Paper No. 44

SCATTERING BY RANDOMLY ORIENTED ELLIPSOIDS: APPLICATION TO AEROSOL AND CLOUD PROBLEMS

Shoji Asano, Makoto Sato and James Hansen, *Goddard Institute for Space Studies, New York, NY 10025*

ABSTRACT

A program has been developed for computing the scattering and absoprtion by arbitrarily oriented and randomly oriented prolate and oblate spheroids. This permits examination of the effect of particle shape for cases ranging from needles through spheres to platelets. We discuss applications of this capability to aerosol and cloud problems. Initial results suggest that the effect of nonspherical particle shape on transfer of radiation through aerosol layers and cirrus clouds, as required for many climate studies, can be readily accounted for by defining an appropriate effective spherical particle radius.

Introduction. Many climate studies involve scattering or absorption of radiation by nonspherical particles. Examples include the climatic impact of natural or man-made changes in stratospheric and tropospheric aerosols and cirrus clouds. The nonspherical shape of such particles has generally been neglected, with the hope that spheres can adequately represent the results for other particles. With improvement of climate modeling capabilities and climate observations, it is becoming increasingly important to obtain a quantitative understanding of the effects of nonspherical particle shape and a prescription for defining the spheres which can be used to approximate nonspherical particles.

A general solution for scattering by infinitely long circular cylinders has been obtained and investigated (Kerker, 1969; Liou 1972), but that case is too special to provide an adequate test of particle shape effects. The next logical particle shape for analysis, in terms of practicality for obtaining a general solution analogous to the Mie solution for spheres, is the spheroid. In fact spheroids, formed by rotation of an ellipse about its major (prolate spheroid) or minor (oblate spheroid) axis, provide an excellent opportunity for investigation of the effect of nonspherical particle shape. As the ratio of the length of the major axis to length of the minor axis becomes large the particle becomes needle-like. At the other extreme it approaches the shape of a platelet. In between it passes through the special case of the sphere.

Asano and Yamamoto (1975) developed the scattering theory for a homogeneous spheroid. The objective of the work described here is to develop a program to implement that theory for particles all the way up to sizes which show the basic features of geometrical optics, and to apply this theory to climate studies.

Single arbitrarily oriented spheroid. Light scattering by a spheroidal particle is specified by the following five physical quantities: 1) particle size relative to the wavelength of incident wave, 2) eccentricity, 3) complex refractive index $\tilde{m}$ relative to that of the surrounding medium, 4) orientation of particle to the incident wave, and 5) the observation direction.

We define the particle size parameter α by $\alpha = 2\pi a/\lambda$, where a is the semi-major axis of the ellipse and λ is the wavelength of the incident radiation. We specify the shape of the spheroid by the ratio of the semi-major axis a to the semi-minor axis b, which thus has a value greater than unity. We use the ratio a/b, rather than the eccentricity, since it is a more direct measure of the elongation of the spheroid.

Fig. 1 shows the geometry of scattering for a linearly polarized plane electromagnetic wave incident at an angle ζ to the z-axis. The origin of the coordinate system is taken at the center of the spheroid with the z-axis as the axis of rotation, i.e., as the major axis for a prolate spheroid and the minor axis for an oblate one. The spherical coordinate system (θ, ϕ) is adopted to represent the direction of a far-field observation point, where θ is the zenith angle measured from the positive z-axis and ϕ is the azimuth angle. The direction of incidence is assumed to be in the $\phi = 0$ plane (x-z plane) and is thus identified by $(\zeta, 0)$.

The efficiency factor for scattering Q_{sca} is defined here as the ratio of the scattering cross section C_{sca} to the area of the geometrical shadow of the spheroid $G(\zeta)$ at the incidence angle ζ. In Fig. 2 Q_{sca} for prolate spheroids with $\tilde{m} = 1.50$ is plotted, as a function of the size parameter α, for various values of the shape parameter from a/b = 1.5 to 5. For small prolate spheroids, $\alpha < 4.0$, the scattering efficiency factors are greater for the spheroids which are closer to being spheres, i.e., for the smaller a/b. Q_{sca} (α) for a/b = 1.5 is very similar to that for spheres, with oscillations which damp out for large size parameters as Q_{sca} approaches the geometrical optics limit of 2. A striking feature is that with increase of a/b the first resonance maximum occurs at large size parameter and the peak value increases.

The major maxima and minima in Fig. 2 are due to interference of light diffracted with light transmitted by the particle. The phase shift for a light ray passing through the particle along the axis in the direction of incidence is $\rho = 2\alpha(\tilde{m}-1)$, which equals α or $2\pi a/\lambda$ for the case $\tilde{m} = 1.5$. Thus successive maxima, caused by constructive interference, occur at intervals of $\sim 2\pi$. The case of a/b = 3 is an exception; however, the average separation of the maxima would also be $\sim 2\pi$ if there were an additional maximum. The results for a/b = 3.5, shown by the dashed curve, in fact suggest that the expected second and third maxima have somehow combined into a single maximum for a/b $\sim$ 3.

