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GENERAL THEORY OF THE DOUBLE FED SYNCHRONOUS

MACHINE

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16. Abstract

Motor and generator operation of a double-fed synchronous machine are studied and physically and mathematically treated. Experiments with different connections, voltages etc. are carried out. It is concluded that a certain degree of asymmetry is necessary for the best utilization of the machine.

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CONTENTS

GENERAL THEORY OF THE DOUBLE-FED SYNCHRONOUS MACHINE

Mohammed G. El-Magrabi

I. INTRODUCTION

The double-fed synchronous machine has some special characteris- /5*
tics and poses an interesting problem from the theoretical as well
the experimental point of view. The term double-fed synchronous ma-
chine (d.s.m.) stands for a three-phase machine, the rotor and stator
of which are fed with currents of the same frequency. The function-
ing of the machine may simply be conceived by the following qualita-
tive considerations. Any chosen type of electrical machines func-
tions by the existence of two circulation waves (m.m.k.) which are
relatively static and constantly remain in step. This condition is
necessary if an energy exchange should take place between the elec-
tric channels and the mechanical channel, otherwise, the mean torque
and as such the transferred power is nil. In machines without direct
current generation, the energy can - according to the respective con-
ditions - be exchanged between one and the other two of the three
channels, i.e., the mechanical and the two electrical channels which
produce the two circulation waves (flux). In the case of the d.s.m.,
rotor as well as stator carry three phase currents of the same fre-
quency. The effect of the stator currents on air-slit flow is iden-
tical to that of a circulation which is roughly of sinus shape with
constant amplitude, and which rotates with a constant velocity

$n = \frac{60 \cdot f}{p}$ r/min (f= frequency, p= amount of polar pairs) in one direction
which depends on the phase sequence of the currents. In a similar
manner a circulation is produced by the rotor currents. It rotates
with the same velocity in relation to the rotor body. The phase se-
quence of stator and rotor currents is chosen in such a way that the
rotor flux is opposite in relation to the stator body. If the rotor
is turned with a velocity of 2n r/min. in the direction of the stator
flux, both flux waves will relatively be static and keep each other
in step. In this case the energy flow between the mechanical and
both electrical channels is possible. Once both flux waves are in
step, the machine will - under stable conditions - rotate with a
fixed velocity of 2n quite independent of the conditions which
are imposed on the mechanical system. These conditions will deter- /6
mine magnitude and direction of power transfer between the machine
and the electrical system.

The general characteristics of the machine show that it forms an
intermediary stage between an induction machine and the normal syn-
chronous machine. Fundamentally, it is similar to the induction ma-

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*Numbers in the margin indicate pagination in the foreign text.

1

chines in the fact that it consists of a three-phase stator and a three-phase rotor. Likewise, the main field of the machine is produced by alternating currents (power-producing currents), and no external direct current source is necessary to produce and maintain the field. In its behavior the machine rather resembles a normal synchronous machine, since it runs with a fixed velocity which is in a fixed relationship to the frequency. Thus, a tendency to oscillate exists and synchronization with a net is necessary.

Literature so far has called the machine a double-fed induction machine. Actually, no energy exchange by induction processes takes place. Strictly speaking, the term thus is incorrect. This is why we use the term double-fed synchronous machine.

A detail of the machine in which it differs from all types of electric machines is the fact that electric power is supplied with one frequency and changed into mechanic power - in the motor case - within the rotor as well as the stator, in the generator case it is produced within both with the same frequency. From this fact a certain saving of material becomes obvious.

The basic idea of the d.s.m. stems from the year 1899. At that time Prof. Kloss mentioned the idea in a German patent[1]). Independently, a short note was published at the same time by H. Grob, which deals with the functioning and characteristics of the d.s.m., but neither an experimental nor an analytical work was mentioned. The next publication was by E. Ziehl (1905) who undertook a theoretical and experimental study of the machine. He also showed for the first time the possibility of generator operation of the machine. The first detailed analysis of the machine was published by J. Tscherdanzev in 1926. The parameter he used was the phase angle between terminal voltage and induced voltage (which in the case of the ordinary synchronous machine is known as output angle), and as a result got the equations of the machine as function of this parameter. He treats only the very simple case of a symmetrical machine.

In the studies of Dr. E. Messing on the common double-fed induction machine, he treats the problem of the symmetrical double-fed machine by using the angle between the two flux waves in space as parameter.

In 1935, Prof. Kloss together with Dr. H.Steudel, edited an article which explained the action of the machine as well as a method to remove its tendency to oscillate whenever operating on heavy current.

All research so far mentioned was limited to the studies of parallel and series connection of the "symmetric machine", i.e., with identical number of turns and identical constants of rotor and stator. This is hypothetical, since this cannot be reached by the best construction of the machine; it can never occur in practice even if planned. The only two analyses of assymetric machines were done by

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1) See bibliography at the end.

T. Herschdorfer (1932) for parallel connection and by Brailowsky (1944) for series connection. Both used the rather traditional parameter, i.e., the space angle between stator and rotor flux. Calculating the machine as to its results takes a large amount of effort. Additionally, the solution of the parallel connection (which is the more difficult case) given by Mr. Herschdorfer was incomplete, since certain approximations were made to calculate the circle diagram and pull-out torque.

In the present research we studied the very general case of double-fed synchronous operation, stator and rotor of the machine having asymetric constants and dissimilar number of turns and fed by voltages of different magnitudes and phases. Both, motor and generator operation are studied. The main equations are developed first and the generation of the common field is studied. After that, combined time and space vector diagrams are set up which enable a clear physical view of the behavior of the machine in various situations. The main equations are solved purely analytically, afterwards equivalent circuits of the machine are given and impedance and current diagrams are constructed. As special cases of the general theory solutions are given which were developed by various authors for the special cases of parallel and series connections. Based on the obtained analytical results, the characteristics of the common d.f.s. operation are explained. It is further explained how they are influenced by asymmetry of the constants and voltages, and improved by a suitable amount of voltage asymmetry. Finally, the theoretical results are confirmed by experiments on various degrees of assymmetry.

2. GENERAL ANALYSIS OF THE MACHINE

(2.1.) Development of Main Equations

Introduction: in every closed circuit of the machine the following law is true at any time:

$$-u = Ri + \frac{d}{dt}(Li + \sum_n M_n i_n) \qquad (1)$$

where:

u = momentary value of terminal voltage, Volt.
i = momentary value of current in the circuit, Ampere.
R = circuit resistance, Ohm.
L = self inductance circuit, Henry.
$M_1, M_2, \ldots M_n$ = counter inductance of the circuit and other circuits, which are magnetically coupled with the circuit considered, Henry.
$i_1, i_2, \ldots i_n$ = currents flowing in the other magnetically coupled circuits.

3

There are six electric circuits in the machine, three in the rotor and three in the stator. The six circuits are coupled magnetically and each one of them has resistance and self inductance. The counter inductance between each phase of the stator and each phase of the rotor varies according to the relative position of both phases. It reaches a maximum when the magnetic axes of both phases coincide, whereas it equals zero when both axes have a shift of $\frac{\pi}{2}$.

Self inductance of a phase of the stator or the rotor as well as the counter inductance of two phases in the stator or rotor are not actually absolute constants, but change due to the change of iron permeability and the fluctuation of reluctance of the magnetic circuit which is caused by stator and rotor slots.

In the following calculations the self inductances as well as the counter inductances between rotor or stator phases are considered to be constants. We also presume that the counter inductance coefficients between each stator and each rotor phase change in the ratio of the cosine of the angle between both respective axes. The iron losses are for the present neglected.

Development of the equations: Fig. 1 represents the general case in which six voltage u_{Su}, u_{Sv}, u_{Sw}, u_{ru}, u_{rv} and u_{rw} are applied to the six phases of the machine. Fig. 2 represents the relative positions of the magnetic axes of the six phases at a chosen point of time $\underline{t}$.
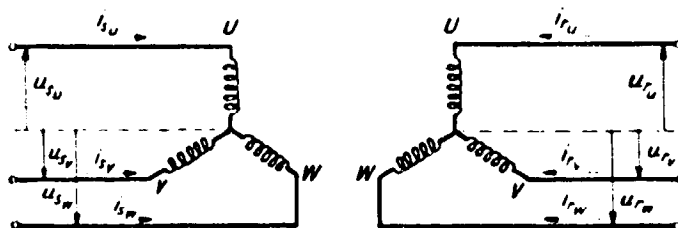
Fig. 1

Using Fig. 2, the application of equation (1) to the six phases results in the following differential equations:

$$-u_{s_u} = i_{s_u} R_s + \frac{d}{dt}\left[i_{s_u} L_{ss} + (i_{s_v}+i_{s_w}) M_{ss} + i_{r_u} M_{rs}\cos\delta \right.$$
$$\left. + i_{r_v} M_{rs}\cos\left(\delta+\frac{2\pi}{3}\right) + i_{r_w} M_{rs}\cos\left(\delta+\frac{4\pi}{3}\right)\right]$$

$$-u_{s_v} = i_{s_v} R_s + \frac{d}{dt}\left[i_{s_v} L_{ss} + (i_{s_u}+i_{s_w}) M_{ss} + i_{r_v} M_{rs}\cos\delta \right.$$
$$\left. + i_{r_w} M_{rs}\cos\left(\delta+\frac{2\pi}{3}\right) + i_{r_u} M_{rs}\cos\left(\delta+\frac{4\pi}{3}\right)\right] \qquad (2)$$

$$-u_{s_w} = i_{s_w} R_s + \frac{d}{dt}\left[i_{s_w} L_{ss} + (i_{s_u}+i_{s_v}) M_{ss} + i_{r_w} M_{rs}\cos\delta \right.$$
$$\left. + i_{r_u} M_{rs}\cos\left(\delta+\frac{2\pi}{3}\right) + i_{r_v} M_{rs}\cos\left(\delta+\frac{4\pi}{3}\right)\right]$$

4

$$-u_{r_u} = i_{r_u} R_r + \frac{d}{dt}\left[i_{r_u} L_{rr} + (i_{r_e} + i_{r_w}) M_{rr} + i_{s_u} M_{sr} \cos\delta \right.$$
$$\left. + i_{s_w} M_{sr} \cos\left(\delta + \frac{2\pi}{3}\right) + i_{s_v} M_{sr} \cos\left(\delta + \frac{4\pi}{3}\right) \right]$$

$$-u_{r_e} = i_{r_e} R_r + \frac{d}{dt}\left[i_{r_e} L_{rr} + (i_{r_u} + i_{r_w}) M_{rr} + i_{s_v} M_{sr} \cos\delta \right.$$
$$\left. + i_{s_u} M_{sr} \cos\left(\delta + \frac{2\pi}{3}\right) + i_{s_w} M_{sr} \cos\left(\delta + \frac{4\pi}{3}\right) \right]$$

$$-u_{r_w} = i_{r_w} R_r + \frac{d}{dt}\left[i_{r_w} L_{rr} + (i_{r_u} + i_{r_e}) M_{rr} + i_{s_w} M_{sr} \cos\delta \right.$$
$$\left. + i_{s_v} M_{sr} \cos\left(\delta + \frac{2\pi}{3}\right) + i_{s_u} M_{sr} \cos\left(\delta + \frac{4\pi}{3}\right) \right]$$

(3)

where:

R_s, R_r = resistance of a rotor or stator phase, Ohm.

L_{ss}, L_{rr} = self inductance of a stator or rotor phase, Henry.

M_{ss}, M_{rr} = counter inductance between two phases of stator or rotor, Henry.

M_{sr}, M_{rs} = maximum of counter inductance between a stator phase and a rotor phase, Henry.
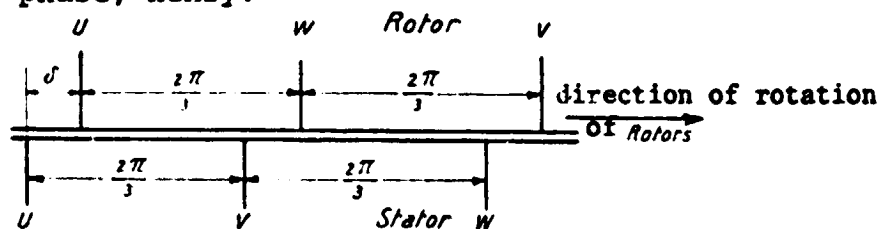
Fig. 2: representation of the axes of the six machine phases (phase sequence: U - V - W for stator and rotor.

The six simultaneous differential equations (2) and (3) refer to the momentary values of voltages, currents and velocity, thus representing the machine in all the various stages.

It is clear that the angle δ has the value

$$\delta = \int \omega_r \, dt \qquad (4)$$

where:

ω_r = electrical angle velocity of the rotation at time t.

For normal operation in a stationary stage as d.s.m. all voltages and currents should be sinusoidal and of identical frequency. They can then be expressed as follow:

5

$$u_{s_u} = U_s \sin(\omega t + \alpha_s) \qquad\qquad i_{s_u} = J_s \sin(\omega t + \lambda_s)$$

$$u_{s_r} = U_s \sin\left(\omega t + \alpha_s - \frac{2\pi}{3}\right) \qquad i_{s_r} = J_s \sin\left(\omega t + \lambda_s - \frac{2\pi}{3}\right)$$

$$u_{s_w} = U_s \sin\left(\omega t + \alpha_s - \frac{4\pi}{3}\right) \qquad i_{s_w} = J_s \sin\left(\omega t + \lambda_s - \frac{4\pi}{3}\right)$$

$$u_{r_u} = U_r \sin(\omega t + \alpha_r) \qquad\qquad i_{r_u} = J_r \sin(\omega t + \lambda_r)$$

$$u_{r_v} = U_r \sin\left(\omega t + \alpha_r - \frac{2\pi}{3}\right) \qquad i_{r_v} = J_r \sin\left(\omega t + \lambda_r - \frac{2\pi}{3}\right)$$

$$u_{r_w} = U_r \sin\left(\omega t + \alpha_r - \frac{4\pi}{3}\right) \qquad i_{r_w} = J_r \sin\left(\omega t + \lambda_r - \frac{4\pi}{3}\right)$$

(5)

Under these conditions the rotor would run with constant velocity 2ω and equation (4) can be expressed as follows:

$$\delta = \int 2\omega \, dt = 2\omega t + \delta_0 \tag{6}$$

where: $\delta_0 = (\delta)_{t=0}$. (an arbitrary constant)

Substituting relations (5) and (6) in the differential equations (2) and (3) and developing the trigonometric functions, we get the following equations for the stationary stage:

$$-u_{s_u} = -U_s \sin(\omega t + \alpha_s)$$

$$= [R_s + j\omega(L_{ss} - M_{ss})] J_s \sin(\omega t + \lambda_s) + j\omega J_r M_{rs}\left[\cos(2\omega t + \delta_0)\sin(\omega t + \lambda_r)\right.$$

$$+ \cos\left(2\omega t + \delta_0 + \frac{2\pi}{3}\right)\sin\left(\omega t + \lambda_r - \frac{4\pi}{3}\right)$$

$$\left. + \cos\left(2\omega t + \delta_0 + \frac{4\pi}{3}\right)\sin\left(\omega t + \lambda_r - \frac{2\pi}{3}\right)\right]$$

$$= [R_s + j\omega(L_{ss} - M_{ss})] J_s \sin(\omega t + \lambda_s) + j\omega J_r \frac{M_{rs}}{2}\left[\sin(3\omega t + \delta_0 + \lambda_r)\right.$$

$$+ \sin(-\omega t - \delta_0 + \lambda_r) + \sin\left(3\omega t + \delta_0 + \lambda_r - \frac{2\pi}{3}\right) + \sin(-\omega t - \delta_0 + \lambda_r)$$

$$\left. + \sin\left(3\omega t + \delta_0 + \lambda_r - \frac{4\pi}{3}\right) + \sin(-\omega t - \delta_0 + \lambda_r)\right]$$

$$= [R_s + j\omega(L_{ss} - M_{ss})] J_s \sin(\omega t + \lambda_s) + j\omega J_r \frac{3 M_{rs}}{2} \sin(-\omega t - \delta_0 + \lambda_r)$$

It can be concluded:

$$-U_s \sin(\omega t + \alpha_s) = [R_s + j\omega L_{s_d}] J_s \sin(\omega t + \lambda_s) \quad - j\omega J_r M_{\Delta} \sin(\omega t + \delta_0 - \lambda_r)$$

6

likewise:

$$-U_s\sin\left(\omega t+\alpha_s-\frac{2\pi}{3}\right) = [R_s+j\omega L_{ss\Delta}]J_s\sin\left(\omega t+\lambda_s-\frac{2\pi}{3}\right)\quad j\omega J_r M_\Delta\sin\left(\omega t+\delta_0-\lambda_r-\frac{2\pi}{3}\right)$$

$$-U_s\sin\left(\omega t+\alpha_s-\frac{4\pi}{3}\right) = [R_s+j\omega L_{ss\Delta}]J_s\sin\left(\omega t+\lambda_s-\frac{4\pi}{3}\right)-j\omega J_r M_\Delta\sin\left(\omega t+\delta_0-\lambda_r-\frac{4\pi}{3}\right)$$ (7)

$$-U_r\sin(\omega t+\alpha_r) = [R_r+j\omega L_{rr\Delta}]J_r\sin(\omega t+\lambda_r) -j\omega J_s M_\Delta\sin(\omega t+\delta_0-\lambda_s)$$

$$-U_r\sin\left(\omega t+\alpha_r-\frac{2\pi}{3}\right) = [R_r+j\omega L_{rr\Delta}]J_r\sin\left(\omega t+\lambda_r-\frac{2\pi}{3}\right)-j\omega J_s M_\Delta\sin\left(\omega t+\delta_0-\lambda_s-\frac{2\pi}{3}\right)$$

$$-U_r\sin\left(\omega t+\alpha_r-\frac{4\pi}{3}\right) = [R_r+j\omega L_{rr\Delta}]J_r\sin\left(\omega t+\lambda_r-\frac{4\pi}{3}\right)-j\omega J_s M_\Delta\sin\left(\omega t+\delta_0-\lambda_s-\frac{4\pi}{3}\right)$$ (8)

where:

$L_{ss\Delta}$ (L_{ss}, M_{ss}) = cyclic inductance of a stator phase
$L_{rr\Delta}$ (L_{rr}, M_{rr}) = cyclic inductance of a rotor phase
M_Δ $\frac{3}{2}M_{sr}$ $\frac{3}{2}M_{rs}$ = mutual cyclic inductance (between stator and rotor)

Taking any two equations of (7) and any two equations of (3) into consideration, these four equations can be solved for the four un- known J_s, J_r, λ_s and λ_r . Since all components in these equations change with the same frequency, it is of advantage to first express them exponentially. We use the following expressions:

$$\mathfrak{U}_s = U_s e^{j\alpha_s} \qquad \mathfrak{J}_s = J_s e^{j\lambda_s}$$
$$\mathfrak{U}_r = U_r e^{j\alpha_r} \qquad \mathfrak{J}_r = J_r e^{j\lambda_r}$$ (9)

Equations (7) and (8) thus are changed into the following:

Stator:

$$-\mathfrak{U}_s e^{j\omega t} = (R_s+j\omega L_{ss\Delta})\mathfrak{J}_s e^{j\omega t} - j\omega M_\Delta \mathfrak{J}_r e^{j\omega t}e^{j(\delta_0-2\lambda_r)}$$

$$-\mathfrak{U}_s e^{j\omega t}e^{-j\frac{2\pi}{3}} = (R_s+j\omega L_{ss\Delta})\mathfrak{J}_s e^{j\omega t}e^{-j\frac{2\pi}{3}} - j\omega M_\Delta \mathfrak{J}_r e^{j\omega t}e^{-j\frac{2\pi}{3}}e^{j(\delta_0-2\lambda_r)}$$ (10)

$$-\mathfrak{U}_s e^{j\omega t}e^{-j\frac{4\pi}{3}} = (R_s+j\omega L_{ss\Delta})\mathfrak{J}_s e^{j\omega t}e^{-j\frac{4\pi}{3}} - j\omega M_\Delta \mathfrak{J}_r e^{j\omega t}e^{-j\frac{4\pi}{3}}e^{j(\delta_0-2\lambda_r)}$$

Rotor:

$$-\mathfrak{U}_r e^{j\omega t} = (R_r+j\omega L_{rr\Delta})\mathfrak{J}_r e^{j\omega t} - j\omega M_\Delta \mathfrak{J}_s e^{j\omega t}e^{j(\delta_0-2\lambda_s)}$$

$$-\mathfrak{U}_r e^{j\omega t}e^{-j\frac{2\pi}{3}} = (R_r+j\omega L_{rr\Delta})\mathfrak{J}_r e^{j\omega t}e^{-j\frac{2\pi}{3}} - j\omega M_\Delta \mathfrak{J}_s e^{j\omega t}e^{-j\frac{2\pi}{3}}e^{j(\delta_0-2\lambda_s)}$$ (11)

$$-\mathfrak{U}_r e^{j\omega t}e^{-j\frac{4\pi}{3}} = (R_r+j\omega L_{rr\Delta})\mathfrak{J}_r e^{j\omega t}e^{-j\frac{4\pi}{3}} - j\omega M_\Delta \mathfrak{J}_s e^{j\omega t}e^{-j\frac{4\pi}{3}}e^{j(\delta_0-2\lambda_s)}$$

The results are the main equations:

$$-\mathfrak{U}_s = (R_s+j\omega L_{ss\Delta})\mathfrak{J}_s - j\omega M_\Delta \mathfrak{J}_r e^{j(\delta_0-2\lambda_r)}$$ (12)

$$-\mathfrak{U}_r = (R_r+j\omega L_{rr\Delta})\mathfrak{J}_r - j\omega M_\Delta \mathfrak{J}_s e^{j(\delta_0-2\lambda_s)}$$ (13)

7

It is clear that equation (12) can represent any equation of (10), likewise equation (13) can represent any equation of (11). Thus, equations (12) and (13) are mathematically sufficient to express the machine under all possible stable operational stages.

(2.2.) Physical Meaning of the Main Equation

The effective and the blind component of stator output per phase equal the real or the imaginary part of the product $\mathfrak{U}_s \mathfrak{J}_s^*$, where $\mathfrak{J}_s^*$ represents the conjugated vector of $\mathfrak{J}_s$. For the stator we thus get:

$$\mathfrak{U}_s \mathfrak{J}_s^* = \mathfrak{U}_s J_s e^{-j\lambda_s} \quad (R_s + j\omega L_{ss\Delta}) J_s^2 + j\omega M_\Delta J_s J_r e^{j(\delta_0 - \lambda_s - \lambda_r)}$$
$$= -R_s J_s^2 - j\omega L_{ss\Delta} J_s^2 + j\omega M_\Delta J_s J_r \cos(\delta_0 - \lambda_s - \lambda_r)$$
$$- \omega M_\Delta J_s J_r \sin(\delta_0 - \lambda_s - \lambda_r)$$
$$\mathfrak{U}_s \mathfrak{J}_s^* = [-J_s^2 R_s - \omega M_\Delta J_s J_r \sin(\delta_0 - \lambda_s - \lambda_r)]$$
$$+ j[-\omega L_{ss\Delta} J_s^2 + \omega M_\Delta J_s J_r \cos(\delta_0 - \lambda_s - \lambda_r)] \tag{14}$$

likewise for the rotor:

$$\mathfrak{U}_r \mathfrak{J}_r^* = [-J_r^2 R_r - \omega M_\Delta J_s J_r \sin(\delta_0 - \lambda_s - \lambda_r)]$$
$$+ j[-\omega L_{rr\Delta} J_r^2 + \omega M_\Delta J_s J_r \cos(\delta_0 - \lambda_s - \lambda_r)] \tag{15}$$

The effective output of the stator per phase thus is:

$$P_{k_s} = -J_s^2 R_s - \omega M_\Delta J_s J_r \sin\Psi \quad \text{Watt}$$

and of the rotor:

$$P_{k_r} = -J_r^2 R_r - \omega M_\Delta J_s J_r \sin\Psi \quad \text{Watt} \tag{16}$$

where:

$$\Psi = \delta_0 - \lambda_s - \lambda_r \tag{17}$$

Since the two quantities $J_s^2 R_s$ and $J_r^2 R_r$ are equal to stator or rotor copper loss, equation (16) results in the fact that <u>the inner or electromagnetic output of the stator and rotor always have the same direction</u>, which is:

$$P_{i_s} \quad P_{i_r} = -\omega M_\Delta J_s J_r \sin\Psi \quad \text{Watt/Phase.}$$

The inner output of the complete machine is:

$$P_i = -6\omega M_\Delta J_s J_r \sin\Psi \quad \text{Watt} \tag{18}$$

For certain values of stator and rotor currents equation (18) shows that the inner output of the machine only depends on the size of angle ψ and not on the various sizes of the three angles δ_0, λ, and λ_r .

For the motor case: $0 < \psi < \pi$

and for the generator case: $\pi < \psi < 2\pi$.

If currents of certain magnitudes flow into the machine, it is always possible to shift the stator currents against the rotor currents in time, more generally expressed this means: λ and λ_r can be varied infinitely and independently from each other without causing any influence on output, torque or power factor of both sides, as long as ψ . $\delta_0 - \lambda - \lambda_r$ remains unchanged. The following study of angle ψ shows how this condition is fulfilled.

The mutual inductance between phase U of the stator and phase U of the rotor is expressed by

$$M_{sr} \cos \delta = M_{sr} \cos (2 \omega t + \delta_0)$$

δ_0 represents the angle (in electrical degrees) which the rotor describes from the time when the magnetic axes of phase U of the stator and phase U of the rotor lie below each other until the time t = 0. Since the mutual inductance changes according to a cosine curve with <u>double</u> net frequency (Fig. 3), the angle $\left(\frac{\lambda_s}{2} - \lambda_s\right)$ equals half of the angle which the rotor describes from the time when the magnetic axes of two respective phases coincide up until the time when i_{s_u} equals 0 (is increasing). Likewise, the angle $\left(\frac{\lambda_s}{2} - \lambda_r\right)$ equals half of the angle which the rotor describes from the time when the magnetic axes of two respective phases coincide up until $i_{r_u} = 0$ (is increasing). The output of the machine only depends on the sum of these angles, i.e. angle ψ . According to equation (17), equations (12) and (13) are written as follows:

$$-U_s = (R_s + j\omega L_{ss\Delta})J_s e^{j\lambda_s} - j\omega M_\Delta J_r e^{j\lambda_s} e^{j\psi} \qquad (19)$$

$$-U_r = (R_r + j\omega L_{rr\Delta})J_r e^{j\lambda_r} - j\omega M_\Delta J_s e^{j\lambda_r} e^{j\psi} \qquad (20)$$

We now assume that the machine runs with a constant torque (and thus a constant output). If angle α_r is changed by $\Delta\alpha_r$, equation (20) is only fulfilled if λ_r changes evenly without variation of J_r, J_s and ψ . Equation (19) then also holds true. Furthermore, the product $U_r \cdot J_r^*$ is not changed and the blind and effective output of rotor and stator are not influenced by it. These stages can be reached in practice when the stator voltage is kept constant and the phase of the rotor voltage α_r is changed. As long as the two sides of the machine are considered separately there is no effect, the phase rather shifts evenly into the rotor current phase λ_r.

9

It is interesting, however, to watch how the machine adapts to the new situation. Since λ_s and ψ remain unchanged, equation (17) shows that δ_0 changes like λ_r . The two angles $\left(\frac{\delta_0}{2} - \lambda_s\right)$ and $\left(\frac{\delta_0}{2} - \lambda_r\right)$ change by $+\frac{\Delta\alpha_r}{2}$ or $-\frac{\Delta\alpha_r}{2}$.

According to the above-mentioned importance of these angles, it /15 is clear that these two changes can only take place due to a momentary change of rotor velocity. Thus, both magnetic axes of the phases U of rotor and stator will coincide at different times in respect to the cyclic change of both currents. The momentary change of rotor velocity results in a shift of the rotor position in respect to a synchronously rotating object. This effect equals the one occurring in normal synchronous machines whenever the generating current is varied. This shift of rotor position shall have the value $\Delta\alpha_r$ in electrical degrees. Fig. 3 shows the phase relation under such conditions before and after phase shift $\Delta\alpha_r$ of rotor voltage.
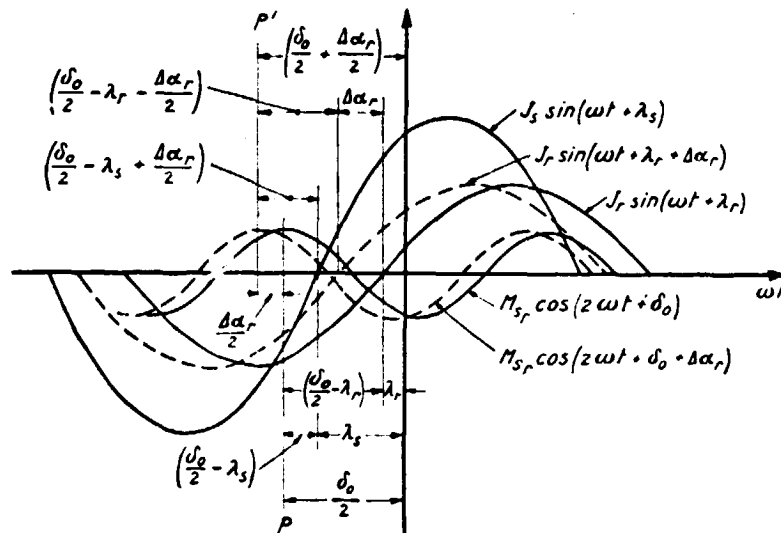
Fig. 3. Influence of phase shift of the rotor voltage.

_____ Before shift _____ After shift

$\Delta\alpha_r$ = Magnitude of shift

P, P'' = The moment when the magnetic axes of the two phases lie below each other before or after shift.

Thus, it is clear that the phase difference between stator and rotor voltage has no influence on magnitude and relative phases of all quantities of stator and rotor. However, it determines the relative position of the rotor body in respect to a synchronously rotating object. This shift of the rotor body is clearly demonstrated when the rotor is supplied with a stroboscope disc, illuminated with net frequency. When the phase angle between stator and rotor

10

voltages is changed, the seemingly immobile figure rotates evenly.

The main equations (12) and (13) express the relationship between machine currents and machine voltages when applying mutual cyclic inductance and the complete self inductances of the turns. Calculating the alternating current machines, usually not the complete self inductances are used, they are rather divided into two components; one which is effected by the mutual flow ("air-slit flow"), and another which is caused by stray flux. Thus, we use these familiar constants.

Where:

L_{σ_s} = stray inductance of stator per phase
L_{σ_r} = stray inductance of rotor per phase
W_s = effective number of turns of stator per phase
W_r = effective number of turns of rotor per phase

The following relationships between the old and new constants are valid:

$$\left.\begin{array}{l} L_{ss\varDelta} - L_{\sigma_s} = \left(\dfrac{W_s}{W_r}\right)^2 \\ L_{rr\varDelta} - L_{\sigma_r} \\ M_\varDelta = \sqrt{(L_{ss\varDelta} - L_{\sigma_s})(L_{rr\varDelta} - L_{\sigma_r})} = (L_{ss\varDelta} - L_{\sigma_s})\dfrac{W_r}{W_s} = (L_{rr\varDelta} - L_{\sigma_r})\dfrac{W_s}{W_r} \end{array}\right\} (21)$$

Thus, equations (12) and (13) are re-written:

$$\left.\begin{array}{l} -\mathfrak{U}_s = (R_s + j X_{\sigma_s})\mathfrak{I}_s - \mathfrak{E}_s \\ -\mathfrak{U}_r = (R_r + j X_{\sigma_r})\mathfrak{I}_r - \mathfrak{E}_r \end{array}\right\} (22)$$

where:

$$\left.\begin{array}{l} \mathfrak{E}_s = -j X_{\mu_s}\left[\mathfrak{I}_s - \mathfrak{I}_r \dfrac{W_r}{W_s} e^{j(\delta_0 - 2\lambda_r)}\right] \\ \mathfrak{E}_r = -j X_{\mu_r}\left[\mathfrak{I}_r - \mathfrak{I}_s \dfrac{W_s}{W_r} e^{j(\delta_0 - 2\lambda_s)}\right] \end{array}\right\} (23)$$

$X_{\sigma_s} = \omega L_{\sigma_s}$
$X_{\sigma_r} = \omega L_{\sigma_r}$
$X_{\mu_s} = \omega(L_{ss\varDelta} - L_{\sigma_s})$ = equivalent mutual or magnetizing reactance reduced to stator.
$X_{\mu_r} = \omega(L_{rr\varDelta} - L_{\sigma_r})$ = equivalent mutual or magnetizing reactance reduced to rotor.

$\mathfrak{E}_s$ and $\mathfrak{E}_r$ are the induced voltages per phase of stator or rotor which induces the mutual air slit flow. They are proportional to the effective number of turns. This can quite simply be recognized from the equation (23):

$$\mathfrak{E}_s = \left[J_s X_{\mu_s} \sin \lambda_s - J_r X_{\mu_s} \frac{W_r}{W_s} \sin(\delta_0 - \lambda_r) \right] - j \left[J_s X_{\mu_s} \cos \lambda_s - J_r X_{\mu_s} \frac{W_r}{W_s} \cos(\delta_0 - \lambda_r) \right]$$

$$E_s = |\mathfrak{E}_s| = X_{\mu_s} \sqrt{J_s^2 + J_r^2 \left(\frac{W_r}{W_s}\right)^2 - 2 J_s J_r \left(\frac{W_r}{W_s}\right) \cos \Psi}$$

likewise:

$$E_r = X_{\mu_r} \sqrt{J_r^2 + J_s^2 \left(\frac{W_s}{W_r}\right)^2 - 2 J_r J_s \left(\frac{W_s}{W_r}\right) \cos \Psi}$$

$$= X_{\mu_r} \cdot \frac{W_s}{W_r} \sqrt{J_s^2 + J_r^2 \left(\frac{W_r}{W_s}\right)^2 - 2 J_s J_r \left(\frac{W_r}{W_s}\right) \cos \Psi}$$
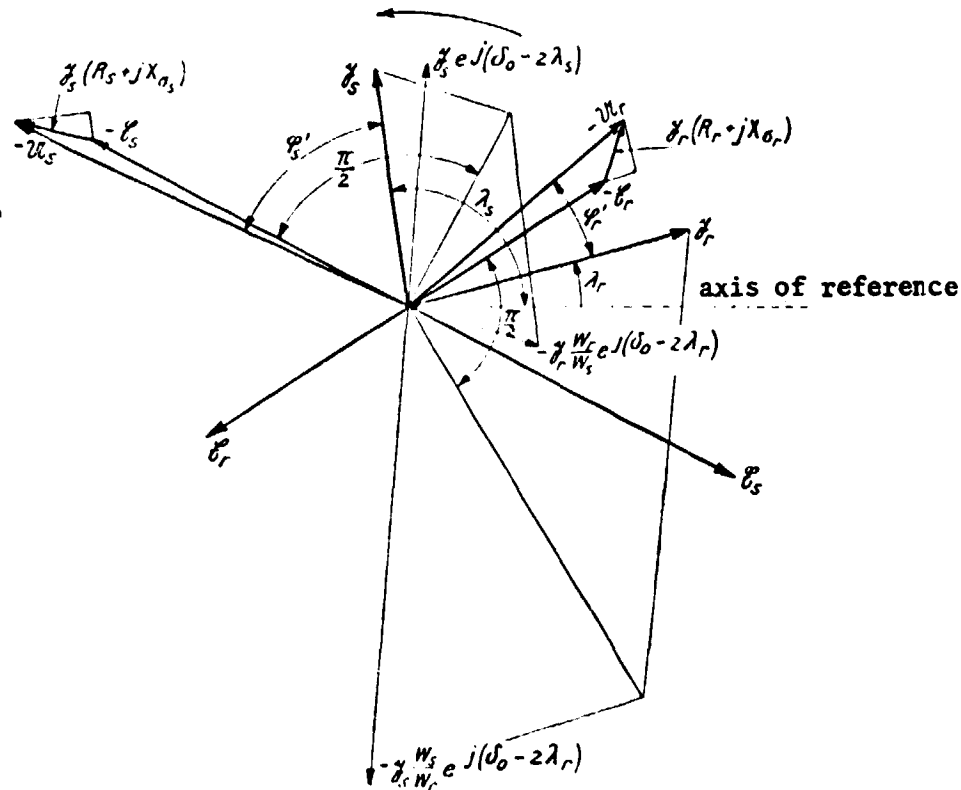
Fig.4. Vector diagram of two corresponding phases of the rotor and stator of an asymmetric machine which is fed with voltages different in phase and magnitude. (motor case, $-\mathfrak{u}_s$ and $-\mathfrak{u}_r$ are terminal voltages drops, $\delta_0 = \pi$ assumed).

/18

thus:

$$\frac{E_s}{E_r} = \frac{X_{\mu_s}}{X_{\mu_r}} \cdot \frac{W_r}{W_s} = \frac{W_s}{W_r} .$$

The vector relationships expressed in equations (22) and (23) are represented for the motor case in Fig.4. Stator and rotor voltages are assumed to be of different magnitude and phase.

12

It is evident from equation (23) that the two parallelograms, the sides of which are:

$$\left[\Im_s, -\Im_r \frac{W_r}{W_s} e^{j(\delta_0 - 2\lambda_r)}\right] \text{ or } \left[\Im_r, -\Im_s \frac{W_s}{W_r} e^{j(\delta_0 - 2\lambda_s)}\right]$$

are similar in a ratio $\frac{W_r}{W_s}$. But it is also important to notice that the five phase angles in Fig. 4, i.e., $\alpha_r, \alpha_s, \lambda_r, \lambda_s$ and δ_0 are measured from any chosen axis. The diagram thus can be simplified by assuming that one of these angles equals zero. It is furthermore possible to reach another simplification: It has been shown earlier that the phase differences between stator and rotor voltages do not have an influence on the magnitudes of the currents and phase differences compared to the corresponding voltages. These magnitude and phase differences only depend on the magnitude of the <u>voltages</u> and the demand conditions. A change of the phase angle between $\mathfrak{U}_s$ and $\mathfrak{U}_r$ thus rotates the two frameworks $(-\mathfrak{U}_s, \Im_s, \mathfrak{E}_s)$ and $(-\mathfrak{U}_r, \Im_r, \mathfrak{E}_r)$ against each other. When we now assume that this phase angle is chosen in a way that $\mathfrak{E}_s$ and $\mathfrak{E}_r$ are in phase, the vector diagram is simplified as shown in Fig.5. In Fig.5 δ_0 is assumed to equal zero.
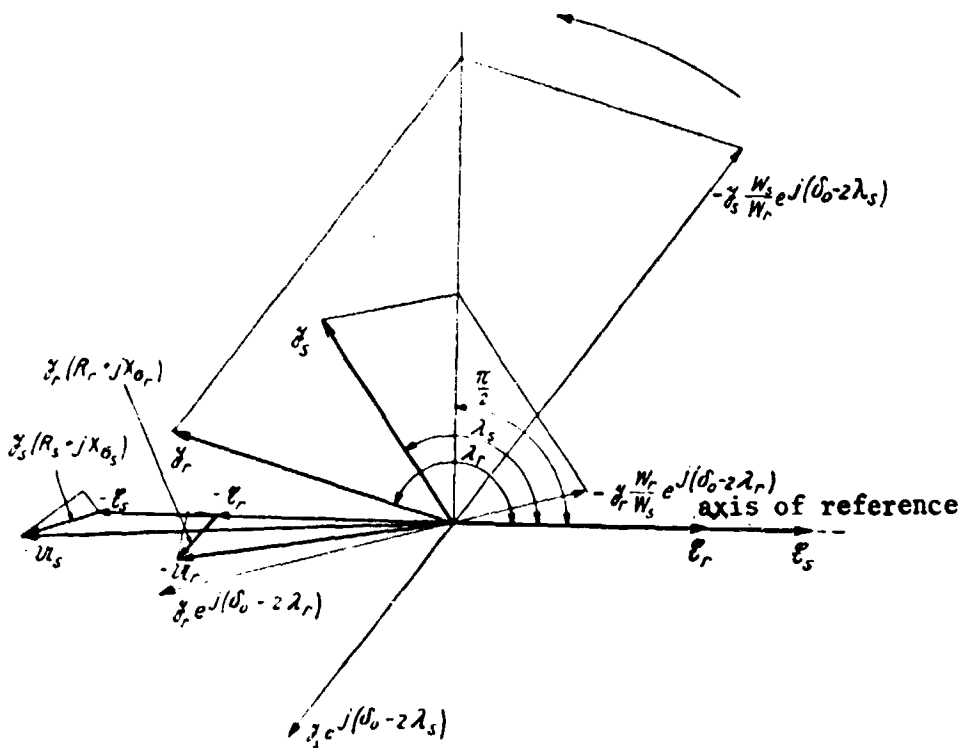

Fig.5. Vector diagram for the same conditions as in Fig.4, with $\mathfrak{E}_s$ and $\mathfrak{E}_r$ in phase and $\delta_0 = 0$.

