

ROOT-MEAN-SQUARE ALTITUDE ERROR IN THE
SINUSOIDAL ALTITUDE COMMAND TASK

Pilot Model Results
Normalized Error

Aircraft Configuration	Filter Configuration		
	No Filter	Nonlinear Filter	Linear Filter
Medium Speed	100	105.5	105.5
High Speed	100	94.	93.
Low Speed	100	90.5	90.

Piloted Results, Subject G
Error in Meters

Medium Speed Aircraft

Trail	Filter Configuration		
	No Filter	Nonlinear Filter	Linear Filter
1	12.2	12.8	11.5
2	9.9	6.9	8.5
3	6.2	7.6	6.8

High Speed Aircraft

1	9.7	8.7	8.8
2	9.7	5.8	8.4
3	8.2	6.6	8.4

Low Speed Aircraft

1	30.0	20.8	25.0
2	24.6	20.2	21.1
3	21.7	21.0	27.7

TABLE III

PILOT RANKING OF THE THREE

FILTER CONFIGURATIONS

Medium Speed Aircraft

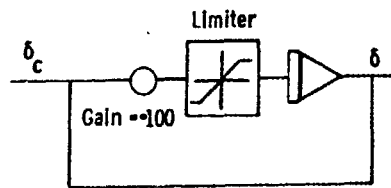
Pilot	Filter Configuration		
	No Filter	Nonlinear Filter	Linear Filter
P	3	1	2
K	2	3	1
E	1	2	3

