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NUMERICAL METHODS FOR METEOROLOGY AND CLIMATOLOGY

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ABSTRACT

Efficient numerical methods for long term weather forecasting are developed. One implicit and one explicit scheme are compared as to accuracy.

INTRODUCTION

It has been estimated that about 30% of the error in meteorological forecasts is attributable to the inaccuracy of numerical schemes currently used. In order to minimize this error, we have developed and tested two new numerical methods. These schemes are particularly designed to perform long term integrations for climatological purposes, since they conserve the global quantities: total mass, total energy, and total potential enstrophy of the flow. One of the schemes is implicit, while the other one is explicit. The greater inherent efficiency of the implicit scheme, for fine spatial grid calculations arises from the fact that its time step Δt may be an integer multiple of $\Delta X/U$, and is not required to be less than $\Delta X/(C+U)$ as is the case for explicit schemes, where ΔX is the smallest spatial interval size, U is the maximum speed of propagation of the flow, while C is the much larger maximum speed of sound waves in the flow. In other words, a much larger time step is permitted for implicit schemes. But, in order to derive this benefit it is essential that the amount of computer time used to solve the implicit equations at any time step, be a small multiple of the time needed to make one time step with an explicit scheme. Our application of these schemes indicates that indeed the implicit scheme requires less computer time than does the explicit scheme, for fine grid sizes.

In section 2, both the implicit and explicit methods are used to solve the shallow water equations for a single-layered atmosphere over a smooth earth and comparisons of the solutions and of the computer time are made. In section 3, with orography present, a comparison is made of the solutions produced by the explicit scheme both with and without the conservation of the global quantities. Here it is observed that the solution produced while conserving the global quantities, is smoother and more accurate. In section 4, a brief description is given of the numerical schemes.

Application of schemes

The technique for devising an implicit scheme and the method for modifying both implicit and explicit schemes so as to conserve the global quantities were applied to the solution of the nonlinear shallow water equations for a single layered, incompressible fluid over a spherical earth. As initial conditions, three subtropical highs were centered symmetrically in each hemisphere on latitudes $\pm 30^{\circ}$, and at longitudes 0°, 120°E, 240°E. The initial wind speed is found geostrophically from the initial height field. The initial hydrostatic "pressure" (i.e. density·height) rose from 1036 (corresponding to 8500 meters) to 1046 at the center of the highs. The maximum of the initially geostrophic velocity is about 13 meters/sec. Contour levels of the initial "pressure" field are plotted, on a stereographic chart of the Northern Hemisphere, along with arrows indicating the initial wind speed and direction in Fig. 1. After 24 hours, we exhibit in Fig. 2, appropriately scaled height fields produced on one third of the Northern Hemisphere with a fine spherical grid using $\frac{360}{128}^{\circ}$ in both longitude and latitude. Fig. 2a was produced by the explicit scheme, while Fig. 2b was produced by the implicit scheme. Note how closely the "high" cells resemble each other after 24 hours. The left most column of numbers represent longitudes, that is row 73 corresponds to longitude $73\frac{360}{128}^{\circ}$. The rightmost column corresponds to the latitude nearest the equator and the second column from the left corresponds to the latitude nearest the North Pole.

For the explicit scheme a time step of 3 minutes was used, while in the implicit scheme an interval of 15 minutes was used. The table below lists the computing times (CPU) required on a 60 bit machine, the CDC-6600, for each method with both the fine grid and a coarse spherical grid of $^{360}/_{64}\,^{\circ}$ for latitude and longitude.

	explicit scheme	implicit scheme	revised implicit scheme
coarse grid	3 min	9 min	3 min
fine grid	24 min	36 min	12 min

Computing Times on CDC-6600

Note that the time step of 15 minutes was used in the implicit scheme for both the fine and coarse grids; whereas in the explicit scheme, 6 minutes was used for the coarse grid and 3 minutes was used for the fine grid. Both schemes produced solutions with comparable accuracy. Furthermore, we have observed that it is possible to revise the implicit scheme to take advantage of the special form of the hydrodynamic equations, so as to cut the computing time of the implicit scheme in half. This feature of the equations of motion is also present in the full hydrodynamic equations used for general circulation models. With this improvement the implicit scheme would run in less time than the explicit scheme, for fine grids.

