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# STEREOGRAPHIC CLOUD HEIGHTS FROM THE IMAGERY OF TWO SCAN-SYNCHRONIZED GEOSTATIONARY SATELLITES 

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Scan synchronization of the sensors of two SMS-GOES satellites yields imagery from which cloud heights can be derived stereographically with a theoretical two-sigma random uncertainty of $\pm 0.25 \mathrm{~km}$ for pairs of satellites separated by $60^{\circ}$ of longitude. Systematic height errors due to cloud motion can be kept below 100 m for all clouds with east-west components of speed below hurricane speed, provided the scan synchronization is within 4 seconds at the mid-point latitude, and the spin axis of each satellite is parallel to that of the earth.

## BACKGROUND

In mid February 1975, while SMS 2 was in its test phase, and still under the control of Goddard Space Flight Center, the spin-scan imaging system of this satellite was synchronized with that of SMS 1 to produce consecutive pairs of stereo images. That is, the spin-scan camera of one satellite was adjusted so that both satellites scanned corresponding portions of a common field of view within a few seconds of each other, for each of a number of pairs of images. For this set of stereo image pairs, SMS 1 was located at a longitude of $75^{\circ} \mathrm{W}$, while SMS 2 was located at longitude $107^{\circ} \mathrm{W}$, resulting in a 32 -degree difference in viewing angle (a $32^{\circ}$ convergence angle). The resulting stereographic pairs of hard-copy images were analyzed photogrammetrically by a specially designed optical-mechanicál instrument to yield cloud heights with a 2 -sigma uncertainty of 500 m (Minzner et al., 1977).

Portions of these same image pairs have also been remapped digitally and combined into two-color anaglyph imagery which, when viewed with anaglyph spectacles, produce 3-D images from hard-copy prints or from the cathode-ray tube of an interactive system, the Atmospheric and Oceanographic Information Processing System (AOIPS).

A second stereo test was accomplished on May 26, 1978, when the two GOES satellites had a longitudinal separation of $60^{\circ}$. The increased separation of the satellites produced no apparent degradation in the stereo imagery, and decreased the uncertainty in the measured cloud heights. The effective area of stereo coverage along the equator was reduced, however, from about $100^{\circ}$ to $70^{\circ}$ as seen in Figure 1.

## UNCERTAINTY ANALYSIS

The study of uncertainty in cloud heights obtained from stereographic pairs is limited in this report to two major sources, image distortion due to each of two causes, and the ability of the eye or other sensor to resolve apparent height differences in the stereo model under various conditions. The word "model" in this paper
signifies the scaled-down system of a pair of stereo images which faithfully reproduce the original viewing parameters of the two GOES VISSR systems. The scale of a model in which the image resolution of one pixel ( 0.9 km ) equals the eye's resolving ability $(38 \mu \mathrm{~m})$ is about $1: 24,000,000$.

Fundamental to the entire stereo analysis are the concepts of base-to-height ratio ( $\mathrm{B} / \mathrm{H}$ ) and vertical-deformation ratio (VDR). Base-to-height ratio is simply explained in a flat-earth model as the ratio of the separation between the two stereo cameras to the height of the cameras above the reference plane. An example of base-to-height ratio $\mathrm{AB} / \mathrm{H}$ is shown in each of two simple flat-earth models of Figure 2. In each of these models, the convergence angle between the nadir-looking rays of the two cameras, one at $\mathrm{A}^{\prime}$ and one at $\mathrm{B}^{\prime}$, is zero.

The concept of vertical-deformation ratio is only slightly more sophisticated than that of $\mathrm{B} / \mathrm{H}$. In the simple model of Figure 2, VDR is nearly the reciprocal of $B / H$, and more exactly is equal to (H-h)/A'B', where (H-h) is the height of the cameras less the target height $h$ which is being measured. It is clear from Figure 2 that $(H-h) / A^{\prime} B^{\prime}=h / C D$, where $C D$ is the apparent horizontal displacement of the image of target T between the two superimposed stereo images. A comparison of the two models of this figure shows that for a fixed target height $h$, the X-displacement CD, parallel to the base line AB , increases as the $\mathrm{B} / \mathrm{H}$ increases. The X -displacement CD is the quantity which provides the apparent vertical relief in the model, and hence permits the recovery of the target height $h$ from the imagery of the two cameras. Any distortion of the apparent horizontal displacement CD introduces a corresponding relative distortion in the measured value of $h$.

