SELF-EXCITED ROTOR WHIRL DUE TO TIP-SEAL LEAKAGE FORCES

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SUMMARY

Self-excited vibrations may cause severe limitation of the performance of turbomachines. Bearing forces, elastic hysteresis and forces from the fluid flow through clearances are known as origin. Forces from the leakage flow became important with increasing performance because they increase with it. Theoretical approaches and extensive experiments were made to determine the dependence of the forces from the leakage losses and from rotating flow in the radial gaps. Therefore evaluation of the exciting forces for the investigated types of turbine stages should be possible.

Former investigations of the vibration system set the mixed damping coefficients of the bearings to zero, the calculation model was partly extremly simplified. A comparison with a symmetrical rotor without negligence of the mixed damping coefficients shows the limits of the way to do so and presents all important parameters for stability. In certain respect the stiffness and damping coefficients itself are not important, only the relation of them to each other. With a given bearing and its coefficients as a function of the so-called Sommerfeld-number the relation between running and critical speed as well as the ratio bearing/rotor stiffness are the determining parameters for stability. Uncertainties in the bearing coefficients, which may result from differences between ideal test bearing and real rotor bearing are able to change the result in stability calculation in a wide range.

INTRODUCTION

Self-excited rotor bending vibrations are of great importance for turbomachinery. While it is possible to reduce vibrations caused by the unbalance of the rotor to a sufficient degree by balancing, the self-excited rotor whirl is only to be eliminated by design corrections. In thermal turbomachinery it seems there are three origins for self-excited rotor whirl. The first kind is the internal damping or elastic hysteresis, the second arises from the lubricating oil-film in the journal bearing (oil whip) and the third is the so called clearance excitation, induced by fluid flow in radial gaps. The two first causes depend on the running speed of the rotor, the vibrations appear quite suddenly at a certain speed limit. The clearance-excitation increases with increasing power of the turbomachine, therefore the vibrations appear suddenly at a certain threshold power. The frequency of these vibrations generally corresponds to the lowest critical speed of the rotor. The following considerations ignore the rarely important influence of elastic hysteresis.

VIBRATION EXCITING FORCES AT THE ROTOR

Bearing- and clearance excitation forces are instationary external forces at the rotor. With x_B and y_B as the displacement in two rectangular coordinates the bearing force follows with

$$B = \begin{bmatrix} B_x \\ B_y \end{bmatrix} = -\begin{bmatrix} c_{xx} & c_{xy} \\ c_{yx} & c_{yy} \end{bmatrix} \begin{bmatrix} x_B \\ y_B \end{bmatrix} - \begin{bmatrix} d_{xx} & d_{xy} \\ c_{yx} & d_{yy} \end{bmatrix} \begin{bmatrix} \dot{x}_B \\ \dot{y}_B \end{bmatrix} \tag{1}$$

The spring- and damping coefficients c and d are defined as functions of the so called Sommerfeld-number. If the coupling values - that are the coefficients with mixed indices - are not zero, a displacement of the journal leads to a force rectangular to the displacement. That force is the cause for self-excited vibrations (oil whip) after reaching a certain running speed (ref. 1 and 2).

The clearance excitation forces do not act on the journal but on the surface of the rotor which is in contact with the working fluid. They result from the unsymmetrical fluid flow through the radial clearances at rotor and blading which appears according to the eccentricity between rotor and casing. Thereby cross forces as well as moments are able to act on the rotor. Some theoretical and experimental investigations however show - with respect to the actual possible accuracy of recording these effects - that it is sufficient to have only regard to the cross force as the essential factor in clearance excitation

$$Q = \begin{bmatrix} Q_x \\ Q_y \end{bmatrix} = \begin{bmatrix} 0 & -q \\ q & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \tag{2}$$

where x and y are the coordinates of rotor displacement.

Primary this effect was explained and calculated based on the leakage losses of blade tips or labyrinths. From eccentric position in the casing non uniform tangential forces result at the turbine wheel, figure 1. The resultant of these forces - the clearance excitation force Q - acts rectangular to the displacement direction. In case of a compatible circumpolar vibration of the rotor the force Q anticipates 90° to the displacement. Is the loss of energy from damping less than the work done by the exciting force Q, then amplitudes will increase. At compressors Q acts in the opposite direction and is therefore able to excite countermoving vibrations. As tangential forces and power of the turbomachine are proportional vibrations begin at a certain threshold power (ref. 3 and 4).

