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USE OF GLOBAL ATMOSPHERIC DATA SETS TO TEST QUASI-GEOSTROPHIC EDDY MOMENTUM FLUX CONVERGENCE

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ABSTRACT

The quasi-geostrophic relation between the fluxes of momentum, potential vorticity, and potential temperature is tested with global sets of atmospheric wind and temperature data by computing the convergence of momentum flux as a residual of the potential temperature and potential vorticity flux and comparing it to the momentum flux convergence computed directly. It is shown that in the troposphere between 18 °N and 74° N the observed momentum flux convergence differs from the quasi-geostrophic convergence by 25%-60%, with the larger errors only occurring where the convergence is small. These results indicate that momentum flux convergence obtained from quasi-geostrophic theory is adequate for qualitative studies of the general circulation, and is comparable in accuracy to values obtained in general circulation models. For simple climate models and qualitative process studies, it can thus provide a useful approach.

Introduction. The poleward eddy transport of westerly momentum, crucial to the maintenance of the zonally averaged distribution of zonal winds, is to a large extent directed against the gradient of zonally averaged angular velocity. This countergradient flux cannot be described by the "diffusion" or "mixing" hypothesis because such a formulation either yields downgradient fluxes, or, if one insists upon countergradient fluxes, negative values of the exchange coefficients. The first option does not fit the data and the second is objectionable on physical grounds.

Various parameterizations of the momentum flux have been propsed. Williams and Davies (1965) parameterized the momentum flux in terms of the mean meridional temperature gradient using an eddy viscosity as the constant of proportionality. From their numerical results they concluded that this type of relation satisfactorily represented the main functions of the large eddies. Saltzman and Vernekar (1968) suggested a more elaborate expression based on the tilting of the trough lines in barotropic flow and a balance between the rate of convergence of momentum and its removal by friction. Their formulation also gave satisfactory agreement with observation.

A different approach to parameterizing the poleward eddy transport of westerly momentum is discussed by Green (1970). The chief characteristic of his theory is that the concept of "mixing" is avoided by finding the transfer of quantities, conserved during the eddy-lifetime, directly in terms of trajectories. He shows that the transfer of entropy is related to the mean gradient and that this transfer can be formalized in terms of observable mean quantities.

An aspect of Green's parameterization that has not been tested is the quasi-geostrophic relation between the fluxes of momentum, potential vorticity and potential temperature. This relation forms the heart of Green's parameterization. It is not clear to what extent it is satisfied in the atmosphere or in a primitive equation model of the atmosphere in which the large-scale eddy motion is explicit. The numerical calculations of Simmons and Hoskins (1976), for example, do show substantial differences between the quasi-geostrophic and primitive equation momentum fluxes accompanying the most unstable baroclinic waves.

Our purpose is to test the quasi-geostrophic relation for atmospheric data by computing the convergence of momentum flux as a residual of the potential temperature and potential vorticity fluxes and comparing it to the momentum flux convergence computed directly. If the large-scale eddy fluxes are nearly quasi-geostrophic, the convergence of momentum flux, computed as a residual of the potential temperature and potential vorticity fluxes, should equal the convergence of eddy momentum flux computed directly.

Equations. From the quasi-geostrophic potential vorticity equation on a β-plane and the geostrophic approximations, one obtains an expression for the convergence of eddy momentum flux as a function of the potential vorticity and potential temperature flux:

$$\frac{\partial u'v'}{\partial y} = -v'q' + f_0^2 \frac{\partial}{\partial p} \left(\overline{\frac{v'}{\sigma} \frac{\partial \psi'}{\partial p}}\right) \tag{1}$$

where q is the quasi-geostrophic potential vorticity.

To test this relationship we must evaluate q in terms of observable quantities. We denote by Q the potential vorticity computed from the observed eastward wind speed, U, and the observed northward wind speed, V. For quasi-geostrophic flow we obtain the following expression which allows us to test the quasi-geostrophy of the momentum flux in terms of observable quantities:

$$\frac{\partial \overline{u'v'}}{\partial y} = \frac{\partial \overline{U'V'}}{\partial y} - \overline{U'\frac{\partial V'}{\partial y}} - f_o \overline{\frac{\alpha'}{\sigma} \frac{\partial V'}{\partial p}} \tag{2}$$

We will refer to the terms in these equations in their β-plane formulation, although our numerical calculations were performed in spherical coordinates.

