Paper No. 62

MAINTENANCE OF QUASI-STATIONARY WAVES IN A 2-LEVEL QUASI-GEOSTROPHIC SPECTRAL MODEL WITH TOPOGRAPHY

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ABSTRACT

The maintenance of the quasi-stationary waves obtained through numerically integrating a 2-level quasi-geostrophic spectral model on a β-plane is investigated. An idealized topography which has only wavenumber n in the zonal direction and the first mode in the meridional direction is used to force the quasi-stationary waves. It is shown that the topographical forcing is not necessarily the mechanism for maintaining the quasi-stationary waves.

Introduction. The earth's topographical forcing is one of the major mechanisms for producing quasi-stationary waves in the atmosphere. Many studies have been made on the role of the topography in forcing these waves, such as Charney and Eliassen (1949), and Derome and Wiin-Nielsen (1971). Saltzman (1968) gives a general review of this kind of study.

There have also been some observational studies on the maintenance of the quasi-stationary component of the atmospheric circulation. Holopainen (1970) studied the energy balance of the quasi-stationary disturbances in the northern hemisphere. He concluded that in winter the quasi-stationary disturbances are usually baroclinic waves which extract available potential energy from the zonally averaged mean flow. These waves then convert part of their potential energy into kinetic energy to offset the destruction of the latter by small-scale turbulent friction, large-scale transient motion and conversion into zonally-averaged mean motion. As far as the energy balance of the quasi-stationary waves is concerned, the effect of mountains seems to be very small. In summer the stationary disturbances appear to form a thermally-driven system.

Thus it is not obvious how the theoretically calculated stationary waves are related to the observed quasi-stationary waves. Our objective is to examine the maintenance of the quasi-stationary waves produced by a simple 2-level quasi-geostrophic truncated spectral model, in which the dependence of the mechanism for maintaining the waves upon parameters, such as the wavenumber n, the imposed thermal equilibrium temperature gradient ΔT_e, and damping coefficients, can be readily obtained.

Two-level quasi-geostrophic truncated spectral model. The planetary-scale waves forced by topography and the cyclone-scale transient waves can be described by the quasi-geostrophic system of equations on a β-plane. We assume that the motion takes place in a cyclic zonal channel of width Y_0 in the y (meridional) direction and of a length X_0 in the X (zonal) direction. Then, we have the following governing equations in p-coordinates:

$$\left(\frac{\partial}{\partial t} + \underset{\sim}{v}_g \cdot \underset{\sim}{\nabla}\right) \zeta_g + \beta v_g - f_o \frac{\partial \omega}{\partial p} = -g \frac{\partial}{\partial p} \underset{\sim}{k} \cdot \underset{\sim}{\nabla} \times \underset{\sim}{\tau}, \tag{1}$$

$$\left(\frac{\partial}{\partial t} + \underset{\sim}{v}_g \cdot \underset{\sim}{\nabla}\right) \frac{\partial \phi}{\partial p} + S_p \omega = - \frac{R}{C_p P} \dot{Q}, \tag{2}$$

$$\underset{\sim}{\nabla} \cdot \underset{\sim}{v} + \frac{\partial \omega}{\partial p} = 0, \tag{3}$$

$$\frac{\partial \phi}{\partial p} = -\alpha, \tag{4}$$

where S_p is static stability, $\underset{\sim}{\tau} = -g\rho_s^2 \nu (\partial \underset{\sim}{v}_g / \partial p)$ the frictional stress, ρ_s horizontally averaged density, ν kinematic coefficient of turbulent eddy viscosity, $\dot{Q} = - \mu C_p(T-T_e)$ diabatic heating, T_e the imposed thermal equilibrium temperature, and μ a constant. Other variables have their usual meaning.

The static stability S_p is assumed to be a constant. The diabatic heating is in the form of Newtonian cooling, where T_e is a function of y only. We assume the following boundary conditions:

$v_g = v = 0$ at $y = 0, Y_o$

$\underset{\sim}{\tau}_T = 0, \ \omega_T = 0$ at $p = p_T$, the top of the model atmosphere,

$\underset{\sim}{\tau}_B = \rho_B C_D \underset{\sim}{v}_g, \ \omega_B = -\rho_B g \underset{\sim}{v}_g \cdot \nabla h$ at $p = p_B$, the bottom of the model atmosphere

where C_D is the drag coefficient, h the topographic function and ρ_B a standard air density at the bottom of the atmosphere.