Tests with shrouded turbine wheels showed considerable larger cross forces than explicable from leakage losses. The cause therefor is a non uniform pressure distribution above the shroudings as a result of the unsymmetrical fluid flow through the sealing gap. Such cross forces arise at shaft glands too (ref. 5). By this means generally for the coefficient q in equation (2) must be put up

$$q = q_S + q_D \tag{3}$$

with q_S as the part from clearance losses and q_D as the part from pressure distribution. With U for the tangential force, p the pressure, φ the circumference angle and e the displacement of the shaft you get according to figure 1

$$q_S = \frac{1}{e} \int_{\varphi=0}^{2\pi} \cos\varphi \, dU; \quad q_D = \frac{1}{e} \int_{z=0}^{b} \int_{\varphi=0}^{2\pi} \sin\varphi \; p \; r \; d\varphi \, dz \tag{4}$$

where r is the radius of the shrouding, z the coordinate in axial direction and b the width of the shrouding. q_S in equation (4) can be developed furthermore by known linear statements on clearance losses. Signifies $U_{is} = \dot{m} \cdot \Delta h_{is}/u$ the isentropic tangential force of the turbine stage with $\dot{m}$ the mass flow, Δh_{is} the isentropic enthalpy difference and u the average circumferential speed you can put up

$$q_S = \frac{U_{is}}{l''} \; K_S \tag{5}$$

l'' is the length of the rotor blade and K_S a factor which takes into account all essential parameters (ref. 4). For the part q_D several numerical procedures were investigated. You can put up (ref. 6)

$$q_D = \frac{\pi}{4s} \; r \; b \; \Delta p_B \; K_D \tag{6}$$

where s is the radial clearance, Δp_B the pressure difference at the shrouding and K_D a factor which depends essential - in case of given design of the sealing - only on the kinetic energy of the flow in circumferential direction divided by Δp_B

$$C_E^* = \frac{\rho \, c_{1u}^2}{2\Delta p_B} \tag{7}$$

ρ is the density and c_{1u} the tangential component of the absolute outlet velocity of the stator blade flow. Figure 2 e.g. shows the theoretical functional relation between K_D and C_E^* for two shrouding designs.

EXPERIMENTAL INVESTIGATION OF EXCITING FORCES

To obtain data as reliable as possible, applicable for design of turbomachines and to state theoretical assumptions, numerous tests at different turbine stage types (impulse and reaction type) were untertaken (ref. 7). Figure 3 shows three investigated stage types. It was possible to measure the cross force Q directly dependent from the adjustable eccentricity, partly the pressure distribution above the shrouding too. Such "nearly-static" force measurement was sometimes completed by real vibration tests. Measurements at shroudless

blading confirm, that the exciting force is calculable using the part q_S. On the other hand tests with shroudings have shown the important influence of q_D for that stage types. Because pressure distribution measurements were not possible for all test series, the results were pointed out according to equation (5) as dimensionless "clearance-excitation-coefficient"

$$K_2 = \frac{Q/e}{U_{is}/1''} = \frac{q \cdot 1''}{U_{is}} \tag{8}$$

K_2 of course combines the effects from leakage losses and pressure distribution. Figure 4 shows results of a 1-stage and 3-stage test rotor with 50 % reaction-blading and a stepped 3-chamber labyrinth at stator and rotor shroudings. K_2 is plotted over the load factor $\psi = 2\Delta h_{is}/u^2$. In figure 5 the results of a chamber-stage with low reaction is plotted. In both cases the part from leakage losses (corresponding K_S) is drawn in for comparison.

The part of pressure distribution at the exciting force is particularly high for impulse- and low-reaction-stages because of the heavy swirl-flow in the gap between stator and rotor wheel. That shows figure 5 and is possible to deduce from figure 2. For that reason further tests were made measuring the pressure distribution too (ref. 8). The large influence of swirl flow suggests to reduce the exciting force by disturbing the circumferential component of flow. Therefore three labyrinth designs were investigated according to figure 6. For disturbing the swirl-flow axial sheet strips were mounted in chamber one (design B) and chamber one to three (design C). In figure 7 the testing results are plotted. As shown it was possible to reduce the exciting force essentially by help of swirl-disturbing-sheets respectively to eliminate it. Anyway it is important to choose an optimal layout; obviously design B leads to considerable exciting forces with changed sign which may excite countermoving vibrations. For the rest at this stage the part of pressure distribution is exceeding; the part from leakage losses does not correspond with theory or is vectorial compensated - which is more probable - by the resultant force from pressure distributions, in case B even over-compensated.