Global data set. The basic data set, covering the period July 1976 through June 1977, consists of twice daily (0000 and 1200 GMT) analyses of the wind and temperature fields for the Northern Hemisphere, as made by the U.S. National Meteorological Center (NMC). A set of this data has been obtained for climate studies at GISS from the archive at the National Center for Atmospheric Research (NCAR). The analyses are based on Flattery's global analysis scheme with 12 h operational forecasts used as the first guess field. With the exception of the zonally averaged meridional wind (which is forced to zero), the values produced by Flattery's analysis are presumed unbiased.

The values of $\partial\overline{U'V'}/\partial y$, $\overline{U'\partial V'}/\partial y$ and $f\overline{\alpha'\partial V'}/\sigma\partial p$ were computed for a 4° x 5° latitude-longitude grid resolution for pressure levels 200, 300, 400, 550 and 700 mb, and then averaged for levels 200 to 400 mb and 400-700 mb.

Results. The main computational results will be presented in terms of seasonal averages: winter (December, January, February), spring (March, April, May), summer (June, July, August), and fall (September, October, November).

Fig. 1a shows the observed and quasi-geostrophic momentum flux convergences as a function of latitude and pressure for pressure levels A for winter. Qualitatively, the observed and quasi-geostrophic convergences are similar during all seasons for both the mid(400 mb - 700 mb) and upper (200 mb - 400 mb) troposphere. Quantitatively, the difference between the observed and quasi-geostrophic momentum flux convergence is largest at latitudes where the convergence is a maximum.

The numerical difference between the observed and quasi-geostrophic momentum flux convergence is given by the sum of the last two terms on the RHS of (2). When these terms are small, the observed momentum flux convergence is quasi-geostrophic. Each of these terms is shown as a function of latitude and pressure in Fig. 1b for winter. During all seasons, the terms are of opposite sign at most latitudes south of 60; at some latitudes between 45° and 25° the terms are both opposite in sign and equal in order of magnitude. By examining only the

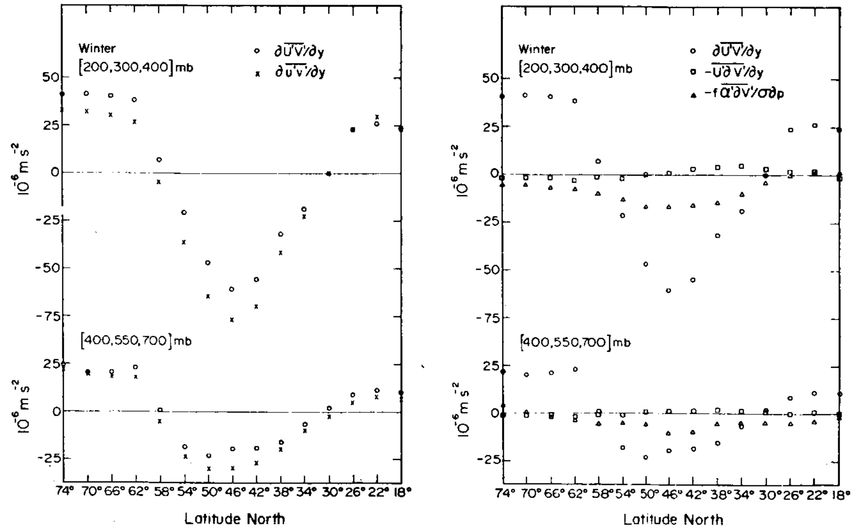

Fig. 1 (a) The winter average of the observed and quasi-geostrophic momentum flux convergence as a function of latitude for the indicated pressure levels. (b) The winter average of the terms in (2) as a function of latitude for the indicated pressure levels.

