Paper No. 7

FORMULAS FOR DETERMINING STORM MOVEMENT FROM THE SURROUNDING FIELDS

R. C. Costen, *Langley Research Center, Hampton, Virginia*

ABSTRACT

Formulas are given for determining a conserved property of a storm and its centroid from any scalar field that has a tendency equation. The velocity of the centroid is also given. These formulas depend only on fields that are external to the storm.

INTRODUCTION

The propagation of storms may be characterized by the movement of local disturbances in many fields, such as density, moisture, internal energy, vorticity, divergence, pressure, stability, convection, etc. It is appropriate, then, to attempt to define a centroid for the disturbance in each of these fields and to characterize the movement and dynamics of a storm by the velocities and relative velocities of these centroids. The purpose of this paper is to show that this can be done for any scalar meteorological quantity s that satisfies a tendency equation of the form

$$\frac{\partial s}{\partial t} + b = 0 \tag{1}$$

where b represents the remaining terms. We shall also show that the centroid and its velocity are determined completely from fields that are outside the disturbance. This is a significant feature since measuring fields within a storm, either in situ or remotely, is often more difficult than measuring the fields that surround a storm. The scalar s may also represent the Cartesian components of vector quantities, such as the convection, and the vertical component of vorticity.

Approach–Throughout this paper, a disturbance in a field is defined by $\partial(\text{field})/\partial t \neq 0$; and a local disturbance is a disturbance that is surrounded by a layer of finite thickness where $\partial(\text{field})/\partial t = 0$. Instead of treating the scalar field s directly, we shall consider its Laplacian

$$L = \nabla^2 s \tag{2}$$

the source of the field s. A storm may be considered as a local disturbance in L, since $\partial L/\partial t$ is much larger within a storm, due to the movement and growth or decay of the storm, than in the quiescent surroundings.

Take V to be the volume of the storm, or, more specifically, the volume of the local disturbance in L, as illustrated in fig. 1. Within V, the disturbance

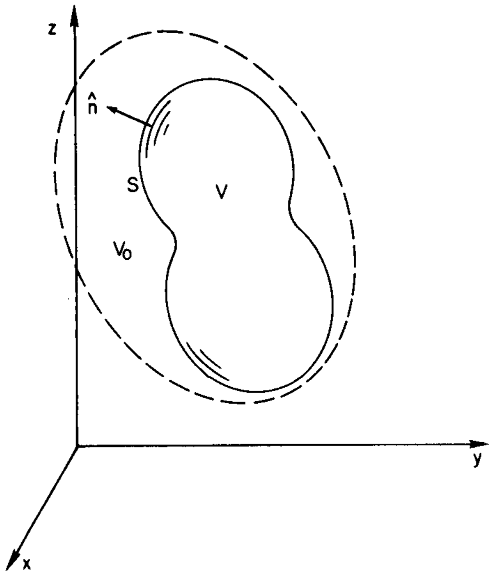

Fig. 1-Volume V that contains a local disturbance in L. Within V, the disturbance $\partial L/\partial t = -\nabla^2 b$ is generally nonzero; in the surrounding layer V_o of finite thickness $\partial L/\partial t = -\nabla^2 b = 0$. Also shown is the closed interface S between V and V_o with its outward unit normal $\hat{n}$.

$\partial L/\partial t = -\nabla^2 b$ is generally nonzero; however, V is surrounded by a layer V_o of finite thickness where $\partial L/\partial t = -\nabla^2 b = 0$. The volumetric integral of L in V is conserved and is termed the <u>efflux</u> E. Its centroid $\vec{R}_E$ may be regarded as a centroid of the storm, and the centroidal velocity $\dot{\vec{R}}_E$ may be regarded as a propagation velocity of the storm.