THE STRUCTURE OF THE VIBRATION SYSTEM

For investigation of rotor stability in the first instance a symmetrical laval-rotor with journal bearings may be suitable because of the large quantity of parameters. Thereby the following questions are to take into consideration:

1. Simplification limits of the rotor model.

2. Influences coming from the bearing properties, especially from data faults or manufacturing tolerances.

Figure 8 presents the vibration system with mass m, rotor-stiffness c_R; both bearings are equal. With equations (1) and (2) as well as the mass forces it is easy to put up the differential equation system for translational motion of the rotor-disk derived from the balance of the forces (ref. 8).

$$a_0\dddot{y} + a_1\ddot{x} + a_2\ddot{x} + a_3\ddot{y} + a_4\dot{x} + a_5\dot{y} + a_6x + a_7y = 0$$
$$b_0\ddot{x} + b_1\ddot{y} + b_2\ddot{y} + b_3\ddot{x} + b_4\dot{y} + b_5\dot{x} + b_6y + b_7x = 0 \qquad (9)$$

The coefficients a and b are combinations of mass, exciting parameter as well as spring- and damping coefficients of the system. The usual starting solution leads to a characteristic equation of 6^{th} degree for the complex natural frequencies which are imaginary at the stability threshold. By iterative solution the maximum of excitation is computed. It at least depends on the relation of stiffness rotor/bearing, to the type of bearing, the Sommerfeld-number and the relation between running speed and ciritical speed of the rigid supported rotor (tuning of system).

A more simplified model (ref. 9) ignores the coupling coefficients of bearing damping. The components of bearing force coming from the coupling coefficients of stiffness are transposed to the centre of mass, the force from damping too. Stiffness of bearings and rotor shaft are considered as spring series. In that way we get the differential equations

$$m\ddot{x} + d_x\dot{x} + c_xx = -(\bar{c}_{xy} + q)\,y$$
$$m\ddot{y} + d_y\dot{y} + c_yy = -(\bar{c}_{yx} - q)\,x \qquad (10)$$

The spring coefficients c respectively $\bar{c}$ and damping coefficients d represent the effect of the parameters of rotor and bearings together. Coupling between x- and y-axis is pointed out by the coupling coefficients of bearings and clearance exciting force alone. For the stability threshold there is a closed solution. Also here the result only depends on relations between the parameters of the system. For sufficient stability it seems to be important - besides the damping - to choose the anisotropy of springs as great as possible.

The more simplified system (10) compared with the exact system (9) shows the usability of the simplified system for qualitative conclusions; on the other hand - dependent on the given conditions - for quantitative investigations large differences may occur. Obvious the simplified calculation is only exact for an inelastic rotor. For real bearings the use of the solution according equation (9) is advisable because of the sensitivity of bearing coefficients as a function of bearing geometry and manufacturing tolerance. Thereby all parameters can change their value the same time, and depending on bearing type and rotor stiffness this might result in very large changes of stability threshold. Unfortunately these changes are small for the not so interesting (because more unstable) cylindrical journal bearing while they are larger for other types of bearings. A clear prediction of the sign of the change is not possible.

Exact calculation from equation (9) using coupling coefficients of bearing damping too gave following results: A larger stiffness ratio rotor/bearing leads to increase or decrease of stability threshold, increasing bearing load acts in the same unpredictable way. With increasing of the ratio running speed/critical speed you will nearly always get a deterioration of the stability threshold. The omission of the coupling coefficients of damping influences the results in different amount and direction, therefore an omission of that bearing

influence is not recommendable; the fault can reach in same case a very large amount, see figure 9, wherein the ordinate S means the ratio of clearance excitation/rotor stiffness: $S = q/c_R$.

The results relating to the influence of Sommerfeld-number and stiffness ratio show that increasing bearing load can improve stability but it must not. In order to answer the question about the reliability of the calculation results dependent on the bearing data systematically only one spring- or damping coefficient was changed by 10 % and the relative change of result was computed. Depending on the ratio of running speed/critical speed and ratio of stiffness for elliptical and three-wedge bearings we got differences up to 300 % to 400 % in stability threshold. These results can be amplified or moderated by taking into account all data faults, see table 1. A good prediction depends essentially on exact geometric bearing conditions - as used at the test bearings - that means exact manufacturing and mounting; on the other hand it is uncertain wether the exactness of bearing data is sufficient. Based on the knowledge of today the calculation looks pretty uncertain. But it is possible to use the tendencies of the results as criteria for design changes at the bearings, as was already stated by experience.