13

The angle between $-\mathfrak{U}_s$ and $-\mathfrak{U}_r$ in Fig.5 is the angle necessary to bring $\mathfrak{E}_s$ and $\mathfrak{E}_r$ in phase. The same diagram can always be used even if the phase difference between $-\mathfrak{U}_s$ and $-\mathfrak{U}_r$ differs from the angle shown in the diagram, as long as the real time phase relations between rotor and stator vector are not deduced from it. If one wants to consider these relationships, the two frameworks constructed from $(-\mathfrak{U}_s, \mathfrak{I}_s, \mathfrak{E}_s)$ and $(-\mathfrak{U}_r, \mathfrak{I}_r, \mathfrak{E}_r)$ can be rotated against each other until the angle between $-\mathfrak{U}_s$ and $-\mathfrak{U}_r$ has reached its real value. Its last position has no influence since it only determines the arbitrary axis of reference.
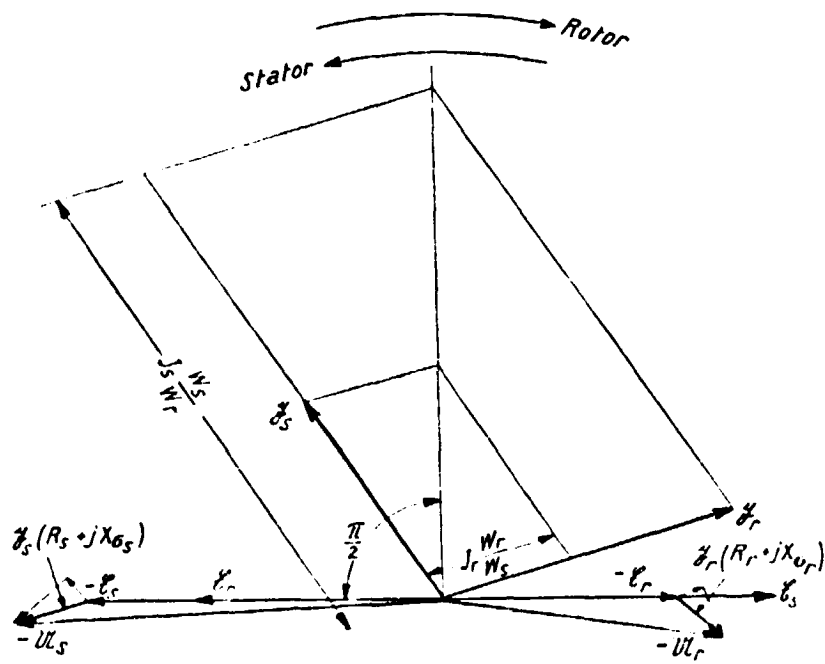
Fig.6. "Two-sequence" vector diagram for motor operation, $-v_s$ or $-v_r$ is the drop in terminal voltage.

A simple time vector diagram which is especially advantageous for connection with the space vector diagram, results from the phase shift of 180° of one side of e.m.k. (sic) and the formation of the other vector of this side in inverse sequence. "Two-sequence" vector diagrams of this kind are shown in Fig.6 and 7 for the motor or generator case.
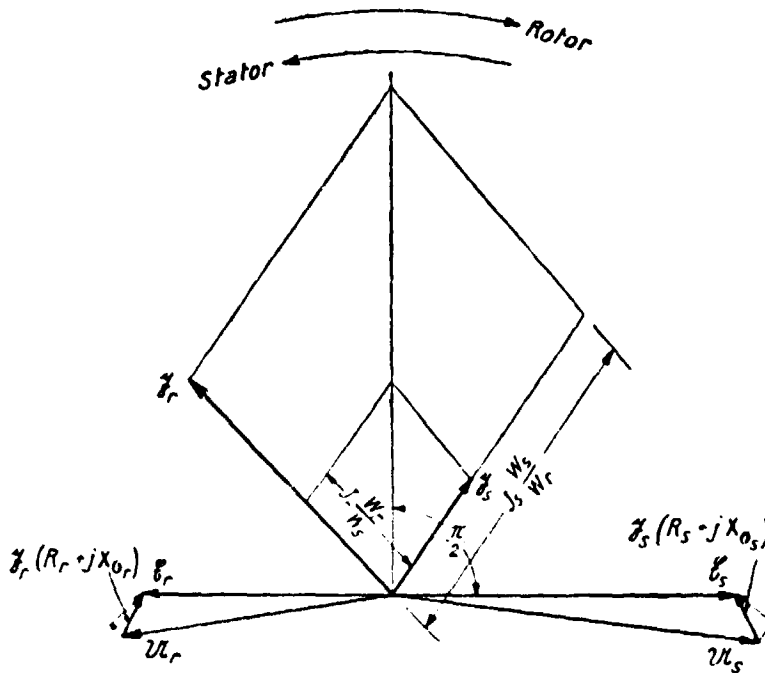
14

Fig.7. "Two-sequence" vector diagrams for generator operation.

(2.3.) Flux Waves and Generation of a Common Field.

Introduction: Here we will study in detail how the "air slit" field is generated.

The ampere turns of the three phases of rotor and stator each generate a synchronously rotating m.m.k. or flux wave . We neglect the harmonic waves and assume that these waves vary in space according to a sine function with a constant amplitude. Since the two induced voltages $|\mathfrak{E}_s|$ and $|\mathfrak{E}_r|$ are proportional to the turn ratio, we consider them as produced by one and only one flow wave having a constant amplitude and rotating in the same direction as the rotor with half its velocity, so that it has the same relative velocity in respect to stator and the rotor. Furthermore: by the selection of phase sequence and rotor velocity the two d.f. waves of rotor and stator are immobile relative to each other. They rotate with the same velocity, i.e., half the rotor velocity in the same direction as the rotor. It is clear that the flux waves of rotor and stator can be treated as space vectors and thus can be combined. This results in a flux wave which produces the mutual or "air slit flow". This flow ϕ is then responsible for the induced voltages $\mathfrak{E}_s$ and $\mathfrak{E}_r$.

Harmonic waves of the flux as well as iron saturation are neglected.

15

Combination of the Flux Waves

The flux wave of a phase in a three-phase machine with distributed turns resembles a trapezoid figure as shown in Fig.8.
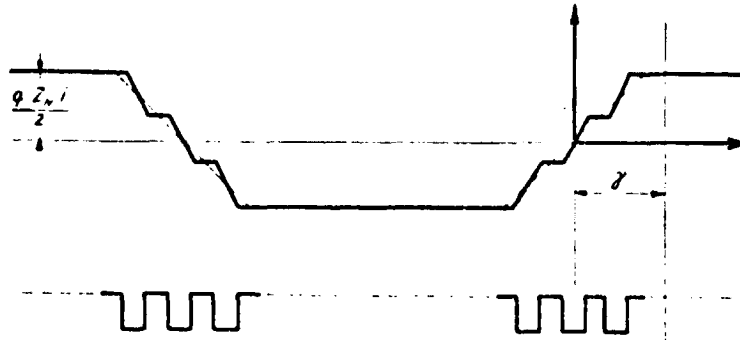

Fig. 8. Flux wave of a phase in a three-phase machine with dis-distributed turns

Its maximum value is:

$$\frac{q \cdot z_N \cdot i}{2} \quad \text{A.W./Pol}$$

where:

i = momentary value of the current of a conductor
q = amount of slots per pole and phase
z_N = amount of conductors per slot

This wave, of course, is stationary and its amplitude varies in time according to the current. If the coordinate system passes through zero, the following Fourier series for the height $\underline{f}$ of the wave at an angle location γ is reached:

$$f = \frac{4}{\pi} \cdot \frac{q \cdot z_N \cdot i/2}{\pi/6} \left(\sin \frac{\pi}{6} \sin \gamma + \cdots + \frac{1}{n^2} \sin \frac{n\pi}{6} \sin n\gamma \right)$$

The fundamental wave of this sequence thus has an amplitude of:

$$\frac{6 q z_N i}{\pi^2} \quad \text{A.W./Pol.} \tag{24}$$

In order to be able to combine the flux waves of stator and rotor we have to consider the fact that these waves move in opposite direction as compared to stator or rotor. The stator flux $\mathfrak{F}_s$ rotates
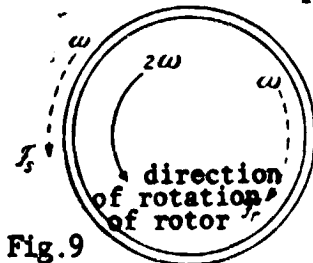

Fig.9

with an angle velocity ω in respect to the static stator, likewise the rotor flux $\mathfrak{F}_r$ rotates with the same angle velocity in the opposite direction in respect to the rotating rotor, whereas the rotor body rotates with an angle velocity 2ω in the same direction as $\mathfrak{F}_s$. These conditions are shown in Fig. 9.

16

It is then clear that in relation to a __stationary__ observer both $\mathfrak{F}_u$ and $\mathfrak{F}_r$ rotate counterclockwise with angle velocity ω . In relation however, to an observer who rotates counterclockwise with angle velocity ω , both $\mathfrak{F}_u$ and $\mathfrak{F}_r$ are stationary, whereas rotor and stator rotate counterclockwise or clockwise with angle velocity ω . These conditions are given in Fig.10. If the two flux waves which are stationary, are combined, one flux wave results. (Fig.10). This produces the air slit flow whose induction B is shown in the sine curve of Fig. 10.

If we let the coordinate system pass through zero (Fig.10), the air slit induction at a place τ drawn from zero, will be:

$$B = B_m \sin \tau \qquad (25)$$

where: B_m = maximal air slit induction.

/23
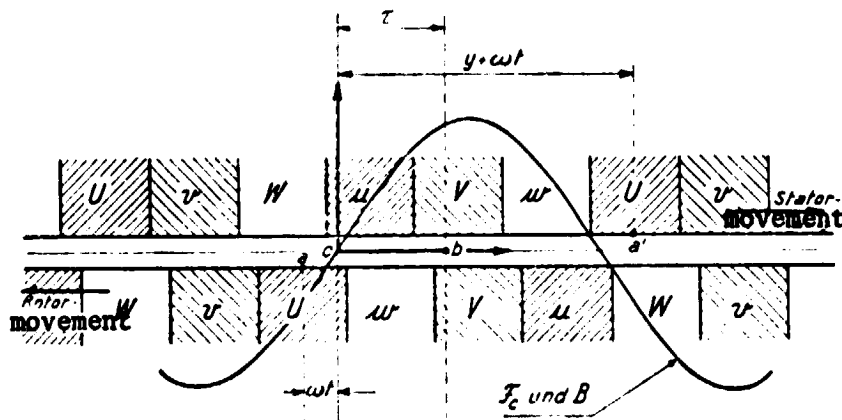
Fig. 10

We now measure the time from the moment on when point $\underline{a}$ in the rotor is directly below 0. When point $\underline{a}$ in the stator is at a place with an angle shift $\underline{y}$ to the right of 0 at the same time, Fig.10 represents the relative positions after $\underline{t}$ seconds. At this very moment (shown in Fig.10), the induced voltages and the currents of the six phases U, V, W of the stator or U, V, W of the rotor shall be expressed as follows:

Induced voltages of the rotor:

$$E_r \sin \omega t, \qquad E_r \sin\left(\omega t - \frac{2\pi}{3}\right), \qquad E_r \sin\left(\omega t - \frac{4\pi}{3}\right) \Biggr\}$$

Rotor currents:

$$J_r \sin(\omega t + \beta_r), \qquad J_r \sin\left(\omega t - \frac{2\pi}{3} + \beta_r\right), \qquad J_r \sin\left(\omega t - \frac{4\pi}{3} + \beta_r\right) \Biggr\} \quad (26a)$$

Induced voltages of the stator:

$$E_s \sin(\omega t + y), \qquad E_s \sin\left(\omega t + y - \frac{2\pi}{3}\right), \qquad E_s \sin\left(\omega t + y - \frac{4\pi}{3}\right) \Biggr\} \quad (26b)$$

17

Stator currents:

$$J_s\sin(\omega t+y+\beta_s),\ J_s\sin\left(\omega t+y-\frac{2\pi}{3}+\beta_s\right),\ J_s\sin\left(\omega t+y-\frac{4\pi}{3}+\beta_s\right)$$

where:

β_r or β_s = phase angles between induced voltage and the current of a rotor or stator phase.

$0<\beta<\pi$ for leading current,

$0>\beta>-\pi$ for lagging current.

Now the peaks of the fundamental wave of each phase lie in the axis of the corresponding coil, and its amplitude is calculated from equation (24). At the considered moment these amplitudes then have the following values:

Rotor:

$$K_r\sin(\omega t+\beta_r),\quad K_r\sin\left(\omega t-\frac{2\pi}{3}+\beta_r\right),\quad K_r\sin\left(\omega t-\frac{4\pi}{3}+\beta_r\right)$$

Stator:

$$K_s\sin(\omega t+y+\beta_s),\ K_s\sin\left(\omega t+y-\frac{2\pi}{3}+\beta_s\right),\ K_s\sin\left(\omega t+y-\frac{4\pi}{3}+\beta_s\right)$$

Where:

$$K_r=\frac{6\,q_r\cdot z_{Nr}\cdot J_r}{\pi^2}\quad\text{and}\quad K_s=\frac{6\,q_s\cdot z_{Ns}\cdot J_s}{\pi^2}$$

This results in the flux produced by the three rotor phases at a position in the air slit:

$$f_r=K_r\left[\sin(\omega t+\beta_r)\sin(\omega t+\tau)+\sin\left(\omega t-\frac{2\pi}{3}+\beta_r\right)\sin\left(\omega t+\tau-\frac{2\pi}{3}\right)\right.$$
$$\left.+\sin\left(\omega t-\frac{4\pi}{3}+\beta_r\right)\sin\left(\omega t+\tau-\frac{4\pi}{3}\right)\right]$$

It can be concluded:

$$f_r=\frac{3}{2}\,K_r\cos(\tau-\beta_r)\tag{27}$$

The flux produced by the three rotor phases at a place <u>b</u> in the air slit also is:

$$f_s=K_s\left[\sin(\omega t+y+\beta_s)\sin(-\omega t-y+\tau)+\sin\left(\omega t+y-\frac{2\pi}{3}+\beta_s\right)\sin\left(-\omega t-y+\frac{2\pi}{3}+\tau\right)\right.$$
$$\left.+\sin\left(\omega t+y-\frac{4\pi}{3}+\beta_s\right)\sin\left(-\omega t-y+\frac{4\pi}{3}+\tau\right)\right]$$

18

This results in:

$$f_s = - \frac{3}{2} K_s \cos(\tau + \beta_s) \qquad (28)$$

The resulting flux f_c at a place $\underline{b}$ then simply equals the sum $(f_r + f_s)$ and according to equations (27) and (28) thus is:

$$f_c = f_r + f_s$$

$$= \frac{3}{2} K_r \cos(\tau - \beta_r) - \frac{3}{2} K_s \cos(\tau + \beta_s)$$

$$f_c = F_r \cos(\tau - \beta_r) - F_s \cos(\tau + \beta_s) \qquad (29)$$

Where: F_r = amplitude of the complete rotor flux wave

$$= \frac{3}{2} K_r$$

$$= \frac{9 q_r \cdot z_{N_r} \cdot J_r}{\pi^2}$$

F_s = amplitude of the complete stator flux wave

$$= \frac{3}{2} K_s$$

$$= \frac{9 q_s \cdot z_{N_s} \cdot J_s}{\pi^2}$$

Presentation of the Field

Equation (29) is independent of time (which is to be expected) and of $\underline{y}$. Under certain ouput conditions, f_c only depends on τ, i.e., the position of the observed point in relation to the observer. It has been shown (22) that the phase shift between rotor and stator voltages has no influence on the quantities $J_s, J_r, \beta_s, \beta_r$. It does, however, influence $\underline{y}$, since $\underline{y}$ is the phase angle between $\mathfrak{E}_s$ and $\mathfrak{E}_r$ (as can be seen in expressions (26a) and (26b)). Equation (29) and the absence of $\underline{y}$ in it show that all quantities in this equation are independent of the phase angle between stator and rotor voltage. It can be concluded that neither the magnitude nor the relative positions in space of stator, rotor and combined flux waves depend on this angle, but rather only on the magnitude of the voltages and output conditions.

Neglecting the magnetic saturation and loss of iron, induction B in every position in the air slit is proportional to the corresponding value of f_c. Since $\underline{B}$ varies in space according to

19

formula (25), f_C can be expressed by the following formula

$$f_c = F_c \sin \tau \qquad\qquad (30)$$

where f_C represents the amplitude of the resulting flux wave. The identity of equations (29) and (30) results in:

$$F_c \sin \tau = F_r \cos (\tau - \beta_r) - F_s \cos (\tau + \beta_s) \qquad\qquad (31)$$

$$F_c \sin \tau = F_r (\cos \tau \cos \beta_r + \sin \tau \sin \beta_r) - F_s (\cos \tau \cos \beta_s - \sin \tau \sin \beta_s)$$

$$= (F_r \cos \beta_r - F_s \cos \beta_s) \cos \tau + (F_r \sin \beta_r + F_s \sin \beta_s) \sin \tau$$

Page 26 and 27 missing in original

20

This excess is necessary in order to compensate the de-magnetizing effect of the lagging currents which the stator supplies to its net.

The space vector diagrams can be combined with the two-sequence "vector" diagram developed in part (2.2.); this is shown in Fig. 12.

Such a combined space and time vector diagram has the advantage of combining all vector quantities in a simple diagram and representing the relative positions in space of the flux waves for any chosen magnitudes and phase positions of the currents. The vectors of the terminals and induced voltages which connect currents and flow Φ of the coil, are time vectors and must be looked at in the right sequence indicated by the arrows. The vectors of the three flux waves and of induction B are mainly space vectors; they can, however, also be regarded as time vectors (in respect to stator or rotor[1]),as long as they are regarded in their right sequence.

3. General Solution of the Main Equations

The main equations (12) and (13) of the machine which were developed in chapter (2.1) can be solved for the magnitude and phase of the currents $\Im_x$ and $\Im_r$. The solution of such equations of the machine has so far only been developed for special cases [2]. The parameter used in these solutions is an angle which almost always corresponds to the angle between the flux waves. The achieved results and calculation of the operational method of the machine are complicated and incomplete in some cases.

In this chapter the solution for the very general case of asymmetric d.s.m. is developed. In addition, the complete calculation of the asymmetric machine will be made possible in a relatively simple way by the use of a new parameter.

The resulting basic equations can, (according to equations (22), (23) and (17)), be written as follows:

/29

$$-\mathfrak{U}_x = \mathfrak{I}_x \left[R_x + j X_{\sigma_x} + j X_{\mu_x}\left(1 - \frac{J_r \cdot W_r}{J_x \cdot W_x} \cdot e^{j\Psi}\right)\right] \qquad (34\,a)$$

$$-\mathfrak{U}_r = \mathfrak{I}_r \left[R_r + j X_{\sigma_r} + j X_{\mu_r}\left(1 - \frac{J_x \cdot W_x}{J_r \cdot W_r} \cdot e^{j\Psi}\right)\right] \qquad (34\,b)$$

---

1) This fact can simply be seen by studying Fig. 10, when equation (31) is superimposed.

2) See bibliography at the end.

21

It can be concluded from the first equation:

$$-\frac{\mathfrak{U}_s}{\mathfrak{J}_s} = \left[R_s + jX_{\sigma_s} + jX_{\mu_s}\left(1 - \frac{J_r \cdot W_r}{J_s \cdot W_s} \cdot e^{j\Psi}\right)\right]$$

$$= \left[R_s + jX_{\sigma_s} + jX_{\mu_s}\left(1 - \frac{J_r \cdot W_r}{J_s \cdot W_s}\{\cos\Psi + j\sin\Psi\}\right)\right]$$

$$= \left(R_s + X_{\mu_s} \cdot \frac{J_r \cdot W_r}{J_s \cdot W_s}\sin\Psi\right) + j\left(X_{\sigma_s} + X_{\mu_s} - X_{\mu_s} \cdot \frac{J_r \cdot W_r}{J_s \cdot W_s}\cos\Psi\right)$$

Which results in:

$$\frac{U_s}{J_s} = \sqrt{\left(R_s + X_{\mu_s} \cdot \frac{J_r \cdot W_r}{J_s \cdot W_s}\sin\Psi\right)^2 + \left(X_{\sigma_s} + X_{\mu_s} - X_{\mu_s} \cdot \frac{J_r \cdot W_r}{J_s \cdot W_s}\cos\Psi\right)^2} \qquad (34c)$$

$$\frac{U_s^2}{J_s^2} = R_s^2 + X_{\mu_s}^2 \cdot \frac{J_r^2 \cdot W_r^2}{J_s^2 \cdot W_s^2} \cdot \sin^2\Psi + 2R_s X_{\mu_s} \cdot \frac{J_r \cdot W_r}{J_s \cdot W_s}\sin\Psi + (X_{\sigma_s} + X_{\mu_s})^2$$

$$+ X_{\mu_s}^2 \cdot \frac{J_r^2 \cdot W_r^2}{J_s^2 \cdot W_s^2} \cdot \cos^2\Psi - 2X_{\mu_s}(X_{\sigma_s} + X_{\mu_s}) \cdot \frac{J_r \cdot W_r}{J_s \cdot W_s}\cos\Psi$$

$$\frac{U_s^2}{J_s^2} = R_s^2 + (X_{\sigma_s} + X_{\mu_s})^2 + X_{\mu_s}^2 \cdot \frac{J_r^2 \cdot W_r^2}{J_s^2 \cdot W_s^2} + 2X_{\mu_s} \cdot \frac{J_r \cdot W_r}{J_s \cdot W_s}[R_s\sin\Psi - (X_{\sigma_s} + X_{\mu_s})\cos\Psi]$$

$$(35a)$$

It holds for the rotor:

$$\frac{U_r^2}{J_r^2} = R_r^2 + (X_{\sigma_r} + X_{\mu_r})^2 + X_{\mu_r}^2 \cdot \frac{J_s^2 \cdot W_s^2}{J_r^2 \cdot W_r^2} + 2X_{\mu_r} \cdot \frac{J_s \cdot W_s}{J_r \cdot W_r}[R_r\sin\Psi - (X_{\sigma_r} + X_{\mu_r})\cos\Psi]$$

$$(35b)$$

substituting:

$$\frac{U_r}{U_r} = u \qquad\qquad \frac{W_s}{W_r} = u$$

$$R_s\left(\frac{W_r}{W_s}\right)^2 = R_s' \qquad X_{\sigma_s}\left(\frac{W_r}{W_s}\right)^2 = X_{\sigma_s}' \qquad J_s \frac{W_s}{W_r} = J_s'$$

$$R_r\left(\frac{W_s}{W_r}\right)^2 = R_r' \qquad X_{\sigma_r}\left(\frac{W_s}{W_r}\right)^2 = X_{\sigma_r}' \qquad J_r \frac{W_r}{W_s} = J_r'$$

By multiplication with u^2, equation (35b) will be:

$$\frac{U_r^2}{J_r'^2} \cdot u^2 = R_r'^2 + (X_{\sigma_r}' + X_{\mu_e})^2 + X_{\mu_e}^2 \cdot \frac{J_s^2}{J_r'^2} + 2 X_{\mu_e} \cdot \frac{J_s'}{J_r'} \, [\, R_r' \sin \Psi' - (X_{\sigma_r}' + X_{\mu_e}) \cos \Psi' \,]$$

(35c)

From equation (35a) and (35c) it can be concluded:

$$\frac{u^2}{u^2} \cdot \frac{J_r'^2}{J_s^2} = \frac{R_s^2 + (X_{\sigma_s} + X_{\mu_e})^2 + X_{\mu_e}^2 \frac{J_r'^2}{J_s^2} + 2 X_{\mu_e} \frac{J_r'}{J_s} \, [\, R_s \sin \Psi - (X_{\sigma_s} + X_{\mu_e}) \cos \Psi \,]}{R_r'^2 + (X_{\sigma_r}' + X_{\mu_e})^2 + X_{\mu_e}^2 \frac{J_s^2}{J_r'^2} + 2 X_{\mu_e} \frac{J_s}{J_r'} \, [\, R_r' \sin \Psi' - (X_{\sigma_r}' + X_{\mu_e}) \cos \Psi' \,]}$$

Which results in:

$$\left(\frac{J_r'}{J_s}\right)^2 \cdot [\, R_r'^2 + (X_{\sigma_r}' + X_{\mu_e})^2 \,] + X_{\mu_e}^2 + \left(\frac{J_r'}{J_s}\right) \cdot [\, 2 X_{\mu_e} \{ R_r' \sin \Psi - (X_{\sigma_r}' + X_{\mu_e}) \cos \Psi \} \,]$$

$$= \left(\frac{J_r'}{J_s}\right)^2 \cdot \left[\frac{u^2}{u^2} \cdot X_{\mu_e}^2 \right] + \frac{u^2}{u^2} \{ R_s^2 + (X_{\sigma_s} + X_{\mu_e})^2 \}$$

$$+ \left(\frac{J_r'}{J_s}\right) \cdot \left[2 \frac{u^2}{u^2} X_{\mu_e} \{ R_s \sin \Psi - (X_{\sigma_s} + X_{\mu_e}) \cos \Psi \} \right]$$

or:

$$\left(\frac{J_r'}{J_s}\right)^2 \cdot \left[R_r'^2 + X_{\sigma_r}'^2 + 2 X_{\sigma_r}' X_{\mu_e} + X_{\mu_e}^2 \left(1 - \frac{u^2}{u^2}\right) \right]$$

$$+ \left(\frac{J_r'}{J_s}\right) \cdot \left[2 X_{\mu_e} \left\{ \left(R_r' - \frac{u^2}{u^2} R_s \right) \sin \Psi - \left(X_{\sigma_r}' - \frac{u^2}{u^2} X_{\sigma_s} + X_{\mu_e} \left[1 - \frac{u^2}{u^2} \right] \right) \cos \Psi \right\} \right]$$

$$+ X_{\mu_e}^2 \left(1 - \frac{u^2}{u^2}\right) - R_s^2 \frac{u^2}{u^2} - X_{\sigma_s}^2 \frac{u^2}{u^2} - 2 X_{\sigma_s} X_{\mu_e} \frac{u^2}{u^2} = 0$$

(36)

Equation (36) is squared in $\frac{J_r'}{J_s}$ and has the following solution:

$$\frac{J_r'}{J_s} = - \frac{2 X_{\mu_e} \left[\left\{ R_r' - \frac{u^2}{u^2} R_s \right\} \sin \Psi - \left\{ X_{\sigma_r}' - \frac{u^2}{u^2} X_{\sigma_s} + X_{\mu_e} \left(1 - \frac{u^2}{u^2}\right) \right\} \cos \Psi \right]}{2 \left\{ R_r'^2 + X_{\sigma_r}'^2 + 2 X_{\sigma_r}' X_{\mu_e} + X_{\mu_e}^2 \left(1 - \frac{u^2}{u^2}\right) \right\}}$$

$$\pm \left\{ \left(\frac{2 X_{\mu_e} \left[\left\{ R_r' - \frac{u^2}{u^2} R_s \right\} \sin \Psi - \left\{ X_{\sigma_r}' - \frac{u^2}{u^2} X_{\sigma_s} + X_{\mu_e} \left(1 - \frac{u^2}{u^2}\right) \right\} \cos \Psi \right]}{2 \left\{ R_r'^2 + X_{\sigma_r}'^2 + 2 X_{\sigma_r}' X_{\mu_e} + X_{\mu_e}^2 \left(1 - \frac{u^2}{u^2}\right) \right\}} \right)^2 \right.$$

$$\left. - \frac{X_{\mu_e}^2 \left(1 - \frac{u^2}{u^2}\right) - \frac{u^2}{u^2} (R_s^2 + X_{\sigma_s}^2 + 2 X_{\sigma_s} X_{\mu_e})}{R_r'^2 + X_{\sigma_r}'^2 + 2 X_{\sigma_r}' X_{\mu_e} + X_{\mu_e}^2 \left(1 - \frac{u^2}{u^2}\right)} \right\}^{\frac{1}{2}}$$

(37)

Setting the following relationship:

$$r_0 = X_{\mu_e}^2 \, \frac{R_s \frac{u^2}{u^2} - R_r'}{R_r'^2 + X_{\sigma_r}'^2 + 2 X_{\sigma_r}' X_{\mu_e} + X_{\mu_e}^2 \left(1 - \frac{u^2}{u^2}\right)} \qquad \text{Ohm}$$

(38a)

23

$$X_{0_s} = X_{\mu_s}^2 \cdot \frac{X_{\sigma_s} \frac{\ddot{u}^2}{u^2} - X_{\sigma_r}' - X_{\mu_s}\left(1 - \frac{\ddot{u}^2}{u^2}\right)}{R_r'^2 + X_{\sigma_r}'^2 + 2 X_{\sigma_r}' X_{\mu_s} + X_{\mu_s}^2\left(1 - \frac{\ddot{u}^2}{u^2}\right)} \qquad \text{Ohm} \qquad (38\text{b})$$

$$r_{0_r} = X_{\mu_r}^2 \cdot \frac{R_r \frac{u^2}{\ddot{u}^2} - R_s'}{R_s'^2 + X_{\sigma_s}'^2 + 2 X_{\sigma_s}' X_{\mu_r} + X_{\mu_r}^2\left(1 - \frac{u^2}{\ddot{u}^2}\right)} \qquad \text{Ohm} \qquad (38\text{c})$$

$$= \frac{X_{\mu_s}^2}{\ddot{u}^2} \cdot \frac{R_s \frac{\ddot{u}^2}{u^2} - R_r'}{X_{\mu_s}^2\left(1 - \frac{\ddot{u}^2}{u^2}\right) - R_s^2 \frac{\ddot{u}^2}{u^2} - X_{\sigma_s}^2 \frac{\ddot{u}^2}{u^2} - 2 X_{\sigma_s} X_{\mu_s} \frac{\ddot{u}^2}{u^2}}$$

$$X_{0_s} = X_{\mu_r}^2 \cdot \frac{X_{\sigma_r} \frac{u^2}{\ddot{u}^2} - X_{\sigma_s}' - X_{\mu_r}\left(1 - \frac{u^2}{\ddot{u}^2}\right)}{R_s'^2 + X_{\sigma_s}'^2 + 2 X_{\sigma_s}' X_{\mu_r} + X_{\mu_r}^2\left(1 - \frac{u^2}{\ddot{u}^2}\right)} \qquad \text{Ohm} \qquad (38\text{d})$$

$$X_{0_r} = \frac{X_{\mu_s}^2}{\ddot{u}^2} \cdot \frac{X_{\sigma_r} \frac{\ddot{u}^2}{u^2} - X_{\sigma_r}' - X_{\mu_s}\left(1 - \frac{\ddot{u}^2}{u^2}\right)}{X_{\mu_s}^2\left(1 - \frac{\ddot{u}^2}{u^2}\right) - R_s^2 \frac{\ddot{u}^2}{u^2} - X_{\sigma_s}^2 \frac{\ddot{u}^2}{u^2} - 2 X_{\sigma_s} X_{\mu_s} \frac{\ddot{u}^2}{u^2}}$$

$$K = -\frac{r_{0_s}}{r_{0_r}} = -\frac{X_{0_s}}{X_{0_r}} \qquad (38\text{e})$$

With the aid of these, equation (37) is written as follows:

$$\frac{J_r'}{J_s} X_{\mu_s} = r_{0_s}\sin\Psi - X_{0_s}\cos\Psi \pm \sqrt{(r_{0_s}\sin\Psi - X_{0_s}\cos\Psi)^2 + \frac{K}{\ddot{u}^2} X_{\mu_s}^2}$$

$$= r_{0_s}\sin\Psi - X_{0_s}\cos\Psi \pm \sqrt{r_{0_s}^2\sin^2\Psi + X_{0_s}^2\cos^2\Psi - 2 r_{0_s} X_{0_s}\sin\Psi\cos\Psi + \frac{K}{\ddot{u}^2} X_{\mu_s}^2}$$

$$= r_{0_s}\sin\Psi - X_{0_s}\cos\Psi \pm \sqrt{r_{0_s}^2(1-\cos^2\Psi) + X_{0_s}^2(1-\sin^2\Psi) - 2 r_{0_s} X_{0_s}\sin\Psi\cos\Psi + \frac{K}{\ddot{u}^2} X_{\mu_s}^2}$$

$$= r_{0_s}\sin\Psi - X_{0_s}\cos\Psi \pm \sqrt{\left(r_{0_s}^2 + X_{0_s}^2 + \frac{K}{\ddot{u}^2} X_{\mu_s}^2\right) - (r_{0_s}\cos\Psi + X_{0_s}\sin\Psi)^2} \qquad (39)$$

We also put:

$$X_{\mu_{0_s}} = \pm \sqrt{r_{0_s}^2 + X_{0_s}^2 + \frac{K}{\ddot{u}^2} X_{\mu_s}^2} \qquad (40\text{a})[1]$$

---

[1] The positive sign is valid when $R_r'^2 + X_{\sigma_r}'^2 + 2 X_{\sigma_r}' X_{\mu_s} + X_{\mu_s}^2\left(1 - \frac{\ddot{u}^2}{u^2}\right) > 0$, and the negative sign is valid when: $R_r'^2 + X_{\sigma_r}'^2 + 2 X_{\sigma_r}' X_{\mu_s} + X_{\mu_s}^2\left(1 - \frac{\ddot{u}^2}{u^2}\right) < 0$ (see chapter (7.3.)). The case when this expression equals zero, is treated in chapter (5.1.).

24

$$\rho_s = \arcsin \frac{r_{0_s}\cos\Psi' + X_{0_s}\sin\Psi'}{+\sqrt{r_{0_s}^2 + X_{0_s}^2 + \frac{K}{\bar{\mu}^2}X_{\mu_s}^2}} \equiv \arcsin \frac{r_{0_s}\cos\Psi' + X_{0_s}\sin\Psi'}{|X_{\mu_{0_s}}|} \quad (40\text{b})$$

$$\left(\frac{\pi}{2} \geqslant \rho_s \geqslant -\frac{\pi}{2}\right)$$

Thus, equation (39) is simplified as follows:

$$\frac{J_r'}{J_s} X_{\mu_s} = r_{0_s}\sin\Psi' - X_{0_s}\cos\Psi' + X_{\mu_{0_s}}\cos\rho_s$$

It can be concluded:

$$-jX_{\mu_s}\frac{J_r'}{J_s}e^{j\Psi} = (r_{0_s}\sin\Psi' - X_{0_s}\cos\Psi' + X_{\mu_{0_s}}\cos\rho_s)(\sin\Psi' - j\cos\Psi')$$

$$= (r_{0_s}\sin^2\Psi' - X_{0_s}\sin\Psi'\cos\Psi' + X_{\mu_{0_s}}\cos\rho_s\sin\Psi')$$
$$+ j(X_{0_s}\cos^2\Psi' - r_{0_s}\sin\Psi'\cos\Psi' - X_{\mu_{0_s}}\cos\rho_s\cos\Psi')$$

$$= (r_{0_s} - r_{0_s}\cos^2\Psi' - X_{0_s}\sin\Psi'\cos\Psi' + X_{\mu_{0_s}}\cos\rho_s\sin\Psi')$$
$$+ j(X_{0_s} - X_{0_s}\sin^2\Psi' - r_{0_s}\sin\Psi'\cos\Psi' - X_{\mu_{0_s}}\cos\rho_s\cos\Psi')$$

$$= [r_{0_s} - (r_{0_s}\cos\Psi' + X_{0_s}\sin\Psi')\cos\Psi' + X_{\mu_{0_s}}\cos\rho_s\sin\Psi']$$
$$+ j[X_{0_s} - (r_{0_s}\cos\Psi' + X_{0_s}\sin\Psi')\sin\Psi' - X_{\mu_{0_s}}\cos\rho_s\cos\Psi']$$

$$= [r_{0_s} - |X_{\mu_{0_s}}|\sin\rho_s\cos\Psi' + X_{\mu_{0_s}}\cos\rho_s\sin\Psi']$$
$$+ j[X_{0_s} - |X_{\mu_{0_s}}|\sin\rho_s\sin\Psi' - X_{\mu_{0_s}}\cos\rho_s\cos\Psi']$$

$$= [r_{0_s} + X_{\mu_{0_s}}\sin(\Psi' \mp \rho_s)] + j[X_{0_s} - X_{\mu_{0_s}}\cos(\Psi' \mp \rho_s)]$$

$$= (r_{0_s} + jX_{0_s}) + X_{\mu_{0_s}}[\sin(\Psi' \mp \rho_s) - j\cos(\Psi' \mp \rho_s)]$$

$$= (r_{0_s} + jX_{0_s}) + X_{\mu_{0_s}}\left[\frac{2}{\text{tg}\frac{\Psi' \mp \rho_s}{2} + \text{ctg}\frac{\Psi' \mp \rho_s}{2}} - j\left(1 - \frac{2}{1 + \text{ctg}^2\frac{\Psi' \mp \rho_s}{2}}\right)\right]$$

Substituting the new parameter:

$$r_s = \pm 2\sqrt{r_{0_s}^2 + X_{0_s}^2 + \frac{K}{\bar{\mu}^2}X_{\mu_s}^2}\,\text{tg}\left(\frac{\Psi' \mp \rho_s}{2}\right) = 2X_{\mu_{0_s}}\text{tg}\left(\frac{\Psi' \mp \rho_s}{2}\right) \quad (41)$$

into the equation we get:

$$-jX_{\mu_s}\frac{J_r'}{J_s}e^{j\Psi} = (r_{0_s} + jX_{0_s}) + X_{\mu_{0_s}}\left[\frac{2}{\frac{r_s}{2X_{\mu_{0_s}}} + \frac{2X_{\mu_{0_s}}}{r_s}} - j\left(1 - \frac{2}{1 + \frac{4X_{\mu_{0_s}}^2}{r_s^2}}\right)\right]$$

$$= r_{0_s} + j(X_{0_s} - X_{\mu_{0_s}}) + \frac{4r_s X_{\mu_{0_s}}^2 + 2jr_s^2 X_{\mu_{0_s}}}{r_s^2 + 4X_{\mu_{0_s}}^2}$$

$$= r_{0_s} + j(X_{0_s} - X_{\mu_{0_s}}) + \frac{2jr_s X_{\mu_{0_s}}}{r_s + 2jX_{\mu_{0_s}}}$$

By putting this relationship into the stator equation (34a) it follows that:

$$-\mathfrak{U}_s = \mathfrak{J}_s \left[(R_s + r_{0_s}) + j(X_{\sigma_s} + X_{i_s}) + \frac{2jr_s X_{\mu_{0s}}}{r_s + 2jX_{\mu_{0s}}} \right] \qquad (42\mathrm{a})$$

where:

$$X_{i_s} = X_{0_s} + X_{\mu_s} - X_{\mu_{0s}}$$

Equation (42a) can be written as follows:

$$\mathfrak{J}_s = \frac{-\mathfrak{U}_s}{R_{e_s} + jX_{e_s} + \dfrac{jr_s X_{\mu_{ss}}}{r_s + jX_{\mu_{ss}}}} \qquad (42\mathrm{b})$$

where:

$$\left. \begin{aligned} R_{e_s} &= R_s + r_{0_s} = R_s + X_{\mu_s}^2 \cdot \frac{R_s \frac{\ddot{u}^2}{u^2} - R_r'}{R_r'^2 + X_{\sigma_r}'^2 + 2X_{\sigma_r}' X_{\mu_s} + X_{\mu_s}^2 \left(1 - \frac{\ddot{u}^2}{u^2}\right)} &&\text{Ohm} \\[2mm] X_{e_s} &= X_{\sigma_s} + X_{i_s} = X_{\sigma_s} + X_{\mu_s} - X_{\mu_{0s}} + X_{\mu_s}^2 \cdot \frac{X_{\sigma_s} \frac{\ddot{u}^2}{u^2} - X_{\sigma_r}' - X_{\mu_s}\left(1 - \frac{\ddot{u}^2}{u^2}\right)}{R_r'^2 + X_{\sigma_r}'^2 + 2X_{\sigma_r}' X_{\mu_s} + X_{\mu_s}^2 \left(1 - \frac{\ddot{u}^2}{u^2}\right)} &&\text{Ohm} \\[2mm] X_{\mu_{ss}} &= 2X_{\mu_{0s}} = \pm 2 \sqrt{r_{0_s}^2 + X_{0_s}^2 + \frac{K}{\ddot{u}^2} X_{\mu_s}^2} &&\text{Ohm} \end{aligned} \right\} \quad (43)$$

which we call "compensation constants" of the circuit. Together with the parameter r_s they form a compensation circuit which completely represents the stator. This circuit is shown in Fig. 13 (after equation (42b))
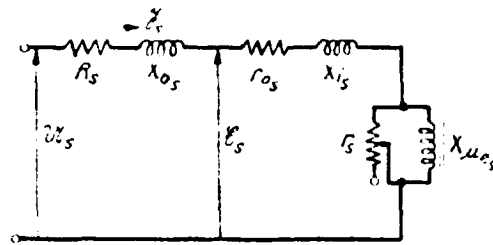

Fig.13. Compensation circuit of the stator of an asymmetric machine.

The algebraic sum of the output in r_s and r_{0s} represents the inner output of the stator (as well as the inner output of the machine). The constant r_{0s} as well as the parameter r_s can be positive or negative. The constant X_{is} can likewise be inductive or capacitive.

26

The compensation constants usually differ from the true constants of the stator $(R_s, X_{\sigma_s}, 2 X_{\mu_s})$. The magnitude of this difference depends on the machine constant and the ratio $_{ii}^u$. The constants r_{0s} and X_{is} equal zero in a symmetric machine [1], the true and compensation constants being identical in this case.