High Speed Aircraft

P	2	2	1
K	1	3	2
E	3	1	2

Low Speed Aircraft

All Filters were ranked equal by all pilots



Sketch. - Nonlinear filter

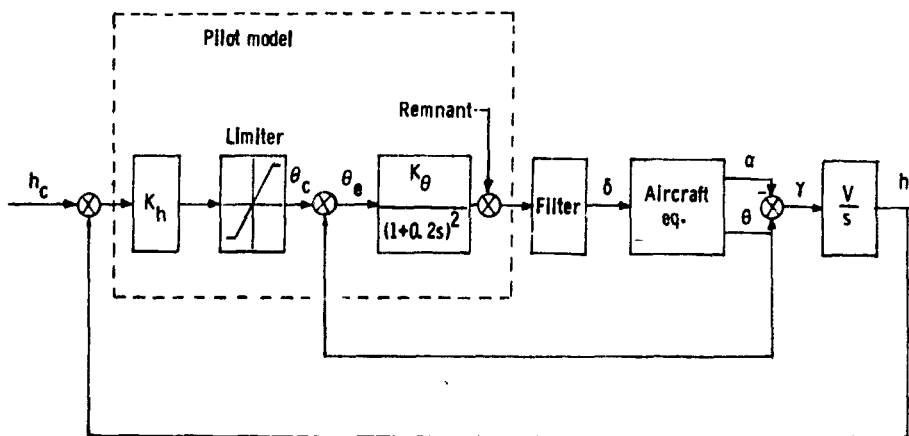
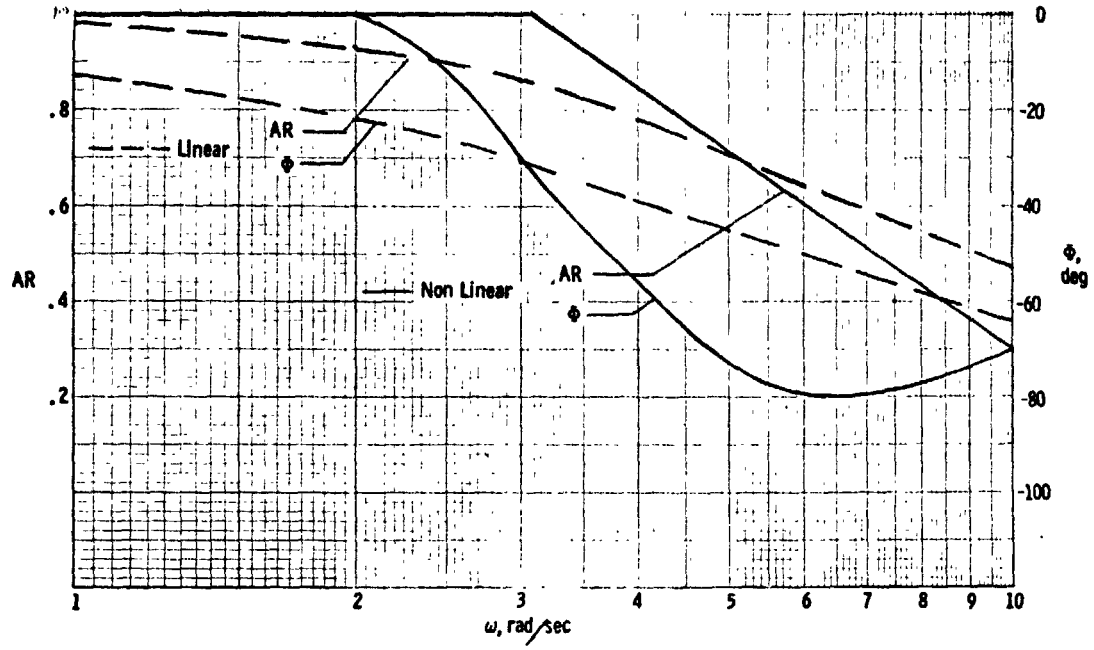
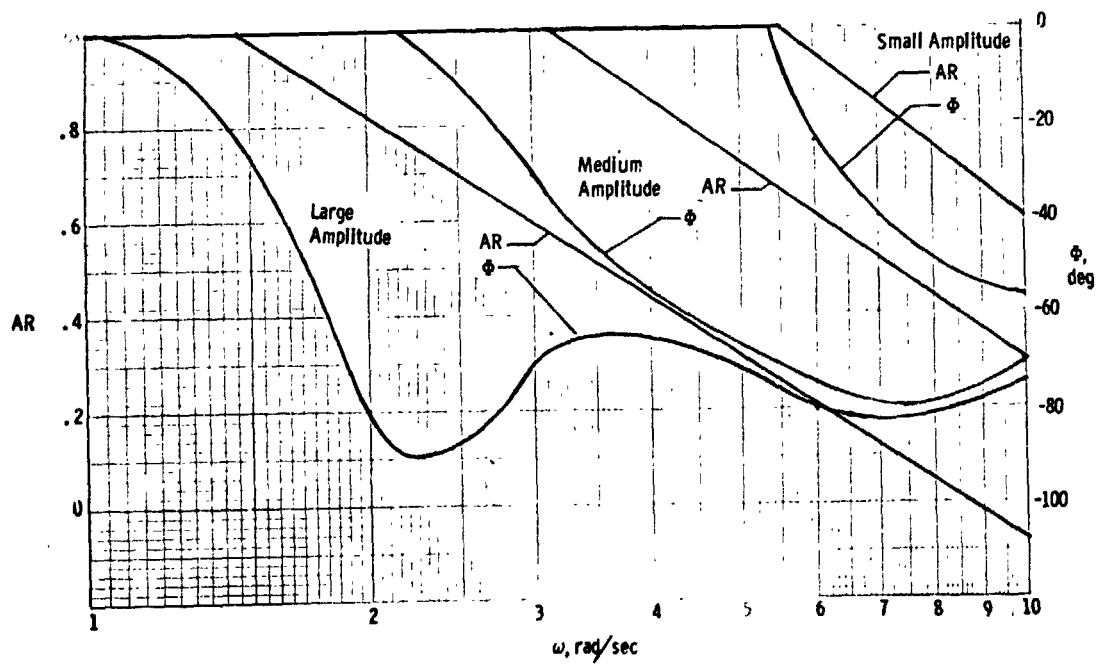


Figure 1 - Block diagram of pilot-model, Aircraft system.