In the vertical direction, we divide the mass of the atmosphere into two equal layers. We carry ϕ and $\underset{\sim}{v}$ at levels 1 (250 mb) and 3 (750 mb), ω and $\underset{\sim}{\tau}$ at the top (0 mb) and bottom of the atmosphere, and ω, $\underset{\sim}{\tau}$ and α at the middle level (500 mb).

Horizontally we expand variables in terms of double Fourier series. The detailed mathematical treatment is given by Yao (1977). In the results discussed below we restrict ourselves to some highly truncated cases where only the first three meridional modes and wavenumbers n and 2n are allowed. n is the lowest existing eddy wavenumber in the x-direction, as well as the wavenumber of the topography. We only have first mode for the topography and the imposed thermal equilibrium temperature. We will restrict ourselves to the cases n = 2 and n = 3, so that both topographical forcing and baroclinic instability are significant, and occur at realistic wavenumbers.

The spectral model was integrated initially to determine the quasi-equilibrium state. Unless otherwise mentioned, we used the following parameter settings: $X_o = 2\pi R_e \cos 45°$, where R_e is the earth's radius, $Y_o = 60°$ latitudinal length, $S_p = 0.03\ m^2 mb^{-2} sec^{-2}$, $f_o = 10^{-4} s^{-1}$, $\beta = 1.6 \times 10^{-11} m^{-1} s^{-1}$, $k_B = 4k_I$, $\mu = 2k_I$, where $k_B \equiv \rho_B g C_D / \Delta p$ and $k_I \equiv g^2 \rho_s^2 \nu / \Delta p^2$. The amplitude used for the topography is 750 m. The densities at level 2 and at the surface are based on $T_{s2} = 250$ K and $T_B = 280$ K, respectively. We define $\Delta T_e = T_e(0) - T_e(Y_o)$.

Maintenance of quasi-stationary waves. In considering the energy balance of the atmospheric flow we employ the kinetic energy and the available potential energy of time-averaged zonal mean flow K_Z and A_Z, of time-averaged eddies (K_S and A_S) and of transient flow (K_T and A_T). Fig. 1 shows the energy exchange processes in an energy cycle diagram. The arrows indicate the direction of the energy conversions when they are positive. $CB(K_Z, K_S)$ is the energy conversion due to topographic effect. This topographical effect is in the form of vertical geopotential flux at the surface.

Case n = 3. Fig. 2 is the stability diagram for n=3. The curve defining the transition between the Hadley and the Rossby regimes is obtained by a stability study (Yao 1977). In the indicated Rossby regime I, the kinetic energy of quasi-stationary waves (K_S) is maintained mainly by the topographical forcing, whereas in the Rossby regime II, K_S is maintained mainly through the energy conversion $A_S \to K_S$. In both Rossby regimes I and II, the available potential energy of the quasi-stationary waves (A_S) is maintained by the energy conversion $A_Z \to A_S$. When ΔTe or $1/K_I$ is sufficiently large, the model's motion becomes highly irregular and a clear distinction between different Rossby regimes can not be made; however, when ΔT_e or $1/K_I$ is very large the assumption of a constant static stability is invalid.

As an example from Rossby regime II, Fig. 3 shows the time-averated energy cycle diagram from the results of integration for the combination $\Delta T_e = 38$ K and $1/k_I = 26$ days. A_S is maintained by the energy conversion $A_Z \to A_S$. A portion of A_S is then converted to the kinetic energy of quasi-stationary waves, K_S. K_S is also produced from the conversion $K_Z \to K_S$ through the topographical effect. It can be seen that $C(A_S, K_S)$ is considerably larger than $CB(K_Z, K_S)$. We conclude that for this case the kinetic energy of the quasi-stationary waves is maintained mainly by the energy conversion $A_S \to K_S$. This is consistent with the findings of Holopainen (1970) for winter.

As an example from Rossby regime I, Fig. 4 shows the energy cycle diagram for the combination of $1/k_I = 26$ days and $\Delta T_e = 34$ K. It is apparent in this case that the quasi-stationary waves are maintained by the topographical effect $CB(K_Z, K_S)$, rather than by the conversion $C(A_S, K_S)$.