CONCLUSIONS

Clearance excitation forces originate from non uniform leakage losses and from non uniform pressure distribution above shroudings or shaft glands as function of the eccentricity of the rotor. Theroretical investigation of these effects was assisted by numerous experiments, so the clearance excitation coefficients should be reliable for the tested stage types. Swirl preventing sheets at shrouded bladings are able to reduce or eliminate the excitation forces.

Considering the structure of the vibration system the great influence of journal bearings on the stability of the rotor is shown. For stability especially the anisotropy of stiffness is essential. The calculation model should be extended for multiple cased turbomachinery respectively shafts with more than two bearings, methods therefor are known. The strong dependence of the stability on bearing parameters leads to the idea to try to obtain the anisotropy not by help of the bearing geometry but by suitable design of the bearing pedestal.

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Table 1. CHANGE OF MAX. SYSTEM EXCITATION (ΔS) IN PERCENT BY CHANGING ONE COEFFICIENT 10 %

width/diameter W/D = 0.8
ratio running/critical speed ω_B/ω_k

		ELLIPTICAL BEARING				THREE WEDGE BEARING			
$c_R/2c_{yy}$		o.2		o.6		o.2		o.6	
Δc_{ik}, Δd_{ik}	ω_B/ω_k	+1o%	-1o%	+1o%	-1o%	+1o%	-1o%	+1o%	-1o%
c_{xx}	o.8	-4	5	-3	3	-5	6	-5	6
	1.6	-11	11	-14	13	-37	54	-61	315
c_{xy}	o.8	3	-3	-27	28	-2	2	-5	5
	1.6	-74	187	-8o	413	-67	63	-84	455
c_{yx}	o.8	-5	4	-2o	22	-3	3	-6	6
	1.6	-69	168	-84	384	-68	69	-81	459
c_{yy}	o.8	4	-6	2o	-27	-6	7	7	-8
	1.6	173	-99	6o4	-93	57	-41	6o7	-87
d_{xx}	o.8	-7	8	34	-37	4	-5	8	-9
	1.6	145	-72	313	-78	62	-67	375	-7o
d_{xy}	o.8	-3	2	-9	8	1	-1	-1	1
	1.6	11	-11	-2o	16	1	-1	62	-62
d_{yx}	o.8	2	-2	-6	5	2	-2	-1	1
	1.6	-2o	16	-23	21	2	-2	-1o	413
d_{yy}	o.8	5	-6	16	-14	7	-7	1o	-11
	1.6	23	-16	-51	123	69	-65	334	-49

FOR EXPLANATION OF THE CLEARANCE EXCITATION

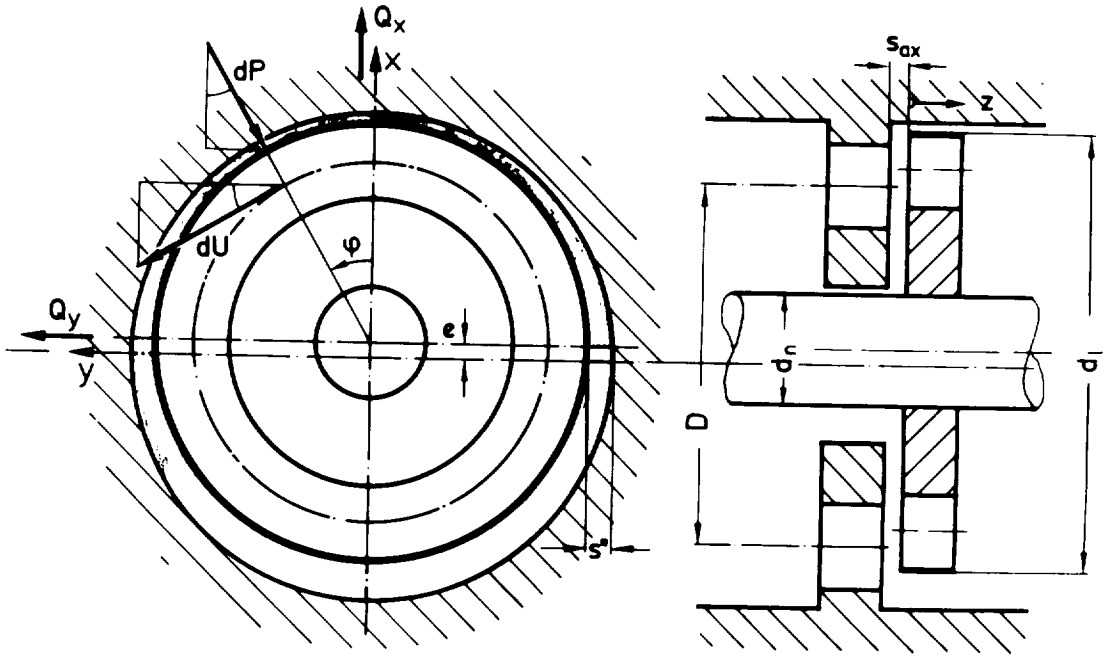


Figure 1.