(a) Comparison of nonlinear and linear filter.

Figure 2. Filter frequency response.



(b) Nonlinear filter with three different input amplitudes.

Figure 2. Concluded.

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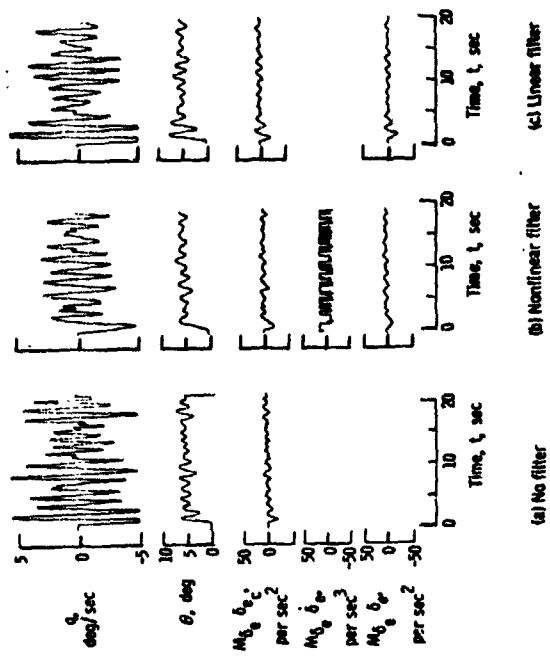
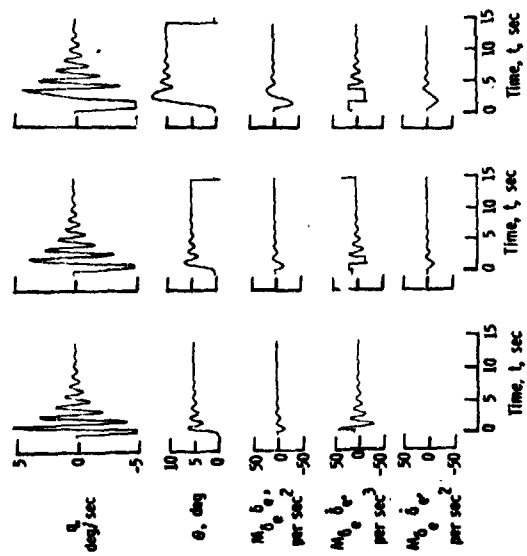


Figure 4 - Step pitch angle changes, medium speed aircraft



(a) No rate limit (b) Rate limit in effect (c) Large command

Figure 3 - Effect of nonlinear rate limit on system stability. Medium speed aircraft.

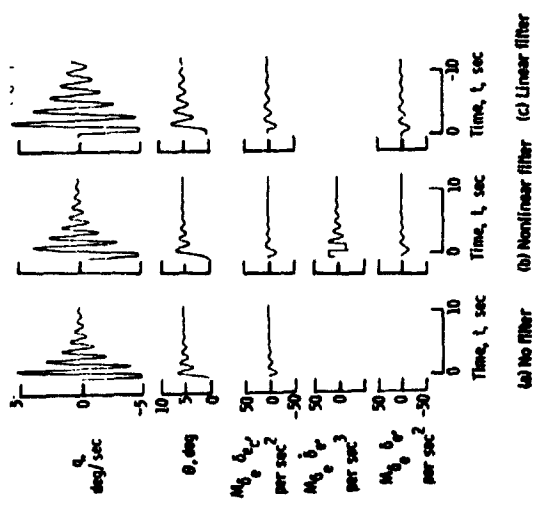


Figure 5. - Step pitch angle change, medium speed aircraft, treatment omitted from pilot model.