It is true for the rotor:

$$\mathfrak{J}_r = \frac{- \mathfrak{u}_r}{R_{e_r} + j X_{e_r} + \dfrac{j r_r X_{\mu_r}}{r_r + j X_{\mu_r}}} \qquad (44)$$

$$R_{e_r} = R_r + r_{0_r} = R_r + X_{\mu_r}^2 \cdot \frac{R_r \dfrac{u^2}{\ddot{u}^2} - R_s'}{R_s'^2 + X_{\sigma_s}''^2 + 2 X_{\sigma_s}' X_{\mu_r} + X_{\mu_r}^2 \left(1 - \dfrac{u^2}{\ddot{u}^2}\right)} \quad \text{Ohm}$$

$$X_{e_r} = X_{\sigma_r} + X_{i_r} = X_{\sigma_r} + X_{\mu_r} - X_{\mu_{0r}} + X_{\mu_r}^2 \frac{X_{\sigma_r} \dfrac{u^2}{\ddot{u}^2} - X_{\sigma_s}' - X_{\mu_r}\left(1 - \dfrac{u^2}{\ddot{u}^2}\right)}{R_s'^2 + X_{\sigma_s}''^2 + 2 X_{\sigma_s}' X_{\mu_r} + X_{\mu_r}^2\left(1 - \dfrac{u^2}{\ddot{u}^2}\right)} \quad \text{Ohm} \qquad (45)$$

$$X_{\mu_{er}} = 2 X_{\mu_{0r}} = \pm 2 \sqrt{r_{0_r}^2 + X_{0_r}^2 + \frac{\ddot{u}^2}{K} X_{\mu_r}^2} \qquad \text{(Ohm [2])}$$

The following relationship can simply be proven by equations (43) and (45):

$$\frac{r_{0_s}}{r_{0_r}} = \frac{X_{0_s}}{X_{0_r}} = - \frac{X_{\mu_{0s}}}{X_{\mu_{0r}}} = - K \qquad (46)$$

where constant K, which we call "dissimilarity factor" equals:

$$K = \frac{1}{u^2} \cdot \frac{R_s^2 + X_{\sigma_s}^2 + 2 X_{\sigma_s} X_{\mu_s} + X_{\mu_s}^2 \left(1 - \dfrac{u^2}{\ddot{u}^2}\right)}{R_r^2 + X_{\sigma_r}^2 + 2 X_{\sigma_r} X_{\mu_s} + X_{\mu_s}^2 \left(1 - \dfrac{\ddot{u}^2}{u^2}\right)} \qquad (46a)$$

and equals $\ddot{u}^2$ in a symmetric machine;

Equations (42b) and (44) are the general solution of the main equations. They provide magnitude and phase of stator or rotor current (and thus output, torque and so on) for any chosen value of the parameter r_s or r_r.

The analysis of Mr. Herschdorfer is essentially limited to the proof of the current circle; the diameter of this circle and the sweep momentum were calculated only approximately.

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1) The term "symmetric machine stands for a machine with $R_r' = R_s$, $X_{\sigma_r}' = X_{\sigma_s}$ and $u = \ddot{u}$ (see chapter (1.7.)).

2) The positive sign before the square root is valid when:

$R_s^2 + X_{\sigma_s}'^2 + 2 X_{\sigma_s}' X_{\mu_r} + X_{\mu_r}^2\left(1 - \dfrac{u^2}{\ddot{u}^2}\right)$ is > 0, and the negative sign is valid when:

$R_s'^2 + X_{\sigma_s}'^2 + 2 X_{\sigma_s}' X_{\mu_r} + X_{\mu_r}^2\left(1 - \dfrac{u^2}{\ddot{u}^2}\right)$ is < 0.

After the equations (42b) and (44) we can, in a very simple way, exactly calculate all characteristic quantities of the machine and also determine the circuit exactly.

Inner Output and Torque:

With the aid of the compensation constants we can write the induced voltage of the stator as follows:

$$\mathfrak{E}_s = -\mathfrak{I}_s \left[r_{0_s} + j X_{i_s} + \frac{j r_s X_{\mu_{es}}}{r_s + j X_{\mu_{es}}} \right] \qquad \text{(Gl. (22), (42a) und (43))}$$

$$\mathfrak{E}_s = -\mathfrak{I}_s \left[\left(r_{0_s} + \frac{r_s X_{\mu_{es}}^2}{r_s^2 + X_{\mu_{es}}^2} \right) + j \left(X_{i_s} + \frac{r_s^2 X_{\mu_{es}}}{r_s^2 + X_{\mu_{es}}^2} \right) \right] \qquad (47)$$

The effective or the blind component of the inner output equals the real or imaginary component of the product $\mathfrak{I}_s{}^* \mathfrak{E}_s$. According to the above it is:

$$\mathfrak{I}_s{}^* \mathfrak{E}_s = -J_s^2 \left[\left(r_{0_s} + \frac{r_s X_{\mu_{es}}^2}{r_s^2 + X_{\mu_{es}}^2} \right) + j \left(X_{i_s} + \frac{r_s^2 X_{\mu_{es}}}{r_s^2 + X_{\mu_{es}}^2} \right) \right]$$

This results in the effective component of the inner output of the stator per phase:

$$P_{i_s} = -J_s^2 \left(r_{0_s} + \frac{r_s X_{\mu_{es}}^2}{r_s^2 + W_{\mu_{es}}^2} \right)$$

From equation (42b) it can be concluded:

$$J_s^2 = \frac{U_s^2}{\left(R_{e_s} + \frac{r_s X_{\mu_{es}}^2}{r_s^2 + X_{\mu_{es}}^2} \right)^2 + \left(X_{e_s} + \frac{r_s^2 X_{\mu_{es}}}{r_s^2 + X_{\mu_{es}}^2} \right)^2}$$

Thus:

$$P_{i_s} = \frac{U_s^2 \left(r_{0_s} + \frac{r_s X_{\mu_{es}}^2}{r_s^2 + X_{\mu_{es}}^2} \right)}{\left(R_{e_s} + \frac{r_s X_{\mu_{es}}^2}{r_s^2 + X_{\mu_{es}}^2} \right)^2 + \left(X_{e_s} + \frac{r_s^2 X_{\mu_{es}}}{r_s^2 + X_{\mu_{es}}^2} \right)^2}$$

$$= \frac{U_s^2 (r_{0_s} r_s^2 + r_{0_s} X_{\mu_{es}}^2 + r_s X_{\mu_{es}}^2)}{r_s^2 [R_{e_s}^2 + (X_{e_s} + X_{\mu_{es}})^2] + X_{\mu_{es}}^2 (R^2 + X_{e_s}^2 + 2 r_s R_{e_s})}$$

Since $P_{is} = P_{ir}$, the complete inner output of the machine is:

$$P_i = -6 U_s^2 \frac{r_{0_s} r_s^2 + X_{\mu_{es}}^2 (r_{0_s} + r_s)}{r_s^2 [R_{e_s}^2 + (X_{e_s} + X_{\mu_{es}})^2] + X_{\mu_{es}}^2 (R_{e_s}^2 + X_{e_s}^2 + 2 r_s R_{e_s})} \quad \text{Watt} \quad (48)$$

28

With the equation (48) the inner output for any chosen value of the parameter r_S can be calculated.

The torque of the machine results, it is:

$$M_d = 0.975 \frac{P_i}{n} \ \text{m·kg}$$

$$M_d = \frac{5.85\, U_s^2}{n} \cdot \frac{r_{0_s} r_s^2 + X_{\mu_{os}}^2 (r_{0_s} + r_s)}{r_s^2 [R_{r_s}^2 + (X_{r_s} + X_{\mu_{os}})^2] + X_{\mu_{os}}^2 (R_{r_s}^2 + X_{r_s}^2 + 2 r_s R_{r_s})} \ \text{m·kg} \quad (49)$$

where: $\qquad\qquad$ n = rotation, t/min.

The sweep momentum of the machine can be calculated, when:

$$\frac{d(M_d)}{d(r_s)} = 0$$

or:

$$(X_{\mu_{os}}^2 + 2 r_s r_{0_s}) \cdot [r_s^2 \{R_{r_s}^2 + (X_{r_s} + X_{\mu_{os}})^2\} + X_{\mu_{os}}^2 \{R_{r_s}^2 + X_{r_s}^2 + 2 r_s R_{r_s}\}]$$

$$- [r_{0_s}(r_s^2 + X_{\mu_{os}}^2) + r_s X_{\mu_{os}}^2] \cdot [2 r_s \{R_{r_s}^2 + (X_{r_s} + X_{\mu_{os}})^2\} + 2 R_{r_s} X_{\mu_{os}}^2] = 0$$

The squared equation in r_S has two solutions:

$$r_{s_{max.\,Mot.}} = -A + B \quad (50a) \qquad\qquad r_{s_{max \cdot Gen.}} = -A - B \quad (50b)$$

$$A = \frac{r_{0_s} X_{\mu_{os}} (X_{\mu_{os}} + 2 X_{r_s})}{R_{r_s}^2 + (X_{r_s} + X_{\mu_{os}})^2 - 2 r_{0_s} R_s - 2 r_{0_s}^2}$$

$$B = \frac{X_{\mu_{os}}}{2} \cdot \frac{\left[\sqrt{(R_{r_s}^2 + X_{r_s}^2)(R_{r_s}^2 + (X_{r_s} + X_{\mu_{os}})^2) + 8 r_{0_s}^2 \left(X_{r_s}^2 + X_{r_s} X_{\mu_{os}} - R_{r_s}^2 + \frac{r_{0_s}^2}{2} \right)} \right]}{R_{r_s}^2 + (X_{r_s} + X_{\mu_{os}})^2 - 2 r_{0_s} R_{r_s} - 2 r_{0_s}^2}$$

Equations (50a) and (50b) give the value of the parameter for maximum torque. Equation (50a) refers to motor operation, whereas equation (50b) refers to generator operation. By substituting these values in equation (48) or (49), the maximum inner output or the sweep momentum of the machine are obtained.

We now want to determine the two values of the parameter for $P_i = 0$. This condition is met when:

$$r_{0_s} r_s^2 + X_{\mu_{os}}^2 (r_{0_s} + r_s) = 0$$

$$r_s^2 r_{0_s} + r_s X_{\mu_{os}}^2 + r_{0_s} X_{\mu_{os}}^2 = 0$$

$$r_s = \frac{- X_{\mu_{os}}^2 \pm \sqrt{X_{\mu_{os}}^4 - 4 r_{0_s}^2 X_{\mu_{os}}^2}}{2 r_{0_s}}$$

$$r_{s_1} = -\frac{X_{\mu_{os}}}{2 r_{0_s}} \left(X_{\mu_{os}} + \sqrt{X_{\mu_{os}}^2 - 4 r_{0_s}^2} \right) \qquad (51a)$$

$$r_{s_2} = -\frac{X_{\mu_{os}}}{2 r_{0_s}} \left(X_{\mu_{os}} - \sqrt{X_{\mu_{os}}^2 - 4 r_{0_s}^2} \right) \qquad (51b)$$

From equations (51a) and (51b) it is evident that r_{s_1} and r_{s_2} always have the same sign which is opposite to that of r_{0_s}. Furthermore, $|r_{0_s}|$ in practice is always much smaller than $|X_{\mu_{os}}|$ and thus we see that $|r_{s_1}|$ is very large whereas $|r_{s_2}|$ is very small. r_{s_1} actually refers to the ideal no-load operation (without iron and friction losses), whereas r_{s_2} refers to a stage of operation during which the two flow waves $\mathfrak{F}_s$ and $\mathfrak{F}_r$ are in exact opposition – in the case of an asymmetric machine – one of them is solely magnetizing and the other solely de-magnetizing, whereas the currents are perpendicular to the respective induced voltages. This state will be treated more explicitly later.

4. The Graphic Theory

With the aid of compensation constants we can develop a useful model of the machine. The solution for stator current is, according to equation (42b):

$$\mathfrak{J}_s = -\frac{-\mathfrak{u}_s}{R_{e_s} + j X_{e_s} + \dfrac{j r_s X_{\mu_{os}}}{r_s + j X_{\mu_{os}}}}$$

substituting:

$$\mathfrak{Z}_{e_s} = \frac{-\mathfrak{u}_s}{\mathfrak{J}_s}$$

we get as compensation impedance of the stator:

$$\mathfrak{Z}_{e_s} = R_{e_s} + j X_{e_s} + \frac{j r_s X_{\mu_{os}}}{r_s + j X_{\mu_{os}}}$$

$$\mathfrak{Z}_{e_s} = R_{e_s} + j X_{e_s} + \cfrac{1}{\cfrac{1}{r_s} + \cfrac{1}{j X_{..}}} \qquad (52a)$$

likewise for the rotor:

$$\mathfrak{Z}_{e_r} = R_{e_r} + j X_{e_r} + \cfrac{1}{\cfrac{1}{r_r} + \cfrac{1}{j X_{\mu_{or}}}} \qquad (52b)$$

30

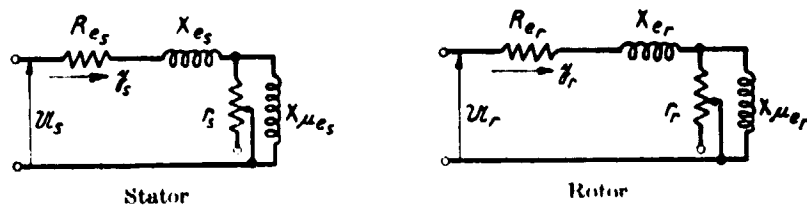It is clear that the circuits given in Fig. 14 represent this impedance completely:

Fig. 14. Compensation circuits of an asymmetric d.s.m. general case.

Since we have reduced the solution of the main equations to this simple form no further proof is needed to show that each of the currents $\Im_s$ and $\Im_r$ have a <u>circle</u> as geometric locus. It is obvious from these known circles that the impedance vector $\Im_{e_s}$ or $\Im_{e_r}$ describes a circle when r_s or r_r varies from $-\infty$ to $+\infty$. It can be concluded that the geometric locus of the current $\Im_s$ or $\Im_r$ also is a circle (as inversion of a circle). This impedance or current circuit of the stator is represented in Fig. 15. The impedance circle is first constructed after equation (52a). By inversion of this circle around zero, the circle of the stator-compensation admittance $\mathfrak{Y}_{e_s} = \dfrac{1}{\Im_{e_s}}$ results. By multiplying the $\mathfrak{Y}_{e_s}$ circle with the vector jU_s , the $\Im_s$ circle results. Of course similar circles apply to the rotor.

The coordinates of the centres or radii of the various circles can be calculated directly. They are:

Stator impedance circuit:

Coordinates:

$$x = R_{e_s}$$

$$y = X_{e_s} + \frac{X_{\mu e_s}}{2}$$

Radius:

$$R = \frac{X_{\mu e_s}}{2}$$

(53a)

Stator circuit:

Coordinates:

$$x = U_s \cdot \frac{X_{e_s} + \dfrac{X_{\mu_{es}}}{2}}{R_{e_s}^2 + X_{e_s}^2 + X_{e_s} X_{\mu_{es}}}$$

$$y = U_s \cdot \frac{R_{e_s}}{R_{e_s}^2 + X_{e_s}^2 + X_{e_s} X_{\mu_{es}}} \qquad (53\,b)$$

Radius:

$$R = U_s \cdot \frac{\dfrac{X_{\mu_{es}}}{2}}{R_{e_s}^2 + X_{e_s}^2 + X_{e_s} X_{\mu_{es}}}$$
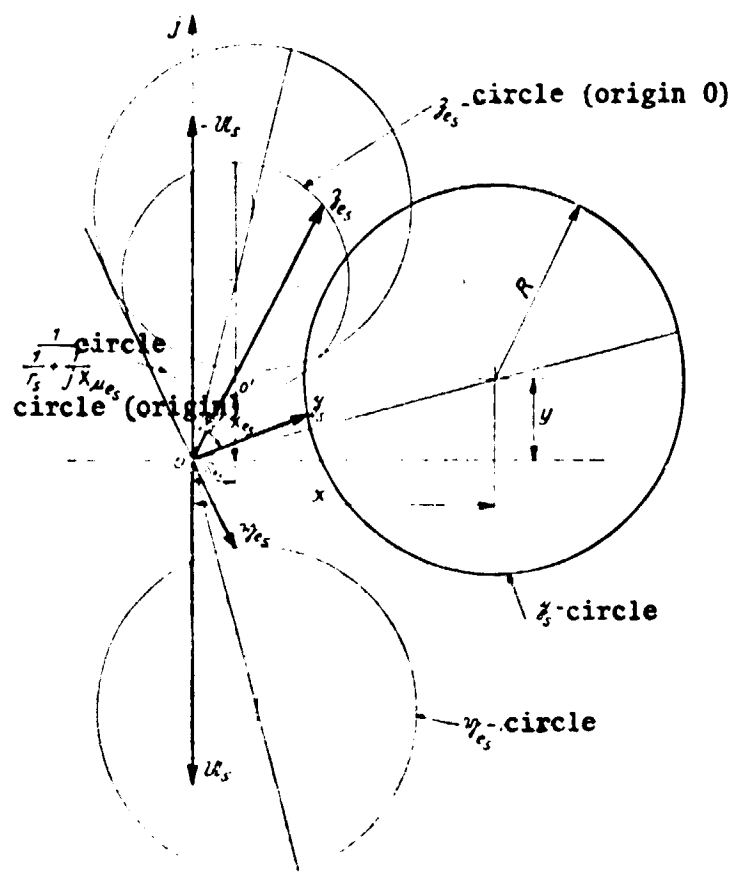
Fig. 15. Impedance and current circle of the stator.

Rotor impedance circle

Coordinates:

$$x = R_{e_r}$$
$$y = X_{e_r} + \frac{X_{\mu_{rr}}}{2}$$

$$\left.\begin{array}{l}\end{array}\right\} \quad (53c)$$

Radius:

$$R = \frac{X_{\mu_{rr}}}{2}$$

Rotor circuit

Coordinates:

$$x = U_r \cdot \frac{X_{e_r} + \dfrac{X_{\mu_{rr}}}{2}}{R_{e_r}^2 + X_{e_r}^2 + X_{e_r} X_{\mu_{rr}}}$$

$$y = U_r \cdot \frac{R_{e_r}}{R_{e_r}^2 + X_{e_r}^2 + X_{e_r} X_{\mu_{rr}}}$$

$$(53d)$$

Radius:

$$R = U_r \cdot \frac{\dfrac{X_{\mu_{rr}}}{2}}{R_{e_r}^2 + X_{e_r}^2 + X_{e_r} X_{\mu_{rr}}}$$

Determination of Inner Output and Torque:

According to equations (16) we write the relationship between terminal output P_{k_s} and inner output P_{i_s} of a stator phase as follows:
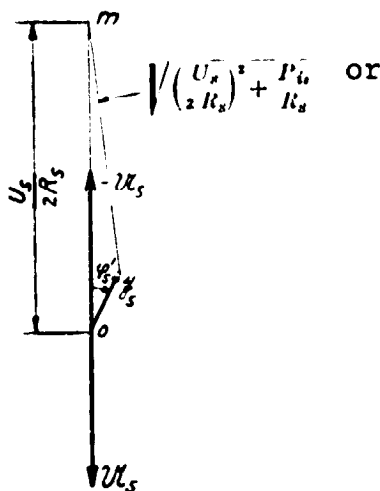
$$P_{k_s} = P_{i_s} - J_s^2 R_s$$

$$P_{i_s} = -U_s J_s \cos\varphi_s' + J_s^2 R_s$$

φ_s' stands for the phase angle between the current vector $\mathfrak{J}_s$ and the terminal voltage drop $-\mathfrak{U}_s$ of the stator. This relationship can also be written:

$$J_s^2 - \frac{U_s}{R_s} J_s \cos\varphi_s' = \frac{P_{i_s}}{R_s}$$

Adding $\left(\dfrac{U_s}{2 R_s}\right)^2$ to both sides we get:

$$J_s^2 + \left(\frac{U_s}{2 R_s}\right)^2 - 2\left(\frac{U_s}{2 R_s}\right) J_s \cos\varphi_s' = \left(\frac{U_s}{2 R_s}\right)^2 + \frac{P_{i_s}}{R_s}$$

Fig. 16.

If a point $\underline{m}$ on vector $\mathfrak{ll}$, in Fig. 16 is determined such that $om = \frac{U_s}{2R_s}$,
the distance between point $\underline{m}$ and the final point of the vector $\mathfrak{J}$,
is $\sqrt{\left(\frac{U_s}{2R_s}\right)^2 + \frac{P_{ii}}{R_s}}$.

If we draw a circle in Fig. 15, whose center is $\underline{m}$ and

$\sqrt{\left(\frac{U_s}{2R_s}\right)^2 + \frac{P_{ii}}{R_s}}$ (for a chosen value of P_{is}) as radius, it forms at its

points of intersection with the circuit those two points where the
inner output equals the assumed value P_{is} .

For $P_{is} = 0$, the radius is $\frac{U_s}{2R_s}$ and the circle passes through
zero and forms two points of intersection P_{00} and P_k where the inner
output equals 0 (Fig. 17). We now consider yet another circle which
is drawn for an inner ouput $P_{is} \neq 0$ and forms two points of inter-
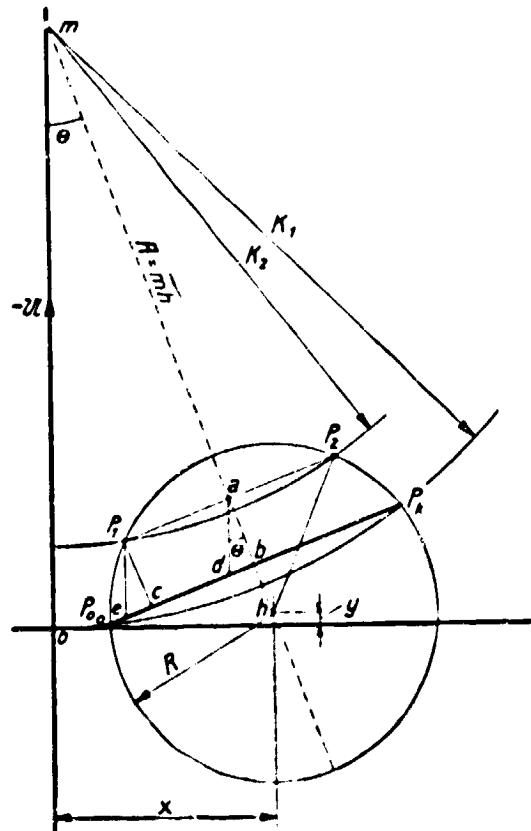section P_1 and P_2.

Fig. 17. Geometric characteristics of the circuit.

The following relationships can be seen from Fig. 17.

$$R^2 = hb^2 + b\,P_k^2 \quad \text{and} \quad R^2 = ha^2 + a\,P_2^2$$
$$= hb^2 + K_1^2 - mb^2 \qquad = ha^2 + K_2^2 - ma^2$$
$$= K_1^2 + A\,(hb - mb) \qquad = K_2^2 + A\,(ha - ma)$$

It can be concluded:

$$K_1^2 - K_2^2 = A\,|(ha + mb) - (ma + hb)|$$
$$= A\,|(A + ab) - (A - ab)|$$
$$= 2\,A\,ab$$

Since $K_1 = \dfrac{U_s}{2R_s}$ and $K_2 = \sqrt{\left(\dfrac{U_s}{2R_s}\right)^2 + \dfrac{P_{is}}{R_s}}$ this relationship can be written as follows:

$$-\frac{P_{is}}{R_s} = 2\,A\,ab$$
$$-P_{is} = 2\,R_s\,A\,ab$$

Since $2\,R_s A$ is a constant, it becomes clear from this relationship that the section <u>ab</u> represents the inner output of a stator phase. (scale $2R_sA$). The straight line P00 Pk is consequently called <u>power line</u>. The relationship also shows that the inner output is negative for positive values of <u>ab</u>, i.e., the upper part of the circle between P00 and Pk refers to a negative output or motor operation, whereas the lower part refers to generator operation. The inner output at the two points P00 and Pk equals 0.

Since the angle θ has a fixed value, it must be concluded that the vertical section <u>ad</u> (which equals $\dfrac{ab}{\cos\theta}$) also represents the inner output (in a scale $2R_sA\cos\theta$), this scale may be written:

$$S_s = 2\,R_s\,A\cos\theta = 2\,R_s\,A \cdot \frac{\dfrac{U_s}{2R_s} - y}{A} = U_s - 2\,R_s\,y \qquad (53e)$$

In practice we may in some cases neglect the quantity y compared to $\dfrac{U_s}{2R_s}$, so that U_s can simply be assumed for the scale.

The necessary quantities for the formation of the circuits of the stator or rotor are $R_s, R_r, X_{\sigma s}, X_{\sigma r}, X_\mu$ and X_μ . These can be measured or calculated by familiar methods. The compensation constants are calculated from equations (43) or (45) and the coordinates of the center and the radius of the circuit of stator or rotor result (equation (53b)/(53d)). Next, the output line of each circle is calculated by constructing a circle with the radius /43 $\dfrac{U_s}{2R_s}$ or $\dfrac{U_r}{2R_r}$, as explained earlier. The circuits then are complete, and current power factors, terminal and inner output as well as torque of stator or rotor, can be deduced . It remains

35

only the question how to determine the <u>corresponding values of stator</u>
<u>and rotor</u>. This can be done in a simple manner with the help of the
fact that P_{i_s} is always equal to P_{i_r}. For this purpose, stator and
rotor circuits can be drawn as in Fig. 18.

a: rotor power line
b: stator power line
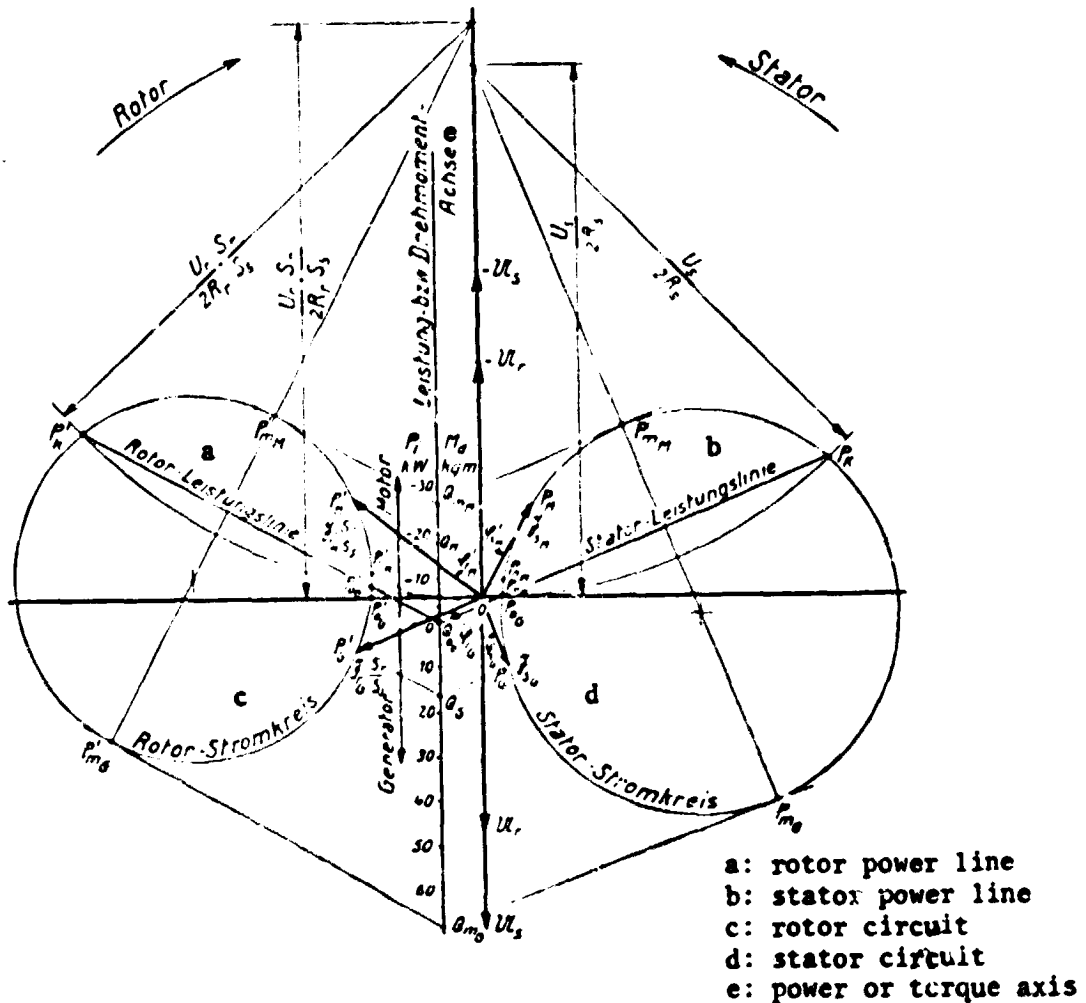c: rotor circuit
d: stator circuit
e: power or torque axis

Fig. 18. Circle diagrams of an asymmetric machine.

The rotor circle is drawn to the left of the origin in inverse se-
quence and its measures are multiplied by quantity $\frac{S_r}{S_s}$. S_s
stands for the scale of the inner output of the stator (as
calculated in equation (53e)) and S_r stands for the similarly
calculated scale of the inner output of the rotor. The scale of the
inner output of both circles thus is S_s. When we extrapolate the
power line of stator or rotor and draw a perpendicular line through
its point of intersection Q_{00}, each point on this straight line,
which we call output axis, represents a certain output or torque.
The corresponding points on the stator or rotor circle are reached
by the two straight lines $Q_{M} P_{M}$ and $Q'_{M} P'_{M}$ through this point,

running parallel to the respective power lines $P_{00}P_k$ and $P'_{00}P'_k$.
The corresponding stator or rotor current is OP_M or $\frac{N_s}{N_r} \cdot OP'_M$. It
is clear that the two circles are coupled with the help of the power
axis. We can then determine the corresponding point on the stator
or rotor circle for any chosen value of the inner output of the
machine or the torque. For any one point on either of the circles,
the power axis provides the corresponding point on the other circle,
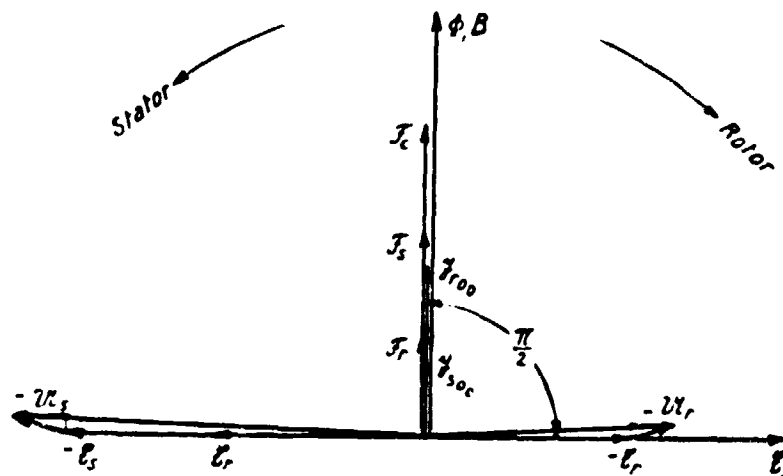it also provides the inner output and the torque of the machine.

Fig. 19. Combined space and time vector diagram of an
asymmetric machine during ideal no-load
operation.

The two points P_{00} and P'_{00} refer to the ideal no-load operation.
These points can practically be reached by working the machine
mechanically and feeding it with a mechanical power covering the
iron, friction and additional losses. Fig. 19 shows the combined
space and time vector diagram for this stage. The inner output
equals zero and the two flux waves coincide. The stator or rotor net
provides an output to stator or rotor which equals the copperloss
during no-load operation in the stator or rotor. This loss is re-
presented by the ordinate of point P_{00} or P'_{00}. The corresponding
value of the stator parameter r_s follows from equation (51a).

The actual no-load operation as motor is represented by points
P_{0M} and P'_{0M}. Naturally, these are slightly higher than P_{00} and
P'_{00}, since the outputs taken up by the nets now also have to cover
the iron, friction and additional losses. The real no-load operation
as generator is represented by point P_{0G} or P'_{0G}.

The inner power equals zero also in point P_k or P'_k. This state somehow resembles the short circuit state of the asynchronous machine, but has another meaning here. This becomes feasible when looking at the combined space and time vector diagram. Fig. 20 shows
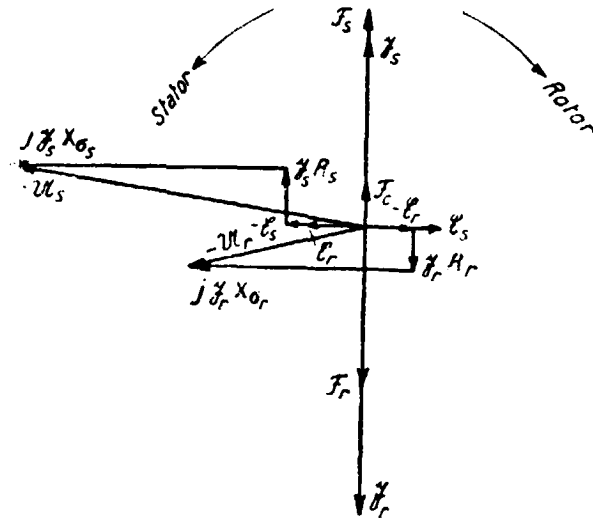

Fig.20. Combined space and time vector diagram of an asymmetric machine for the state represented by points P_k or P'_k.

this diagram for an asymmetric machine.

Stator or rotor current is perpendicular to the induced voltage $\mathfrak{C}_s$ and $\mathfrak{C}_r$ the one leads its respective voltage, the other lags. Both flux waves $\mathfrak{F}_s$ and $\mathfrak{F}_r$ are in exact opposition, the resulting wave $\mathfrak{F}_c$ then is very small. The power taken in by the net is translated in the machine as copper loss. The induced voltages are much smaller during no-load operation or normal operation due to the large inductive voltage drops. This fact is also true for the short cir-
induction machine. In this case, however, the machine theoretically is still <u>running</u>. Another difference is the torque which here is zero.

5. The Solution of the Main Equation for the Cases K= ∞ and K=0

(5.1.) The Unilateral Rotor Magnetization

$$K = \infty \quad \text{or} \quad R_r^2 + X_{\sigma_r}^2 + 2 X_{\sigma_r} X_{\mu_r} + X_{\mu_r}^2 \left(1 - \frac{u^2}{u^2}\right) = 0 \tag{46a}$$

Under these condition and ideal no-load operation, the rotor takes in all the magnetizing current needed for the generation of the machine field; the stator current remains zero[1]. This case we call "unilateral rotor magnetization". The constants r_{0_s}, X_{0_s} and $X_{\mu_{0_s}}$ are in this case infinitely large (equations (38a), (38b) and (40a)). Substituting equations (40a) and (40b) in equation (39) is not valid, since now p_s is undetermined. This is due to the fact that the solution (equation 37) of the squared equation (36) also provides an undetermined value for $\frac{J_r'}{J_s}$ in this case. The solution for $\frac{J_r'}{J_s}$ shall be gained directly from equation (36) in this case, and the solution for the stator current shall be expressed anew. It is, however, clear, that the rotor compensation constants or rotor circuit result from the above-mentioned general expressions (equation 45).

The solution of equation (36) under these conditions is as follows:

$$\frac{J_r'}{J_s} = \frac{X_{\mu_s}^2\left(1 - \frac{u^2}{u^2}\right) - R_s^2 \frac{u^2}{u^2} - X_{\sigma_s}^2 \frac{u^2}{u^2} - 2 X_{\sigma_s} X_{\mu_s} \frac{u^2}{u^2}}{2 X_{\mu_s}\left\{\left(R_r' - \frac{u^2}{u^2} R_s\right)\sin\Psi - \left(X_{\sigma_r}' - \frac{u^2}{u^2} X_{\sigma_s} + X_{\mu_s}\left[1 - \frac{u^2}{u^2}\right]\right)\cos\Psi\right\}}$$

/47

With the help of the introduced constant defined by equations (38c) and (38d) this expression can be simplified:

---

1) See chapter (7.3.)

$$\frac{J_r'}{J_s} \cdot \frac{2\ddot{u}^2}{X_{\mu_s}} = \frac{2\ddot{u}^2}{X_{\mu_s}} \cdot \frac{X_{\mu_s}^2\left(1-\frac{\ddot{u}^2}{u^2}\right) - R_s^2\frac{\ddot{u}^2}{u^2} - X_{\sigma_s}^2\frac{\ddot{u}^2}{u^2} - 2X_{\sigma_s}X_{\mu_s}\frac{\ddot{u}^2}{u^2}}{2X_{\mu_s}\left\{\left(R_s\frac{\ddot{u}^2}{u^2}-R_r'\right)\sin\Psi - \left(X_{\sigma_s}\frac{\ddot{u}^2}{u^2}-X_{\sigma_r}'-X_{\mu_s}\left[1-\frac{\ddot{u}^2}{u^2}\right]\right)\cos\Psi'\right\}}$$

$$= \frac{X_{\mu_s}^2\left(1-\frac{\ddot{u}^2}{u^2}\right) - R_s^2\frac{\ddot{u}^2}{u^2} - X_{\sigma_s}^2\frac{\ddot{u}^2}{u^2} - 2X_{\sigma_s}X_{\mu_s}\frac{\ddot{u}^2}{u^2}}{\frac{X_{\mu_s}^2}{\ddot{u}^2}\left(R_s\frac{\ddot{u}^2}{u^2}-R_r'\right)\sin\Psi - \frac{X_{\mu_s}^2}{\ddot{u}^2}\left(X_{\sigma_s}\frac{\ddot{u}^2}{u^2}-X_{\sigma_r}'-X_{\mu_s}\left[1-\frac{\ddot{u}^2}{u^2}\right]\cos\Psi'\right)}$$

$$= \frac{1}{r_{0_r}\sin\Psi - X_{0_r}\cos\Psi'}$$

or:

$$\frac{J_r'}{J_s} = \frac{\frac{X_{\mu_s}}{2\ddot{u}^2}}{r_{0_r}\sin\Psi - X_{0_r}\cos\Psi}$$

$$-jX_{\mu_s}\frac{J_r'}{J_s}e^{j\Psi} = -j\frac{X_{\mu_s}^2}{2\ddot{u}^2}\cdot\frac{\cos\Psi + j\sin\Psi}{r_{0_r}\sin\Psi - X_{0_r}\cos\Psi}$$

$$= \frac{X_{\mu_s}^2}{2\ddot{u}^2}\cdot\frac{\sin\Psi - j\cos\Psi}{r_{0_r}\sin\Psi - X_{0_r}\cos\Psi'}$$

By substituting this expression in equation (34a) the following results:

$$-\mathfrak{U}_s = \mathfrak{J}_s\left[R_s + j(X_{\sigma_s}+X_{\mu_s}) + \frac{X_{\mu_s}^2}{2\ddot{u}^2}\cdot\frac{\sin\Psi - j\cos\Psi}{r_{0_r}\sin\Psi - X_{0_r}\cos\Psi}\right]$$

It results the stator compensation impedance:

$$\mathfrak{Z}_{e_s} = \frac{-\mathfrak{U}_s}{\mathfrak{J}_s}$$

$$= R_s + j(X_{\sigma_s}+X_{\mu_s}) + \frac{X_{\mu_s}^2}{2\ddot{u}^2}\cdot\frac{\sin\Psi - j\cos\Psi}{r_{0_r}\sin\Psi - X_{0_r}\cos\Psi}$$

$$= R_s + j\left(X_{\sigma_s}+X_{\mu_s}+\frac{X_{\mu_s}^2}{2\ddot{u}^2 X_{0_r}}\right) + \frac{X_{\mu_s}^2}{2\ddot{u}^2}\cdot\left(\frac{\sin\Psi - j\cos\Psi}{r_{0_r}\sin\Psi - X_{0_r}\cos\Psi} - \frac{j}{X_{0_r}}\right)$$

$$= R_s + j\left(X_{\sigma_s}+X_{\mu_s}+\frac{X_{\mu_s}^2}{2\ddot{u}^2 X_{0_r}}\right) + \frac{X_{\mu_s}^2}{2\ddot{u}^2}\cdot\left(\frac{X_{0_r}\sin\Psi - jX_{0_r}\cos\Psi - jr_{0_r}\sin\Psi + jX_{0_r}\cos\Psi}{X_{0_r}(r_{0_r}\sin\Psi - X_{0_r}\cos\Psi)}\right)$$

$$= R_s + j\left(X_{\sigma_s} + X_{\mu_s} + \frac{X_{\mu_s}^2}{2\,u^2 X_{0_r}}\right) + \frac{X_{\mu_s}^2}{2\,u^2} \cdot \left(\frac{\sin\Psi}{r_{0_r}\sin\Psi - X_{0_r}\cos\Psi} - j\frac{r_{0_r}\sin\Psi}{X_{0_r}(r_{0_r}\sin\Psi - X_{0_r}\cos\Psi)}\right)$$

$$= R_s + j\left(X_{\sigma_s} + X_{\mu_s} + \frac{X_{\mu_s}^2}{2\,u^2 X_{0_r}}\right) + \frac{X_{\mu_s}^2 \cdot \sqrt{r_{0_r}^2 + X_{\mu_s}^2} \cdot \sin\Psi}{2\,u^2 X_{0_r}(r_{0_r}\sin\Psi - X_{0_r}\cos\Psi)}\left[\frac{X_{0_r}}{\sqrt{r_{0_r}^2 + X_{0_r}^2}} - j\frac{r_{0_r}}{\sqrt{r_{0_r}^2 + X_{0_r}^2}}\right]$$

Substituting the new parameter:

$$Z_{0_s} = \frac{X_{\mu_s}^2 \cdot \sqrt{r_{0_r}^2 + X_{0_r}^2} \cdot \sin\Psi}{2\,u^2 X_{0_r}(r_{0_r}\sin\Psi - X_{0_r}\cos\Psi)} \tag{54a}$$

and the constant angle

$$\Theta_s = \text{arc tg}\ \frac{-r_{0_r}}{X_{0_r}} \qquad [0 \leqslant \Theta_s \leqslant \pi] \tag{54b}$$

for $\mathfrak{Z}_{e_s}$ in this expression, we obtain:

$$\mathfrak{Z}_{e_s} = R_s + j\left(X_{\sigma_s} + X_{\mu_s} + \frac{X_{\mu_s}^2}{2\,u^2 X_{0_r}}\right) + Z_{0_s}\,e^{j\Theta_s}$$

or:

$$\mathfrak{Z}_{e_s} = R_{e_s} + j X_{e_s} + Z_{0_s}\,e^{j\Theta_s} \tag{55}$$

where:

$$R_{e_s} = R_s \qquad\qquad X_{e_s} = X_{\sigma_s} + X_{i_s} = X_{\sigma_s} + X_{\mu_s} + \frac{X_{\mu_s}^2}{2\,u^2 X_{0_r}}$$

$$\Theta_s = \text{arc tg}\ \frac{-r_{0_r}}{X_{0_r}} \qquad [0 \leqslant \Theta_s \leqslant \pi]$$

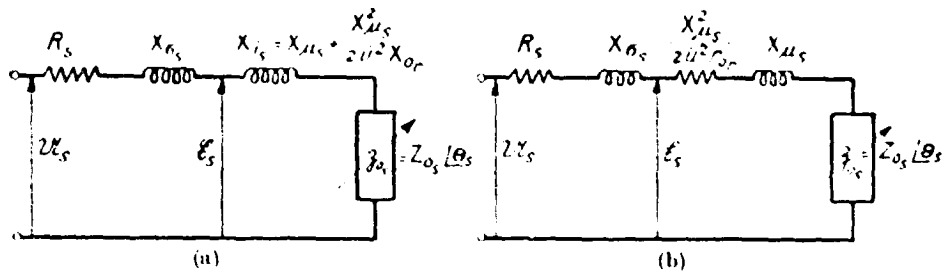
Fig. 21. Two possible compensation circles for the stator for $K = \infty$
Z_{0_s} = parameter, variable between $-\infty \to +\infty$
$\Theta_s = \text{arc tg}\ \frac{-r_{0_r}}{X_{0_r}} \qquad [0 \leqslant \Theta_s \leqslant \pi]$

41

The expression (55) or Fig. 21a now represent compensation impendance or compensation circuit of the stator for the case K = ∞. . The last member obviously is an impendance with a **constant angle**

θ_x and a variable quantity Z_{0s} which corresponds to the angle Ψ or the conditions of demand. By changing Z_{0s} from $-\infty$ and $+\infty$, $\mathfrak{Z}_{c_s}$ completely represents the stator circle for all theoretically possible operational stages.