CALCULATED COEFFICIENT K_D OF CLEARANCE EXCITATION FORCE RESULTING FROM PRESSURE DISTRIBUTION

a flat shrouding; b stepped shrouding with 3 sealing tips

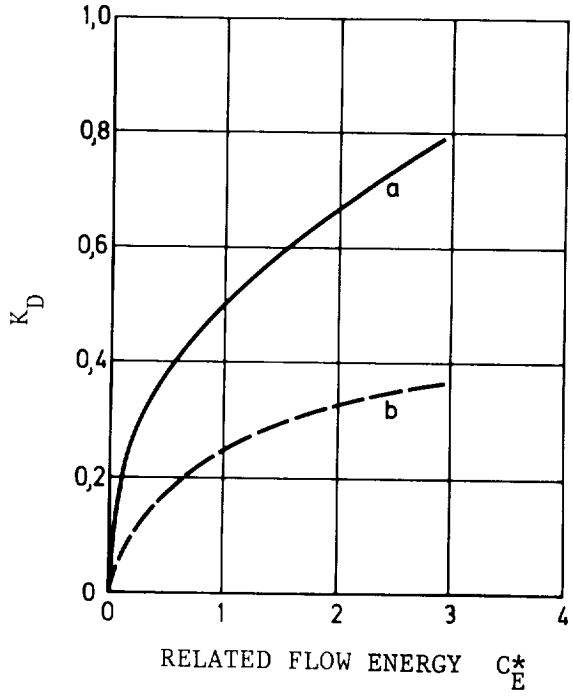


Figure 2.

INVESTIGATED STAGE TYPES

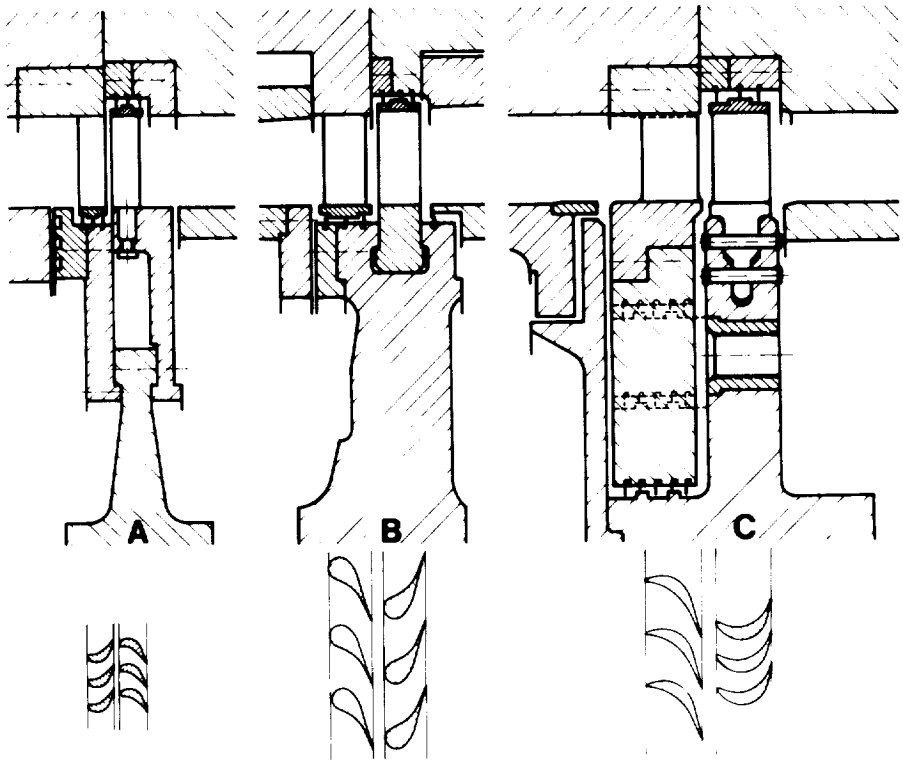


Figure 3.