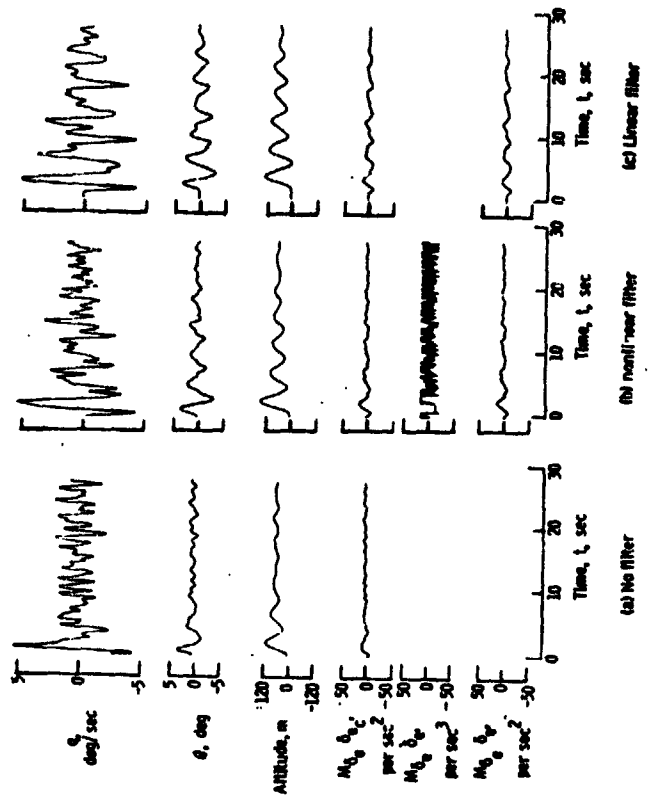


Figure 6. - Step altitude change, major, ep'd aircraft.

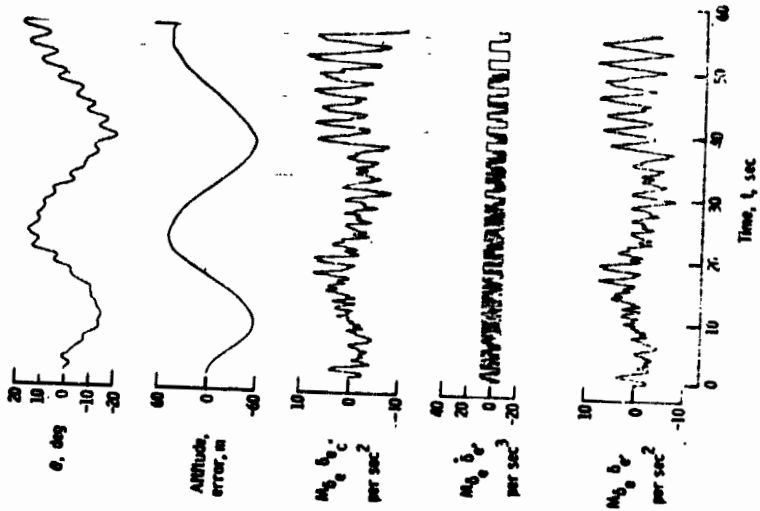


Figure 8. - Sinusoidal altitude command, low speed aircraft.
 $h_c = 120$ cos. $2\pi t - 120$