The behavior of $\mathfrak{Z}_{c_s}$ in the complex plane is represented in Fig. 22. It is a straight line with the steepness angle θ_x . Without out further proof it can be concluded that the locus of the stator current is a circle since it is obviously derived by inversion of $\mathfrak{Z}_{c_s}$.
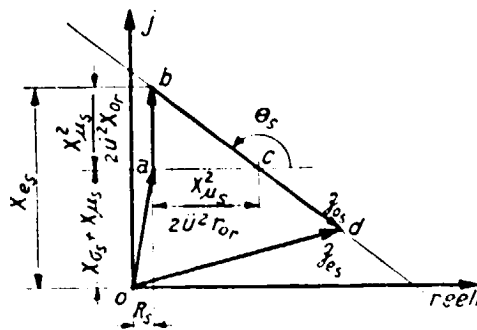

Fig. 22. The behavior of stator compensation impedance $\mathfrak{Z}_{c_s}$ for the case K = ∞ .

It is interesting to note - what can be proven quite simply - that the blind resistance member $j \dfrac{X_{\mu_s}^2}{2\,\bar{u}^2 X_{0r}}$ can be substituted in equation (55) by a Ohm member $\dfrac{X_{\mu_s}^2}{2\,\bar{u}^2 r_{0r}}$ without any further change in equation (55). This can also clearly be seen in Fig. 22 by considering impedance vector $\mathfrak{Z}_{c_s}$ to be the sum of the vectors oa + ac + cd instead of the sum oa + ab + bd. In the first case the compensation circle of Fig. 21b results, in the second case the compensation circle of Fig. 21a as defined by equation (55) is obtained. It is of advantage, however, to use the compensation circle of 21a or equation (55) since the inner output in this case is represented by the output of a single resistance, namely the Ohm part of impedance $\mathfrak{Z}_{0_s}$. This fact causes a certain simplification in the analytical solution after the compensation circles.

42

The Inner Output and the Torque

For a chosen value of the parameter Z_{0_s}, the induced voltage of the stator can be written as follows:

$$\mathfrak{C}_s = -\mathfrak{J}_s [j X_{i_s} + \mathfrak{Z}_{0_s}]$$
$$= -\mathfrak{J}_s [Z_{0_s} \cos\Theta_s + j(X_{i_s} + Z_{0_s} \sin\Theta_s)] \qquad (56)$$

The effective component of the inner output then equals the real component of the product $\mathfrak{J}_s^* \mathfrak{C}_s$.

Combined with equation (56):

$$\mathfrak{J}_s^* \mathfrak{C}_s = -J_s^2 [Z_{0_s} \cos\Theta_s + j(X_{i_s} + Z_{0_s} \sin\Theta_s)]$$

Thus, the inner output of the stator per phase is: /50

$$P_{i_s} = -J_s^2 Z_{0_s} \cos\Theta_s$$

The magnitude of stator current J_S is, according to the compensation circle:

$$J_s = \frac{U_s}{Z_{e_s}} = \frac{U_s}{\sqrt{(R_{e_s} + Z_{0_s} \cos\Theta_s)^2 + (X_{e_s} + Z_{0_s} \sin\Theta_s)^2}}$$

Thus:

$$P_{i_s} = -\frac{U_s^2 Z_{0_s} \cos\Theta_s}{(R_{e_s} + Z_{0_s} \cos\Theta_s)^2 + (X_{e_s} + Z_{0_s} \sin\Theta_s)^2}$$

The complete inner output of the machine then is:

$$P_i = -6 U_s^2 \cdot \frac{Z_{0_s} \cos\Theta_s}{R_{e_s}^2 + X_{e_s}^2 + Z_{0_s}^2 + 2 Z_{0_s}(R_{e_s} \cos\Theta_s + X_{e_s} \sin\Theta_s)} \quad \text{Watt} \qquad (57)$$

The torque is:

$$M_d = 0{,}975 \frac{P_i}{n} \ \text{m} \cdot \text{kg}$$

$$M_d = \frac{5{,}85 \, U_s^2}{n} \cdot \frac{Z_{0_s} \cos\Theta_s}{R_{e_s}^2 + X_{e_s}^2 + Z_{0_s}^2 + 2 Z_{0_s}(R_{e_s} \cos\Theta_s + X_{e_s} \sin\Theta_s)} \ \text{m} \cdot \text{kg} \qquad (58)$$

where: n = number of revolutions, r/min.

43

Equation (57) or (58) gives the amount of inner output or torque of the machine for any value of the parameter Z_{0s}.

The sweep momentum of the machine can be calculated when:

$$\frac{d(M_d)}{d(Z_{0_s})} = 0$$

or:

$$Z_{0s_m} \cdot \cos\Theta_s \cdot \{2\, Z_{0s_m} + 2(R_{c_s}\cos\Theta_s + X_{c_s}\sin\Theta_s)\}$$
$$= \{R_{c_s}^2 + X_{c_s}^2 + Z_{0s_m}^2 + 2\, Z_{0s_m}(R_{c_s}\cos\Theta_s + X_{c_s}\sin\Theta_s)\}\cos\Theta_s$$

which results in:

$$Z_{0s_m} = \pm \sqrt{R_{c_s}^2 + X_{c_s}^2} \qquad\qquad (59)$$

These two values of the parameter correspond to the sweep stage and give the sweep output or momentum of the machine by substituting them in equation (57) or (58).

After equation (57), it is clear that the magnitudes of the parameter Z_{0s}, which correspond to the two stages of $P_i = 0$, have the values 0 and ∞ .

The Circuit Diagram

As already mentioned, in this case too, the final point of the stator current vector is on a circuit. The dimensions of this circle can easily be determined by inversion of the impedance straight line (Fig. 22), as shown in Fig. 23.

The coordinates of the center are:

$$x = \frac{U_s}{2(X_{c_s} - R_{c_s}\,\mathrm{tg}\,\Theta_s)}$$

$$y = \frac{U_s}{2(R_{c_s} - X_{c_s}\,\mathrm{ctg}\,\Theta_s)}$$

the radius of the circle is:

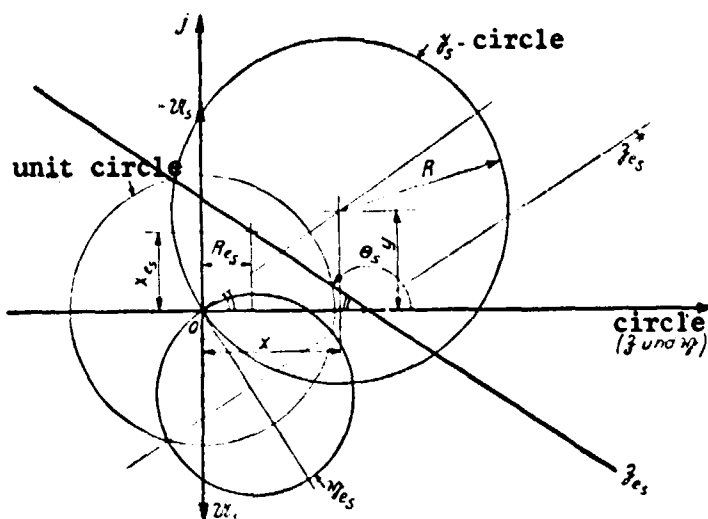$$R = \frac{U_s}{2(R_{c_s}\sin\Theta_s - X_{c_s}\cos\Theta_s)} = \sqrt{x^2 + y^2}$$

$$(60)$$

44

Fig. 23. Calculation of stator circuit for case K = ,_ .

The expressions (60) serve to construct the circle diagram of the stator for case K = η . In this case the general expressions (53d) for the dimensions of the circuit, as well as the results of the general solution and graphic theory (chapters 3 and 4) are also valid. The graphic theory in this case is also valid for the stator when the stator circle is constructed after expressions (60).

(5.2.) The Unilateral Stator Magnetization

$$K = 0 \quad \text{or} \quad R_s^2 + X_{\sigma_s}^2 + 2\,X_{\sigma_s}\,X_{\mu_s} + X_{\mu_s}^2\left(1 - \frac{u^2}{\ddot{u}^2}\right) = 0 \qquad (46\,\text{a})$$

Given the condition K = 0, the stator will also take in all of the magnetizing current during ideal no-lead operation and the rotor current will remain zero. This case will thus be called "unilateral stator magnetization " (see chapter (7.3.)). Likewise, the rotor circuit is in this case undetermined by the expressions of the general solution, whereas the stator circuit is determined by these expressions. The stator circuit is constructed after these expressions and the rotor circuit is treated as above and has similar expressions. The rotor compensation impedance is:

$$\mathfrak{Z}_{c_r} = R_{c_r} + j\,X_{c_r} + Z_{0_r}\,e^{j\theta_r} \qquad (61)$$

45

where:

$$R_{e_r} = R_r$$
$$X_{e_r} = X_{\sigma_r} + X_{i_r}$$
$$= X_{\sigma_r} + X_{\mu_r} + \frac{X_{\mu_r}^2 \cdot \ddot{n}^2}{2 X_{0_s}}$$

Z_{0_r} = Parameter of the rotor circle, variable between $-\infty$ and $+\infty$.

$$Z_{0_r} = \frac{\ddot{n}^2 X_{\mu_r}^2 \cdot \sqrt{r_{0_s}^2 + X_{0_s}^2} \cdot \sin \Psi}{2 X_{0_s}(r_{0_s} \sin \Psi - X_{0_s} \cos \Psi)}$$

$$\Theta_r = \operatorname{arc tg} \frac{-r_{0_s}}{X_{0_s}} \qquad [0 \leqslant \Theta_r \leqslant \pi]$$

R_0, X_0 = the respective stator compensation constants as defined by equations (38a) and (38b).

Similar to the stator equations (56) to (59) for $K = \infty$, rotor equations are valid for K = 0. The dimension of the rotor circuit diagram are:

Coordinates:

$$x = \frac{U_r}{2(X_{e_r} - R_{e_r} \operatorname{tg} \Theta_r)}$$

$$y = \frac{U_r}{2(R_{e_r} - X_{e_r} \operatorname{ctg} \Theta_r)} \qquad\qquad (62)$$

Radius:

$$R = \frac{U_r}{2(R_{e_r} \sin \Theta_r - X_{e_r} \cos \Theta_r)} = \sqrt{x^2 + y^2}$$

6. Special Cases

The above developed analysis is very general and valid for all kinds of operational stages of the d.s. machine. The individual special cases which one would want to use in practice can be developed directly from the above theory. In the first place, there are the parallel and series connections. For these special cases the above results are simplified more or less according to the conditions. We now treat these two special cases for the asym-metric and symmetric machine.

46

Case 1: Parallel connected asymmetric machine

$$\ddot{u} \neq 1, \quad u = 1, \quad \mathfrak{U}_s = \mathfrak{U}_r = \mathfrak{U}, \quad R_s \neq R_r', \quad X_{\sigma_s} \neq X_\sigma'$$

It holds for the dissimilarity factor:

$$K = \frac{R_s^2 + X_{\sigma_s}^2 + 2\,X_{\sigma_s}\,X_{\mu_s} + X_{\mu_s}^2\left(1 - \frac{1}{\ddot{u}^2}\right)}{R_r^2 + X_{\sigma_r}^2 + 2\,X_{\sigma_r}\,X_{\mu_r} + X_{\mu_r}^2(1 - \ddot{u}^2)}$$

and the compensation constants of the stator (equation (43)):

$$R_{c_s} = R_s + r_{0_s} = R_s + X_{\mu_s}^2 \cdot \frac{R_s\,\ddot{u}^2 - R_r'}{R_r'^2 + X_{\sigma_r}'^2 + 2\,X_{\sigma_r}'\,X_{\mu_s} + X_{\mu_s}^2(1 - \ddot{u}^2)}$$

$$X_{c_s} = X_{\sigma_s} + X_{i_s} = X_{\sigma_s} + X_{\mu_s} - X_{\mu 0_s} + X_{0_s}$$

$$= X_{\sigma_s} + X_{\mu_s} - X_{\mu_{cs}} + X_{\mu_s}^2 \cdot \frac{X_{\sigma_s}\,\ddot{u}^2 - X_{\sigma_r}' - X_{\mu_s}(1 - \ddot{u}^2)}{R_r'^2 + X_{\sigma_r}'^2 + 2\,X_{\sigma_r}'\,X_{\mu_s} + X_{\mu_s}^2(1 - \ddot{u}^2)}$$

$$X_{\mu_{cs}} = 2\,X_{\mu 0_s} = 2\sqrt{r_{0_s}^2 + X_{0_s}^2 + K \cdot \frac{X_{\mu_s}^2}{\ddot{u}^2}}$$

and the compensation constants of the rotor:

$$R_{c_r} = R_r + r_{0_r} = R_r - \frac{r_{0_s}}{K}$$

$$X_{c_r} = X_{\sigma_r} + X_{i_r} = X_{\sigma_r} + X_{\mu_r} - X_{\mu 0_r} + X_{0_r}$$

$$= X_{\sigma_r} + \frac{X_{\mu_s}}{\ddot{u}^2} - \frac{1}{K}(X_{\mu_{cs}} + X_{0_s})$$

$$X_{\mu_{cr}} = 2\,X_{\mu 0_r} = \frac{X_{\mu_{cs}}}{K}$$

The compensation circuit is shown in figure 24a.

In order to calculate the induced voltages $\mathfrak{C}_s$ and $\mathfrak{C}_r$ Fig. 24b is used.

The equations (53a) to (53d) are valid for the calculation of the dimensions of impedance or current circles.

Control: Mr. Herschdorfer treats the simple case for $u = u = 1$ and reached the same results for the coordinates of the centers of impendance circles as we did from equations (53a) and (53c), assuming $u = 1$. The radii of the impedence circles he only calculated roughly.

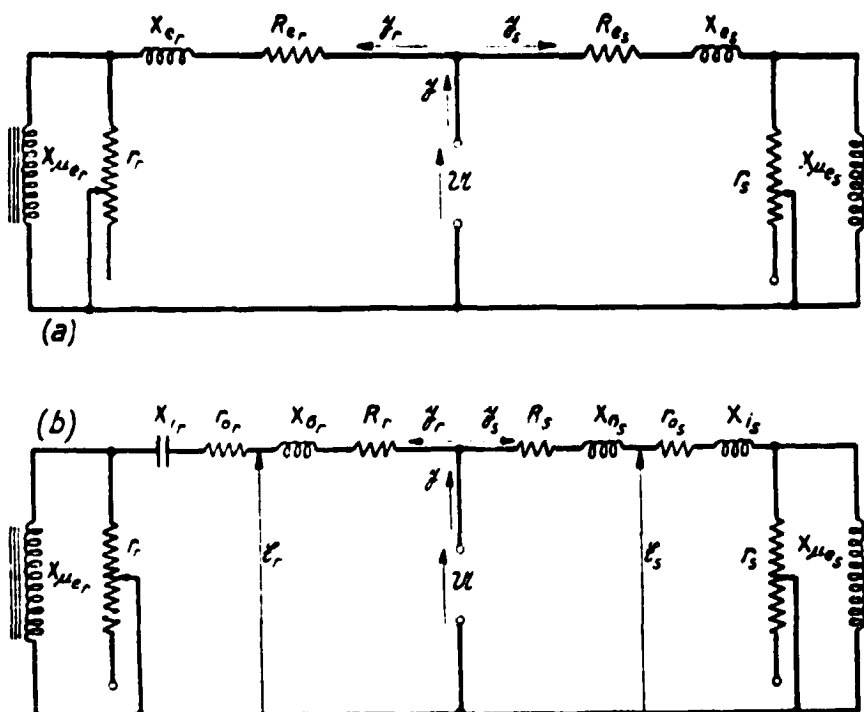Fig. 24. Compensation circuit of a parallel connected asymmetric d.s. machine.

The net current $\Im \ \Im_r + \Im_r$ can easily be calculated or graphically determined after Fig. 18. It has to be considered, however, that its vector does not describe a circle, but a 4th degree curve because the circuits of $\Im_s$ and $\Im_r$ are different.

Case 2: Series connected asymmetric machine

$$\Im_s = \Im_r = \Im. \quad u \neq 1, \quad R_s \neq R_r', \quad X_{\sigma_s} \neq X_{\sigma_r}'$$

$\mathfrak{U}_s + \mathfrak{U}_r = \mathfrak{U}$ = Net voltage.

48

From equations (34a) and (34b) it results:

$$-\mathfrak{U} = \mathfrak{J}\left[(R_s + R_r) + j(X_{\sigma_s} + X_{\sigma_r}) + jX_{\mu_s}\left(1 - \frac{1}{\mathfrak{u}}e^{j\Psi}\right) + jX_{\mu_s}(1 - \mathfrak{u}e^{j\Psi})\right]$$

if we substitute for $e^{j\Psi}$

$$e^{j\Psi} = \cos\Psi + j\sin\Psi$$

$$= 1 - 2\sin^2\frac{\Psi}{2} + 2j\sin\frac{\Psi}{2}\cos\frac{\Psi}{2}$$

$$= 1 - \frac{2}{1 + \operatorname{ctg}^2\frac{\Psi}{2}} + j\frac{2}{\operatorname{tg}\frac{\Psi}{2} + \operatorname{ctg}\frac{\Psi}{2}}$$

the result is:

$$\frac{-\mathfrak{U}}{\mathfrak{J}} = (R_s + R_r) + j(X_{\sigma_s} + X_{\sigma_r}) + jX_{\mu_s}\left[\left(1 + \frac{1}{\mathfrak{u}^2} - \frac{2}{\mathfrak{u}}\right) + \frac{2}{\mathfrak{u}}\left(\frac{2}{1 + \operatorname{ctg}^2\frac{\Psi}{2}} - j\frac{2}{\operatorname{tg}\frac{\Psi}{2} + \operatorname{ctg}\frac{\Psi}{2}}\right)\right]$$

we also substitute:

$$X_{\mu_r} = \frac{X_{\mu_s}}{\mathfrak{u}}$$

$$r = X_{\mu_s}\operatorname{tg}\frac{\Psi}{2} = \frac{4X_{\mu_r}}{\mathfrak{u}}\operatorname{tg}\frac{\Psi}{2}$$

$$\mathfrak{Z}_e = \frac{-\mathfrak{U}}{\mathfrak{J}}$$

the compensation impedance of the machine becomes:

$$\mathfrak{Z}_e = (R_s + R_r) + j(X_{\sigma_s} + X_{\sigma_r}) + jX_{\mu_s}\left[\left(1 + \frac{1}{\mathfrak{u}^2} - \frac{2}{\mathfrak{u}}\right) + \frac{2}{\mathfrak{u}}\left(\frac{2}{1 + \frac{X_{\mu_s}^2}{r^2}} - j\frac{r}{X_{\mu_s} + \frac{X_{\mu_s}}{r}}\right)\right]$$

$$= (R_s + R_r) + j\left(X_{\sigma_s} + X_{\sigma_r} + X_{\mu_s}\left\{1 + \frac{1}{\mathfrak{u}^2} - \frac{2}{\mathfrak{u}}\right\}\right) + j\frac{X_{\mu_s}}{1 + \frac{X_{\mu_s}^2}{r^2}} + \frac{X_{\mu_s}}{\frac{r}{X_{\mu_s}} + \frac{X_{\mu_s}}{r}}$$

$$\mathfrak{Z}_e = (R_s + R_r) + j\left(X_{\sigma_s} + X_{\sigma_r} + X_{\mu_s}\left\{1 + \frac{1}{\mathfrak{u}^2} - \frac{2}{\mathfrak{u}}\right\}\right) + \frac{jrX_{\mu_s}}{r + jX_{\mu_s}}$$

$$\mathfrak{Z}_e = R_e + jX_e + \frac{jrX_{\mu_s}}{r + jX_{\mu_s}}$$

where the compensation constants have the following simple values:

$$R_c = R_s + R_r$$

$$X_c = X_{\sigma_s} + X_{\sigma_r} + X_{\mu_s}\left(1 + \frac{1}{n^2} - \frac{2}{n}\right)$$

$$X_{\mu_c} = \frac{4 X_{\mu_s}}{n}$$

The compensation circuit of the machine per phase (rotor and stator combined) is shown in Fig. 25.
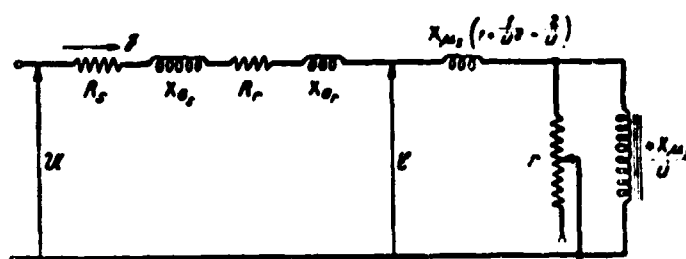

Fig. 25. The compensation circuit of a series connected asymmetric d.s. machine.

Thus, the compensation circuit is sufficient in this special case. Furthermore, the compensation resistance R_C in this case equals the real resistance of the machine $(R_s + R_r)$. The coordinates of the centers and the radius of the impedance and current circles are (after chapter (4)):

<u>Impedance circle:</u>

  Coordinates:

$$x = R_c$$

$$y = X_c + \frac{X_{\mu_c}}{2}$$

  Radius:

$$R = \frac{X_{\mu_c}}{2}$$

<u>Current circle:</u>

  Coordinates:

$$x = U \cdot \frac{X_c + \frac{X_{\mu_c}}{2}}{R_c^2 + X_c^2 + X_c X_{\mu_c}}$$

$$y = U \cdot \frac{R_c}{R_c^2 + X_c^2 + X_c X_{\mu_c}}$$

50

Radius:

$$R - U \cdot \frac{\frac{X_{\mu_r}}{2}}{R_r{}^2 + X_\sigma{}^2 + X_r X_{\mu_r}}$$

Control: Dr. Brailowsky deals with the case of the series connected machine and reached the same results for the dimensions of the impedance and current circle as we did from the above expressions.

Case 3: Parallel connected asymmetric machine

$$a = u = 1, \quad R_s = R_r = R, \quad X_{\sigma_s} = X_{\sigma_r} = X_\sigma, \quad X_{\mu_s} = X_{\mu_r} = X_\mu, \quad U_s = U_r = U$$

$$2\mathfrak{J}_s = 2\mathfrak{J}_r = \mathfrak{J} \quad = \text{net current}$$

$$R_{e_s} = R_{e_r} = R \qquad\qquad r_{\sigma_s} = 0 = r_{\sigma_r}$$

$$X_{e_s} = X_{e_r} = X_\sigma \qquad\qquad X_i = 0 = X_{i_r}$$

$$X_{\mu_s} = X_{\mu_r} = 2 X_\mu$$

$$r_s = r_r$$

$$\mathfrak{J}_s = \mathfrak{J}_r = R + j X_\sigma + \frac{2 r_s j X_\mu}{r_s + 2 j X_\mu}$$

$$\frac{U}{\mathfrak{J}/2} = R + j X_\sigma + \frac{2 r_s j X_\mu}{r_s + 2 j X_\mu}$$

$$\frac{U}{\mathfrak{J}} = \frac{R}{2} + j \frac{X_\sigma}{2} + \frac{r_s j X_\mu}{r_s + 2 j X_\mu}$$

If we write:

$$\frac{U}{\mathfrak{J}} = \mathfrak{J}_r \qquad \text{and} \qquad r = \frac{r_s}{2}$$

the compensation impedance of a phase of the complete machine is:

$$\mathfrak{J}_r = R_r + j X_r + \frac{r j X_{\mu_r}}{r + j X_{\mu_r}}$$

where the compensation constants of a phase of the complete machine are:

$$R_r = \frac{R}{2}$$

$$X_r = \frac{X_\sigma}{2}$$

The compensation circuit of the machine is shown in Fig. 26.
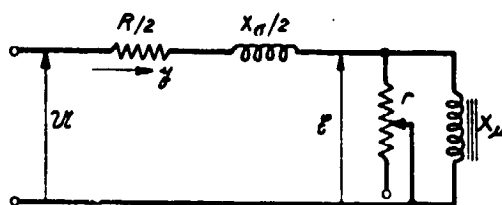
$$X_{\mu_r} = X_\mu$$

51

Fig. 26. Compensation circuit of a parallel connected asymmetric d.s. machine.

Case 4: <u>Series connected symmetric machine</u>

$$n = 1, \quad R_{s_r} = R_r = R, \quad X_{\sigma_s} = X_{\sigma_r} = X_\sigma, \quad X_{\mu_s} = X_{\mu_r} = X_\mu, \quad \mathfrak{F}_s = \mathfrak{F}_r = \mathfrak{F}, \quad u = 1$$

$$2\,\mathfrak{U}_s = 2\,\mathfrak{U}_r = \mathfrak{U} \quad = \text{net voltage.}$$

$$
\begin{aligned}
R_{c_s} &= R_{c_r} = R & r_{0_s} &= 0 = r_{0_r} \\
X_{c_s} &= X_{c_r} = X_\sigma & X_{i_s} &= 0 = X_{i_r} \\
X_{\mu_{cs}} &= X_{\mu_{cr}} = 2\,X_\mu & r_s &= r_r
\end{aligned}
$$

$$\mathfrak{F}_{c_s} = \mathfrak{F}_{c_r} = R + j\,X_\sigma + \frac{2\,r_s\,j\,X_\mu}{r_s + 2\,j\,X_\mu}$$

$$-\frac{\mathfrak{U}}{2} = R + j\,X_\sigma + \frac{2\,r_s\,j\,X_\mu}{r_s + 2\,j\,X_\mu}$$

$$-\frac{\mathfrak{U}}{\mathfrak{F}} = 2\,R + 2\,j\,X_\sigma + \frac{4\,r_s\,j\,X_\mu}{r_s + 2\,j\,X_\mu}$$

we now write:

$$\mathfrak{F}_c = \frac{-\mathfrak{U}}{\mathfrak{F}}$$

$$r = 2\,r_s$$

the compensation impedance of a phase of the <u>complete machine</u> is:

$$\mathfrak{F}_r = R_c + j\,X_c + \frac{r\,j\,X_{\mu_r}}{r + j\,X_{\mu_s}}$$

where the compensation constants of a phase of the complete machine are:

$$
\begin{aligned}
R_c &= 2\,R \\
X_c &= 2\,X_\sigma \\
X_{\mu_r} &= 4\,X_\mu
\end{aligned}
$$

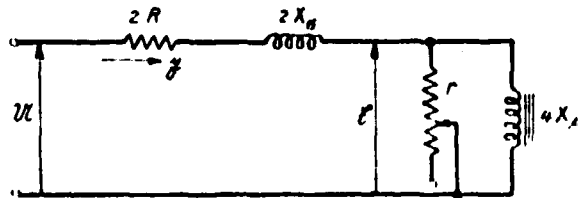The compensation circuit of the machine is shown in Fig. 27.

Fig. 27. Compensation circuit of a series connected symmetric d.s. machine.

It is interesting to note that the above results are in cases (3) and (4) also valid for a machine with $u \neq 1$, if it is parallel or series connected by an ideal transformer with a transformation ratio equal to u ,provided that its constants fulfill the condition $R_s = R_r'$, $X_{\sigma_s} = X_{\sigma_r}'$.

7. The Characteristics of the Common Double-Fed
Synchronous Machine

Under the various operational conditions

(7.1.) Definitions

The results which were gained in chapters (3) to (6) enable the quantitative calculation of all quantities of the machine under any chosen operational condition. Based on the gained analytical results this chapter deals with the characteristics of the double-feeding synchronous operation and its physical explanations as well as the influence of the various possible operational environments and conditions.

Therefore, we subdivide the conditions which can occur in practice into three groups according to the following definitions.

A. The "symmetric machine":

$$\frac{R_s}{R_r} = \frac{X_{\sigma_s}}{X_{\sigma_r}} = \left(\frac{W_s}{W_r}\right)^2 = u^2 \quad \text{and} \quad \frac{U_s}{U_r} = \frac{W_s}{W_r} = u$$

$$R_s = R_r' \quad X_{\sigma_s} = X_{\sigma_r}' \quad u = u \quad | \text{ or } U_s = U_r'|$$

The "symmetric machine" thus is a machine with equal constants and terminal voltages - referring to one and the same side.

B. The "constructional asymmetric machine":

$$\frac{U_s}{U_r} = \frac{W_s}{W_r} \qquad u = \ddot{u} \quad [\text{or} \quad U_s = U_r']$$

The condition $R_s = R_r'$ and $X_{\sigma_s} = X_{\sigma_r}'$ shall be partly or completely unrealized:

C. The "essentially asymmetric machine":

$$u \neq \ddot{u} \qquad |\text{or} \quad U_s \neq U_r'|$$

The term symmetric machine as defined above, differs from the one existing in literature[1] by not needing an equal number of turns of rotor and stator (i.e., $u = 1$) . It only needs the realization of two conditions; first, the equality of the constants of both machines parts when they refer to one and the same part, and second, the voltage applied to the two parts must have the ratio u . It is obviously very difficult to satisfy the first condition in practice; however, it is not necessary in order to operate the machine satisfactorily. The term symmetric machine, however, is very useful from the theoretical point of view since it describes the basic laws which rule the behavior of the machine in a very simple way. But if one or the other of the conditions of symmetry is not satisfied (which in practice is always the case) a certain inequality of the functioning of both machine parts results; this we call asymmetry. The non-satisfaction of the first condition we call constructional asymmetry since it is due to the construction of the machine and not its operation. An important example for the constructional asymmetric machine is the parallel connected machine with $\ddot{u} = 1$. Since the machine constants of the normal construction type do not differ much from the condition $\frac{R_s}{R_r} = \frac{X_{\sigma_s}}{X_{\sigma_r}} = \ddot{u}^2$, the prerequisite of symmetry can be used for approximate calculations and plots.

By not satisfying the second condition, however, quite a considerable asymmetry results. The slightest change of the ratio of the applied voltages (i.e. the change of the $\frac{\ddot{u}}{u}$ ratio) or the turn ratio for the parallel connection, has a considerable influence on asymmetry. The degree of asymmetry in this case actually depends on the quantity $\left(1 - \left(\frac{\ddot{u}}{u}\right)^2\right)$. This case, i.e., the non-satisfaction of the second condition $(\ddot{u} = u)$ of symmetry we thus call "essential asymmetry".

---

[1]:see: E. Messing "Information on the double fed induction machine" dissertation, Karlsruhe 1931.

54

(7.2.) Characteristics of the symmetric machine

The two conditions of symmetry are:

$$\frac{R_s}{R_r} = \frac{X_{\sigma_s}}{X_{\sigma_r}} = \left(\frac{W_s}{W_r}\right)^2 = \ddot{u}^2 \quad \text{and} \quad \frac{U_s}{U_r} = \frac{W_s}{W_r} = \ddot{u}$$

or:

$$R_s = R_r', \quad X_{\sigma_s} = X_{\sigma_r}' \quad \text{or} \quad U_s = U_r' \quad \text{or} \quad u = u.$$

thus:

$$r_{0_s} = 0 = r_{0_r} \quad \text{(Equations (38a) and (38c)}$$
$$X_{0_s} = 0 = X_{0_r} \quad \text{(Equations (38b) and (38d)}$$
$$K = \ddot{u}^2 \quad \text{(Equations (46a)}$$

The compensation constants here equal the real constants of the machine because:

$$R_{c_s} = R_s = \ddot{u}^2 R_r = R_r' \qquad R_{c_r} = R_r = \frac{R_s}{\ddot{u}^2} = R_s'$$

$$X_{c_s} = X_{\sigma_s} = \ddot{u}^2 X_{\sigma_r} = X_{\sigma_r}' \qquad X_{c_r} = X_{\sigma_r} = \frac{X_{\sigma_s}}{\ddot{u}^2} = X_{\sigma_s}'$$

$$X_{\mu_{cs}} = 2 X_{\mu_s} = 2 \ddot{u}^2 X_{\mu_r} = 2 X_{\mu_r}' \qquad X_{\mu_{cr}} = 2 X_{\mu_r} = \frac{2 X_{\mu_s}}{\ddot{u}^2} = 2 X_{\mu_s}'$$

The parameters of the compensation circuits are:

$$r_s = 2 X_{\mu_{0s}} \operatorname{tg}\left(\frac{\Psi - \rho_s}{2}\right) = 2 X_{\mu_s} \operatorname{tg}\frac{\Psi}{2} \quad [\rho_s = 0] \quad (\text{eq (40a), (40b) and (41))}$$

$$r_r = 2 X_{\mu_{0r}} \operatorname{tg}\left(\frac{\Psi - \rho_r}{2}\right) = 2 X_{\mu_r} \operatorname{tg}\frac{\Psi}{2} \quad [\rho_r = 0]$$

Which results in:

$$\frac{r_s}{r_r} = \ddot{u}^2$$

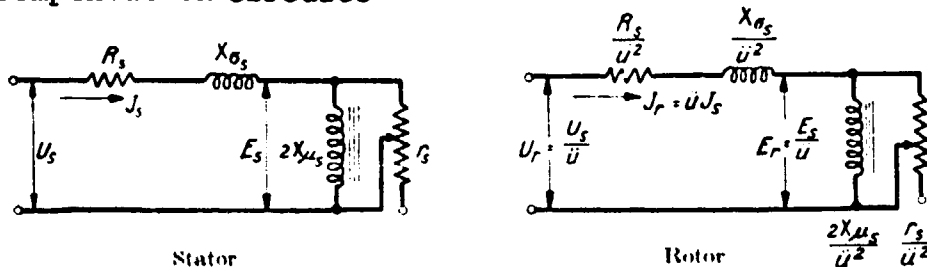The compensation circuits of the machine are shown in Fig. 28.

Fig. 28. Compensation circuits of a symmetric machine.

It can thus be shown that the stator compensation constants or the stator parameter r_s is n^2 times as large as the rotor constants or the respective rotor parameter r_r. It can be concluded that the stator always has the <u>same power factor</u> as the rotor. Furthermore, since the stator voltage is n times as large as the rotor voltage, the rotor current always is n times as large as the stator current. This results in the fact that both machine parts always take in the same <u>blind and the same effective power</u>, and that both circuit diagrams of the machine are similar (in the ratio n). The dimensions of the stator circuit are:

/62

$$x = U_s \cdot \frac{X_{\sigma_s} + X_{\mu_s}}{R_s^2 + X_{\sigma_s}^2 + 2 X_{\sigma_s} X_{\mu_s}} \quad \text{Ampère}$$

$$y = U_s \cdot \frac{R_s}{R_s^2 + X_{\sigma_s}^2 + 2 X_{\sigma_s} X_{\mu_s}} \quad \text{Ampère}$$

$$R = U_s \cdot \frac{X_{\mu_s}}{R_s^2 + X_{\sigma_s}^2 + 2 X_{\sigma_s} X_{\mu_s}} \quad \text{Ampère}$$
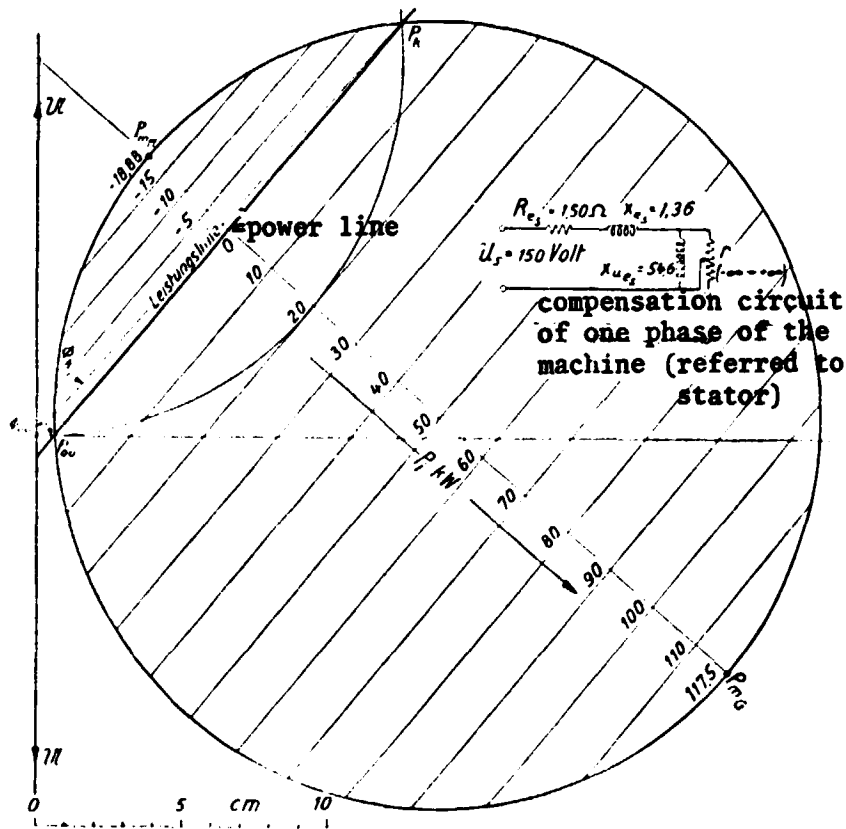

Fig. 29. Circle diagram of the symmetric machine. $U_s = 150$ Volt, $U_r = 75$ Volt, $n = 2$, R_s, R_r' 1.50 Ω, X_{σ_s}, X_{σ_s}' 1.36 Ω, X_{μ_s} 27.3 Ω 25 Hz, 1500 r/min.

Scales: stator current 1cm=4amp; rotor current 1cm=8amp; terminal output 1cm=3.6kW (total); inner output: measured perpendicular to output line 1cm=0.869kW/ph./side or 5.213kW (total).

The dimensions of the rotor circuit are u- times as large. It /63
is then sufficient to draw only one of both circles (fig.29). As
explained earlier (chapter 4), stator terminal output, stator current
and power factor for any chosen value of inner output are then
calculated. The respective terminal output and power factor of the
rotor are the same, the rotor current, however, results from multi-
plication of the stator current with u.