CLEARANCE EXCITATION COEFFICIENT FOR STAGE B (50 % REACTION)

axial clearance s_{ax} = 3.3 mm

○, △ stage displacement
□ rotor displacement
× stator displacement

◇ ● ⧖ } calculated from leakage losses

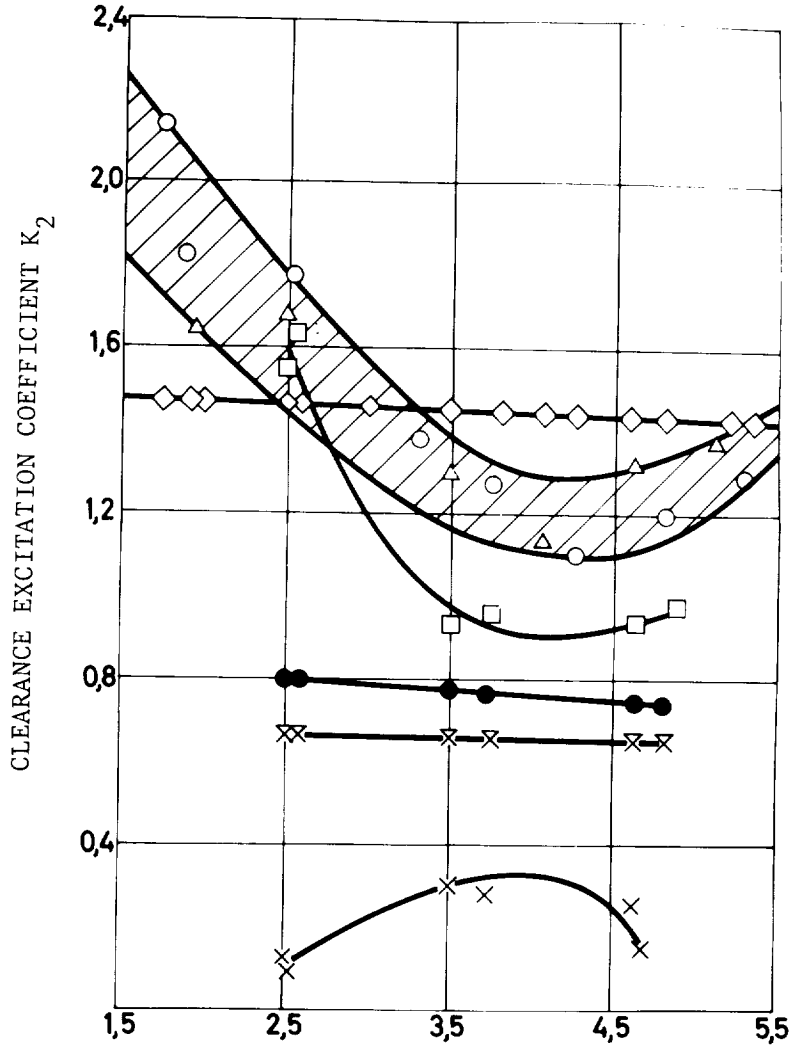


Figure 4.

CLEARANCE EXCITATION COEFFICIENT FOR STAGE C (LOW REACTION TYPE)