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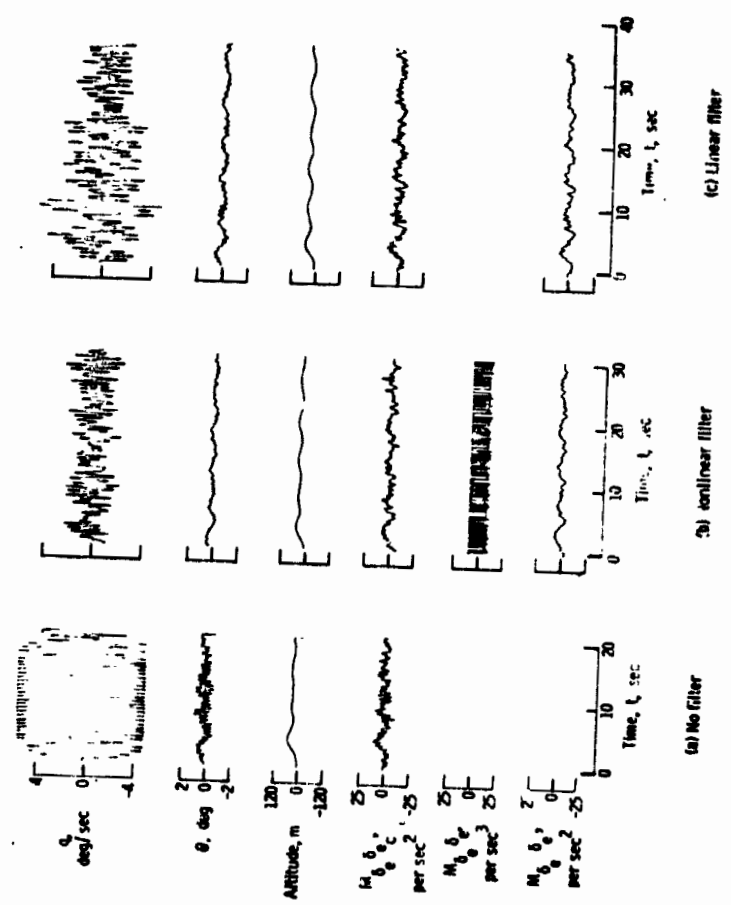


Figure 7. - Step altitude change, high speed aircraft.