<u>Centroidal Equations</u>-In a previous paper (ref. 1), the movement of <u>a local disturbance</u> in a <u>vector</u> field $\vec{v}$ was treated, where $\vec{v}$ satisfied the equation $\partial\vec{v}/\partial t + \vec{f} = 0$. The vector field $\vec{v}$ was characterized by its curl and its divergence, as the scalar field s is now characterized by its Laplacian. The scalar formulation can be derived from the previous vector formulation by taking the gradient of eq. (1), and making the identifications $\vec{v} = \nabla s$ and $\vec{f} = \nabla b$ in the previous vector formulas. This gives for the

conserved efflux

$$E = \iiint \nabla^2 s \, dV = \oiint \frac{\partial s}{\partial n} \, dS \qquad (3a)$$

its centroid

$$\vec{R}_E = E^{-1} \iiint \vec{r} \nabla^2 s \, dV = E^{-1} \oiint \left(\vec{r} \frac{\partial s}{\partial n} - \hat{n} s\right) dS \qquad (3b)$$

and the velocity of the centroid

$$\dot{\vec{R}}_E = E^{-1} \oiint \hat{n}(b - \chi) dS \qquad (3c)$$

where χ is a scalar field that must be constructed by solving Laplace's equation

$$\nabla^2 \chi = 0 \text{ (in } V) \qquad (3d)$$

with the Neuman boundary condition

$$\frac{\partial \chi}{\partial n} = \frac{\partial b}{\partial n} \text{ (on } S) \qquad (3e)$$

where S is the surface that encloses volume V and has an outward unit normal $\hat{n}$, as shown in fig. 1.

According to these formulas, it is only necessary to know s, $\partial s/\partial n$, b, and $\partial b/\partial n$ on the enclosing surface S in order to determine the conserved efflux E, its centroid $\vec{R}_E$, and the centroidal velocity $\dot{\vec{R}}_E$. This contrasts with the previous vector formulation (ref. 1), which included volume integrals that could not be transformed to surface integrals.

Classical Center of Mass- Eqs. (3) also contrast with the classical formulas for the center of mass and its movement, as derived from the continuity equation

$$\frac{\partial \rho}{\partial t} + \text{div } \rho\vec{u} = 0 \qquad (4)$$

where ρ is the density and $\vec{u}$ is the fluid velocity. The classical formulas apply to an arbitrary volume τ and give for the conserved mass

$$M = \iiint \rho d\tau \qquad (5a)$$

for the center of mass

$$\vec{R}_M = M^{-1} \iiint \vec{r} \rho d\tau \qquad (5b)$$

and for the centroidal velocity

$$\dot{\vec{R}}_M = M^{-1} \iiint \rho \vec{u} d\tau \qquad (5c)$$

The volumetric integrals in eqs. (5) cannot, in general, be transformed to surface integrals. Therefore, in order to determine the center of mass and its movement, we must know ρ and $\vec{u}$ throughout the volume τ.

Comparative Example–Since eq. (4) has the same form as eq. (1), we may apply eqs. (3) to the volume V of a local disturbance in the Laplacian of the density field ρ by making the identifications $s = \rho$ and $b = \mathrm{div}\, \rho\vec{u}$, as follows:

$$E = \oiint \frac{\partial\rho}{\partial n}\, dS \tag{6a}$$

$$\vec{R}_E = E^{-1} \oiint \left(\vec{r}\,\frac{\partial\rho}{\partial n} - \hat{n}\rho\right) dS \tag{6b}$$

$$\dot{\vec{R}}_E = E^{-1} \oiint \hat{n}(\mathrm{div}\, \rho\vec{u} - \chi)\, dS \tag{6c}$$

where χ satisfies

$$\nabla^2\chi = 0 \quad (\text{in } V) \tag{6d}$$

$$\frac{\partial\chi}{\partial n} = \frac{\partial}{\partial n}\, \mathrm{div}\, \rho\vec{u} \quad (\text{on } S) \tag{6e}$$

Only surface integrals appear in eqs. (6), in contrast to eqs. (5) where volume integrals occur. Also, E represents the conserved efflux of $\nabla\rho$ for a storm, $\vec{R}_E$ its centroid, and $\dot{\vec{R}}_E$ its propagation velocity. Caution is appropriate, however, for although ρ is positive, $\nabla^2\rho$ may take on positive and negative values, and the centroid may conceivably lie or move outside the storm.

CONCLUSION

Any scalar meteorological quantity for which a tendency equation exists may be used to define a conserved property of a storm and its centroid. The movement of each such centroid is determined wholly from fields that are external to the storm.

REFERENCE

1. Costen, R. C.: General solutions for the movement of storms. NASA CP 2029, 1977, pp. 219-223.