axial clearance

△ 2.2 mm
□ 2.5 mm
× 3.9 mm
◇ 4.2 mm

○ calculated from leakage losses

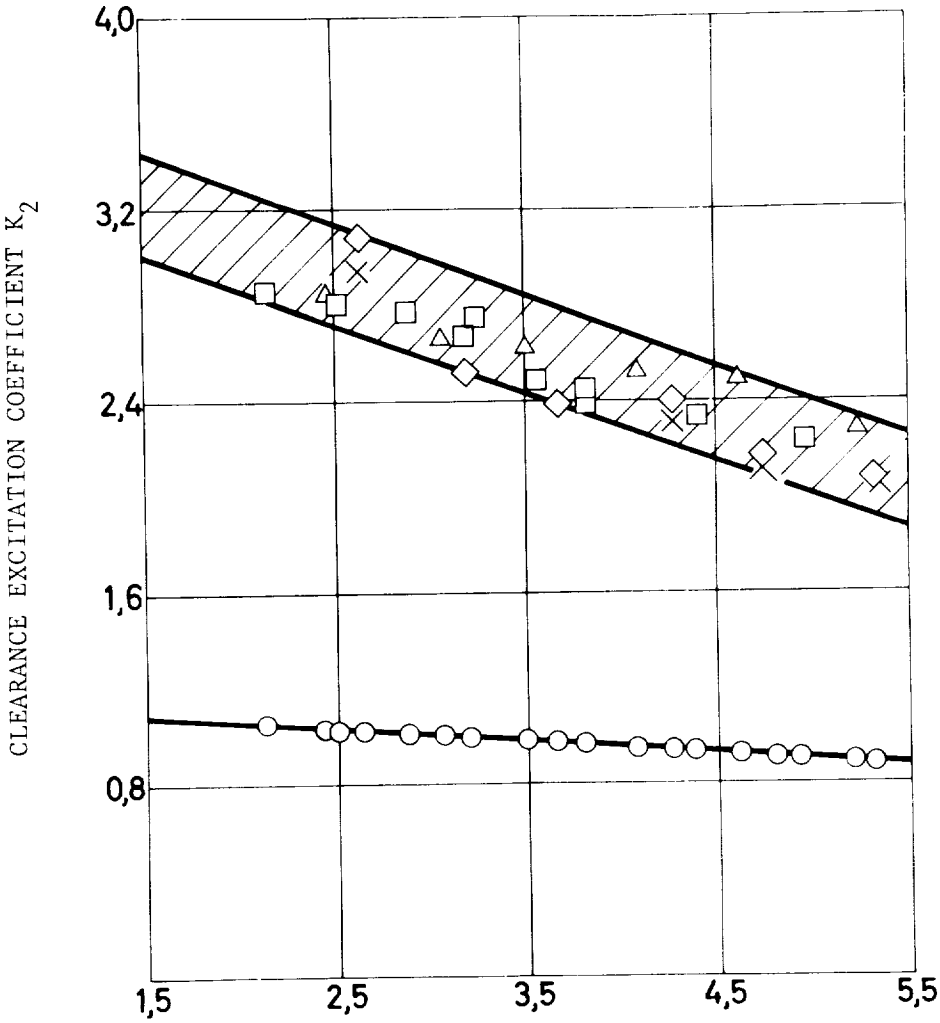


Figure 5.

INVESTIGATED LABYRINTH DESIGNS

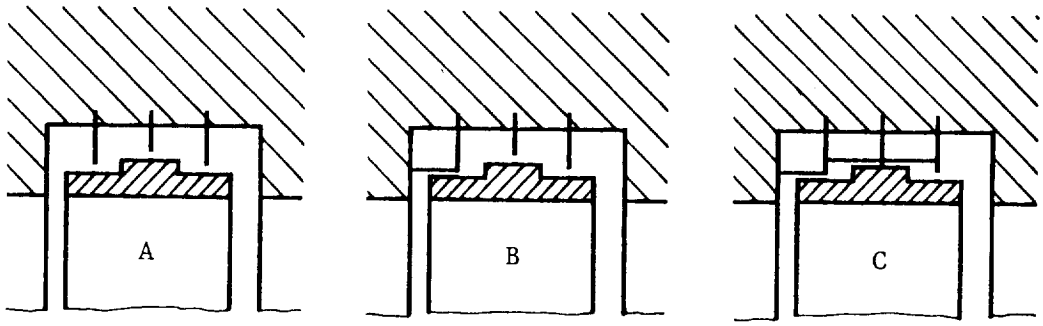


Figure 6.

TESTING RESULTS FOR 3 LABYRINTH DESIGNS (A,B,C)

K_2 clearance excitation coefficient
Ψ load factor
C_E^* related flow energie

□ ——— from force measuring
Δ —·— integration of pressure distribution
— — — calculation from leakage losses

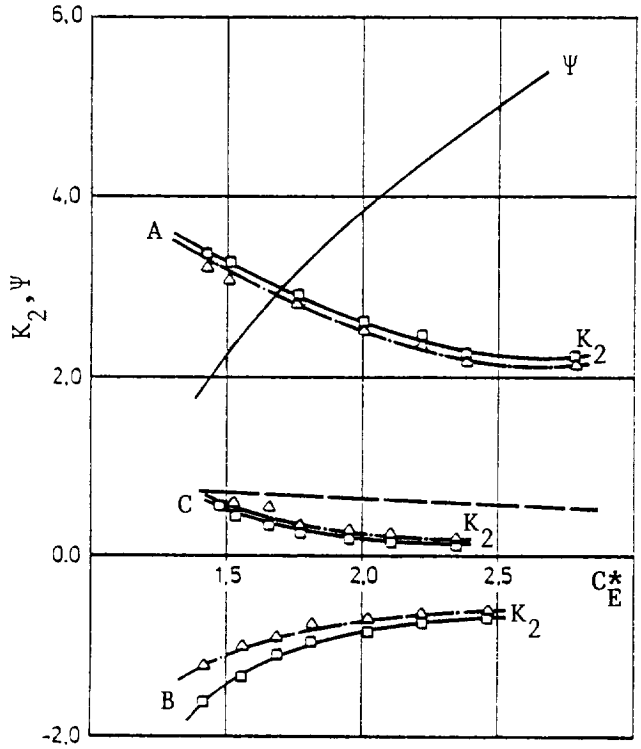


Figure 7.