Case n = 2. For n = 2 a delineation between different Rossby

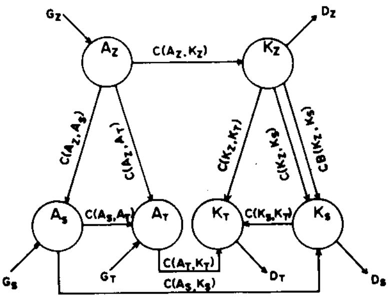

Fig. 1. Energy cycle diagram. The arrows indicate the positive direction of individual processes as defined.

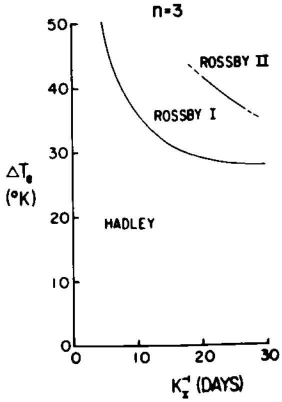

Fig. 2. Stability diagram for the case n = 3.

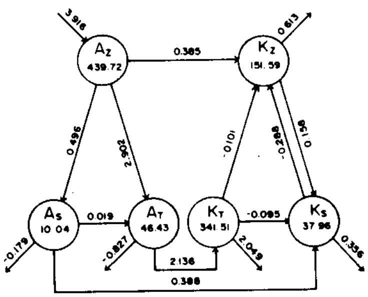

Fig. 3. Energy cycle of the quasi-steady state with n = 3, $1/K_I = 26$ days and $\Delta T_e = 38$ K. Energy is in m^2/s^2, while conversion rates are in $10^{-4} m^2 s^3$.

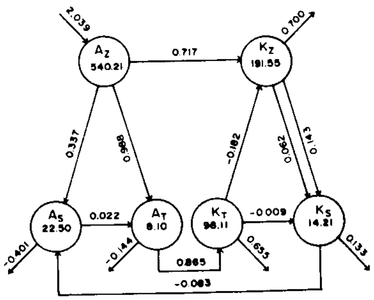

Fig. 4. Same as Fig. 3, but with $\Delta T_e = 34$ K.

regimes is not well defined. For relatively small values of ΔT_e and $1/K_I$, typical of atmospheric values, K_S in the Rossby regime is maintained by topographical forcing, whereas A_S is maintained by energy conversion $A_Z \rightarrow A_S$. When ΔT_e or $1/K_I$ is sufficiently large, the motion becomes highly irregular and no systematic change of regimes is observed.

Discussion. The finding that quasi-stationary waves are maintained by energy conversions $A_Z \rightarrow A_S$ and $A_S \rightarrow K_S$ when n=3 with ΔT_e and $1/K_I$ moderately large implies that in Rossby regime II these quasi-stationary waves are generated mainly by baroclinic instability. However, the baroclinic waves generated are also stationary. According to Stone (1977), this is a plausible explanation of the baroclinic nature associated with quasi-stationary waves as observed by Holopainen (1970) in winter. It is noted, though, that topography is needed to trigger the generation of available potential energy A_S; if there is no topography $C(A_Z, A_S)$ is negligible.

Acknowledgements. The paper reported here is an extension from the author's Ph.D. dissertation (Yao, 1977). He is indebted to Professor Akio Arakawa of the University of Los Angeles, for his guidance through that dissertation work.

REFERENCES

Charney, J.G. and A. Eliassen, Tellus, 1, 38-54, 1949.

Derome, J., and A. Wiin-Nielson, Mon. Wea. Rev., 99, 564-576, 1971.

Holopainen, E.O., Quart. J. Roy. Meteor. Soc., 96, 626-644, 1970.

Saltzmann, B., Meteor. Monogr., 8, Amer. Meteor. Soc., 4-19, 1968.

Stone, P.H., Contribution of the U.S. Delegation to the Yalta Polymode Theoretical Institute. P. Rhines, ed., U.S. Polymode Organizing Committee, Cambridge 22 pp. (Available from the US PMOC).

Yao, M.-S., Ph.D. Thesis, University of California, Los Angeles, 161 pp.