Fig. 30a or 30b show the combined space and time vector
diagram for generator or motor operation during full load operation.
The amplitudes of the flux waves F_s and F_r are equal (because
$\frac{J_r}{J_s} = u$) and thus have the same angle with the combined flux wave;
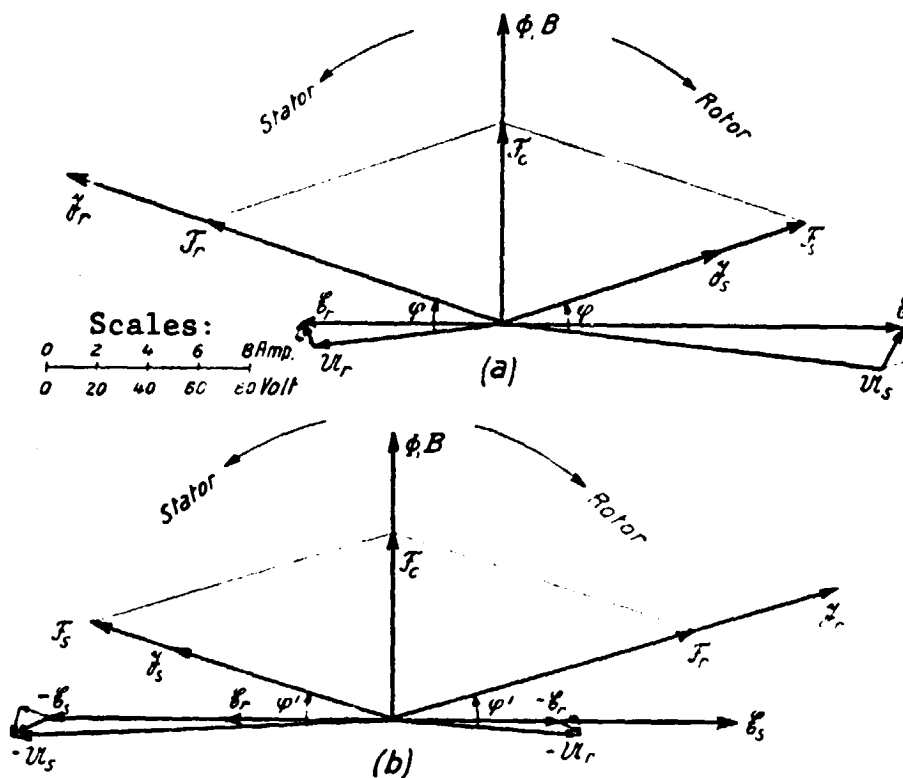the currents have the same angle with the respective induced voltages.

Fig. 30. Combined space and time vector diagram of a symmetric
machine. a) generator operation. b) motor operation.

It has been shown that both machine parts of a symmetric machine /64
always function in exactly the same manner and play the same role in
all respects.

Calculations of currents, terminal output and power factor was

carried out for all theoretically possible functions[1] with the aid
of the circle diagram. In Fig. 31 the calculation is shown to be
a function of the inner output of the machine. The motor operation
obviously corresponds to the upper part of the circle diagram above
the power line or negative inner output of Fig. 31. It can be
noticed that motor operation is possible only in a very small area
of the whole theoretical operating area. Compared to induction
machines, motor operation of the d.s. machine corresponds to a
smaller part of the total operating area[2]. Point P_{mM} or P_{mG} in
Fig. 29 corresponds to the state of maximal sweep output of the
machine during motor or generator operation. Normal stable opera-
tion then is only possible in the lower half of the circuit
diagram between P_{mM} and P_{mG}. This corresponds to the heavy lines
in Fig. 31. The practically applicable operating field limited
by the maximum values of the currents, only covers a small part
of the whole stable field. It has to be noted that the charac-
teristic line of terminal output as function of the inner output
$(P_k = f(P_i))$ is an ellipse.

The two points where the inner output equals zero (P_{00} and P_k)
evidently correspond to the two values of the paramer ∞ or 0. In
both cases the machine receives an effective power from the net
which equals the copper losses. The first point P_{00} corresponds to
the parameter $r = \prime$ and refers to the ideal no-load operation
which can be simply reached in practice by operating the machine
mechanically and feeding it an amount of mechanical power which
exactly covers it mechanical and iron losses. The combined space and
time vector diagram of Fig. 32 corresponds to the ideal no-load
operation. Both flux waves coincide in space. Each side of the
machine takes in an effective output from its net which corresponds
to its copper loss. The effective output is represented by the
ordinate of P_{00}.

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1) The calculations were carried out for an experimental machine
(machine I, see chapter (8.1.) assuming symmetry. The machine has
the following constants:

$$R_x = 1.14 \text{ Ohm}, \quad X_{a_x} = 1.30 \text{ Ohm}, \quad R_r = 0.465 \text{ Ohm},$$
$$X_{a_r} = 0.355 \text{ Ohm}, \quad \ddot{u} = 2.00 \text{ Ohm}, \quad X_{\mu_x} = 27.3 \text{ Ohm}. \quad \text{Daher}$$
$$R_r' = 0.465 \cdot 4 = 1.86 \text{ Ohm} \quad \text{und} \quad X_{a_r}' = 0.355 \cdot 4 = 1.42 \text{ Ohm}.$$

The assumed constants of the symmetric machine are:

$$R_x \quad R_r' = \frac{1.14 + 1.86}{2} = 1.50 \text{ Ohm}, \quad X_{a_x} = X_{a_r}' = \frac{1.30 + 1.42}{2} = 1.36 \text{ Ohm}.$$
$$X_{\mu_x} = 27.3 \text{ Ohm}, \quad U_x = 150 \text{ Volt}, \quad U_r = 75 \text{ Volt}.$$

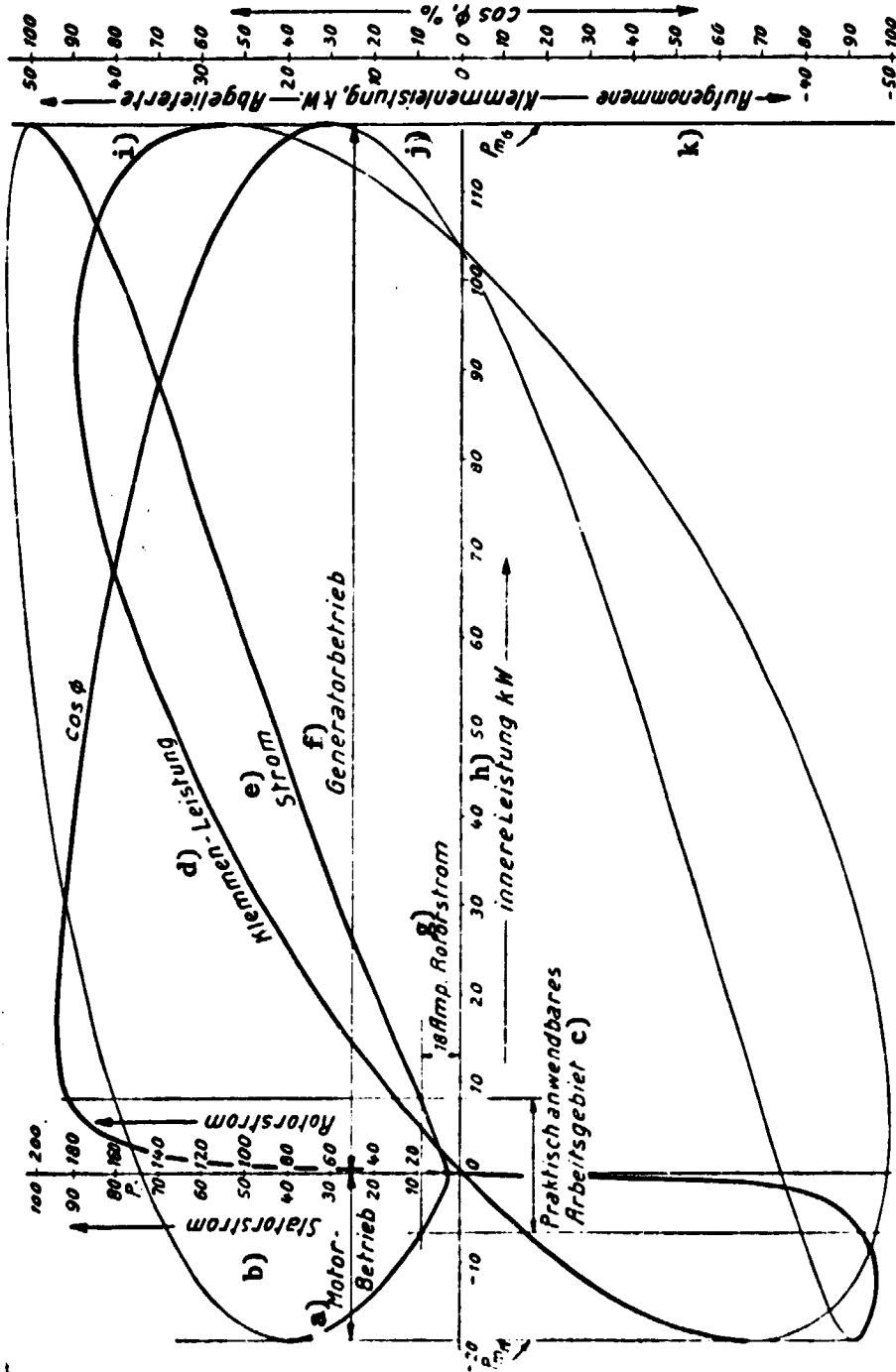2) See the comparison with induction machines at the end of this
chapter.

Fig. 31. Characteristic lines of the symmetric machine in the theoretically possible operating field.

$U_s = 130$ Volt. $U_r = 75$ Volt. $u = \bar{u} = 2$, $R_s = R_r' = 1.50\,\Omega$. $X_{\sigma_s} = X_{\sigma_r} = 1.36\,\Omega$. $X_\mu = 27.3\,\Omega$. 25 Hz. 1500 r/min.

a):motor operation; b):stator current; c):practically applicable operating field; d):terminal power;
e):current; f):generator operation; g):rotor current; h):inner output; i):supplied- j):terminal
power; k): received-;

A point slightly higher on the circle correponds to the real no-load operation as motor since the machine now has to additionally take in power from the net in order to cover its mechanic and iron losses.
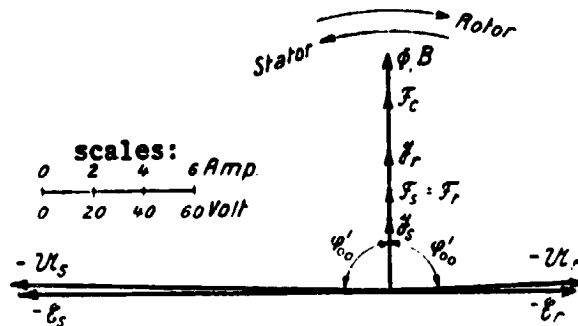

Fig. 32. Combined space and time vector diagram for the ideal no-load operation of a symmetric machine.
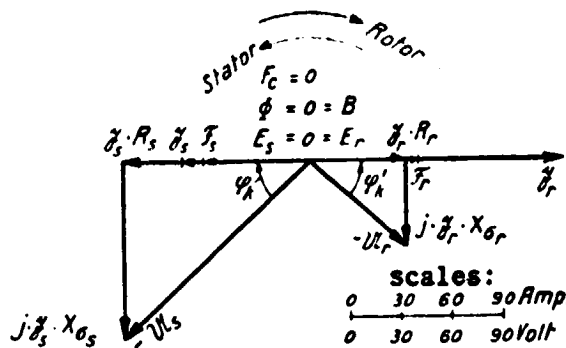

Fig. 33. Combined space and time vector diagram for the condition corresponding to point P_k (symmetric machine).

The second point P_k corresponds in a certain sense to the short circuit point of the asynchronous machine, but has a different meaning. In this case, the value of the parameter is zero, thus the induced voltage also equals zero. The air slit flow also equals zero, thus the flux waves of stator and rotor in space must be in opposition. This condition is given in Fig. 33. It is then clear that the machine under this condition is theoretically still operating. Another difference between P_k and the short circuit point of an asynchronous machine is the fact that the torque of the d.s. machine equals zero at P_k because the air slit flow equals zero. A short circuit asynchronous machine, however, still has a small air slit flow and thus also a torque.

The expressions for the sweep output of the machine (equation (50a), (50b) and (48)) are simplified under symmetric conditions as follows:

60

Generator operation:

$$P_{im_G} = \frac{\dfrac{3\,U_s^2}{R_s}}{\sqrt{1 + \left(\dfrac{R_s}{X_{\mu_s}} + \dfrac{X_{\sigma_s}^2}{R_s X_{\mu_s}} + \dfrac{X_{\sigma_s}}{R_s}\right)^2} - 1} \quad \text{Watt}$$

Motor operation:

$$P_{im_M} = \frac{-\dfrac{3\,U_s^2}{R_s}}{\sqrt{1 + \left(\dfrac{R_s}{X_{\mu_s}} + \dfrac{X_{\sigma_s}^2}{R_s X_{\mu_s}} + \dfrac{X_{\sigma_s}}{R}\right)^2} + 1} \quad \text{Watt}$$

(63)

for approximate calculations it can be assumed that:

$$P_{im_G} \simeq \frac{3\,U_s^2}{\sqrt{R_s^2 + X_{\sigma_s}^2} - R_s} \quad \text{Watt}$$

$$P_{im_M} \simeq \frac{-3\,U_s^2}{\sqrt{R_s^2 + X_{\sigma_s}^2} + R_s} \quad \text{Watt}$$

(63a)

It is interesting to draw a comparison between sweep output or sweep momentum of the d.s.machine and the asynchronous machine. If the same machine during asynchronous operation was using the same net (i.e., same voltage and frequency) for sweep output during gener-ator or motor operation the following expressions are valid[1]:

$$(P_{im_G})_{Asyn.} \simeq \frac{\dfrac{3\,U_s^2}{2}}{\sqrt{R_s^2 + 4 X_{\sigma_s}^2} - R_s} \quad \text{Watt}$$

$$(P_{im_M})_{Asyn.} \simeq \frac{-\dfrac{3\,U_s^2}{2}}{\sqrt{R_s^2 + 4 X_{\sigma_s}^2} + R_s} \quad \text{Watt}$$

(63b)

From expressions (63a) and (63b) the ratios between sweep output for d.s. and asynchronous operation results:

$$S \simeq 2 \cdot \frac{\sqrt{R_s^2 + 4 X_{\sigma_s}^2} \mp R_s}{\sqrt{R_s^2 + X_{\sigma_s}^2} \mp R_s}$$

---

[1] see E Arnold, Technology of three phase current, v V,1 p 71

For larger machines the ratio $\frac{R_s}{X_{\sigma_s}}$ is rather small and, as we know, increases to about 1 for very small machines.

For $\frac{R_s}{X_{\sigma_s}} \ll 1$ is $N \simeq 4$ for motor and generator operation

For $\frac{R_s}{X_{\sigma_s}} \simeq 1$ is $N \simeq 6$ for generator operation

and $N \simeq 2.7$ for motor operation.

It can thus be shown that <u>sweep output and sweep momentum during d.s. operation always are considerably larger than during asynchronous operation. The ratio between both of them depends on the capacity of the machine as well as the operational manners (motor or generator)</u>. The sweep output of larger machines is about four times as large (i.e., the sweep momentum is double), the smaller the machine, the larger this ratio is for generators and the smaller for motors. In very small machines it is about 6 for generators and 2.7 for motors (thus the sweep momentum ratio is 3 or 1.35).

Another advantage of d.s. operation is the need of a smaller blind output, i.e., an improved power factor. If, for example, $R = 1$ in a machine it can be directly parallel or series connected. The conditions of symmetry will then in both cases be satisfied when $R_s = R_r = R$ and $X_{\sigma_s} = X_{\sigma_r} = X_\sigma$. The compensation circuit of a phase of the whole machine (i.e., for net circuit) is shown in Fig. 34 for all three considered modes of operation, i.e., d.s. operation with parallel and series connection and asynchronous operation. The reactance X_μ as defined above, represents the ratio $\frac{E}{J_\mu}$, where E is the induced voltage and J_μ the current component perpendicular to it during feeding of one side whereas the other one remains open.

The nominal voltage or nominal current per phase of stator or rotor (assumed to be equal) shall be U or J. If the machine was supplied with nominal voltage from the nominal frequency in every case, nominal output in case (a) has a voltage U and a current 2J and in case (b) a voltage 2U and a current J. These two cases are obviously similar. In both cases the machine has the same blind and effective output and the same power factor for the same mechanical output. During asynchronous operation, however, the nominal output is reached with voltage U and current J. If the power factor in all three cases was the same, the output in (c) would be half of (a) or (b). Obviously, this is not an advantage

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1) Neglecting R and X_μ , the blind output will in every case be the same, namely $\frac{U^2}{X_\mu}$ . .

since the machine works with half the rotation of (a) or (b) in
case (c). Considering, however, the blind input, it becomes
obvious from Fig. 34 that the blind input in case (c) is about the
same[1] as in case (a) or (b). This results in the important
characteristic of double-fed synchronous operation: the d.s.
machine needs a blind input which equals about half of the blind
input necessary for an asynchronous machine of the same nominal
output and the same number of poles, the power factor being better
during d.s. operation. The same machine thus produces considerably
more than twice the output during d.s. operation as it would
produce during asynchronous operation with half the number of
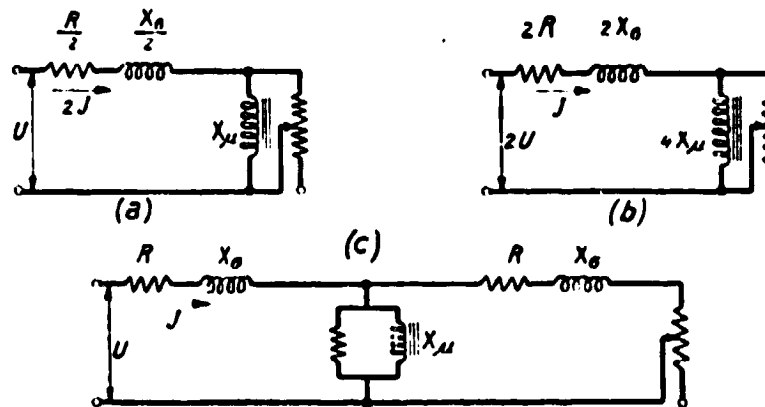revolutions.

Fig. 34. Compensation circuit per phase of a machine with

$a = 1,\ R_a = R_r = R,\ X_{\sigma_a} = X_{\sigma_r} = X_\sigma.$

a) double fed synchronous machine, parallel connection
b) double fed synchronous machine, series connection
c) asynchronous operation

Neglecting magnetizing current, the drop in voltage in case
(a) or (b) will equal $\dfrac{J\sqrt{R^2+X_\sigma^2}}{U}$ of the voltage, whereas it will
be $\dfrac{2J\sqrt{R^2+X_\sigma^2}}{U}$ of the voltage in case (c), i.e., twice as large.
It can be concluded that a d.s. machine has about half the voltage
drop of a similar asynchronous machine (actually it is a little
more). This fact has, as we know, another more important success
which is that the sweep output or sweep momentum of a d.s. machine
are much larger than that of an asynchronous machine, as shown above
For the same reason the diameter of a d.s. machine is about twice
as large as that of a similar asynchronous machine.

As Fig. 34 also shows, the copper losses of the machine are about 6 J^2R in case (c), i.e., equal to the copper losses in case (a) or (b). Iron losses during d.s. operation evidently are larger than during asynchronous operation. On the one side iron losses during d.s. operation occur also in the rotor with net frequency, on the other side pulsation losses are considerable, due to the higher velocity of the rotor body. But since the output during d.s. operation is double and copper losses remain unchanged, the <u>effectiveness still is considerably higher</u>, even for an asynchronous machine of conventional construction with a rotor consisting of commercial, thick laminations.

These advantages of d.s. operation were explained based on this simple example, but they hold for any symmetric machine, as well as the practical machines which do not have a very prominent asymmetry.

(7.3.) Influence of asymmetry

In the last chapter it was shown that the symmetric machine represents the ideal case of double-fed synchronous operation, when both machine parts share blind and effective output equally under all operating conditions. The most favorable construction of the machine supplying the largest sweep output as well as largest effectiveness for a certain kind of material, would be a symmetric machine with the smallest possible values of effective and stray resistances and the same maximum values of J_s and J_r'. Because of the relatively limited slot volume of the rotor, it is impossible to fulfill the conditions of symmetry in practice. Because of that, neither the constants nor the allowable values of the currents can be the same and the difference is the larger the smaller the machine is. In practice, therefore, there is always asymmetry, which causes a dissimilarity of functioning which results in increasing the current in the one part and decreasing in the other. The effectiveness and sweep output are also lessened. In this chapter, the influence of asymmetry shall be discussed.

During double-fed synchronous operation two voltages U_s and U_r are applied to both machine parts. The induced voltages E_s and E_r always have a ratio u . If the applied voltages also had the ratio u and the constants R_s and R_r or $x_{..}$ and $x_{..}$ had the ratio u^2 (which is a symmetric machine) then both circles of the machine (stator and rotor), as referred to one and only one side, would be identical. The necessary magnetizing current as well as the effective current would distribute evenly as we have already seen. If there is, however, any inequality of the respective voltages or costants, this asymmetry must be compensated by a difference between both currents J_s and J_r (generally in the quantities and phase shift angles). Both machine parts, however, always have the same inner effective output independent of symmetry, as shown in chapter (2.2.). Consequently, there can only be a difference between the inner <u>blind outputs</u>. The asymmetry is then compensated by reception or release of dissimilar blind

currents.

Thus, in asymmetric machines (both, constructional and essential asymmetry), there is an operational dissimilarity which merely consists in the contribution of each side of magnetization current necessary for the common field (chapter (2.3.). The larger the asymmetry, the larger the difference between the blind outputs becomes. The inner effective outputs always stay the same, and thus the terminal effective outputs also are essentially the same[1]. The processes which are effected by asymmetry appear completely during no-load operation as well as during operation. They can be better recognized during no-load operation since the effective currents which are always symmetric are superimposed on the asymmetric magnetizing currents during operation and thus somewhat cover the influence of asymmetry. In order to get a good idea of it, it is then reasonable to study the influence of asymmetry during ideal no-load operation $(P_i = 0)$. The following study is based on the ideal no-load operation.

Let us first consider a constructional asymmetric machine $(u = u)$ during ideal no-load operation. Fig. 35 represents the conditions of the machine. Each side receives lagging current[2] (supplies leading current) and supports the maintainance of the common field with a purely magnetizing current. The resulting flux wave of the machine is supported by both sides. The amplitude of this wave is:

$$F_e = |J_s| \cdot W_s + |J_r| \cdot W_r$$

If we assume that the rotor voltage is increased whereas the stator voltage remains unchanged, i.e., $u > u$, it becomes obvious from the same figure, that, in order to maintain the equilibrium of the voltages, rotor current is increased, the induced voltages increase and stator current decreases. Under these new conditions, the rotor thus has a larger part of the magnetizing current. The change of one of the terminal voltages thus causes a transfer of the magnetizing current from one side to the other. Increase of one voltage causes an increase of the current of the respective side and a decrease of the current of the other side.

/72

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1) At least within the practical operating field where the copper losses are small compared to the output.
2) According to the general terminology we here call a current "lagging" when it is magnetizing, i.e., when it lags the net voltage.
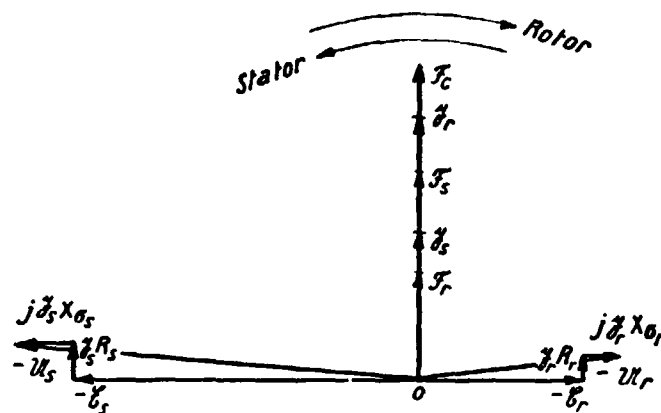
Fig. 35. Combined space and time vector diagram of a construc-
tional asymmetric machine during ideal no-load operation.

If rotor voltage is continuously increased, rotor current
continues to increase and stator current decreases. Both currents
however, remain purely magnetizing until the stator current has
reached the value zero, and the rotor takes over all of the
magnetizing current. We call this case "unilateral rotor magne-
tization". This case obviously exists when the induced voltage
of the stator equals its terminal voltage. Thus, the following
condition is valid for the unilateral rotor magnetization:

$$U_s = E_s = \ddot{u} \cdot E_r = \frac{U_r \cdot X_{\mu_r} \cdot \ddot{u}}{\sqrt{R_r^2 + (X_{\sigma_r} + X_{\mu_r})^2}}$$

It corresponds to the terminal voltage ratio:

$$u_{e_r} = \left(\frac{U_s}{U_r}\right)_{e_r} = \ddot{u}\,\frac{X_{\mu_r}}{\sqrt{R_r^2 + (X_{\sigma_r} + X_{\mu_r})^2}} \tag{64}$$

It can be concluded:

$$X_{\mu_r}^2\left(1 - \frac{\ddot{u}^2}{u_{e_r}^2}\right) = -(R_r^2 + X_{\sigma_r} + 2 X_{\sigma_r} X_{\mu_r})$$

Substituting this equation in expression (46a), we obtain:

$$K = \infty$$

The unilateral rotor magnetization corresponds to the dissimi-   /73
larity factor K = ∞.

66

By increasing the rotor voltage above a value corresponding to the unilateral rotor magnetization, the rotor current continues to increase and a stator current exists again. Now, however, the stator receives a leading current and has an induced voltage which is larger than its terminal voltage. The stator current is then obviously purely de-magnetizing. The amplitude of the resulting flux wave can be calculated by:

$$F_e = |J_r| \cdot W_r - |J_s| \cdot W_s$$

The dissimilarity factor now has a negative value.

It can be seen that by decreasing the rotor voltage below the value corresponding to the constructional asymmetry $\left(U_r = \frac{U_s}{ii}\right)$ the rotor current has a **smaller** and the stator current a larger value than those shown in Fig. 35. By decreasing the rotor voltage somewhat, the rotor current reaches the value zero, and the stator receives all of the current necessary for the production of the field. This case we also call a "unilateral stator magnetization". For this it is also valid:

$$U_r = E_r = \frac{E_s}{ii} = \frac{U_s \cdot X_{\mu_s}}{ii \sqrt{R_s^2 + (X_{\sigma_s} + X_{\mu_s})^2}}$$

It corresponds to the terminal voltage ratio:

$$u_{e_s} = \left(\frac{U_s}{U_r}\right)_{e_s} = ii \cdot \frac{\sqrt{R_s^2 + (X_{\sigma_s} + X_{\mu_s})^2}}{X_{\mu_s}} \tag{65}$$

It can be concluded:

$$X_{\mu_s}^2 \left(1 - \frac{u_{e_s}^2}{ii^2}\right) = -(R_s^2 + X_{\sigma_s}^2 + 2 X_{\sigma_s} X_{\mu_s})$$

where:

$$K = 0$$

The unilateral stator magnetization thus corresponds to the dissimilarity factor K = 0.

By further decreasing the rotor voltage, the rotor current again increases; it is now, however, leading (de-magnetizing). The stator current increases evenly in order to compensate this demagnetizing influence of the rotor current. For the amplitude of the resulting flux wave under these conditions the following expression is valid:

$$F_e = |J_s| \cdot W_s - |J_r| \cdot W_r$$

The dissimilarity factor here also has a negative value.

The above considerations show that during ideal no-load operation stator or rotor current can have any chosen value, it can lag or lead depending on the ratio $\ddot{u}/u$. A certain field, however, exists with borders corresponding to the unilateral magnetizations. In this field the ratio can change, while both currents remain lagging (magnetizing). This field we call "magnetizing-similarity-field". Its borders correspond to the values 0 and ∞ of the dissimilarity factor and within this field the dissimilarity factor is positive, outside of it it is negative. If the ratio $\ddot{u}/u$ is outside of this field a current is demagnetizing. The two currents of a constructional asymmetric machine are always lagging. The sum of stator and rotor k.v.a. is, however, lagging in any case since it is almost equal to the k.v.a. necessary for the production of the field which obviously is always lagging.

After the above physically explained influence of asymmetry we now want to deduce it mathematically. At first, we make use of a well-based simplification by neglecting the resistances R_s and R_r since they do not have an important influence on the magnitude of the induced voltages during no-load operation and thus also do not considerably influence their equilibrium and the currents. By neglecting these resistances, rather simple expressions for the currents of the machine during ideal no-load operation result. Thus, the influence of the essential asymmetry can be studied directly by assuming the ratio $\ddot{u}/u$ to be a variable.

According to the expressions (equation (38a), (38b), (40a) and (46a)) given in chapter (3), the compensation constants of the stator and the dissimilarity factor neglecting the Ohm-resistances can be written as follows:

$$r_{0_s} = 0$$

$$X_{0_s} = X_{\mu_s}^2 \cdot \frac{X_{\sigma_s}\frac{\ddot{u}^2}{u^2} - X_{\sigma_r}' - X_{\mu_s}\left(1 - \frac{\ddot{u}^2}{u^2}\right)}{X_{\sigma_r}'^2 + 2 X_{\sigma_r}' X_{\mu_s} + X_{\mu_s}^2\left(1 - \frac{\ddot{u}^2}{u^2}\right)}$$

$$X_{\mu_{0_s}} = \pm \sqrt{X_{0_s}^2 + \frac{K}{\ddot{u}^2} X_{\mu_s}^2}$$

$$K = \frac{1}{u^2} \cdot \frac{X_{\sigma_s}^2 + 2 X_{\sigma_s} X_{\mu_s} + X_{\mu_s}^2\left(1 - \frac{u^2}{\ddot{u}^2}\right)}{X_{\sigma_r}^2 + 2 X_{\sigma_r} X_{\mu_s} + X_{\mu_s}^2\left(1 - \frac{\ddot{u}^2}{u^2}\right)}$$

which results in:

$$\frac{K}{\ddot{u}^2} \cdot X_{\mu_s}^2 = X_{\mu_s}^2 \cdot \frac{\frac{\ddot{u}^2}{u^2} X_{\sigma_s}^2 + 2\frac{\ddot{u}^2}{u^2} X_{\sigma_s} X_{\mu_s} - X_{\mu_s}^2 \left(1 - \frac{\ddot{u}^2}{u^2}\right)}{X_{\sigma_r}'^2 + 2 X_{\sigma_r}' X_{\mu_s} + X_{\mu_s}^2 \left(1 - \frac{\ddot{u}^2}{u^2}\right)}$$

and thus:

$$X_{\mu_{0s}} = \pm \sqrt{\frac{X_{\mu_s}^4 \left[X_{\sigma_s} \frac{\ddot{u}^2}{u^2} - X_{\sigma_r}' - X_{\mu_s}\left(1 - \frac{\ddot{u}^2}{u^2}\right)\right]^2}{\left[X_{\sigma_r}'^2 + 2 X_{\sigma_r}' X_{\mu_s} + X_{\mu_s}^2\left(1 - \frac{\ddot{u}^2}{u^2}\right)\right]^2} + \frac{X_{\mu_s}^2\left[\frac{\ddot{u}^2}{u^2} X_{\sigma_s}^2 + 2\frac{\ddot{u}^2}{u^2} X_{\sigma_s} X_{\mu_s} - X_{\mu_s}^2\left(1 - \frac{\ddot{u}^2}{u^2}\right)\right]}{X_{\sigma_r}'^2 + 2 X_{\sigma_r}' X_{\mu_s} + X_{\mu_s}^2\left(1 - \frac{\ddot{u}^2}{u^2}\right)}}$$

This expression can be directly simplified to[1]:

$$X_{\mu_{0s}} = X_{\mu_s} \cdot \frac{\frac{\ddot{u}}{u}\left[X_{\mu_s} X_{\sigma_s} + X_{\mu_s} X_{\sigma_r}' + X_{\sigma_s} X_{\sigma_r}'\right]}{X_{\sigma_r}'^2 + 2 X_{\sigma_r}' X_{\mu_s} + X_{\mu_s}^2\left(1 - \frac{\ddot{u}^2}{u^2}\right)}$$

Thus the stator compensation circuit looks like Fig. 36.

Fig. 36. Compensation circuit of the stator neglecting the Ohm-resistance.

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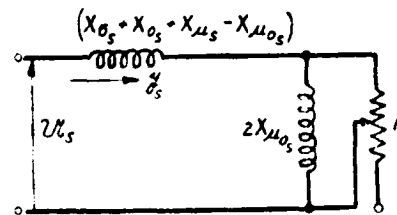
1) The signs in front of the square root can be omitted for $X_{\mu_{0s}}$ in this expression since the sign of $X_{\mu_{0s}}$ is always the same as the one of the quantity $\left[X_{\sigma_r}'^2 + 2 X_{\sigma_r}' X_{\mu_s} + X_{\mu_s}^2 \cdot \left(1 - \frac{\ddot{u}^2}{u^2}\right)\right]$. This can be seen in equation (66). If we assume the opposite sign of the magnitude $\left[X_{\sigma_r}'^2 + 2 X_{\sigma_r}' X_{\mu_s} + X_{\mu_s}^2\left(1 - \frac{\ddot{u}^2}{u^2}\right)\right]$ for $X_{\mu_{0s}}$, we obtain the following expression instead of equation (66):

$$\mathfrak{Z}_{e_s} = j \frac{X_{\mu_s} X_{\sigma_s} + X_{\mu_s} X_{\sigma_r}' + X_{\sigma_s} X_{\sigma_r}'}{X_{\sigma_r}' + X_{\mu_s}\left(1 + \frac{\ddot{u}}{u}\right)}$$

This impedance always is an inductive blind resistance and decreases with increase of $\ddot{u}/u$ which is not true for any value of $\ddot{u}/u$ as we have already seen physically. The sign of $X_{\mu_{0s}}$ is in the general case the same as the one of the quantity

$$\left[R_r'^2 + X_{\sigma_r}'^2 + 2 X_{\sigma_r}' X_{\mu_s} + X_{\mu_s}^2\left(1 - \frac{\ddot{u}^2}{u^2}\right)\right]$$

During ideal no-load operation, $r_s = \infty$ and the complete compensation impedance is purely blind, it is:

$$\mathfrak{Z}_{e_s} = j(X_{\sigma_s} + X_{0_s} + X_{\mu_s} + X_{\mu_{0_s}})$$

$$= j\left[X_{\sigma_s} + X_{\mu_s}\left|1 + \frac{X_{\mu_s}\left[X_{\sigma_s}\frac{\ddot{u}^2}{u^2}X_{\sigma_r} - X_{\mu_s}\left(1 - \frac{\ddot{u}^2}{u^2}\right)\right] + \frac{\ddot{u}}{u}\{X_{\mu_s}X_{\sigma_s} + X_{\mu_s}X_{\sigma_r}' + X_{\sigma_s}X_{\sigma_r}'\}}{X_{\sigma_r}'^2 + 2X_{\sigma_r}'X_{\mu_s} + X_{\mu_s}^2\left(1 - \frac{\ddot{u}^2}{u^2}\right)}\right|\right]$$

This expression can then directly be simplified to:

$$\mathfrak{Z}_{e_s} = j\frac{X_{\mu_s}X_{\sigma_s} + X_{\mu_s}X_{\sigma_r}' + X_{\sigma_s}X_{\sigma_r}'}{X_{\sigma_r}' + X_{\mu_s}\left(1 - \frac{\ddot{u}}{u}\right)} \qquad (66)$$

Likewise, the following expression holds for rotor compensation impedance neglecting the resistances and during ideal no-load operation:

$$\mathfrak{Z}_{e_r} = j\frac{X_{\mu_r}X_{\sigma_r} + X_{\mu_r}X_{\sigma_s}' + X_{\sigma_r}X_{\sigma_s}'}{X_{\sigma_s}' + X_{\mu_r}\left(1 - \frac{u}{\ddot{u}}\right)} \qquad (67)$$

If we call the lagging (magnetizing) current positive and the leading current negative, the following expressions are valid for the magnitudes of the currents:

$$J_s = U_s \cdot \frac{X_{\sigma_r}' + X_{\mu_s}\left(1 - \frac{\ddot{u}}{u}\right)}{X_{\mu_s}X_{\sigma_s} + X_{\mu_s}X_{\sigma_r}' + X_{\sigma_s}X_{\sigma_r}'} \qquad (68)$$

$$J_r = U_r \cdot \frac{X_{\sigma_s}' + X_{\mu_r}\left(1 - \frac{u}{\ddot{u}}\right)}{X_{\mu_r}X_{\sigma_r} + X_{\mu_r}X_{\sigma_s}' + X_{\sigma_r}X_{\sigma_s}'} \qquad (69a)$$

In order to be able to draw a direct comparison between stator and rotor current the currents have to be referred to one and the same location. The rotor current referred to the stator is:

$$J_r' = \frac{J_r}{\ddot{u}}$$

$$= U_r \cdot \ddot{u} \cdot \frac{X_{\sigma_s} + X_{\mu_s}\left(1 - \frac{u}{\ddot{u}}\right)}{X_{\mu_s}X_{\sigma_r}' + X_{\mu_s}X_{\sigma_s} + X_{\sigma_r}'X_{\sigma_s}}$$

$$= U_s \cdot \frac{\frac{\ddot{u}}{u}X_{\sigma_s} - X_{\mu_s}\left(1 - \frac{u}{\ddot{u}}\right)}{X_{\mu_s}X_{\sigma_s} + X_{\mu_s}X_{\sigma_r}' + X_{\sigma_s}X_{\sigma_r}'} \qquad (69b)$$

70

If the stator voltage is kept constant and the rotor voltage
or the ratio u/u are changed, the impedances Z_{es} and $Z_{er}^1 = u^2$
Z_{er} as well as the current J_s and J_r^1 will change according to the
expressions (66), (67), (68) and (69b). Fig. 37 shows these
quantities as function of the ratio u/u or the rotor voltage.
The half of this figure which lies above the abscissa corresponds
to lagging currents or inductive compensation impedances, the
lower half corresponds to leading currents or capacitive compen-
sation impedancies.

Two right-angled hyperbolas are obtained for compensation im-   /77
pedances, each obviously has a vertical asymptote through a
border of the magnetizing-similarity-field. The shape of each
current is a straight line. One part lies above and another
below the abscissa which separates the magnetizing from the de-
magnetizing current. The two borders of the magnetizing-
similarity-field (the unilateral magnetizations) correspond to

the ratios u/u equal to $\left(\frac{X'_{\sigma r} + X_{\mu s}}{X_{\mu s}}\right)$ and $\left(\frac{X_{\mu s}}{X'_{\sigma r} + X_{\mu s}}\right)$.

a): lagging (magn.)
    current, or
    inductive resistance

b): leading (de-magn.)
    current, or
    capacitive resistance

c): unilateral stator
    magnetization

d): constructional
    asymmetry

e): unilateral rotor
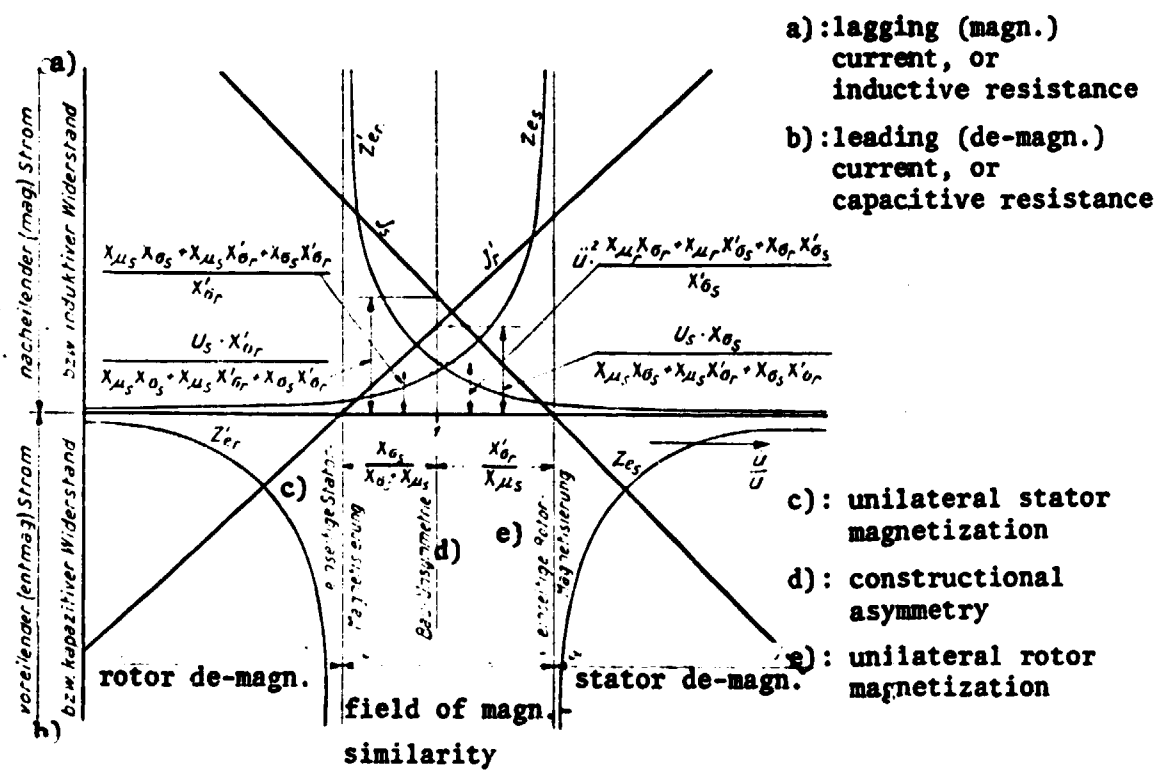   magnetization

Fig. 37. Influence of asymmetry on compensation impedance and
         currents during ideal no-load operation and neglectance
         of the Ohm resistances

Within this field both currents are magnetizing. Their ratio $\frac{J_s}{J_r'}$ can
have any value between $+\infty$ and $-\infty$ according to the ratio
$\frac{u}{u}$. The constructional asymmetry $(u \quad u)$ corresponds to a
ratio $\frac{J_s}{J_r'}$ which does not differ much from +1. It is interesting

71

to note, that the ratio $\frac{J_s''}{J_r''}$ of constructional asymmetry equals the
ratio $\frac{X_{\sigma_r}'}{X_{\sigma_s}}$. This means: <u>The ratio of the currents referred to</u>   /78
<u>one and the same side is inversely proportional to the stray</u>
<u>reactances.</u>[1])

The characteristic lines of Fig. 37 were calculated for an
experimental machine (machine I, see chapter (8.1.) and shown in
Fig. 38. It can be seen from it that the field of magnetization
similarity is rather limited in practice. Although figure 38 has
been calculated for a small machine (and thus with a relatively

large ratio $\frac{X_{\sigma_s} + X_{\sigma_r}}{X_{\mu_s}}$), the machine is already outside of this
field at a value of $\ddot{u}$ equal to about 1 ± 0,05. <u>The distribu-</u>
<u>tion of the magnetizing current between the two machine parts</u>
<u>thus strongly depends on the ratio</u> $\frac{\ddot{u}}{u}$. For the considered
machine, the total magnetizing current is transferred to one side
at a difference of about 5% between voltage ratio and transfer
ratio. At a difference of about 17% or 12%, the nominal current
is reached already during no-load operation due to this strong
asymmetry. The algebraic sum $(J_s + J_r')$ (a lagging current is
assumed to be positive) is also shown in Fig. 38. It increases
in a straight line with the ratio $\frac{\ddot{u}}{u}$ or U_r. This sum obviously
represents the effective magnetizing current (referred to the
stator). Thus, the air slit voltage referred to the stator is:

$$E_s = X_{\mu_s} \cdot (J_s + J_r')$$

and thus the function $E_s = f\left(\frac{\ddot{u}}{u}\right)$ also is a straight line.