Orography effects

A major source of computational errors is found to be the way in which current numerical methods treat flow over and around high and extensive mountain ranges, e.g. Rocky Mountains, Andes Mountains, Himalayan Mountains, etc. In order to eliminate this difficulty, Arakawa has advocated using difference schemes that conserve the total potential enstrophy, total energy, and total mass. We have found it possible to modify any difference scheme, so as to make the new scheme conserve these global quantities. In particular, we show the effect that this conservative modification has when we introduced orography into the shallow water model previously described in section 2. That is, we introduced three identical mountain ridges centered respectively along the three longitudes 60°E, 180°E, and 300°E. The maximum height above sea-level of the mountain ridge is $\frac{8500}{16}$ meters, at the equator. The mountain height decreases to zero at the poles. The initial velocity is again found from geostrophic balance on the sphere. The fluid tends to flow around the ridges, by veering to the pole. In Fig. 3a, we see that the solution produced after 24 hours without the conservation of the global quantities has larger velocities and steeper height gradients near the pole, than does the solution shown in Fig. 3b, which is produced with the conservation of total mass, total energy, and total potential enstrophy. In the Table below we indicate the relative change from the initial values of these global quantities, after t=24 hours and after t=48 hours, for the explicit scheme. In the first column the larger relative deviations are found without the use of the conservative modification method; whereas the smaller relative deviations in the second column are produced with the use of the conservative modification method.

quantity	without conservative modification	with conservative modification
total mass (24 hours)	$140 \cdot 10^{-6}$	$0.2 \cdot 10^{-6}$
total energy (")	$203 \cdot 10^{-6}$	$0.5 \cdot 10^{-6}$
total pot.enstrophy	$5766 \cdot 10^{-6}$	$10.0 \cdot 10^{-6}$
total mass (48 hours)	$235 \cdot 10^{-6}$	$0.5 \cdot 10^{-6}$
total energy (")	$260 \cdot 10^{-6}$	$1.1 \cdot 10^{-6}$
total pot.enstrophy	$17{,}750 \cdot 10^{-6}$	$16.0 \cdot 10^{-6}$

Relative errors in total quantities.

The Numerical Schemes

The explicit scheme is a so-called leapfrog scheme in which the solution is advanced from $(t-\Delta t)$ and t to the time $(t+\Delta t)$. The first order time derivatives are replaced by differences centered at time t while the first order spatial derivatives in latitude and longitude, are replaced by fourth order, five point, centered difference expressions. At the latitudes closest to the pole, the values of the solution determined by this difference scheme are smoothed by using the fast Fourier transform. This program has been adapted from the ideas of Kreiss and Oliger [4] and Williamson and Browning [6], and is described in Isaacson and Stoker [5].

The implicit scheme is roughly speaking a Crank-Nicolson type scheme in which the solution is advanced from time t to time (t+Δt). Here the fourth order accuracy is obtained by using a Padé rational fraction in the spatial difference operators. After clearing fractions, by multiplying all terms by the denominator, we obtain a compact three point spatial difference expression. The resulting simultaneous difference equations are simplified by writing the spatial operator as the product of a longitudinal factor and a latitudinal factor. These factors have a block tri-diagonal matrix representation, that is easily invertible. This idea and its implementation were proposed by A. Harten based on the methods used by Beam and Warming [1,2] in aerodynamics. Considerable effort was needed to find an efficient and stable factorization method. Finally, it was found necessary to use a fourth order Shapiro filter at each time step in order to maintain stability.

The conservative modification method is described in general terms in Isaacson [3]. Here the work involved in modifying the solution found at time (t+Δt) is of the order of the amount of calculation used in one time step of the explicit scheme.

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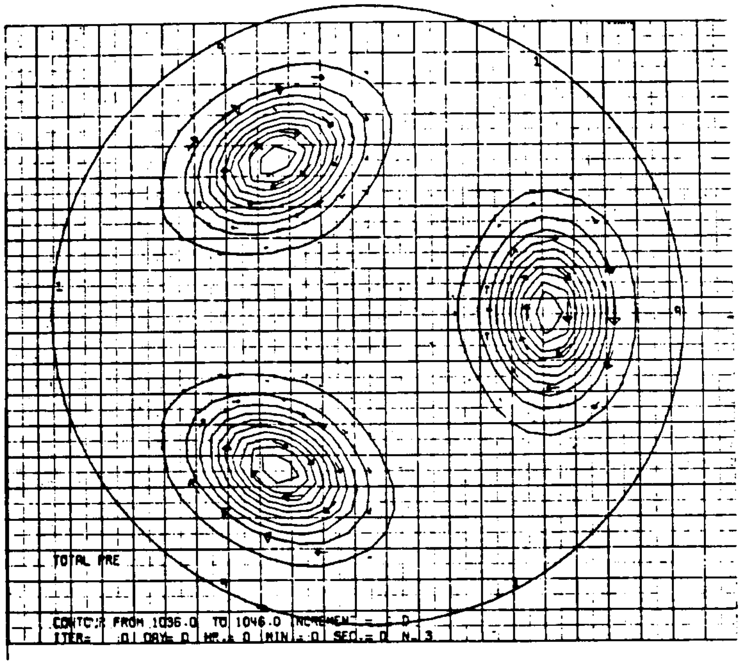

Fig. 1. Contour levels of the initial pressure field.