Application of the anomalous diffraction approximation of van de Hulst (1957) to scattering by prolate spheroids at the parallel incidence gives extinction efficiency curves identical to those for spheres, without regard to a/b, but with the phase lag parameter $\rho = 2\alpha(\tilde{m}-1)$. However, in fact, the position and peak value of the first maximum of the scattering efficiency curves in Fig. 2 are strongly dependent on a/b. Furthermore, the mean periods of the major oscillations of the scattering efficiency curves are larger for the prolate spheroids than for spheres. These discrepancies from the expectation of the anomalous diffraction approximation can be attributed to effects of edge phenomena for grazing reflection,

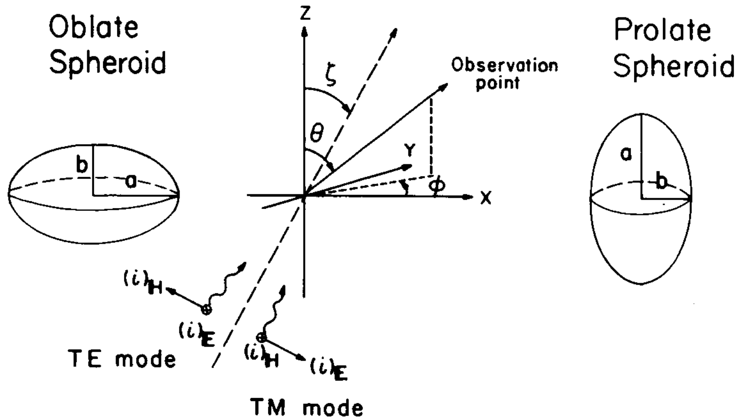


Fig. 1. Scattering geometry. A spherical coordinate system is adopted to represent the scattered field in the far-field zone. The origin of the coordinate system and the z-axis are the center and axis of revolution of the spheroid, respectively. The angle of incidence ζ is the angle in the plane of incidence (the x-z plane)between the direction of incidence and the z-axis.

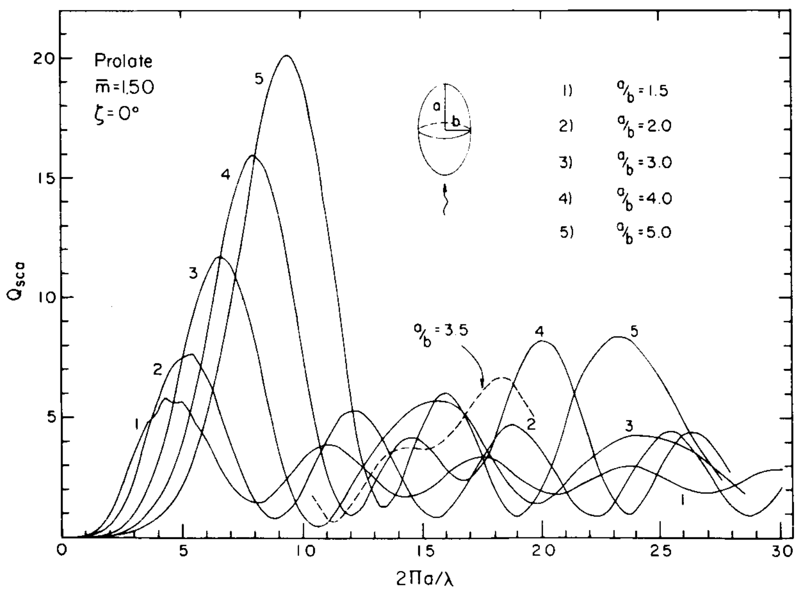


Fig. 2. Scattering efficiency factors Q_{sca} at $\zeta = 0$ as a function of the size parameter $2\pi a/\lambda$ for the prolate spheroids with $\bar{m} = 1.50$, for several values of the shape parameter a/b.

as discussed by Asano (1979).

Randomly oriented spheroids. Fig. 3 compares the scattering by randomly oriented prolate spheroids with scattering by spheres of 'equivalent' size. For the spheres integration over a narrow distribution of particle sizes was performed in order to smooth out the oscillations which occur for a single size. For the spheroids this was not necessary since the integration over random orientations served the same function.

We define an equivalent spherical particle radius for any particle shape to be the radius of the circle which would give the same area as that of the particle's shadow. Since the complete scattering behavior of spheres is available in efficient subroutines (e.g. Hansen and Travis, 1974) this prescription represents a simple approximation for scattering by nonspherical particles.

Parts (a) and (b) of Fig. 3 compare the efficiency factor for scattering (Q_{sca}) and the asymmetry parameter of the scattering diagram ($\langle\cos\theta\rangle$) for spheres and prolate spheroids. Two principal conclusions are: (1) spheres provide a good approximation for the nonspherical particles for these parameters which define the gross radiative properties, when the equivalent radius we have defined is employed, (2) there is no evidence that the nonspheres are less forward scattering than spheres.