The increase of the sum $(J_s + J_r')$ with the ratio $\frac{\ddot{u}}{u}$ is very
small since it is not caused by asymmetry but by the increase of
the rotor terminal air slit flow or E_s because of the increase of
the rotor terminal voltage. If the ratio $\frac{\ddot{u}}{u}$ is changed con-
sidering that the air slit voltage remains unchanged, the sum
$(J_s + J_r')$ remains unchanged and positive since it actually

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1) This fact is shown by an exact procedure to separate the stray
reactance of a slip ring rotor machine. The machine is d.s.
operated and stator and voltage are adjusted according to the
ratio $u = \ddot{u}$. The mechanic output is also adjusted such that
the machine covers its copper losses by the net. Under these con-
ditions, the ratio J_r'/J_s thus equals the ratio $X_{\sigma_s}/X_{\sigma_r}'$. As will
be shown later (Fig. 38) the Ohm-resistances have no considerable
influence on the magnitude of the currents in this experiment.
By a common short circuit experiment, the sum $X_{\sigma_s} + X_{\sigma_r}'$ is cal-
culated and thus the individual stray reactances can be calcu-
lated. This procedure is used to separate the stray reactances
of experimental machines (see chapter (8.1.)).

represent the magnetizing current necessary for the production of
the air slit flow. Consequently, the asymmetry does not cause an
increase of the complete blind current at constant air slit
voltage. Due to the blind output of the stray reactances

$(J_n^2 X_{\sigma_s}$ or $J_r^2 X_{\sigma_r})$ an increase of the total blind terminal

voltage of the machine at constant air slit voltage is caused by
the asymmetry. In Fig. 38 the output (obviously purely blind) of
each side as well as the total output of the machine are shown.

/80

/79

a): lagging
   (magnetizing)

b): leading,
   (de-magn.)

c): stator blind
   power

d): total blind
   power

e): rotor blind
   power

f): total blind
   power/V

g): unilateral
   stator magn.

h): constructional
   asymmetry

i): unilateral
   rotor magn.

j): blind power

k): current

————— Ohm resistance neglected
— — — — Ohm resistance considered
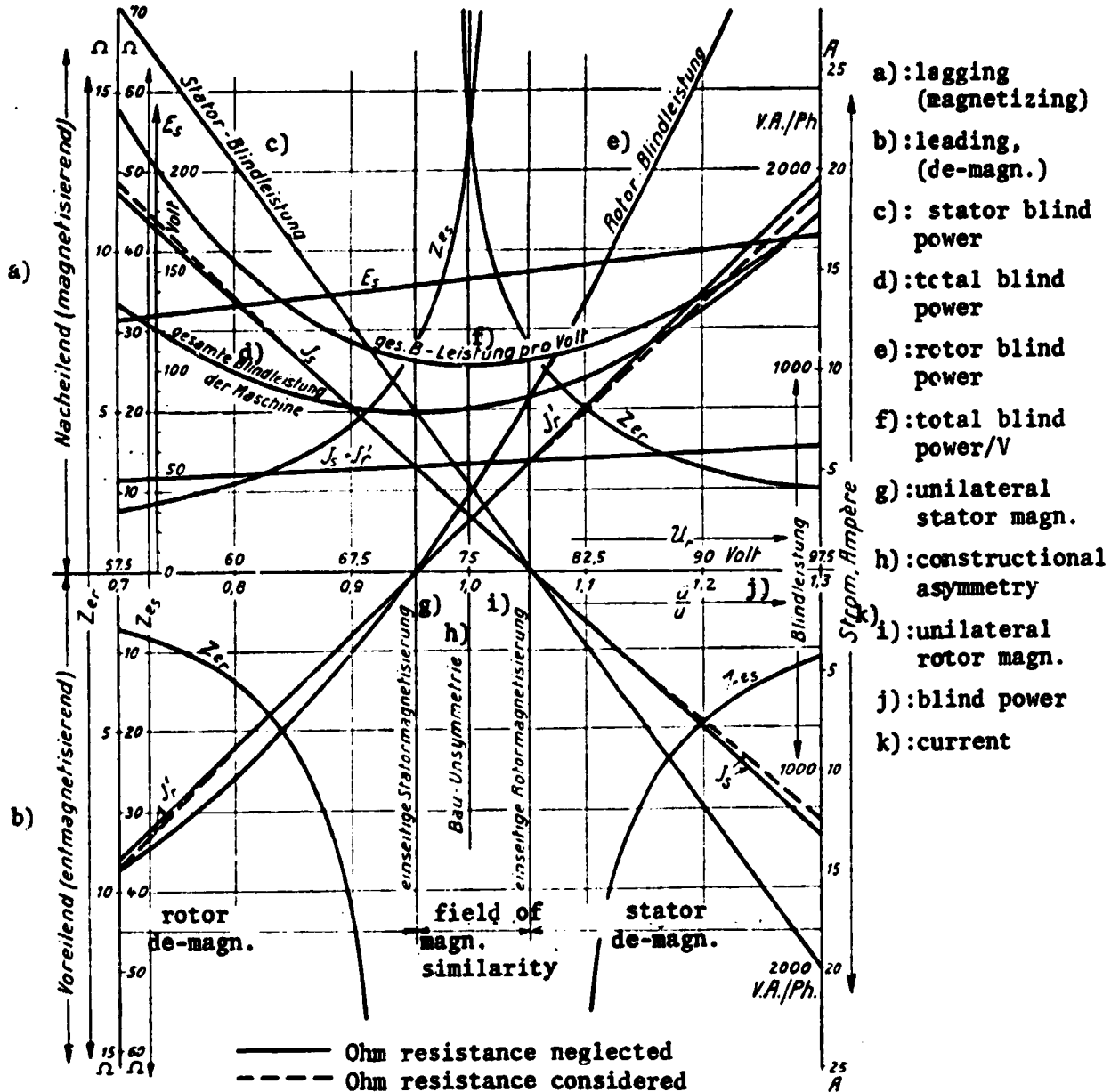
Fig. 38. Influence of asymmetry during ideal no-load operation.

$[U_n = 150$ Volt $n = 2$ $X_{\sigma_s} = 1.30$ $X_{\sigma_r} = 0.355$ $X_{\mu_s} = 27.3$ 25 Hz 1500 r/min.

73

The increase of the total output of the machine lies quite
far outside of the field of magnetizing similarity and it is
stronger to the right than to the left because the air slit
voltage and thus the magnetizing blind output increases with the
increase of U_r. For the same reason the total output does not
have its minimum near $\frac{u}{u} = 1$, but rather far to the left of it.
In order to compensate this influence of change of the air slit
voltage for the total output so that only the influence of
asymmetry remains, the total output at every point is divided by
the corresponding value of $E_s{}^2$. This results in the <u>total output
per volt of the air slit voltage</u> (Fig. 38). It can be concluded
<u>that the asymmetry causes an increase of the total blind output
of the machine, which increases the faster the more the ratio</u>

$\frac{u}{u}$ deviates from 1. <u>Within the field of magnetizing simi-
larity the increase of blind output caused by the asymmetry is
essentially insignificant</u>.

The above results were derived without considering the Ohm
resistances. In order to show the influence of the resistances
on these results, the currents J_s and J_r' were calculated after
the equations (34c) and (36b) for ideal no-load operation. The
dashed line in Fig. 38 represents the calculated results with
the consideration of the resistances. No considerable differ-
ence exists between the calculated results with or without con-
sideration of the Ohm resistances for constructional asymmetry
and small degrees of essential asymmetry, only for large degrees
of essential asymmety a small difference exists.

In order to limit the blind power intake, the machine should
function within the field of magnetizing similarity. In this
respect it is most favorable to chose $\frac{u}{u} = 1$ (i.e., $n = 1$ for parallel
connection) since this case corresponds to the minimum blind out-
put per volt (Fig. 38). It will be shown in chapter (7.6.), however,
that the ratio has to be $n < u$, in order to make best use of the
machine, i.e., to maintain the maximum continuous output of a
certain machine.

(7.4.) <u>Characteristics of the constructional</u>
<u>asymmetric machine</u>

In Fig. 39 the compensation circles and circuits diagrams of a
constructional asymmetric machine are shown. They are calculated
for the same machine I as in chapter (7.2.) for the symmetric ma-
chine without the prerequisite of symmetry of the constants. The
sum of the constants $(R_s + R_r')$ or $(X_{\sigma_s} + X_{\sigma_r})$ is the same in both
cases and thus the difference between both cases is only caused by
the <u>constructional asymmetry</u>. The circle diagram obtained assuming
asymmetry is drawn with lines in Fig. 39 for each side in the respective
scales. It can be seen from it that the <u>influence of constructional
asymmetry during demand shows mainly in a very considerable shift of</u>

74

the center of the one circle above and of the other below the
abscissa. The ratio and the x-components of the centers, however,
are only insignificantly influenced. This shift of the circles in
two opposite directions obviously causes a certain dissimilarity of
the currents as well as a considerable dissimilarity of the power
factors. The circle of the side with the larger Ohm resistance
(rotor) is shifted above the abscissa and thus this side will have
a considerably larger and the other side a smaller output during
motor operation. Quite in the contrary, during generator operation
the first side will have a smaller and the other a larger power
factor.
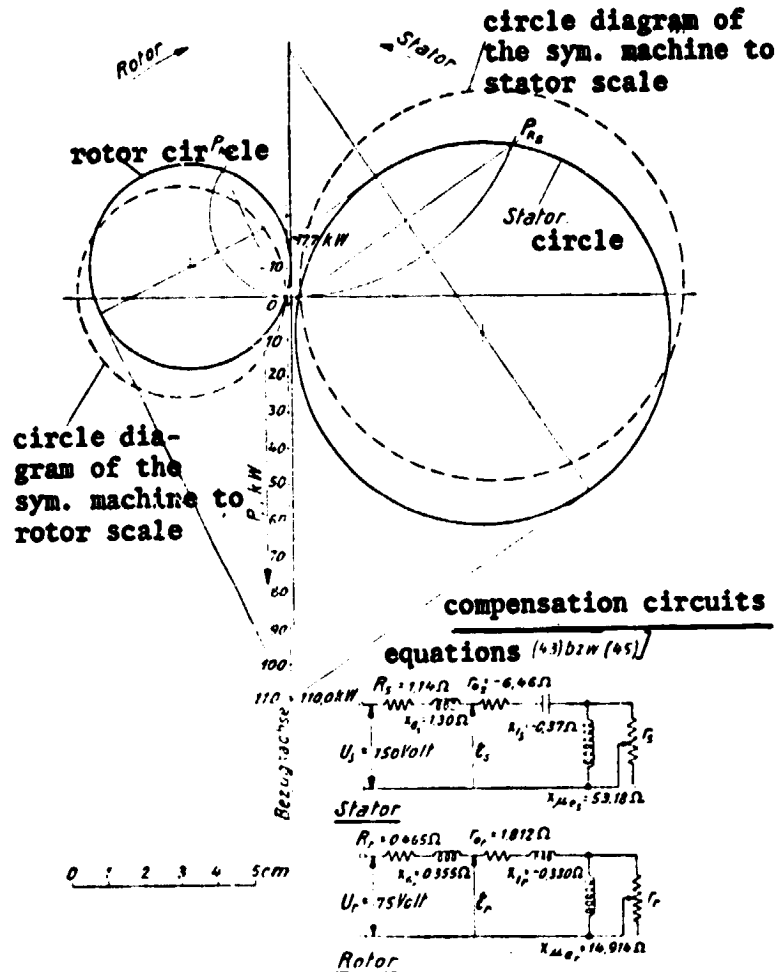
Fig. 39. Compensation circuits and current diagrams of a construc-
tional asymmetric machine.

Scales: stator current:1cm.=10,0 Amp./ph., rotor current:1cm.=37,35
Amp./ph., inner output:1cm=10,3ºkW (total)- stator terminal
output:1cm.=4,5kW (total), rotor terminal output:1cm.=8,40
kW (total).

75

This shift of the circuits caused by constructional asymmetry has another important consequence on the load capacity of the machine or its continuous output[1]. Since the terminal output of the stator differs only slightly[2] from that of the rotor, and the power factors are dissimilar, the supplied current of the side with larg $\cdot$ resistance is considerably smaller than that of the other side during normal motor operation and considerably larger during normal generator operation. This fact can also be seen from Fig. 40, where the characteristic lines of the machine are drawn as function of the inner output in the complete possible operating field. They were calculated after the circuit diagrams (Fig. 39). The currents J_s and J_r' are the same only at a very low demand, otherwise they differ in the sense that $J_n > J_r'$ during motor operation and $J_r' > J_n$ during generator operation at stable run.

The characteristic lines of the machine (Fig.40) have the same character as the symmetric machine. The characteristic lines of the stator, however, differ from those of the rotor. Everywhere in the theoretically possible field, the currents J_s and J_r' do not differ much, although they do show a large deviation within the applicable field. The deviation of the currents is about the same everywhere. The difference between the terminal outputs as well as between the power factors is the larger, the larger the inner output is. Stator or rotor terminal output as function of the inner output is an ellipse and thus the function of the complete terminal output of the machine $P'_{kv} = P'_{k_s} + P'_{k_r} = f(P'_i)$ also is an ellipse, as shown in Fig.40. An important comparison between the symmetric and the constructional asymmetric machine is obtained by comparing the energy-ellipse $P'_{kv} = f(P'_i)$ of both cases. These are again shown in Fig.48 (chapter (7.5.)). The ellipse B corresponds to the studied case of a constructional asymmetric machine and ellipse A corresponds to the same machine with symmetry presumed (chapter (7.2.)). The comparison between both ellipses shows in the first place that a _decrease of sweep output or momentum of motor and gene-_ _rator operation is caused by the constructional asymmetry_. The efficiency which, of course, is only important for the practically applicable field, is not considerably decreased by the constructional asymmetry since both ellipses almost coincide within this field.

Another theoretically important consequence of asymmetry is the state corresponding to the points P_{ks} or P_{kr} (Fig.39). This state differs somewhat from the corresponding one of the symmetric machine. As Fig.40 shows, the currents J_s and J_r' are not the same at this state and thus the amplitude of the resulting flux wave does not equal zero. This way a small induced voltage proportional to the difference $|J_n - J_r|$ exists in every part and every current is

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1) See chapter (7.6.)
2) Only by the same amount as the difference between the copper losses of the two sides.

76
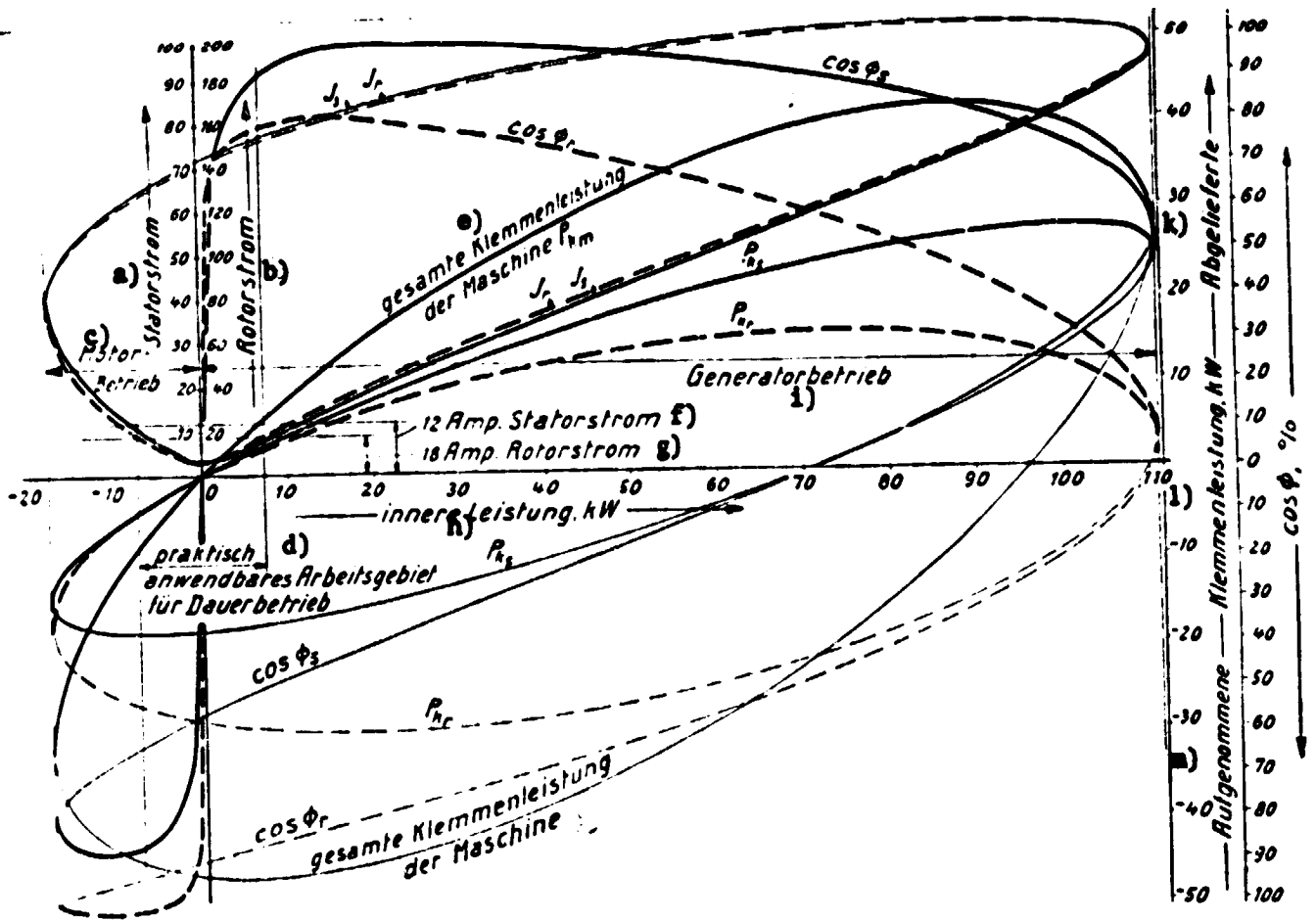
a):stator current; b):rotor current; c):motor operation; d):practically applicable
operating field for continuous demand; e):total terminal power; f):stator current;
g):rotor current; h):inner power; i):generator operation; j):total terminal power;
k):supplied- ; l):terminal power; m):received- ;

Fig. 40. Characteristic lines of the constructional asymmetric
machine in the theoretically possible working field.
U_s = 150 V., U_r = 75 V., u = ü = 2 25 Hz, 1500 r/min.,
R_s = 1.14 R_r = 0.465 x_o = 1.30 x_o = 0.355 x_μ = 27.3

———; characteristic lines of the stator and total output

– – –; characteristic lines of the rotor.

perpendicular to the corresponding induced voltage since the inner output equals zero. One of the currents then is purely magnetizing, the other purely de-magnetizing. The compensation circles and the combined space and time vector diagram of the machine at this state are shown in Fig. 41.
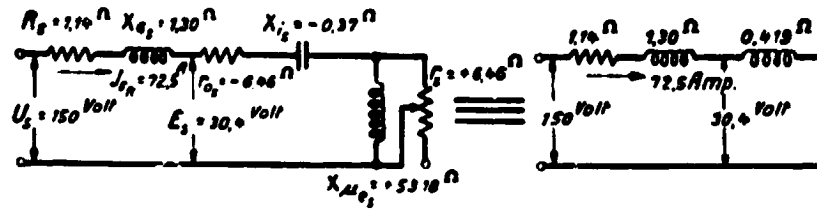
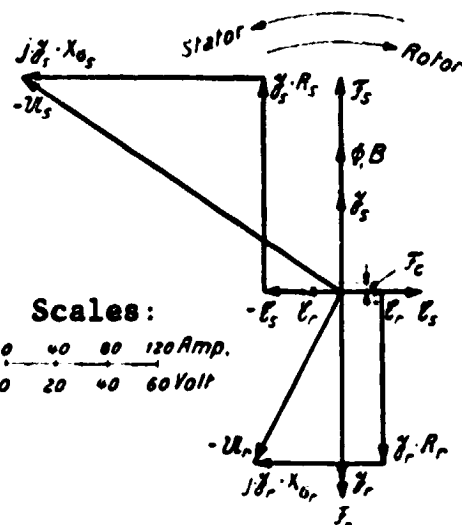Stator compensation circuit
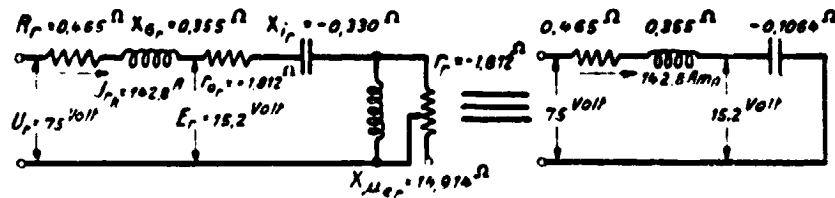
Rotor compensation circuit

Fig. 41. Compensation circuits and combined space and time vector diagram of a constructional asymmetric machine for the state represented by point P_{ks} or P_{kr} (machine I).

The magnitude of the parameter r_s results from equation (51b) and that of the parameter r_r from a similar expression[1].

(7.5.) Characteristics of the essentially
asymmetric machine

The characteristics of the essentially asymmetric machine shall be explained with three examples which were calculated for the experimental machine I[2]. The machine has a transmission ratio $n = 2.00$ and its normal voltage is 150 V per stator phase or 75 V per rotor phase. The calculations were done for three different degrees of essential asymmetry. They are:

Case (1): $U_n = 147$ Volt, $U_r = 76$ Volt.

Substituting these values in expression (46a) for the dissimilarity factor, we obtain:

$$K = \frac{1}{u^2} \cdot \frac{R_s^2 + X_{\sigma_s}^2 + 2 X_{\sigma_s} X_{\mu_s} + X_{\mu_s}^2 \left(1 - \frac{u^2}{u^{\cdot 2}}\right)}{R_r^2 + X_{\sigma_r}^2 + 2 X_{\sigma_r} X_{\mu_r} + X_{\mu_r}^2 \left(1 - \frac{u^2}{u^{\cdot 2}}\right)}$$

$$= + 10.52$$

Thus, this case corresponds to a positive dissimilarity factor and lies within the field of magnetizing similarity.

Case (2): $U_n = 153.5$ Volt, $U_r = 73.2$ Volt.

This results in: K = 0.00

The dissimilarity factor equals zero and thus this case corresponds to a border of the field of magnetizing similarity, namely, the unilateral stator magnetization.

Case (3): $U_n = 143$ Volt, $U_r = 79$ Volt.

This results in: K = -12.50

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1) In this case the parameter r_s or r_r practically has the value r_{0s} or r_{0r}.
2) The assumed constants of the machine are: $R_s = 1.14$ Ohm, $X_{\sigma_s} = 1.30$ Ohm, $R_r = 0.465$ Ohm, $X_{\sigma_r} = 0.355$ Ohm, $X_{\mu_s} = 27.3$ Ohm.

This case corresponds to a <u>negative</u> dissimilarity factor and thus lies outside of the field of magnetizing similarity. It corresponds to a <u>de-magnetizing stator</u>, since the denominator of the expression of $\overline{K}$ is negative.

In order to draw a real quantitative comparison between all three cases, we chose the voltages U_s and U_r so that the sum U_s+U_r' is about the same in all three cases (about 300 volt), thus the air slit induction also is about the same. A calculation example of the compensation circles and current diagram is given in annex I.

Case (1): <u>Within the field of magnetizing similarity</u>:

Fig.42. shows the compensation circles of the machine and its combined circuit diagrams under the conditions of this case. The dashed circles in the same figure show the circuit diagrams of the machine operating as a constructional asymmetric machine (i.e., with voltages of 150 and 75 volt) (chapter (7.4.)). As already mentioned, this case lies within the field of magnetizing similarity and thus both machine parts accept magnetizing currents during ideal no-load operation (point P_{0_s} and P_{0_r}, Fig.42.). But since the ratio is $u < u$, this state is close to the unilateral rotor magnetization or in other words, because of this essential asymmetry, the rotor accepts a larger part of the necessary magnetizing current during ideal no-load operation than the constructional asymmetric machine - as also indicated by the points P_{0_s} and P_{0_r}. <u>This way the stator current is shifted close to the origin and the rotor current far away from it and an increase of the power factor of the stator as well as a decrease of that of the rotor during motor as well as generator operation is caused</u>.

The characteristic lines of the machine in the complete theoretic operating field were calculated with the help of the circuit diagram and are shown in Fig. 43. In their general character they do not differ from that of a symmetric or constructional asymmetric machine. The characteristic lines of the currents only show a considerable change in the practically applicable field where the rotor current is slightly larger and the stator current smaller than in the constructional asymmetric machine, which is obviously caused by the transfer of magnetizing current from rotor to stator. Thus the difference between the currents J_s and J_r' at maximum demand is smaller during motor operation and larger during generator operation than it is for the constructional asymmetric machine. The rotor power factor of the constructional asymmetric machine at maximum demand during motor generation was much larger than that of the stator. Since the rotor in this case, however, has a considerably larger part of the magnetizing current - compared to the constructional asymmetric machine - its power factor is smaller than that of the constructional asymmetric machine and relieves the stator of a part of the blind current.
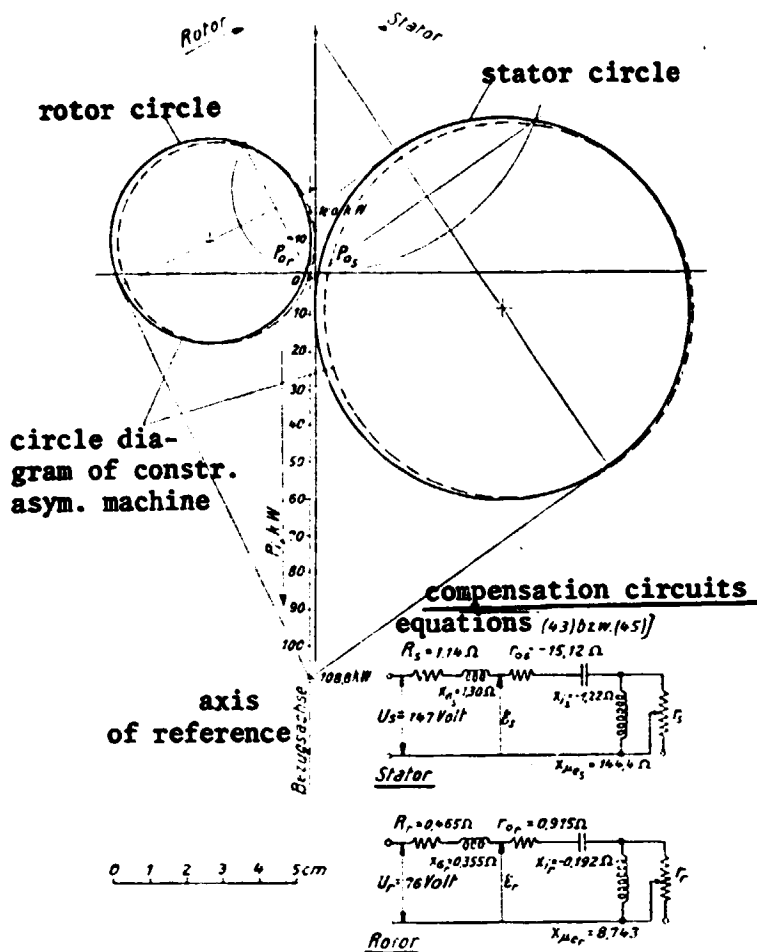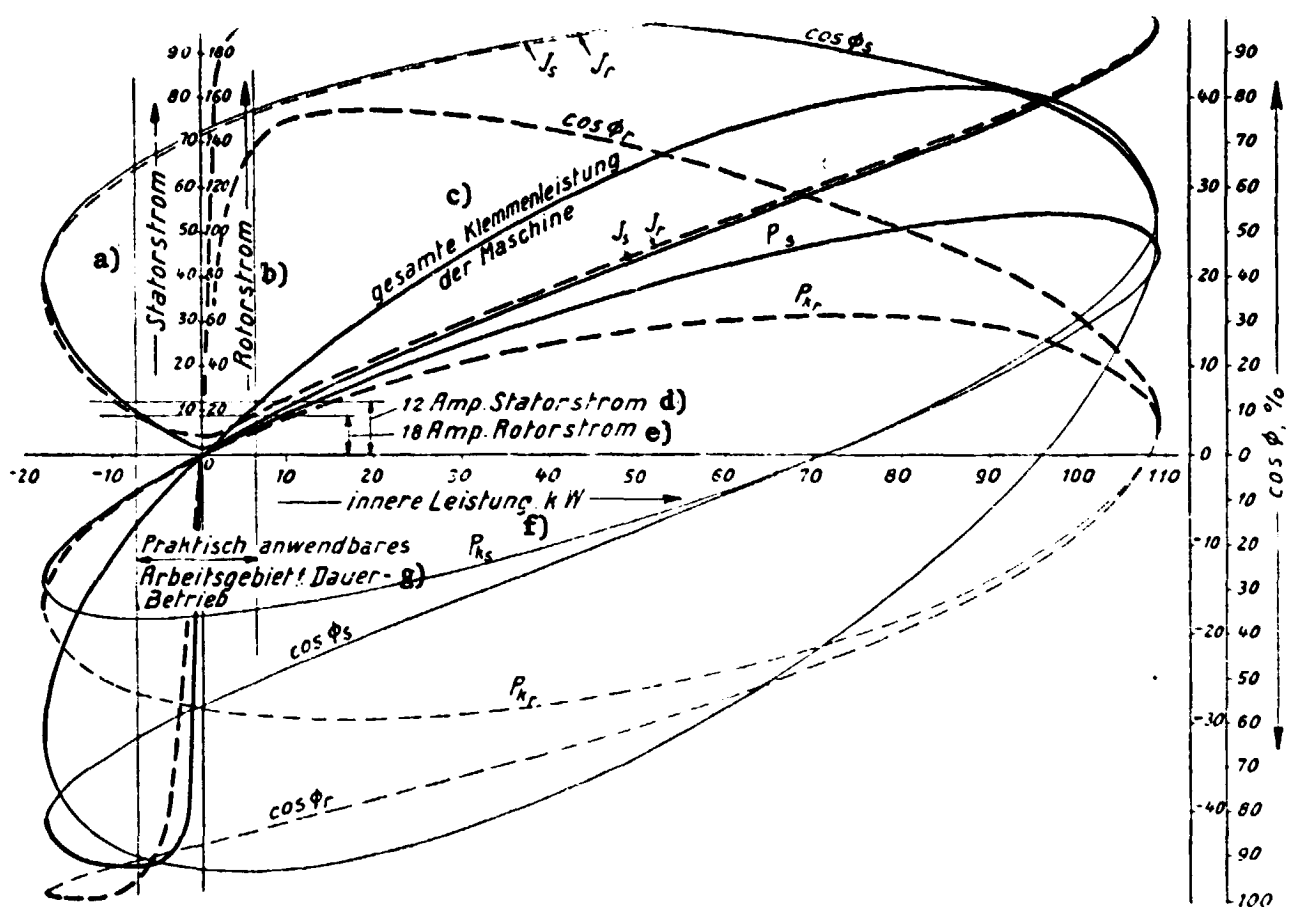
Fig. 42.   Compensation circles and current diagrams of an essentially asymmetric machine.

Case (1): Within field of magnetizing similarity.

$U_s = 147$ Volt, $U_r = 76$ Volt, $\ddot{u} = 2$ 25 Hz, 1500 r/min.
$R_s = 1,14\ \Omega$, $X_{\sigma_s} = 1,30\ \Omega$, $R_r = 0,465\ \Omega$, $X_{\sigma_r} = 0,355\ \Omega$, $X_{\mu_s} = 27,3\ \Omega$

Scales: stator current:1cm =10Amp /Ph., rotor current:1cm = 36.2Amp /Ph., inner output:1cm =10.18kW (total), stator terminal output:1cm =4.41kW (total), rotor terminal output:1cm =8.25kW (total).

a):stator current; b):rotor current; c):total terminal power; d):stator current;
e):rotor current; f):inner power; g):practically applicable operating field for
continuous demand;

Fig. 43. Characteristic lines of the essentially asymmetric machine
in the theoretically possible operating field.

Case 1: U_s = 147 V , U_r = 76 V , u = 2 25 Hz, 1500 r/min. (within
the field of magnetizing similarity).

____ Characteristic lines of stator and output of the
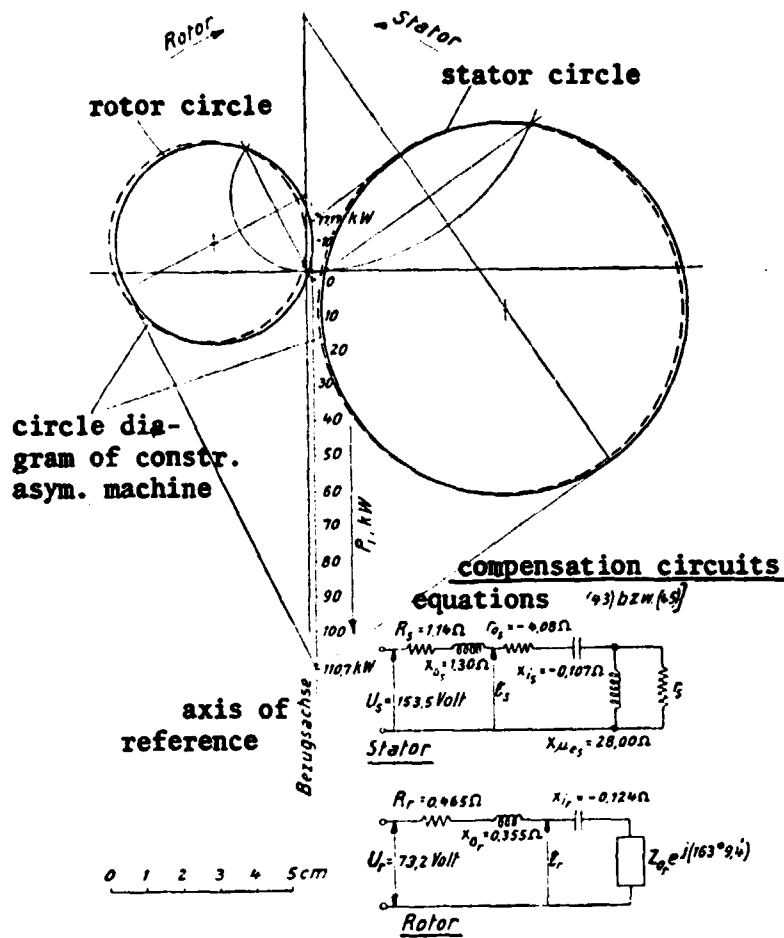machine.

____ Characteristic lines of rotor.

Fig.44. Compensation circles and current diagrams of an essentially asymmetric machine

Case 2: Unilateral stator magnetization

$U_s = 153.5$ Volt, $U_r = 73.2$ Volt, $\ddot{u} = 2.25$ Hz, 1500 r/min.
$R_s = 1.14\ \Omega$, $X_{\sigma_s} = 1.30\ \Omega$, $R_r = 0.465\ \Omega$, $X_{\sigma_r} = 0.355\ \Omega$, $X_{\mu_s} = 27.3\ \Omega$.

Scales: stator current:1cm.=10.0Amp./ph., rotor current:1cm= 39.0 Amp./ph., inner output:1cm.=10.63kW (total), stator terminal voltage:1cm.=4.605kW (total), rotor terminal voltage:1cm=8.56kW (total).

Case 2: <u>Unilateral stator magnetization</u>

In this case $U_s > U_r'$, i.e., $u > \dot{u}$ and thus the stator takes over a larger part of the magnetizing current during ideal no-load operation. As already mentioned, the voltages are chosen such that the stator takes over all of the magnetizing current during ideal no-load operation. Thus, the stator circle shall be shifted far from the origin parallel to the abscissa, and the rotor circle shall be shifted in the same direction until the latter passes through the origin. The compensation circles and circle diagrams were calculated and are shown in Fig.44 where the shift of the circles becomes obvious. By this shift <u>a decrease of the power factor of the stator and an inrease of that of the rotor during generator as well as motor operation is caused</u>. The increase of the power factor of the rotor is large enough in this case for the rotor to accept a leading current in a part of the motor field as shown in Fig.44. This intake of leading current (de-magnetizing current) is unfavorable especially since it also exists during maximum demand because it is balanced by an additional lagging current of the other side as shown previously. <u>Both currents then are larger during motor generation and the load capacity of the machine is limited</u>.

The characteristic lines of the machine were calculated after the circuit diagrams and are shown in Fig. 45. It shows the increase of the power factor of the rotor and the decrease of the stator power factor during motor as well as generator operation as compared to the constructional asymmetric machine and thus they deviate more from each other during motor generation and less during generator operation. Both power factors which differ significantly in the constructional asymmetric machine during generator operation, are thus almost equal in this case during maximum demand as generator due to the asymmetry which relieves the rotor from a part of the blind current. Thus both currents J_s and J_r' become almost equal during maximum demand of generator operation. During motor generation the difference between them is, because of the chosen essential asymmetry, larger than in the constructional asymmetric machine. /93

/90

/92

Case 3: <u>Outside of the magnetizing similarity field</u>:

The compensation circuit and circuit diagrams of the machine in this case are given in Fig. 46. The influence of essential asymmetry can be seen which causes the <u>shift of the rotor circle far from the origin and of the stator circle almost parallel to the abscissa</u> since $U_r' > U_s$. The rotor thus should take over a large part of the magnetizing current as compared to the constructional asymmetric machine. The extent of the essential asymmetry is in this case large enough for the stator to take over de-magnetizing (leading) current already during no-load operation (point P_{0s}, Fig.46). Thus, the stator circle has a part to left of the origin in the motor and generator field. In this part of the circle diagrams the rotor takes in a blind current which is larger than the blind current necessary for field generation and coverage of stray k.v.a. in order to compen-
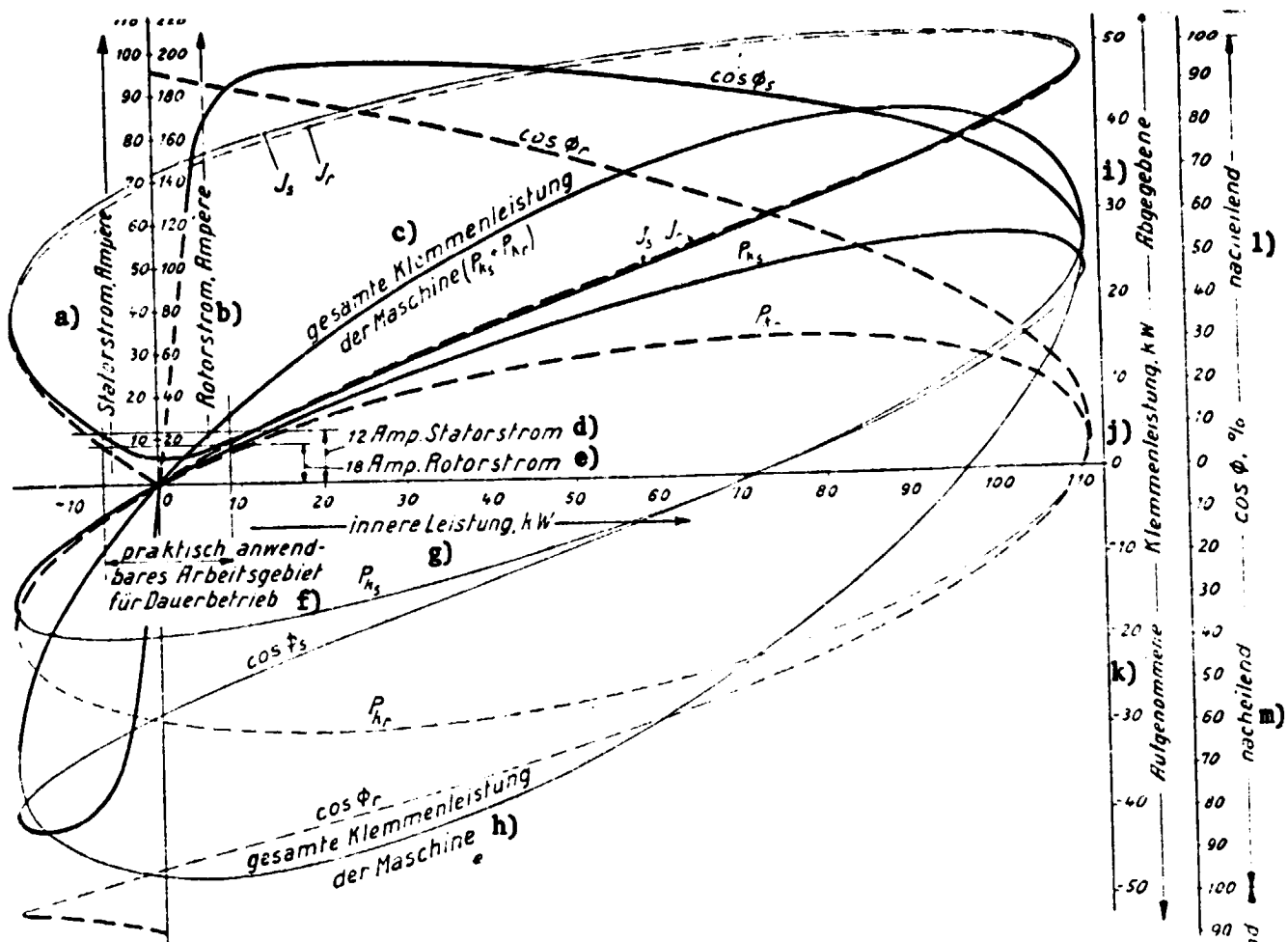
84

a):stator current; b):rotor current; c):total terminal power; d):stator current;
e):rotor current; f):practically applicaple operating field for continuous demand;
g):inner power; h):total terminal power; i):supplied- ; j):terminal power;
k):received- ; l):lagging; m):lagging; n):leading;

Fig. 45. Characteristic lines of the essentially asymmetric
machine in the theoretically possible operating field.