SIMPLIFIED ROTOR MODEL
(coupling springs and dampers not drawn)

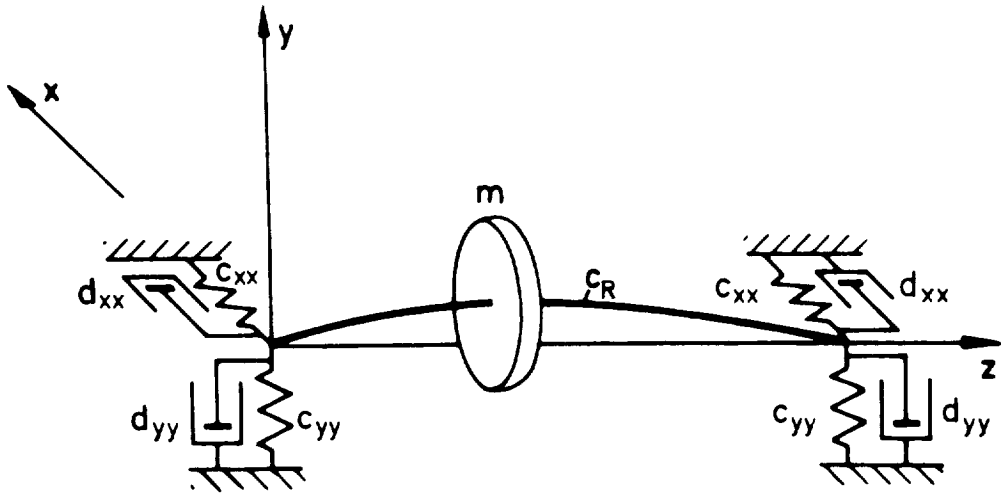
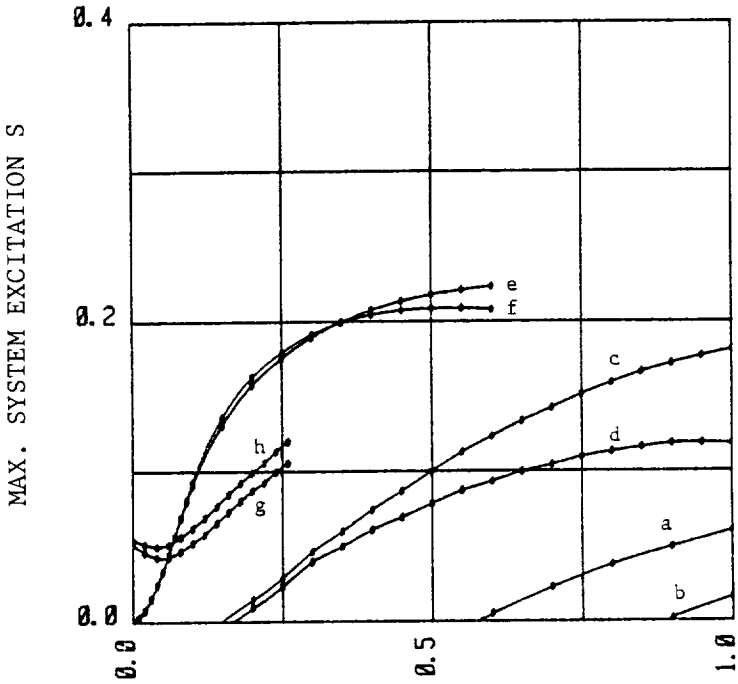


Figure 8.

MAX. SYSTEM EXCITATION FOR 4 BEARING TYPES



SOMMERFELD-NUMBER So

				$d_{xy}=d_{yx}=0$	$d_{xy}\neq d_{yx}\neq 0$
width/diameter	W/D	= 0.8	cylindrical journal bearing	a	b
stiffness ratio	$c_R/2c_{yy}$	= 0.6	elliptical bearing	c	d
running/ critical speed	ω_B/ω_k	= 1.6	three-wedge bearing	e	f
			tilting pad bearing	g	h

Figure 9.