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Fig. 2a. Height field after 24 hours - explicit scheme

73	2	3	1	2	3	2	4	4	4	5	6	6	6	7	7	7	7	7	7	7	7	7	7	7	6	6	6	6	6	6	7	7
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98	2	2	2	1	1	0	-2	-2	-4	-4	-5	-5	-5	-5	-4	-4	-3	-1	1	2	3	3	3	4	3	3	2	1	-2	-3	-4	-4
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100	2	2	2	1	1	1	-1	-1	-2	-3	-3	-3	-3	-1	1	3	4	5	5	5	5	5	6	6	5	5	5	4	4	3	3	2
101	2	2	2	1	1	1	0	-1	-2	-2	-2	-2	0	2	3	5	5	6	6	6	6	6	6	6	6	6	6	5	5	5	4	4
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109	2	1	2	2	2	3	3	4	4	4	5	5	6	6	6	6	7	7	7	7	7	7	7	7	6	6	6	6	7	7	7	7
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111	2	1	2	2	2	3	3	4	5	5	6	6	6	6	6	7	7	7	7	7	7	7	7	7	6	6	6	6	7	7	7	7
112	2	0	2	2	2	3	3	4	5	5	6	6	6	6	7	7	7	7	7	7	7	7	7	7	6	6	6	6	7	7	7	7
113	2	0	2	2	2	3	3	4	5	5	6	6	6	7	7	7	7	7	7	7	7	7	7	7	6	6	6	6	7	7	7	7
114	1	0	2	2	2	3	3	4	5	5	6	6	6	7	7	7	7	7	7	7	7	7	7	7	6	6	6	6	6	7	7	7
115	1	0	3	2	2	3	3	4	5	5	6	6	6	7	7	7	7	7	7	7	7	7	7	7	6	6	6	6	6	7	7	7
116	1	0	3	2	2	3	3	4	5	5	6	6	6	7	7	7	7	7	7	7	7	7	7	7	6	6	6	6	6	6	7	7

Fig. 2b. Height field after 24 hours - implicit scheme

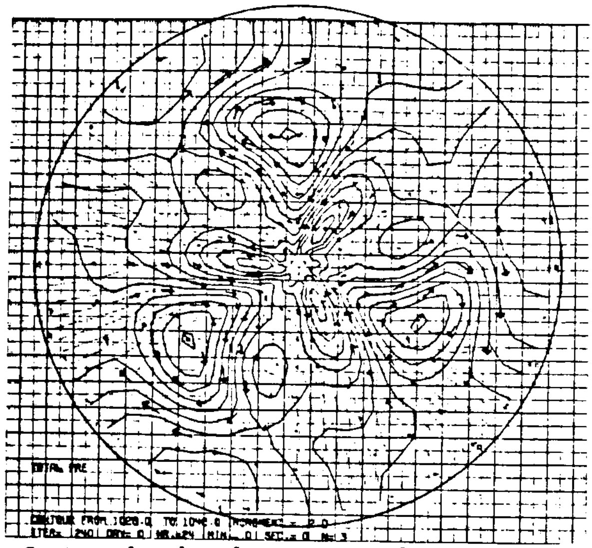

Fig. 3a. Contour levels of pressure, after 24 hours, found without the conservation of the total mass, total energy, and total potential enstrophy.

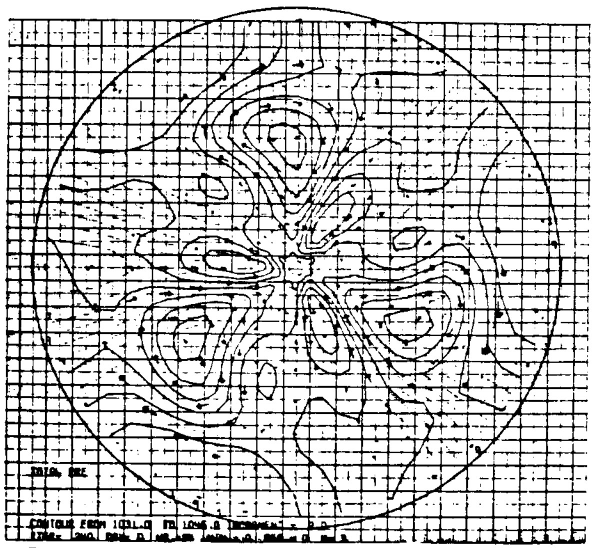

Fig. 3b. Contour levels of pressure after 24 hours, found with the conservation of the total mass, total energy, and total potential enstrophy.