sum of these terms, we could falsely conclude that the observed fluxes are more quasi-geostrophic than they actually are. We use $|\alpha' \partial \overline{v'}/\sigma\partial p| / |\partial \overline{U'V'}/ \partial y|$ as a measure of the degree of quasi-geostrophy, and define the relative error as the ratio $|\alpha' \partial V'/\sigma\partial p|_{Max}/|\partial U'V'/\partial y|_{Max}$ and the absolute error as $|\alpha' \partial V'/\sigma\partial p|_{Max}$. The subscript "Max" refers to the maximum values of these quantities for a given tropospheric level and season.

The relative and absolute errors for pressure levels A are listed as a function of tropospheric level and season in Table 1. As can be seen, the degree of quasi-geostrophy of the observed momentum flux convergence, as measured by the relative error, ranges from 25%-41% in the upper troposphere and 26%-60% in the mid troposphere. In the upper troposphere the relative error for winter, spring, and fall is about 30% less than for summer. A comparison of mid and upper tropospheric relative errors indicates that the relative errors in the mid troposphere are 50% greater than the relative errors of the upper troposphere during all seasons except winter. However the relative errors only exceed 30% at levels or in seasons when the momentum convergence is small and relatively unimportant.

SEASON / PRESSURE		Winter		Spring		Summer		Fall		Annual	
		Abs. Error 10^{-6} m s^{-2}	Rel. Error	Abs. Error 10^{-6} m s^{-2}	Rel. Error	Abs. Error 10^{-6} m s^{-2}	Rel. Error	Abs. Error 10^{-6} m s^{-2}	Rel. Error	Abs. Error 10^{-6} m s^{-2}	Rel. Error
Pressure levels A	200,300,400	17	.29	10	.25	9	.41	11	.26	11	.29
	400,550,700	6	.26	8	.38	6	.60	7	.41	5	.29
Pressure levels B	150,200,250	11	.17	6	.16	4	.20	10	.25	7	.18
	200,250,300	14	.20	11	.24	10	.40	9	.19	10	.24
Pressure levels A with constant σ	200,300,400	17	.29	6	.15	7	.32	8	.19	7	.18
	400,550,700	8	.35	5	.24	5	.50	5	.29	4	.24

Table 1. Relative and absolute errors as a function of season for pressure levels A, B and pressure levels A with constant σ.

In the numerical calculations above, the term $\alpha' \partial V'/\sigma\partial p$, which has been a measure of quasi-geostrophy, was computed with a pressure dependent σ. In quasi-geostrophic theory, however σ is often assumed independent of pressure. Thus $\alpha' \partial V'/\sigma \partial p$ was also computed from pressure levels A data with σ artificially fixed at its mid-tropospheric value of $3.4 \times 10^{-6} m^4 s^2 kg^{-2}$. Table 1 shows that in the mid and upper troposphere the absolute error for constant σ is less than for the corresponding variable σ values in the annual mean and during all seasons except winter. The constant σ results are the same as those with variable σ in the sense that mid-tropospheric relative errors are still 50% greater than the upper-tropospheric relative errors during spring, summer, and fall.

To determine the sensitivity of the momentum flux convergence to the horizontal resolution, the NMC data were interpolated to an 8°x10° latitude-longitude grid. The results were identical to those described above and are thus not shown here.

Summary. With our measure of relative error we have shown that the observed momentum flux convergence averaged seasonally is quasi-geostrophic within 25%-40% in the upper troposphere and within 25%-60% in the mid troposphere. The observed momentum flux convergence averaged annually is quasi-geostrophic within 30% in the mid and upper troposphere. The degree of quasi-geostrophy is slightly greater for a constant static stability than for a static stability dependent on pressure.

Since the relative errors do not exceed 30% except when the absolute values are small and relatively unimportant, the momentum flux convergence computed from quasi-geostrophic theory is a satisfactory approximation for qualitative studies of the general circulation. In fact, the accuracies are as good as provided by current general circulation models. It thus can be usefully employed in simple climate models and qualitative process studies.

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