The models of Figure 2 are unrealistically simple when compared with the situation of two geostationary satellites separated by $60^{\circ}$ of longitude, each viewing about $1 / 3$ of the surface of a spherical earth. The nadir-looking rays of the two satellites are no longer parallel, as in Figure 2, but rather, form a convergence angle V at the earth's center. This convergence angle is equal to the difference in the longitudinal positions of the two satellites. For a spherical-earth model the $\mathrm{B} / \mathrm{H}$ is a function of convergence angle. When the sphericity of the earth, the height of the satellites, and the convergence angle $V$ are all considered, one obtains values of VDR shown in the three-dimensional graph of Figure 3.

Figure 3 shows that for any one convergence angle, VDR maximizes at the midpoint longitude between the two subsatellite points. The greatest maximum of VDR plotted in this figure is 4.9 for a convergence angle of $10^{\circ}$. As the convergence angle increases successively from $10^{\circ}$ to $60^{\circ}$, the corresponding maximum values of VDR decrease from 4.9 to 0.71 . For a unit target height $h$, it is evident that the $X$ displacement CD measured along a spherical-earth model increases with increasing convergence angle, and hence provides an improved basis for the determination of model height ( h ).

For any single convergence angle, e.g., $60^{\circ}$, the VDR decreases as the radial distance between the target and the midpoint longitude (ML) increases along any direction from the ML. Figure 3 shows this decrease only along the equator. The value of VDR becomes very small, about 0.02 , at a longitudinal distance ( $80-\mathrm{V} / 2$ ) degrees from the ML. This graph shows that for a unit model height $h$, the related X-displacement distance CD along the model sphere at this longitude becomes very large, i.e., 50 units. A longitudinal distortion of one unit at this longitude along the equator of the model sphere would introduce a distortion of only $2 \%$ in h . A longitudinal distortion of one unit at the ML would introduce a distortion of $141 \% \mathrm{in} \mathrm{h}$. (The longitudinal distortion is measured in terms of longitudinal position along the equator of a hypothetical model sphere, and not in terms of a linear distance along the image plane.)

There are two possible sources of serious error in cloud-height determination due to errors or distortions in ground-scale position of the cloud. The first of these is due to remapping distortions; the second is due to possible cloud motion occurring between the sensing of corresponding pairs of stereo images by the two satellites.

Since each satellite views a given land mass, e.g., North America, from very different locations, the image of the land mass viewed from one satellite does not conform with that viewed from a second satellite. In order to perform the photogrammetrical analysis for cloud heights, it is necessary to properly remap a region of interest on one satellite image to the same region of interest on the other satellite image. This remapping is required so that all sea-level features of one image are congruent with those of the other image. The higher land features and the cloud features will then show horizontal displacements proportional to the value of the VDR at the location of the features. If the remapping introduces east-west distortions in any particular target, the related target height will be in error in the amount of the local VDR times the east-west distortion.

For the hard-copy photogrammetrical analyses, it has been found desirable to remap imagery so as to compress image information rather than to stretch image information. Thus, for analyses of areas east of the ML, the imagery of the eastern satellite is remapped and compressed to fit that of the western satellite. This procedure retains the model resolution of the western satellite image, and increases the model resolution of the eastern satellite image. A related procedure applies to areas west of the ML.

An error in equatorial cloud height due to cloud motion may occur when some time increment $\Delta t$ exists between the scanning of a particular cloud feature, first by the sensor of one satellite, and then by the sensor of the second satellite. The magnitude of the height error may be calculated by multiplying $\Delta t$ by $u$, the east-west component of the cloud motion, and multiplying this result by the VDR for the particular equatorial longitude.

By a reversal of the above procedure, a set of values of longitudinal departure from the ML, and the corresponding set of values of VDR were used to calculate a related set of values of $\Delta t$ which would produce a constant cloud-height error of 100 meters, for each of four east-west cloud speeds. These data are longitudinally symmetrical about the ML, and one half of each of these four data sets is plotted in Figure 4. These curves all show a minimum value at the ML, which for the current GOES East and GOES West satellites, lies at $105^{\circ} \mathrm{W}$. The successive minima for eastwest cloud speeds of $10,20,30$, and $40 \mathrm{~m} / \mathrm{s}$ are seen to be $14,7.0,4.7$, and 3.5 sec onds respectively.

When the separation between exposure times is less than 3.5 seconds at the ML, any clouds at the ML which have an east-west speed of less than $40 \mathrm{~m} / \mathrm{s}$ will have a height error of less than 100 m due to cloud motion. For clouds at latitudes remote from the ML, the 100 m error limit will be retained with much larger time separations, provided the spin axis of both satellites are sufficiently well aligned with that of the earth. For east-west cloud speeds less than hurricane speeds ( $33 \mathrm{~m} / \mathrm{s}$ ), a time separation of 4 seconds at the ML will guarantee cloud-height errors of less than 100 m at all locations, when the spin-axis alignment is suitable.