Case 2: Unilateral stator magnetization. U_s = 153.5 Volt,
U_r = 73.2 Volt, ü = 2 25 Hz, 1500 r/min.

———┤ characteristic lines of stator and total output

_-_-┤ characteristic lines of rotor.

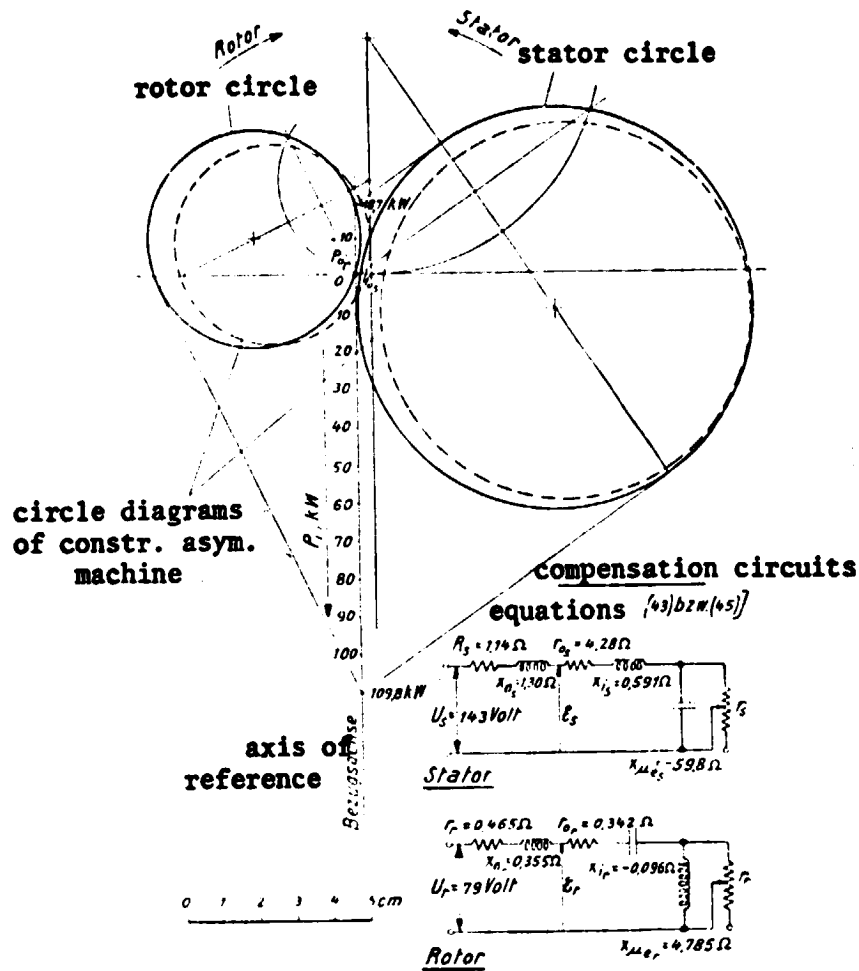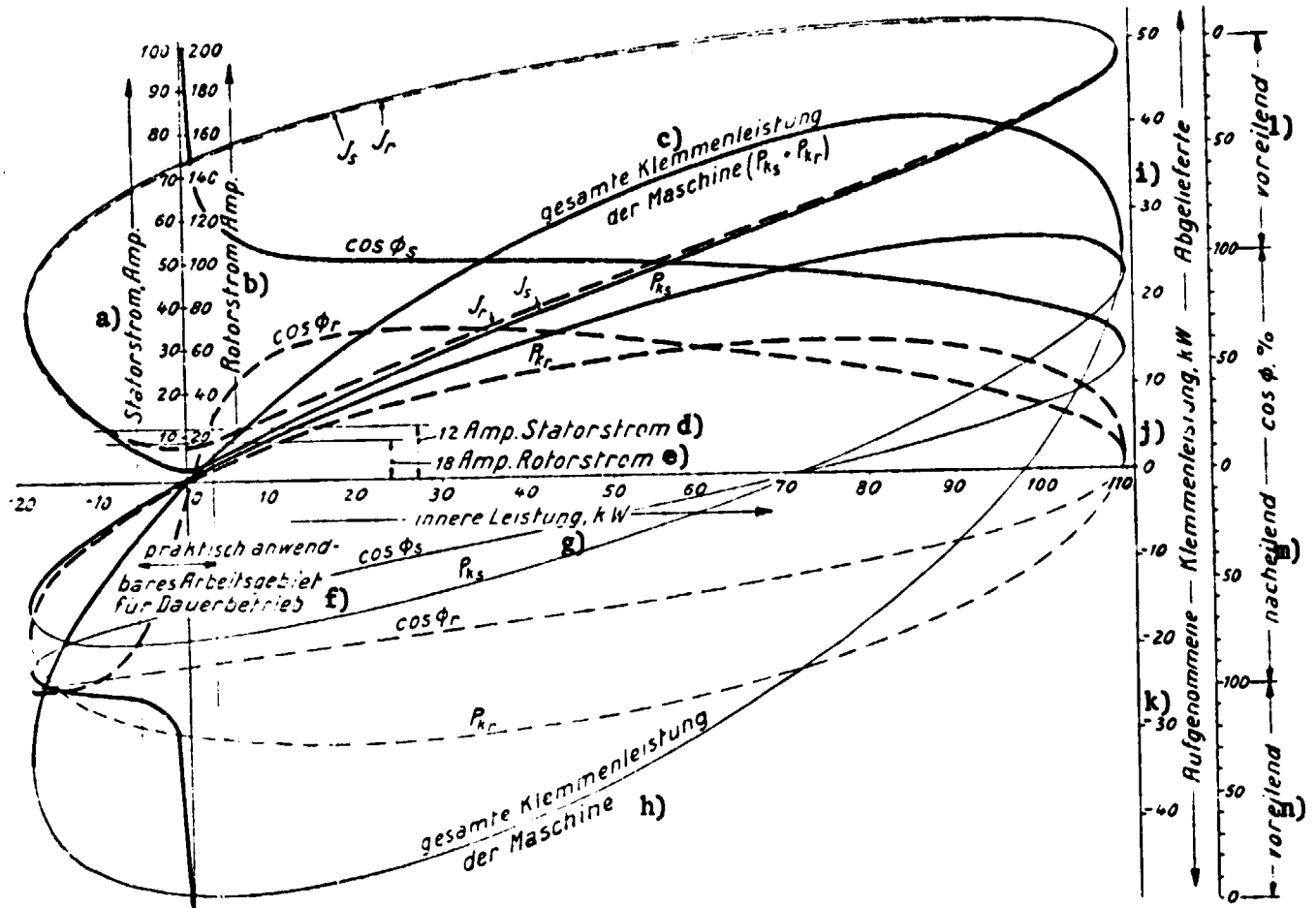sate the de-magnetizing effect of the leading current.

Fig.46. Compensation circles and current diagrams of an essentially
        asymmetric machine.

Case 3: Outside of the magnetizing similarity field; stator de-
         magnetizing

$U_s = 143$ Volt, $U_r = 79$ Volt, $\bar{n} = 2\ 25$ Hz, 1500 r/min.

$R_s = 1.14\ \Omega.\ X_{a_s} = 1.30\ \Omega.\ R_r = 0.465\ \Omega.\ X_{a_r} = 0.355\ \Omega.\ X_{\mu_s} = 27.3\ \Omega.$

Scales: stator current:1cm =10Amp./ph., rotor current:1cm =33.87
         Amp./ph., inner output:1cm=9.903kW (total), stator terminal
         voltage:1cm.=4.29kW (total), rotor terminal voltage:1cm=
         8.03kW (total).

86

a):stator current; b):rotor current; c):total terminal power; d):stator current;
e):rotor current; f):practically applicable operating field for continous demand;
g):inner power; h):total terminal power; i):supplied- ; j):terminal power; k):received- ;
l):leading; m):lagging; n): leading;

Fig. 47. Characteristic lines of the essentially asymmetric
machine in the theoretically possible operating field.
Case 3:   Outside of the magnetizing similarity field (stator
de-magnetizing).
U_s = 143 V., U_r = 79 V., ü = 2 25 Hz, 1500 r/min.

_____) characteristic lines of stator and total output.

_ _ _) characteristic lines of rotor.

As Fig.47 shows, the rotor current is much larger than the stator current in this part of the circle diagrams. Because of this pronounced asymmetry of the currents the continuous output of the machine is strongly limited especially during generator operation. The asymmetry is much smaller during motor operation as can be seen from the circle diagram, and the currents J_s and J_r' do not differ much, especially for large outputs.

Considering these three examples it can be concluded that the influence of essential asymmetry consists mainly (and to a very large extent) in the unequal distribution of the magnetizing currents among both machine parts as explained in chapter (7.3.). During operation, this influence becomes evident essentially in a shift of both current circles of the machine in the same direction, almost parallel to the abscissa. The circle of the side with the larger voltage is shifted far from the origin, the other one close to it. This is the case because the first side has to take over the magnetizing current of the second side in order to compensate the essential asymmetry (chapter 7.3.). This shift of the circles evidently causes an increase of the power factor of the second side, during both motor and generator operation. We have shown, however, in chapter (7.4.) that the constructional asymmetry causes a shift of both circles in opposite direction perpendiculat to the abscissa; Thus, the power factor of the one side increases. The power factor of the other side decreases during motor generation and vice-versa during generator operation. The influence of essential asymmetry thus differs fundamentally from that of constructional asymmetry, although we deal in both cases with merely an unequal distribution of the magnetizing current among both machine parts. By any kind of asymmetry the magnitudes of the currents are influenced due to the take over of the magnetizing current from one side to the other. Thus, the essential asymmetry provides the possibility to transfer any ratio of the blind current from one side to the other and thus to control the magnitude of the currents within wide ranges in a very simple way. This fact has an important success, namely, the complete utilization of the machine by chosing an extent of essential asymmetry in a manner that both currents J_s and J_r' reach their allowable values at the same time during operation (see chapter (7.6.)).

The influence of asymmetry on the sweep output and the effectiveness of the machine can be demonstrated by the comparison of the "energy ellipses" of the machine ($P_{kM} = f(P_i)$). They are shown in Fig. 48 for all three discussed cases of the essential asymmetry as well as for the symmetric and constructional asymmetric machine. The maximum inner output during continous operation as motor and the maximum terminal output during continous operation as generator are also given for every case. Ellipse A of the ideal d.s. operation (assuming symmetry) everywhere lies outside of the others and thus corresponds to the maximum sweep output and the best effectiveness during both motor and generator operation. As a comparison between ellipse B and A shows, a considerable decrease
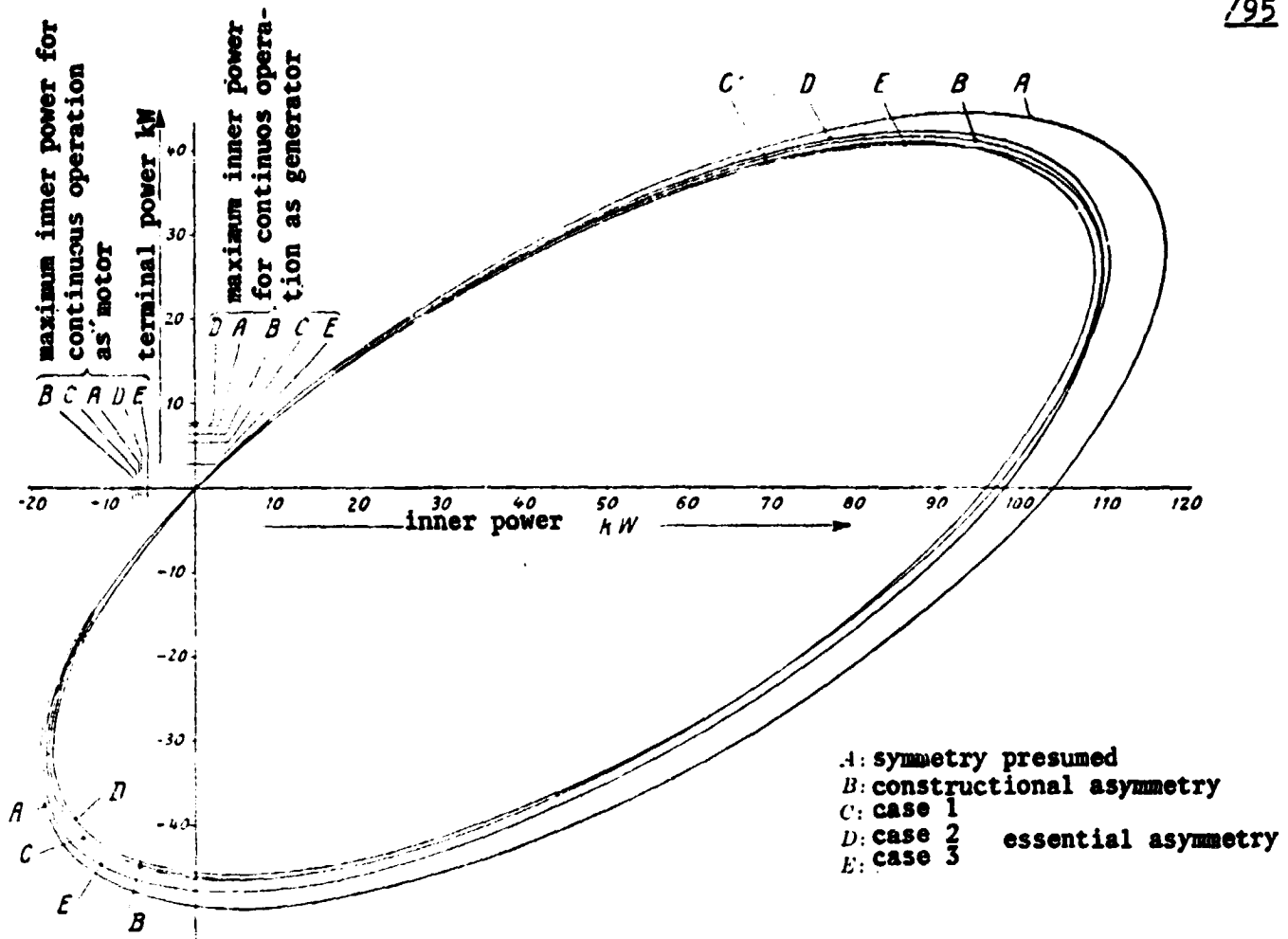
Fig. 48. Energy ellipses of the machine for various degrees
of asymmetry.

A: symmetry presumed
B: constructional asymmetry
C: case 1
D: case 2   essential asymmetry
E: case 3

89

of sweep output and a slight decrease of effectiveness is caused by
the constructional asymmetry. The comparison between the three
ellipses C, D and E with ellipse B shows that <u>a decrease or increase
of sweep output and effectiveness is caused by the essential asym-
metry according to its direction</u>. The change of sweep output and
effectiveness caused by the essential asymmetry in all three cases
(i.e., not exaggerated unpractically) is not as significant as the
one caused by the constructional asymmetry. It is interesting to
note that the extent of essential asymmetry which compensates the
influence of constructional asymmetry and which causes the equality
of the currents J_s and J_r' during nominal load of an operating mode,
also causes an increase of sweep output and effectiveness during the
respective mode of operation. This fact can be seen from a com-
parison of ellipse B with ellipses D and C. Ellipse D which refers
to case (2) of essential asymmetry (where the currents J_s and J_r'
have about the same values during normal demand of generator
operation) has a larger generator sweep output as well as a larger
effectiveness (as compared to the constructional asymmetric machine) /97
during generator operation since it lies higher than ellipse B in
this field. By comparison of ellipses A and B it is also shown that
case (1) of the essential asymmetry where both currents J_s and J_r'
almost have the same values during normal demand of motor generation,
also has a larger sweep output as well as a larger effectiveness
during motor operation.

It is, however, important to note that the influence of the
essential asymmetry (if not unpractically exaggerated) on sweep out-
put and the effectiveness of the machine is very small and of no
considerable significance in practice.

(7.6.) <u>Essential asymmetry as means of enhancing</u>
<u>continuous output</u>

In chapter (7.2.) and (7.5.) it has been shown that the symmetric
machine represents the ideal model of d.s. operation, where both ma-
chine parts always play exactly the same role in the production of
the common field and supplying effective output. Consequently, both
machine parts always have the same (supplied) current and the same
power factor, and the machine has the maximum sweep and effective
output. The continuous output of the machine which is merely deter-
mined by the allowable values of the currents during continuous
operation, would also be largest for a symmetric machine if the al-
lowable values of the currents had the ratio " i.e., the currents
J_s and J_r' <u>were of the same</u> allowable value. In practice this con-
dition, however, is by far not fulfilled. The rotor slots always
have a smaller volume than the stator slots and thus the allowable
value of J_s is considerably larger than that of J_r'. It is simple to
prove that:

$$\frac{J_{s}}{J_{r}} = \frac{j_s \cdot F_s \cdot l_s}{j_r \cdot F_r \cdot l_r}$$

where:

90

J_{sz} = allowable value of stator current for continuous operation

J_{rz} = allowable value of J_r' for continuous operation

= $\frac{1}{\ddot{u}}$ X (allowable value of rotor current for continuous operation)

j_s, j_r = allowable current density of stator or rotor for continuous operation

F_s, F_r = complete area of stator or rotor slots

= cross section of a slot X amount of slots

f_s, f_r = slot filling factor of stator or rotor
copper volume of a slot

= volume of a slot

The ratio $\frac{F_s}{F_r}$ is evidently larger than 1 and depends on the size of the machine. The smaller the machine, the larger this ratio is. It can increase to about 1.6 - 1.8 (according to the number of re- volutions) for small machines with about 5 H.P. For still smaller machines it might be yet larger. The ratio $\frac{j_s}{j_r}$ generally is smaller than 1 due to the better heat emission of the rotor, its value is about 0.9. The ratio $\frac{f_s}{f_r}$ depends on the voltages of both machine parts and generally is about 0.9-1 for induction machines of common construction (with the rotor being purposely coiled for a smaller voltage). As an example we consider the experimental machine (see chapter (8.1.)). For the mentioned quantities it is of the following values:

$$F_s = 112 \times 10^2 \text{ mm}^2 \qquad F_r = 75{,}1 \times 10^2 \text{ mm}^2$$
$$f_s = 0{,}33 \qquad f_r = 0{,}32$$
$$j_s = 4{,}0 \text{ A/mm}^2 \qquad j_r = 4{,}5 \text{ A/mm}^2$$

consequently:

$$\frac{F_s}{F_r} = \frac{112 \times 10^2}{75{,}1 \times 10^2} = 1{,}49$$

$$\frac{j_s}{j_r} = \frac{4}{4{,}5} = 0{,}89$$

$$\frac{f_s}{f_r} = \frac{0{,}33}{0{,}32} = 1{,}03$$

thus the ratio becomes:

$$\frac{J_{sz}}{J_{rz}} = 1{,}49 \times 0{,}89 \times 1{,}03 = 1{,}36$$

The rounded off values of the allowable currents are

$$J_{s} = 12 \text{ Ampère}$$
$$J_{r} = 18 \text{ Ampère, also} \quad J_{r}' = 9 \text{ Ampère}$$

and thus:

$$\frac{J_{s}}{J_{r}'} = \frac{12}{9} = 1.33$$

It is also important to notice that the given values are valid for the ratios $\frac{v_s}{v_r}$, $\frac{j_s}{j_r}$ and $\frac{I_s}{I_r}$ for asynchronous machines of common construction. The ratio would be larger yet for machines which are specifically qualified for d.s. operation. This is due to the fact that rather a larger induction is allowed in the rotor teeth than in the stator teeth of an induction machine since no considerable loss of iron is produced in the rotor and more space is consequently available for the rotor slots. This can not be done during d.s. operation because these obviously are iron losses both in rotor and stator with net frequency.   /99

We see that the allowable value of stator current is rather higher than that of the supplied rotor current. The total utilization of the machine requires that both currents J_s and J_r reach their allowable values simultaneously, otherwise one side of the machine is under-loaded. A symmetric machine with identical currents J_s and J_r' does not fulfill this condition and thus its load capacity is strongly limited by the rotor current. In the above-mentioned example the stator current only reaches 75% of its allowable value when the rotor current reaches its allowable value. This way a significant ratio of the continuous output of the machine is lost. Regarding the continuous output of the machine the symmetric machine thus does not represent the best model of a d.s. machine. If a suitable amount of the magnetizing current was transferred from rotor to stator, rotor current would decrease, stator current would increase, and the load capacity of the machine would be increased. By applying a suitable degree of essential asymmetry, this very goal can be achieved.

The ratios of a constructional asymmetric machine are about the same as those of a symmetric machine, but the load capacity is slightly greater during motor operation and slightly less during generator operation[1]. Load capacity of a symmetric and a constructional asymmetric machine is generally limited by rotor current.

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1) Since the rotor slots have a smaller volume than the stator slots and also because of the brush voltage drop, the supplied rotor resistance R_r' is always greater than stator resistance. This is why the center of the rotor circle is always shifted above, and the center of the stator circle below the abscissa. Thus $J_s > J_r'$ during motor operation and $J_s < J_r'$ during generator operation (see Fig.40).

92

Consequently, there is a need for a certain degree of essential asymmetry so that both currents J_s and J_r reach their allowable values simultaneously with the load, and the machine can be completely utilized and and its continuous output increased. The voltage ratio $u = U_s/U_r$, has to be greater than the transmission ratio $\ddot{u} = W_s/W_r$, for the stator to take over a greater part of the magnetizing current while the rotor current decreases. The necessary degree of essential asymmetry usually depends on the size of the machine and its operation (motor or generator). It is larger in smaller machines because the difference between the allowable value of J_s and J_r' is larger in small machines than in larger ones; it is also larger for generator than motor operation because of the influence of constructional asymmetry (which always prevails).

We have to consider, however, that the ideal case when both currents reach their allowable values simultaneously with the load cannot always be achieved. Since the influence of essential asymmetry on the decrease of rotor current is only caused by the stator taking over blind current from the rotor, it is limited by the condition when the stator takes over the total necessary blind current of the machine during full load and the rotor has a purely effective current. A still larger degree of asymmetry causes a greater rotor current (with leading power factor). In special cases with a very large difference between J_s and J_r' and exceptionally small magnetizing current, it might happen that the rotor current reaches its allowable value at a power factor equal to 1 whereas the stator current stays smaller than its allowable value. In such cases the stator current cannot be essentially smaller than its allowable value since the stator takes over all the necessary blind current and thus has rather a small power factor.

For practical application of d.s. operation with the machine functioning on a single net, mainly series and parallel connection have to be considered. In the present literature both connections are considered to be equivalent and it is assumed that the most favorable operation of the machine is reached at $u = \ddot{u}$ (i.e., $\ddot{u} = 1$ for parallel connection)[1]. Both assumptions, however, are not right. Regarding continuous output, series connection is of great disadvantage, since the current of the machine must in this case not exceed the allowable value of the rotor current during continous operation, thus the stator is underloaded. A very considerable ratio of continous output is lost as compared to a parallel connected machine with a suitable degree of essential asymmetry. The ratio of continuous output loss by applying series connection might amount to 30% for small machines. The most favorable model of the machine can consequently be achieved with parallel connection; this way the total utilization of the machine can be attained by using essential asymmetry. In this case the voltage ratio is $u = 1$ and this is why the necessary essential asymmetry is attained when $\ddot{u} < 1$ i.e., the effective number of turns of the rotor coil is larger than that of

1) See e.g., B.B.J.Herschdorfer, Arch f. Elektrot., Bd.XX,1932,S620/625.

the stator coil. As we have seen the best value of n depends on
the difference between the constants of stator and rotor (the con-
structional asymmetry), as well as on the allowable values of the
currents and the mode of operation of the machine (motor or generator).

8. Comparison of Theory with Measurements on /101
Experimental Machines

8.1. Machine data and experimental connections

To carry out experiments the following two machines were avail-
able:

Machine I: A slip ring induction motor of common construction
with the following coil and power data for normal asynchronous
operation:

        M.F. Oerlikon, Type -18,   4 Pole, 500 Volt
        8 Hp,   1460 T/min., 50 Hz., 11,4 Amp.

Stator: circuit: star, amount of slots: 48, wires in series/slot:
           25, maximum allowable current for continuous operation =
           12 Amp./ph.

Rotor: circuit: triangle, amount of slots: 60, wires in series/
           slot: 10 maximum allowable current for continous operation =
           18 Amp./ph.

For the experiments the machine is fed by a synchronous generator
with 25 Hz so that the number of revolutions does not greatly exceed
its normal value. For practical reasons regarding the transformation
of voltage, the stator is connected in a triangle, consequently the
effective turn ratio is:

$$\ddot{u} = \frac{W_s}{W_r} = \frac{400 \cdot 0{,}958}{200 \cdot 0{,}957} = 2{,}00$$

The assumed normal stator or rotor terminal voltage then is 150 or 75
Volt. The constants and losses of the machine were calculated to be
25 Hz.

Stator: $X_{\sigma_s} = 1{,}30$ Ohm, $R_s = 1{,}14$ Ohm   (warm alternating current)

Rotor: $X_{\sigma_r} = 0{,}355$ Ohm, $R_r = 0{,}465$ Ohm   (warm alternating current)

        (including brush resistance at about 50% of nominal current)

Mechanic losses + friction losses = 260 Watt (n= 1500r/min).

The magnetizing reactance X_μ and the iron losses evidently change
with saturation and/or size of air slit induction and are given as
function of the air slit voltage as referred to the stator (fig.49).

94

C-2

The sum of the stray reactances was calculated by normal short circuit experiments. The separation of the stray reactances is carried out by the method suggested in chapter (7.3.). The machine is d.s. operated with the voltage U_s = 150 V. and U_r = 75 V. The mechanic power supplied to the machine is then adjusted in a way that the received electric output equals the copper losses of the machine (i.e. ideal no-load operation). The currents of the six phases of the machine are measured under these conditions and the mean value of J_s and J_r is calculated. It is very important, however, that the voltages are adjusted exactly since a small deviation of their ratio U_s/U_r, from the transmission ratio U_s/U_r has a considerable influence on the currents during no-load operation: the current of one side is increased, the current of the other side is decreased (chapter 7.3.) and consequently a large error arises in the calculated ratio J_r'/J_s.
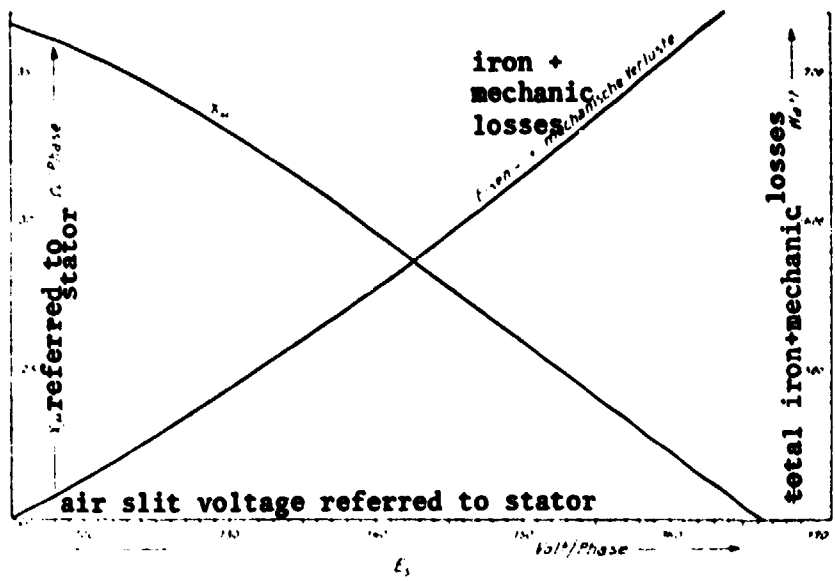
Fig. 49. Change of X_μ and losses with saturation (Machine I).

The applied voltmeters have to be accurately calibrated and the three voltages of each side have to be measured in order to achieve satisfying accuracy. Under these conditions the mean values of the currents are:

$$J_s = 2.78 \text{ Amp.} \qquad\qquad J_r = 5.09 \text{ Amp.}$$

consequently, the ratio is:

$$\frac{X_{\sigma_s}}{X_{\sigma_r}'} = \frac{J_r'}{J_s} = \frac{5.09}{2 \cdot 2.78} = 0.915$$

The short circuit experiments yielded the following mean value for the sum of the stray reactances:

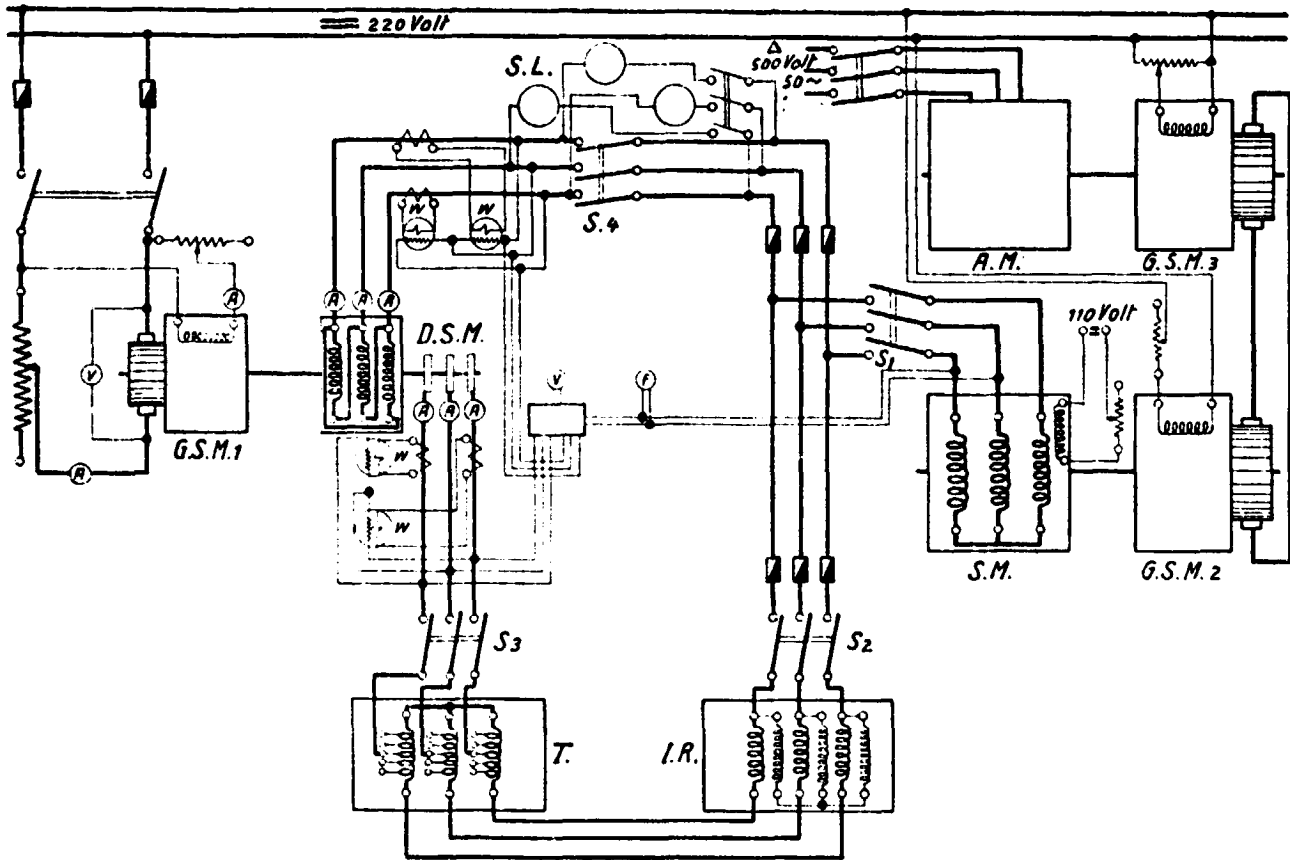$$X_{\sigma_s} + X_{\sigma_r}' = 2.72 \text{ Ohm.}$$

95

Fig. 50. Circuit diagram for the experiments with machine I

D.S.M. = double-fed synchronous machine. G.S.M.1 = drive or load
direct current machine. S.M. = synchronous machine. G.S.M.2 and
G.S.M.3 = direct current machine. A.M. = asynchronous machine.
I.R. = induction regulator. T = autotransformer. S.L. = synchroniz-
ing lamps.

Consequently:

$$X_{\sigma_s} = 2{,}72 \times \frac{0{,}915}{1{,}915} = 1{,}30 \ \text{Ohm}$$

and

$$X'_{\sigma_r} = 2{,}72 \times \frac{1}{1{,}915} = 1{,}42 \ \text{Ohm} \quad \text{or} \quad X_{\sigma_r} = \frac{1{,}42}{4} = 0{,}355 \ \text{Ohm}.$$

In order to carry out the experiments, the machine is directly coupled with a direct current machine which serves as brake for the motor experiments and as driving motor for the generator experiments. Fig. 50 shows the circuit diagram of the experimental set-up. The three machines a.m., g.s.m.3, and g.s.m.2 serve as Ward-Leonard drive for the synchronous machine s.m. the voltage of which is supplied directly to the stator and via the induction regualtor i.r. and the autotransformer I.R.to the rotor. For synchronization the machine is driven by the direct current machine g.s.m.1 until its numbers of revolutions reaches twice the synchronous value (1500 r/min). The voltage of the synchronous machine is then fed to the rotor by turning on the switches S_1, S_2 and S_3. Now the stator is synchronized with the synchronizing lamp s.l. with the synchronous machine by switch S_4.

This switch enables the exact adjustment of the voltage on each side of the machine and its frequency. The magnitude and sense of load of the d.s.machine are essentially adjusted by exciting the direct current machine g.s.m.1, since the group s.m. - g.s.m.2 is much larger than the group d.s.m.-g.s.m1. The d.s. machine works as motor or generator whenever the induced voltage of the direct current machine g.s.m.1 is larger or smaller than the voltage of the direct current net. Since the group s.m.-g.s.m.2 is much larger than the group d.s.m.-g.s.m.1, the frequency of the synchronous machine (and consequently the number of revolutions of the group d.s.m.-g.s.m.1) will not change much during demand. The induced voltage of the direct current machine g.s.m.1-and consequently the demand - thus mainly depends upon exciting the direct current machine g.s.m.1.

Machine II: For the experiments with series and parallel connections a special machine has been coiled with a turn ratio $a = 1$ for 50Hz and the corresponding number of revolutions during d.s. operation. It has the following data and constants:

380/√3 Volt/Phase/Seite, 50 Hz., 2000 T/Min., 6 Poles.

Stator: Circuit: star, number of slots: 36, wires in series/slot: 30, maximum allowable current for continuous operation = 10 Amp./ph.

Rotor: Circuit: star, number of slots: 54, wires in series/slot: 20, maximum allowable current for continuous operation = 7 Amp./ph.

$X_{\sigma_s} = 1,75$ Ohm, $R_s = 0,99$ Ohm **warm, alternating current**

$X_{\sigma_r} = 2,30$ Ohm, $R_r = 1,68$ Ohm **warm, alternating current, including**

brush resistance at ca·50% of nominal current.

Mechanic losses + air friction losses = 180 Watt

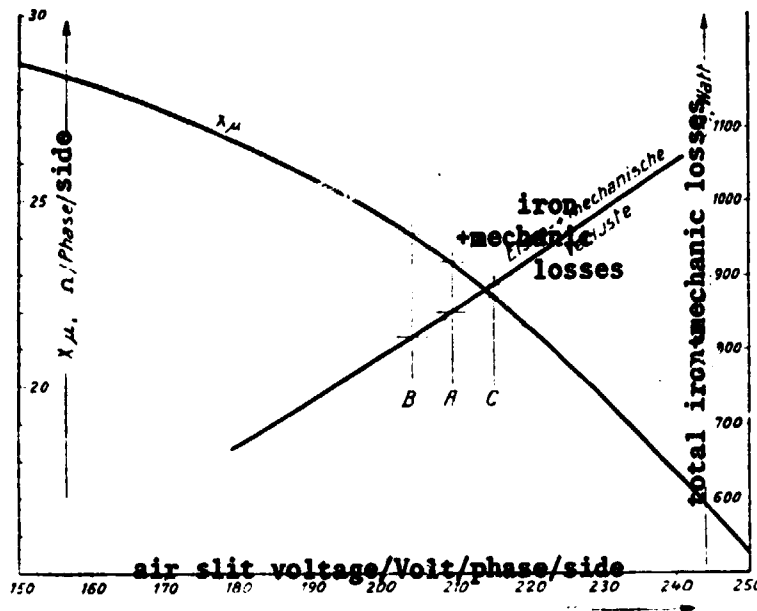Magnetizing reactance and iron losses are also given as function of air slit voltage in Fig. 51.

Fig. 51. Change of X_μ and of losses with saturation (Machine II)
A = no-load operation as motor, B = motor operation,
50% nominal output, C = generator operation, 50% nominal
output.

The stray reactances were calculated after the same method used for the preceding machine. Since the ratio is $\ddot{u} = 1$, the condition $u = \ddot{u}$ is satisfied by parallel connection, consequently the procedure is much simpler in this case. Only the currents of the parallel connected machine have to be measured during ideal no-load oeration with normal voltage. A very accurate adjustment of the voltage is unnecessary in this case. The following mean values of the currents under these conditions were obtained:

$J_s = 5.15$ Amp. $J_r = 3.92$ Amp.

consequently:

$$\frac{X_{\sigma_s}}{X_{\sigma_r}} = \frac{J_r}{J_s} = \frac{3,92}{5,15} = 0,761$$

The sum of stray reactance was calculated from the short circuit experiment. It is:

98

$$X_{\sigma_s} + X_{\sigma_r} = 4{,}05\ \Omega$$

This results in:

$$X_{\sigma_s} = 4{,}05 \cdot \frac{0{,}761}{1{,}761} = 1{,}75\ \Omega$$

$$X_{\sigma_r} = 4{,}05 \cdot \frac{1}{1{,}761} = 2{,}30\ \Omega$$

The circuit diagrams for the experiments on this machine are shown in Fig. 52; a = parallel connection, b = series connection. The synchronizing procedure for parallel connection is the same as explained previously for machine I. For series connection, however, the d.s.m. is first switched to the stationary synchronous generator s.g. with switch S₂. The generator is excited. Both will then slowly be driven by the machine g.s.m.2 of the latter until the nominal number of revolutions is reached and then synchronized with the net by switch S₁. Switch S₂ is then switched off. In both cases a) and b) the demand evidently is only determined by exciting the direct current machine g.s.m.1.