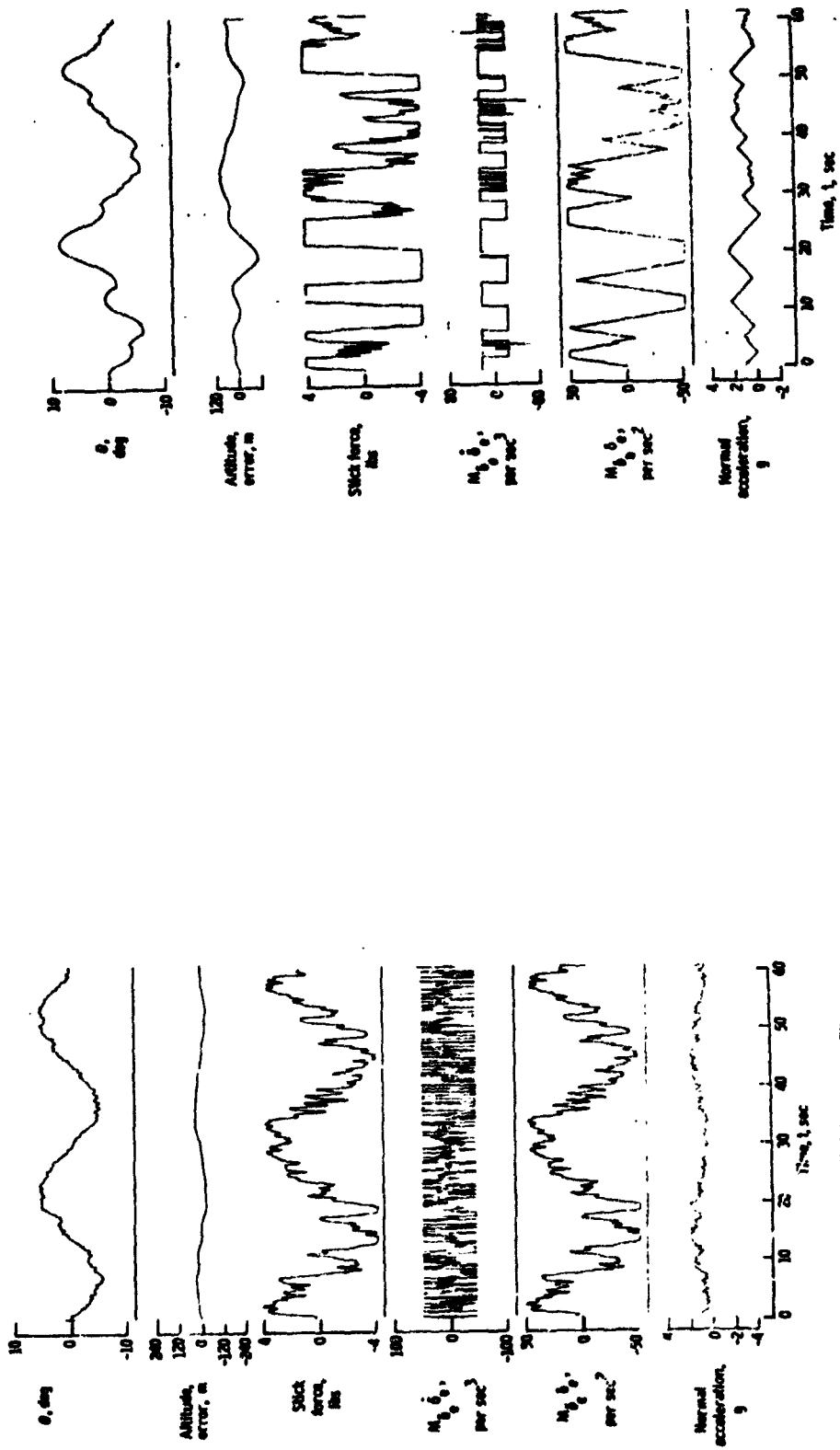
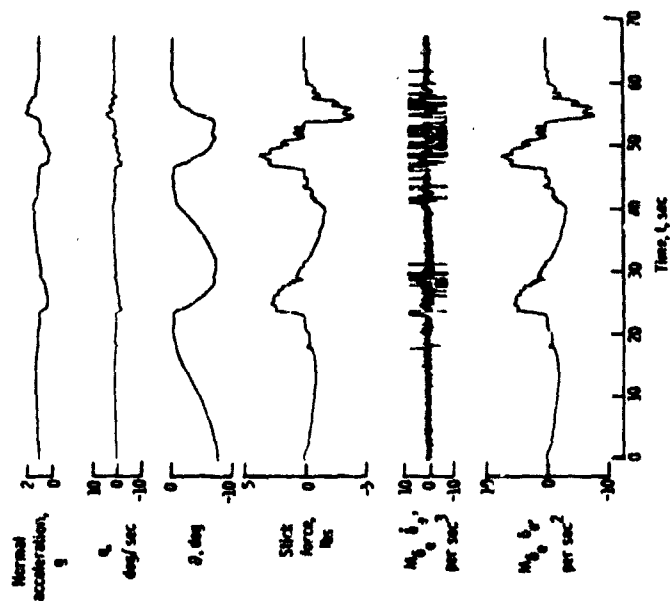
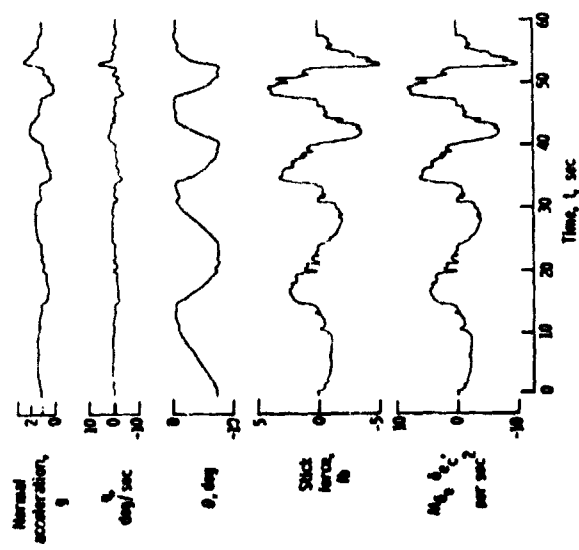