The limiting factor in the determination of the target height in the stereo model is the ability of the sensor or operator to correlate the two images of the target in the model by converging the corresponding rays at the model height $h$ of the target, and then to measure $h$ with an appropriate scale. In the analysis of hard-copy pairs of stereo images, this process consists of having the operator move a floating dot from the reference level to the apparent height of the target in the model. The accuracy with which this process can be accomplished depends upon a number of
factors: (1) the operator's eye resolving ability, (2) the base-to-height ratio of the model, and (3) the linearity and smoothness of motion of the floating-dot reference mark.

The mean value of the eye's resolving ability (era) expressed at the two-sigma level is about $38 \mu \mathrm{~m}$ at 10 inches from the image (Sidgwick, 1961). Figure 2, which was used above to illustrate $\mathrm{B} / \mathrm{H}$, also illustrates the relationship of the eye's resolving ability to the model-height uncertainty $2 \delta \mathrm{~h}$ for each of two values of $\mathrm{B} / \mathrm{H}$. The twosigma value of era implied in Figure 2 also implies a two-sigma value for 28 h . It is apparent that for a fixed value of era, the value of $2 \delta \mathrm{~h}$ decreases as $\mathrm{B} / \mathrm{H}$ increases. A graph of the two-sigma value of $2 \delta \mathrm{~h}$ as a function of $\mathrm{B} / \mathrm{H}$ is shown in Figure 5. This graph, which was developed from the geometrical relationship shown in Figure 2, also shows the relationship between $\mathrm{B} / \mathrm{H}$ and convergence angle for pairs of geostationary satellites. One can see that $2 \delta \mathrm{~h}$, the two-sigma value of uncertainty in the perception of the vertical height of the model has been decreased from about .043 mm to .022 mm by increasing the convergence angle of the satellite pair from $32^{\circ}$ to $60^{\circ}$. The relationship between $2 \delta \mathrm{~h}$ and $\mathrm{B} / \mathrm{H}$ applies independently of image resolution as long as the image resolution is equal to or below the value of era.

With the two images of a target feature merged at the model height $h$, it is necessary to measure this model height accurately with an appropriate scale. In hard-copy stereography, this process is accomplished by means of a floating dot somewhere in the optical train. This dot is attached to a precision lead screw which serves as a scale. The dot is designed to be moved continuously and smoothly, with a high degree of linear precision along this scale which provides a measure of the distance between the apparent position of the reference surface and the apparent target height in the model. The ground-scale height of the target feature is then obtained by multiplying both the measured model height h and its uncertainty $2 \delta \mathrm{~h}$ by the viewing scale of the imagery. For satellites separated by 60 and 32 degrees, the theoretical uncertainty $\pm \delta \mathrm{h}$ is found to be $\pm 0.25 \mathrm{~km}$ and $\pm 0.48 \mathrm{~km}$, respectively. The latter value is in agreement with measured uncertainty values (Minzner et al., 1978).

Image contrast and continuity of the target are also important factors in the ability of the eye or other sensor to determine the model height of the feature. No attempt is made to discuss these topics in this limited paper.

## CONCLUSIONS

Theoretical considerations as well as actual measurements show that random cloud-height uncertainty and effective area of stereo coverage both decrease with an increase in longitudinal separation of the scan-synchronized satellites. For the present $60^{\circ}$ separation, the theoretical two-sigma height uncertainty is about $\pm 0.25 \mathrm{~km}$ even though the finest resolution of the GOES imagery is about 0.9 km . The present location, $135^{\circ} \mathrm{W}$, of the western member of the scan-synchronous pair of satellites eliminates from effective stereo coverage a large region which includes a significant portion of northeastern United States. The coverage for this important area of our country could be restored to an operational stereo cloud-height system, while retaining the coverage of the remainder of the contiguous United States, by either of two methods: (1) move the western satellite eastward by $15^{\circ}$, or (2) place a third active GOES satellite either at $120^{\circ} \mathrm{W}$ or $30^{\circ} \mathrm{W}$. Each of these approaches would have its own set of advantages and disadvantages, none of which are discussed in this paper because of space limitations.

For the current satellite locations, a scan synchronization within four seconds at the midpoint latitude would keep systematic cloud-height errors due to cloud
motion within 100 m for all clouds with an east-west component of speed below hurricane speed, provided the spin axis of each satellite is nearly parallel to that of the earth.

## REFERENCES

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