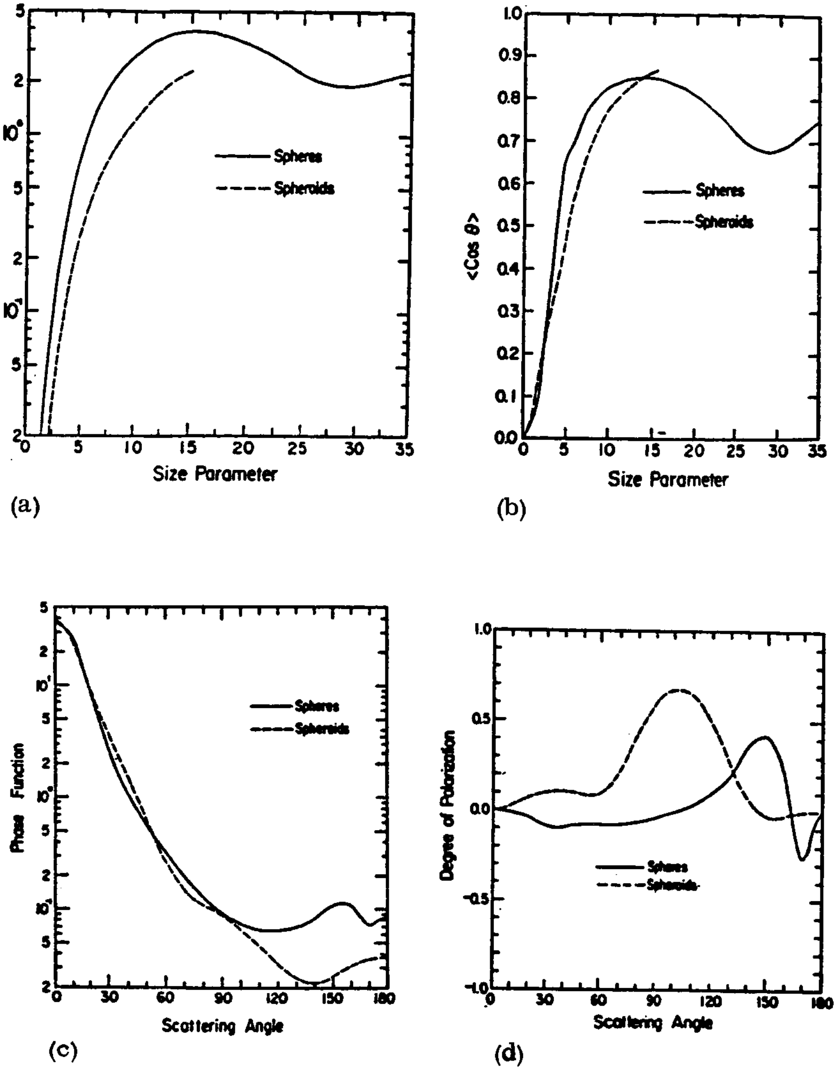
Parts (c) and (d) of Fig. 3 compare the scattering diagram and linear polarization for spheres and prolate spheroids. It is apparent that the scattering at a specific angle is not in general well approximated by the simple prescription of using spheres of equivalent cross-sectional area. There is no evidence in the scattering by spheroids of this size parameter for special spherical particle geometrical optics features such as the rainbow and glory.

Discussion. These initial results are encouraging with regard to climate applications. The principal parameters defining the radiative transfer characteristics of aerosols and clouds are the efficiency factors for scattering and absorption and the asymmetry parameter. Our very limited sample of calculations for randomly oriented nonspherical particles suggests that it may be possible to accurately approximate these parameters with results for equivalent spheres. With regard to remote sensing applications which require knowledge of scattering behavior at a specific angle, the indications are that it will be necessary to take account of the specific particle shape involved. The calculations presented are only a small fraction of what will be needed to draw any firm conclusions; calculations are underway for larger sizes and for randomly oriented oblate spheroids.

Acknowledgements. During the course of this research S.A. held a NAS-NRC Post-doctoral Research Associateship at the Goddard Institute for Space Studies.

REFERENCES

Asano, S., Appl. Opt., 18 (in press), 1979.
Asano, S. and G. Yamamoto, Appl. Opt., 14, 29, 1975.
Hansen, J.E. and L.D. Travis, Light Scattering in Planetary Atmospheres, GISS Report, New York, 97 pp., 1976.
Kerker, M., Scattering of Light, Academic Press, NY, 666 pp., 1969.
Liou, K.N., Appl. Opt., 11, 667, 1972.
van de Hulst, H.C., Light Scattering by Small Particles, Wiley, New York, 470 pp., 1957.



g. 3. Comparison of scattering by randomly oriented prolate spher-
ds with a/b = 5 with scattering by equivalent spheres. All calculations
e for a real refractive index 1.33. (a) is the efficiency factor for
attering, (b) is the asymmetry parameter of the phase function,
) is the phase function and (d) is the degree of polarization.