Figure 8. - Concluded
 (a) Large rate limit in nonlinear filter
 (b) Sinusoidal altitude command, $h_c = 120$ cos. 2 $\pi t - 120$
 $h_c = 120$ cos. 2 $\pi t - 120$



(b) Nonlinear filter
Figure 10. - Continued



(a) No filter
Figure 10. - Step pitch angle change and P. meslin's speed aircraft

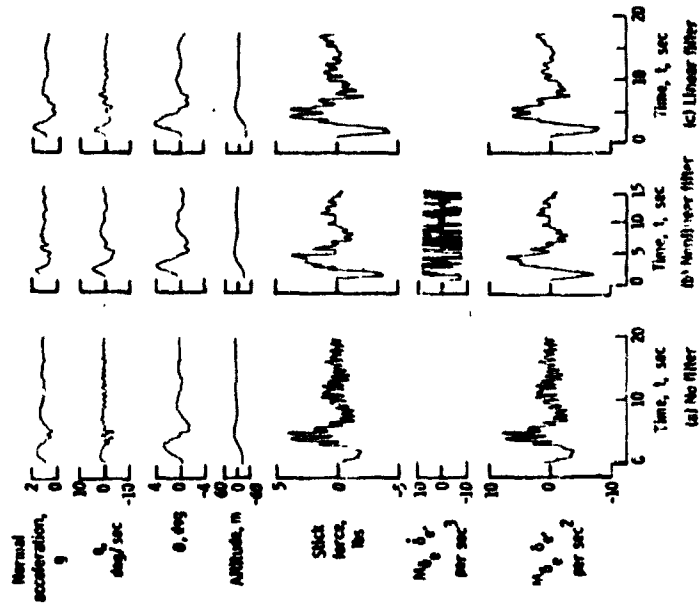
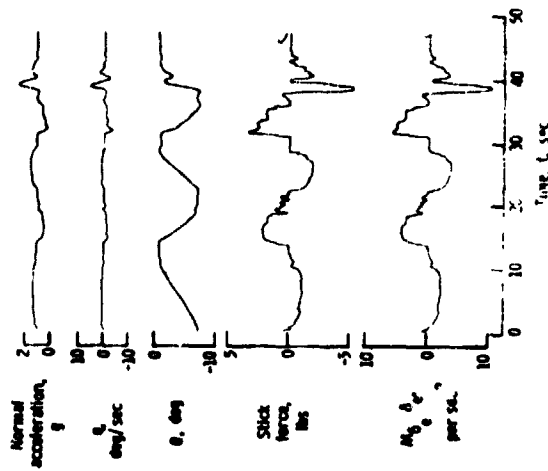


Figure 11. - Ship altitude control, pilot P, medium speed aircraft.



(a) Linear filter

Figure 12. - Concluded.

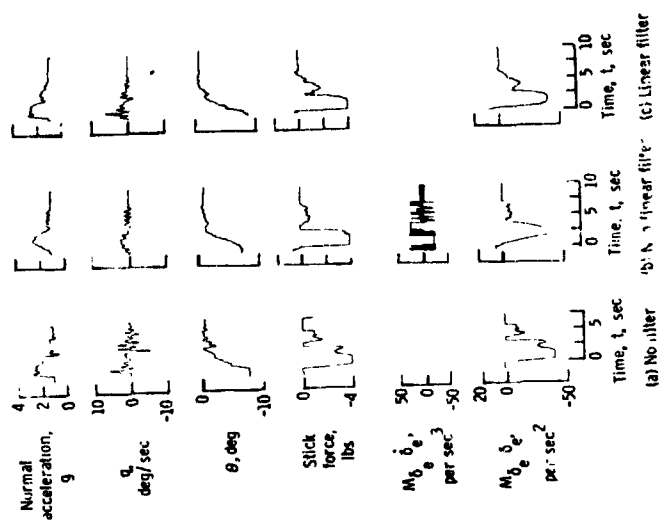


Figure 12. - Step pitch angle change, pilot P, high speed aircraft.

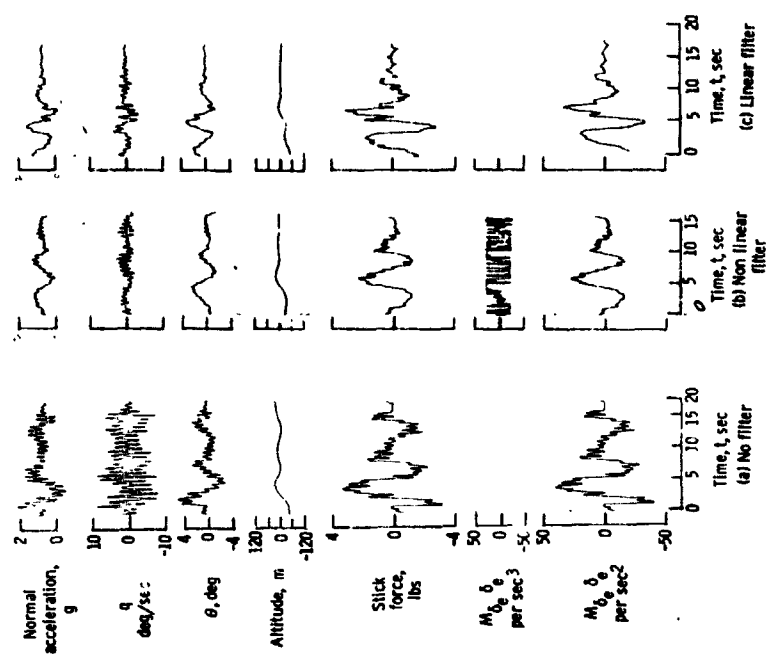


Figure 13. - Step altitude change, pilot P, high speed aircraft.

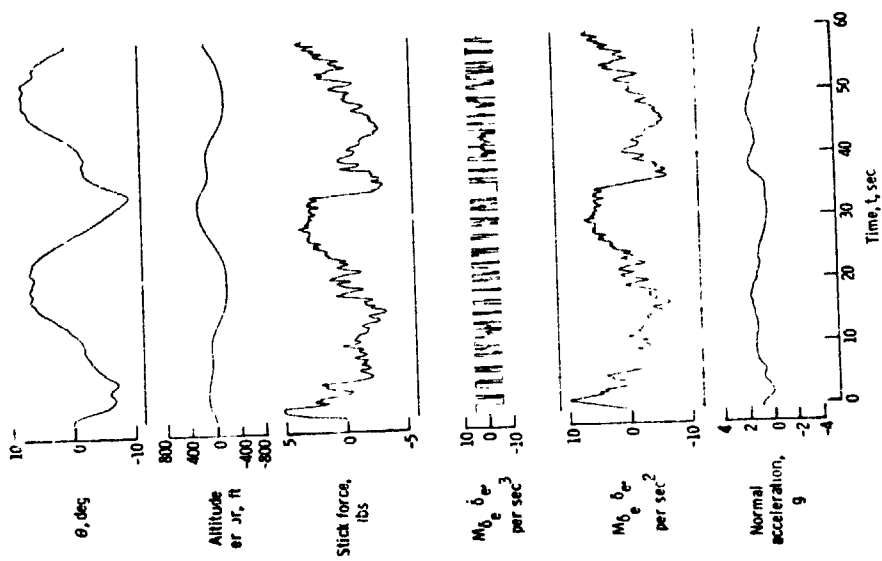


Figure 14. - Sinusoidal altitude command, low speed aircraft.