(8.2.) Operating curves

Load experiments were carried out for motor and generator operation with the following voltages:

Machine I: <u>motor operation</u>

1. U_s = 150 Volt, U_r = 75 Volt (constructional asymmetry)

2. Us = 143 Volt, U_r = 75 Volt (approximately unilateral rotor magnetization)

3. U_s = 160 Votl, U_r = 70 Volt (distinct essential asymmetry, rotor de-magnetizing)

<u>generator operation</u>

1. U_s = 150 Volt, U_r = 75 Volt (constructional asymmetry)

2. U_s = 150 Volt, U_r = 72 Volt (approximately unilateral rotor magnetization)

3. U_s = 160 Volt, U_r = 70 Volt (distinct essential asymmetry, rotor de-magnetizing)

Machine II:

parallel connection, 380 Volt, motor and generator operation
series connection, 760 Volt, motor and generator operation

a) Experiments with/city net parallel connections

city net△ 500Volt, 50Hz.

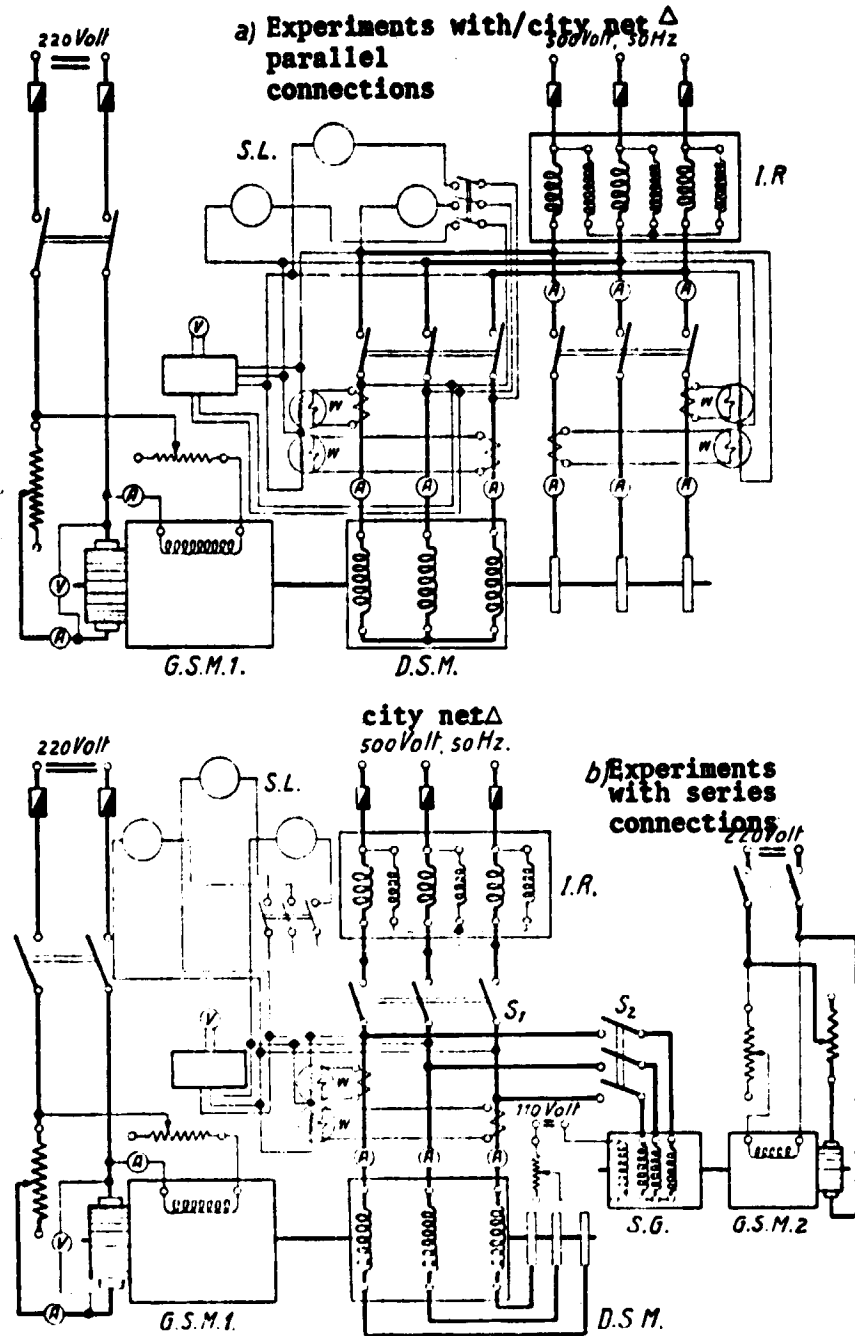**b) Experiments with series connections**

Fig. 52. Circuit diagram for the experiments with machine II.

For each of the mentioned cases operating curves (currents, received power, power factors and efficiency as function of the supplied power) were theoretically calculated and controlled by measurements in a load experiment. In order to find out the supplied or received mechanical power, the losses of the direct current machine were calculated and its supplied or received electric power was measured in the experiment.

The theoretical calculations were carried out as follows: first the compensation constants are calculated and then the various quantities from the compensation circles for various values of the parameters within the practical operating field are calculated[1]. An example for these calculations is given in annex II.

The change of saturation during load causes a considerable deviation between the measured results and the results calculated for constant saturation. As Fig. 51 shows, the magnetizing reactance X_μ of machine II changes between no-load and half-load operation by about 3% for motor operation and by 4% for generator operation. The ratios of machine I are more unfavorable, the corresponding values are about 8% for constructional asymmetry. In order to remove this influence of saturation change on the comparison between measured and calculated results we assumed a value for X_μ (and for the iron losses) which corresponds to the air slit voltage at 50% of nominal power for each calculated case. Also the resistance of the rotor circle changes with the current due to the brush voltage drop. The assumed and above-mentioned values of rotor resistance correspond to half of the allowable value of the rotor current. The constants of the machine which were assumed for the calculation of the operating curves are consequently valid for about half of the continuous output.

Figures 53 to 62 show the calculated operating curves for all eight above-mentioned cases. The measured points are also plotted in the same figures. A good correspondance resulted in all cases between the measurements and the theoretically calculated results. Figures 53 to 58 of the first machine show clearly the extent of influence on the behavior of the machine during operation by the relationship of its terminal voltages (the essential asymmetry). They differ essentially (to a very large extent) in their currents. Figures 53 and 56 show the character of constructional asymmetry, i.e., a smaller (compared to stator current) supplied rotor current during motor operation and a larger during generator operation. The difference between both currents is almost the same in both cases in the total operating field. The influence of essential asymmetry on currents and power factors as explained in chapter (7.5.) can be seen in figures 54, 55, 57 and 58.

Figures 59 and 60 of the parallel connection (machine II) obviously also have the same character of constructional asymmetry. Figures 61 and 62 represent the characteristic lines of the machine

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1) In order to obtain sufficient accuracy, graphic determination was avoided.

for series connection. A comparison between the operating curves of both connections shows the important fact that there is no difference between the characteristic lines of both connections as long as power and current of the total machine are considered. For a certain supplied power, the machine always takes in the same total power and double net current and consequently has the same total power factor during parallel as during series connection.

There is a very considerable difference, however, between the currents J_s and J_r or the power factors $\cos \varphi_s$ and $\cos \varphi_r$ of parallel connection. The constructional asymmetry consequently has no considerable influence on net current and output of the complete machine during parallel connection; it is only effective (very clearly) in the sharing of this net current among both machine parts. This result agrees with the above-explained fact that asymmetry only considerably affects the division of blind current among both machine parts.

The increase of continuous output caused by this influence of constructional asymmetry during parallel connection - as compared to series connection - for motor generation and its decrease for generator operation (chapter 7.6.) is also shown in figures 59 to 62. The continuous output for parallel connection is:

motor operation: 7.15kW (supplied mechanic output)

generator operation: 6.05kW (supplied electric output)

and for series connection:

motor operation: 6.00kW (supplied mechanic output)

generator operation: 6.20kW (supplied electric output)

In all these cases the machine is not totally utilized, the stator is underloaded in any case (Fig. 59 to 62). As we have seen in chapter (7.6.), the total utilization of the machine can be reached by parallel connection with a suitably chosen transmission ratio $\ddot{u} \sim 1$.
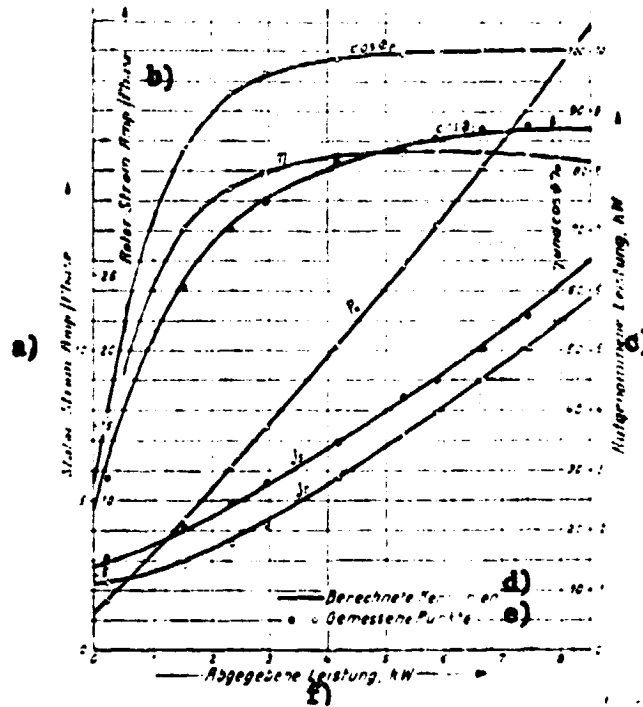
Fig. 53. Operating curves; motor operation, 25 Hz, 1500 r/min.

U_s = 150 V, U_r = 75 V, u = ü = 2 (machine I)

a):stator current; b):rotor current; c):received power; d):calculated characteristic lines; e):measured points; f):supplied power;
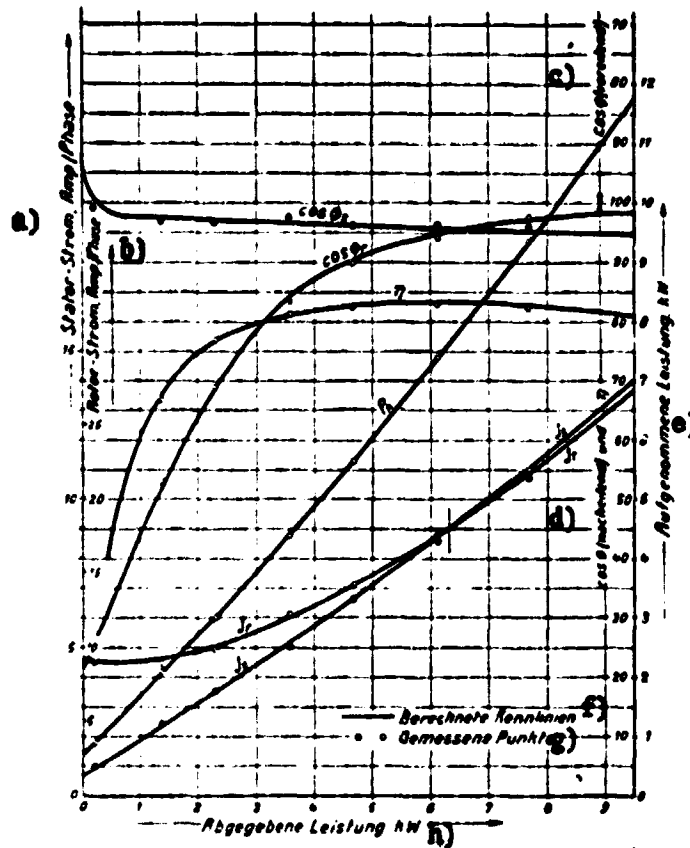
Fig. 54. Operating curves; motor operation, 25 Hz, 1500
r/min.

U_s = 153 V, U_r = 75 V, ü = 2 (machine I)

a):stator current; b):rotor current; c):leading;
d):lagging; e):received power; f):calculated
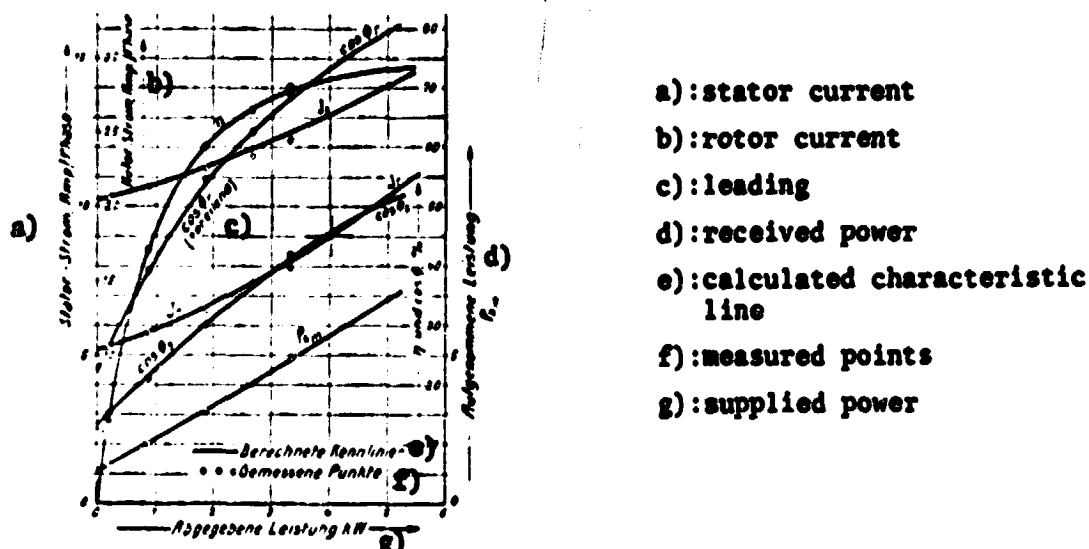characteristic lines; g):measured points;
h):supplied power;

a): stator current

b): rotor current

c): leading

d): received power

e): calculated characteristic
   line

f): measured points

g): supplied power

Fig. 55. Operating curves; motor operation
25 Hz, 1500 r/min.
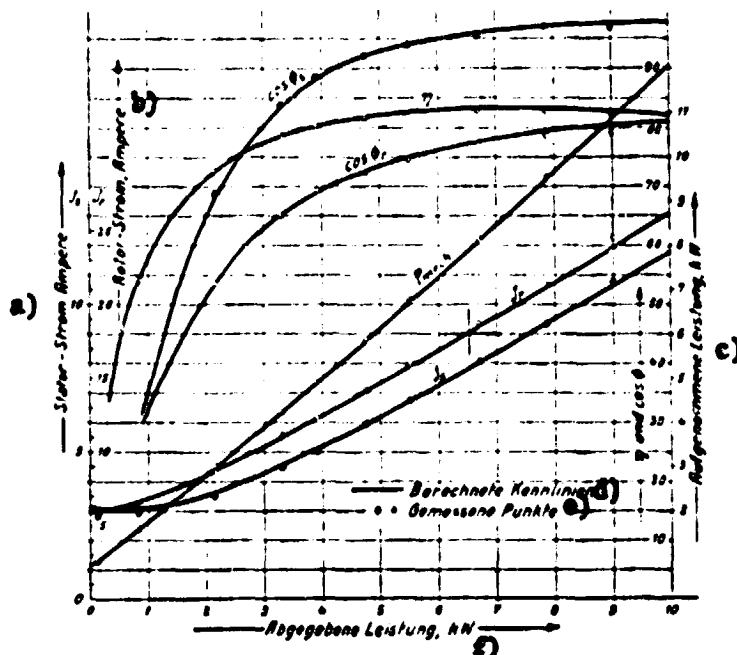$U_s = 160$ V, $U_r = 70$ V,   ü = 2
(rotor de-magnetizing, machine I)

Fig. 56. Operating curves; generator operation
25 Hz, 1500 r/min.
$U_s = 150$ V, $U_r = 75$ V,   u = ü = 2
(machine I)

a): stator current; b): rotor current; c): received power; d): calculated
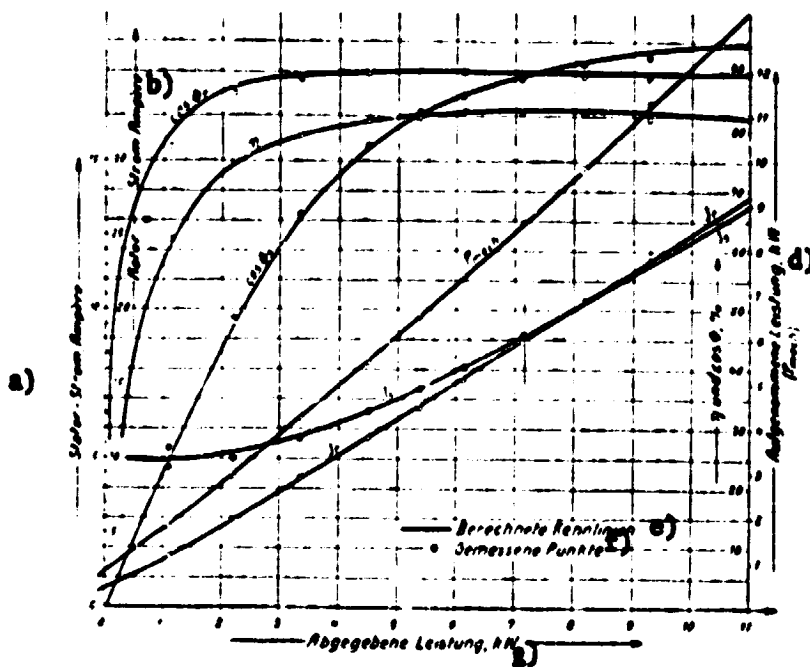characteristic lines; e): measured points; f): supplied power;

105

Fig. 57. Operating curves; generator operation
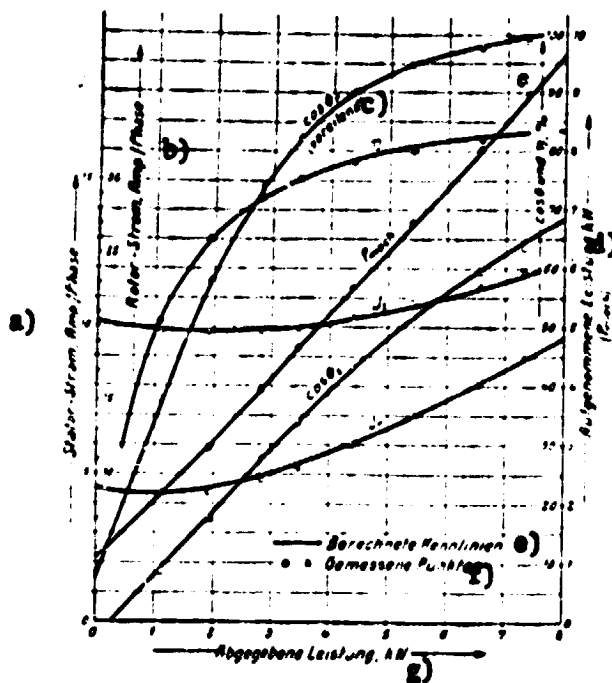25 Hz, 1500 r/min.
U_s = 150 V, U_r = 72 V, ü = 2 (machine I)

Fig. 58. Operating curves; generator operation
25 Hz, 1500 r/min.
U_s = 160 V, U_r = 70 V, ü = 2
(rotor de-magnetizing, machine I)

a):stator current; b):rotor current; c):leading; d):received power; e):calculated
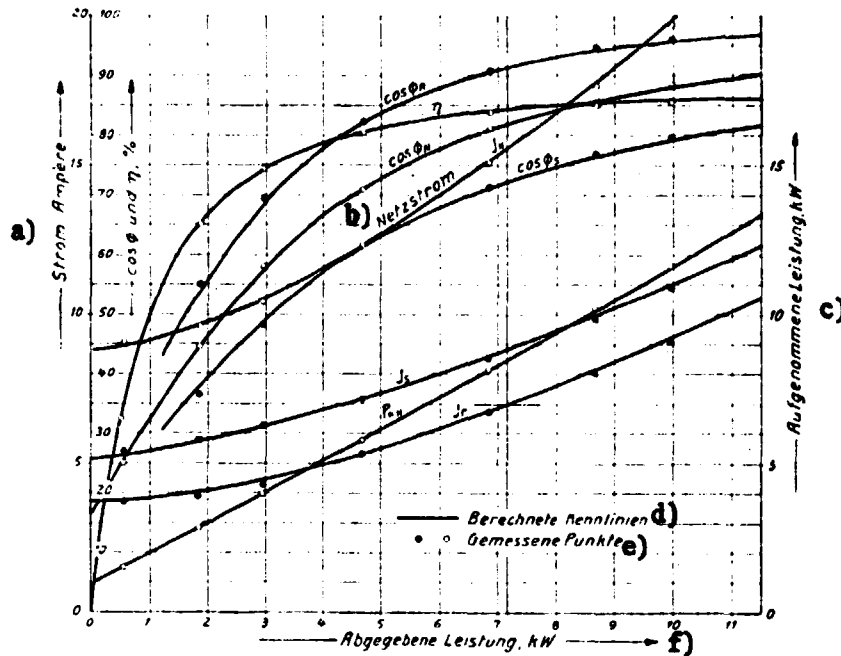characteristic lines; f):measured points; g):supplied power;

106

Fig. 59. Operating curves; parallel connection,
motor operation.
ü = 1, 380 V, 50 Hz, 2000 r/min. (machine II)
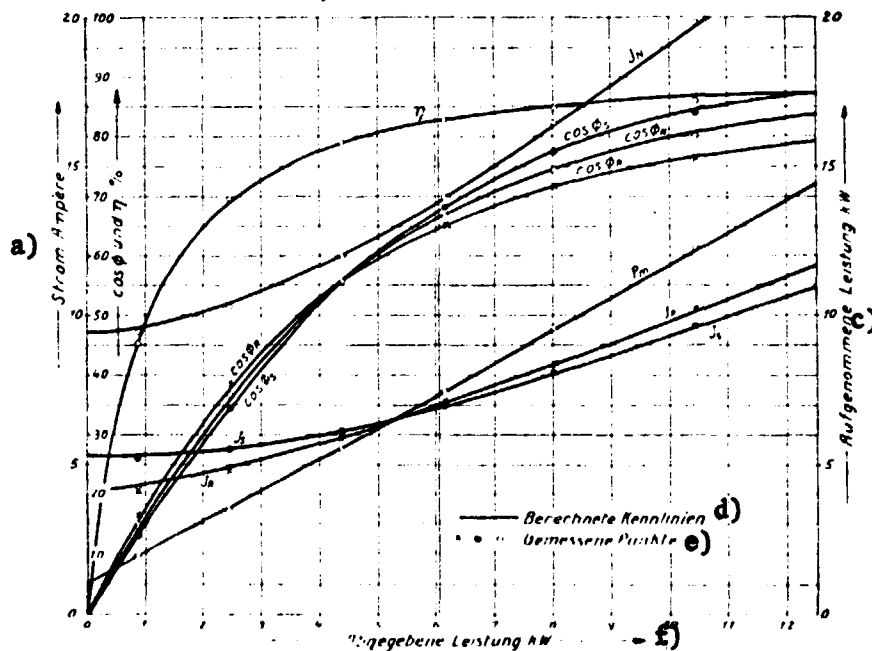

Fig. 60. Operating curves; parallel connection,
generator operation.
ü = 1, 380 V, 50 Hz, 2000 r/min. (machine II)

a):current; b):net current; c):received power; d):calculated characteristic lines;
e):measured points; f):supplied power;

Fig. 61. Operating curves; series connection; motor
operation.
ü = 1, 760 V, 50 Hzm 2000 r/min. (machine II)

Fig. 62. Operating curves; series connection;
generator operation.
ü = 1, 760 V, 50 Hz, 2000 r/min. (machine II)

a):current; b):received power; c):calculated characteristic lines; d):measured
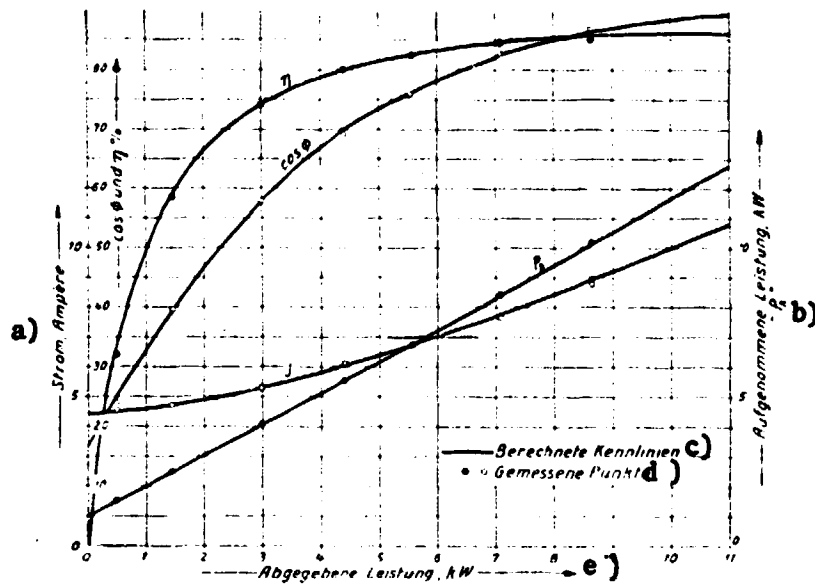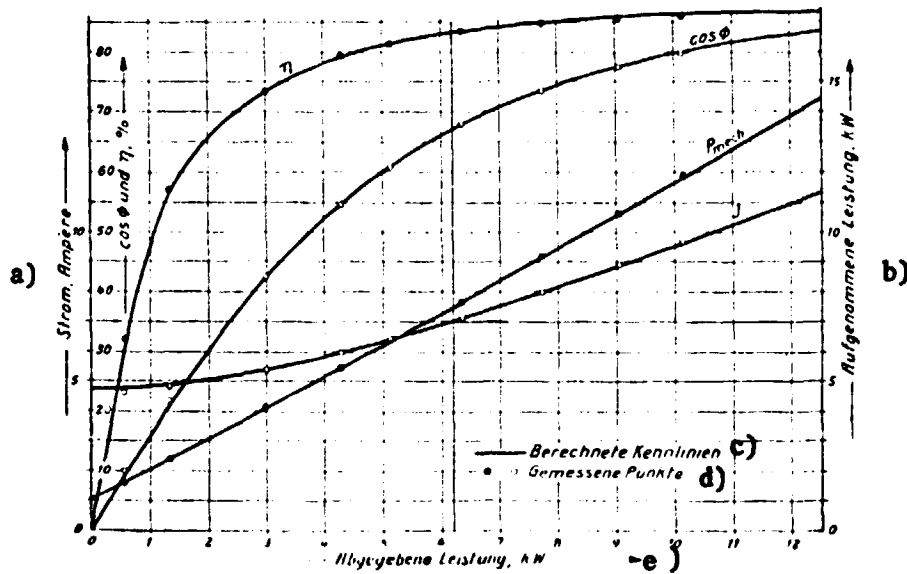points; e):supplied power;

108

The very common case of the double-fed synchronous machine was treated in this study, stator and rotor of a machine having asymmetric constants and dissimilar number of turns; they are fed by voltages of different magnitudes and phases. Both motor and generator operation are studied. The main equations are set up and solved by observing the periodic change of the counter inductances between stator and rotor. The following results were obtained:

1. The general differential equations of the machine.

2. The vector equation for stable operation.

3. Combined space and time vector diagram for the relations between the various space and time vectors.

4. The general solution of the main equations and exact calculation of current and output of each part of the machine for any chosen degree of asymmetry.

5. Compensation circuits of the machine.

6. Impedance and current locus curves.

7. The characteristics of double-feeding synchronous operation under the various operational conditions and the influence of asymmetry of constants and voltages on the behavior of the machine.

The studies on various degrees of asymmetry as well as series and parallel connections were carried out on two experimental machines and they conform well with theory.

A method for exact separation of stray reactances of stator and rotor is suggested and applied to both experimental machines.

It was found that a certain degree of asymmetry is necessary for total utilization of the machine. It could be shown accordingly that the parallel connection with $\ddot{u} < 1$ is more favorable than series connection since it makes a saving of material possible which might amount to more than 40%.

Calculation examples after the circle diagrams

Data: (Case (3) of the essentially asymmetric machine, chapter (7.5.))

$$R_x = 1,14 \text{ Ohm}, \quad X_{o_s} = 1,30 \text{ Ohm}, \quad R_r = 0,465 \text{ Ohm}, \quad X_{o_r} = 0,355 \text{ Ohm}$$
$$X_\mu = 27,3 \text{ Ohm}, \quad \ddot{u} = 2,00 \quad , \quad U_x = 143 \text{ Volt}, \quad U_r = 79 \text{ Volt}.$$

Calculations:

$$X_\mu = 27,3/(2)^2 = 6,825 \text{ Ohm}.$$

The dissimilarity factor is derived from equation (46a)

$$K = -12.501$$

After equations (38a) to (38d):

$$r_{0_s} = + \ 4,280 \text{ Ohm}$$
$$X_{0_s} = -56,617 \text{ Ohm}$$
$$r_{0_r} = + \ 0,342 \text{ Ohm}$$
$$X_{0_r} = - \ 4,529 \text{ Ohm}$$

$$(\text{control}: \quad \frac{r_{0_s}}{r_{0_r}} = +12,501 \quad \frac{X_{0_s}}{X_{0_r}} = +12,501) \quad)$$

It can be concluded from equation (40a):

$$X_{\mu_{0_s}} = -118,32 + 3205,48 - 2329,3 = 29,908 \text{ Ohm}$$

Likewise:

$$X_{\mu_{0_r}} = + \sqrt{0,1172 + 20,5109 - 14,904} = +2,393 \text{ Ohm}$$

$$(\text{control}: \quad \frac{X_{\mu_{0_s}}}{X_{\mu_{0_r}}} = -12,501 \quad)$$

Consequently the compensation constants are:

Stator: (equation (43))

$$R_{c_s} = 1,140 + 4,280 = + \ 5,420 \text{ Ohm}.$$
$$X_{c_s} = 1,300 + 27,300 + 29,908 - 56,617 = + \ 1,891 \text{ Ohm}$$
$$X_{\mu_{c_s}} = 2 \cdot (-29,908) = - \ 59,816 \text{ Ohm}.$$

Rotor: (equation (45))

$$R_{c_r} = 0,465 + 0,342 = + \ 0,807 \text{ Ohm}$$
$$X_{c_r} = 0,355 + 6,825 - 2,392 - 4,529 = + \ 0,259 \text{ Ohm}$$
$$X_{\mu_{c_r}} = 2 \cdot 2,393 = + \ 4,786 \text{ Ohm}.$$

The dimensions of the current circles consequently are:

Stator: (equation (53b)):

$$x = +50.0 \text{ Amp.}$$
$$y = -9.67 \text{ Amp.}$$
$$R = +53.4 \text{ Amp.}$$

Radius of the circle determining the power line:

$$\frac{U_s}{2 R_s} = 143/2 \cdot 1{,}14 = 62{,}7 \text{ Ampère}$$

Scale of the inner output in vertical direction:

$$S_s = U_s - 2 \cdot R_s \cdot y = 143 + 2 \cdot 1{,}14 \cdot 9{,}67 = 165{,}05 \text{ Volt}$$

Rotor: (equation (53d)):

$$x = +107.0 \text{ Amp.}$$
$$y = +32.6 \text{ Amp.}$$
$$R = +96.6 \text{ Amp.}$$

Radius for the circle determining the power line:

$$\frac{U_r}{2 R_r} = 79/2 \cdot 0{,}465 = 85{,}0 \text{ Ampère}$$

Scale of the inner output in vertical direction:

$$S_r = U_r - 2 \cdot R_r \cdot y = 79 - 2 \cdot 0{,}465 \cdot 32{,}6 = 48{,}7 \text{ Volt.}$$

Reduction factor (=ratio of scales) = S_s/S_r = 165.05/48.7 = 3.387

Consequently the reduced dimensions of rotor circuit are:

$$x = 107.0/3.387 = 31.60 \text{ Amp.}$$
$$y = 32.6/3.387 = 9.62 \text{ Amp.}$$
$$R = 96.6/3.387 = 28.53 \text{ Amp.}$$

Radius of the circle determining the power line:
$$85.0/3.387 = 25.1 \text{ Amp.}$$

The figure (page 110) shows the circle diagrams with a scale 1 cm = 4 Ampere. The various scales are:

Scale: 1cm =	Stator	Rotor	Machine
current, Ampere/phase	4	13.55	-
inner output, total, kW	1.98	1.98	3.96
terminal output, total, kW	1.716	3.210	-

circle

circle

compensation circuits:
(equations (43) or (45))
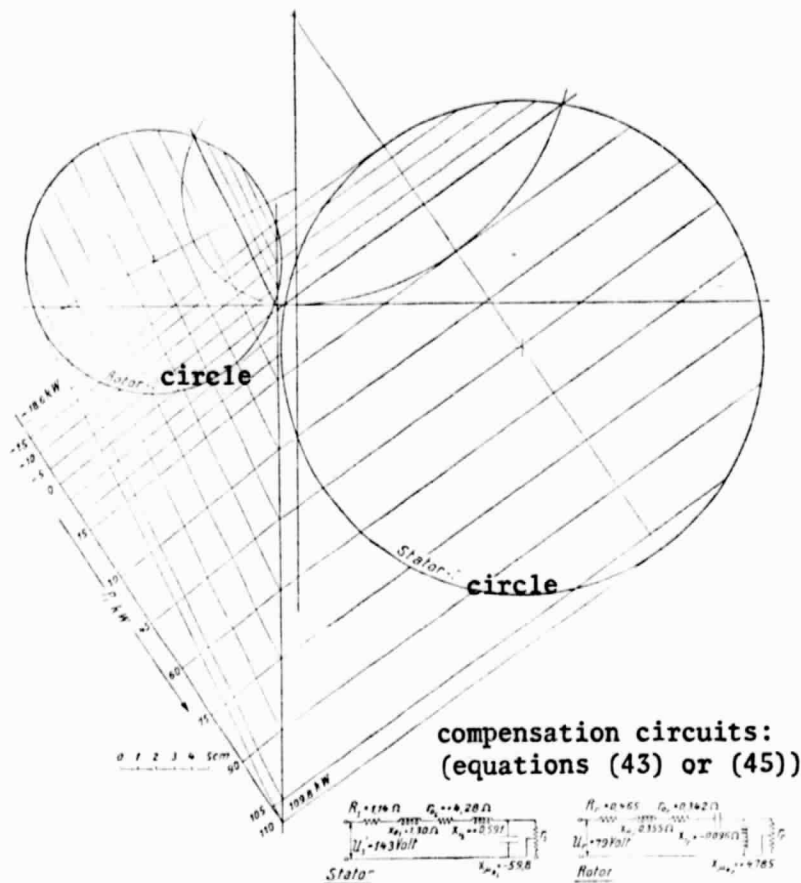
Circle diagram of the essentially asymmetric machine, case (3).

$U_s = 143$ Volt, $U_r = 79$ Volt, $u = 2$ 25 Hz. 1500 r/min.
$R_s = 1.14 \, \Omega$, $R_r = 0.465 \, \Omega$, $X_{\sigma_s} = 1.30 \, \Omega$, $X_{\sigma_r} = 0.355 \, \Omega$, $X_{p_s} = 27.3 \, \Omega$

For the given points and multiplication by the corresponding scales, the following data from measurements of the circle diagrams are obtained:

112

Stable operating field

P_{i_m} kW	J_s Amp.	P_{k_s} kW	$\cos\phi_s$ %	J_r Amp.	P_{k_r} kW	$\cos\phi_r$ %	P_{k_m} kW
18,7	39,6	−14,8	86,8	77,9	17,8	96,2	32,5
15	22,3	9,1	95,2	43,9	10,0	96,3	19,1
10	13,2	5,6	98,8	27,1	−5,9	91,5	11,5
−5	6,1	−2,6	98,7	16,7	2,7	68,3	5,3
0	2,4	0,0	3,3	15,3	−0,3	8,9	0,4
15	15,9	6,7	98,3	37,4	5,7	64,2	12,4
30	28,6	12,3	100,0	61,3	9,9	68,4	22,2
45	40,1	17,0	98,9	83,0	12,8	64,9	29,8
60	51,2	21,1	96,1	104,8	14,6	58,6	35,7
75	62,1	24,4	91,5	125,9	15,2	51,0	39,6
90	73,5	26,6	84,5	147,4	14,3	40,8	40,9
105	87,2	26,6	71,1	174,8	9,6	23,3	36,2
109,8	96,7	23,0	55,5	194,0	2,4	5,2	25,4

Unstable operating field

P_{i_m} kW	J_s Amp.	P_{k_s} kW	$\cos\phi_s$ %	J_r Amp.	P_{k_r} kW	$\cos\phi_r$ %	P_{k_m} kW
18,7	39,6	14,8	86,8	77,9	17,8	−96,2	32,5
15	55,6	18,0	75,5	110,2	24,3	93,1	42,3
10	63,4	18,7	68,8	125,9	26,9	90,2	45,6
−5	68,9	18,7	63,3	137,0	28,4	87,7	47,1
0	73,2	18,4	58,5	145,0	29,4	85,5	47,7
15	83,4	16,2	45,3	165,3	30,6	78,0	46,8
30	90,6	13,0	33,6	180,0	30,1	70,7	43,2
45	95,9	9,1	22,1	190,8	28,4	62,9	37,5
60	100,2	4,3	10,1	199,2	25,6	54,3	29,9
75	103,0	1,3	2,9	205,0	21,5	44,3	20,3
90	104,4	7,7	17,3	207,7	16,1	32,7	8,4
105	102,5	16,6	37,7	205,0	6,5	13,4	10,1
109,8	96,7	23,0	55,5	194,0	2,4	5,2	25,4

After these results the characteristic lines of the machine are represented as function of the inner output in Fig. 47.

113

Calculation examples from the compensation circuits

Data: As an example we take the calculations of the operating curves of one of the cases mentioned in chapter (8.2.). We are taking the first case, generator operation of machine I with the following values of the voltages of the machine:

$$U_s = 150 \text{ Volt}, \quad U_r = 75 \text{ Volt}.$$

The induced voltage referred to the stator at 50% of the nominal output is about 153.9 Volt after the results of the load experiment. The corresponding value of $X_{\mu.}$ is, after Fig. 49:

$$X_{\mu.} \quad 24.55 \text{ Ohm}$$

The data of the calculation being:

$$R_s = 1.14 \quad X_{\sigma_s} = 1.30 \quad R_r = 0.465 \quad X_{\sigma_r} = 0.355 \text{ Ohm}$$
$$X_{\mu.} = 24.55 \text{ Ohm}, \quad U_s = 150 \text{ Volt}, \quad U_r = 75 \text{ Volt}$$
$$u - \ddot{u} = 2.00 \text{ (constructional asymmetry)}$$

The compensation constants are obtained in the same way as in Annex I. They are:

Stator:

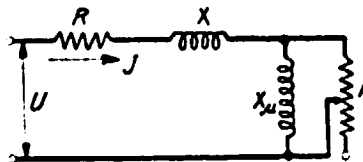$$R_{e_s} = -4.63 \text{ Ohm}$$
$$X_{e_s} = -1.03 \text{ Ohm}$$
$$X_{\mu_s} = +47.72 \text{ Ohm}$$

Rotor:

$$R_{e_r} = +2.088 \text{ Ohm}$$
$$X_{e_r} = +0.048 \text{ Ohm}$$
$$X_{\mu_r} = +13.43 \text{ Ohm}$$

For the given circuit it is simple to prove that the following relations are valid for any chosen value of the parameter.

$$J = \frac{e}{\sqrt{(r + A)^2 + B^2}} \cdot \sqrt{1 + \left(\frac{r}{X_\mu}\right)^2}$$

$$r = \frac{-e^2 \cdot r}{(r + A)^2 + B^2}$$

Terminal output: /121

$$\frac{e^2 \cdot r}{(r + A)^2 + B^2} - J^2 R$$

114

where:

$$e = \frac{U \cdot X_\mu}{\sqrt{R^2 + (X + X_\mu)^2}} \quad \text{Volt}$$

$$A = \frac{R \cdot X_\mu^2}{R^2 + (X + X_\mu)^2} \quad \text{Ohm}$$

$$B = \frac{X X_\mu (X + X_\mu) + R^2 X_\mu}{R^2 + (X + X_\mu)^2} \quad \text{Ohm}$$

Using this relation a considerable simplification in the calculations is obtained. For various values of the parameter r_s the corresponding values J_s, P_{1s} and P_{ks} are calculated. For various values of the parameter r_r of the rotor circle the corresponding values of J_r and P_{1r} are calculated and J_r is drawn as function of the inner output P_{1r}. Since now P_{1s} always equals P_{1r} the values of the rotor current corresponding to the chosen values of the paramer r_s can be determined from the curve $J_r = f(P_{1r})$. Then the calculations of the rotor circle for the obtained values of rotor current can be completed. The calculations are given in the following tables and the connecting curve is drawn next to it.

Rotor:
$\qquad$ e = 74.0 Volt
$\qquad$ A = +2.03 Ohm
$\qquad$ B = 0.3623 Ohm

Calculations:

r_r, Ohm	J_r, Amp.	P_{1r}, Watt Phase
– ∞	5.505	– 49.2
– 26	6.72	174.6
– 14	8.92	404.7
– 10.5	11.08	601
– 8.3	13.82	837.4
6.5	18.3	1221
5.5	22.9	1619

Stator:
$\qquad$ e = 146.2 Volt
$\qquad$ A = – 4.4 Ohm
$\qquad$ B = 1.426 Ohm

115

Calculation of the operating curves, 150/75 Volt,
generator operation. Connection of both sides.
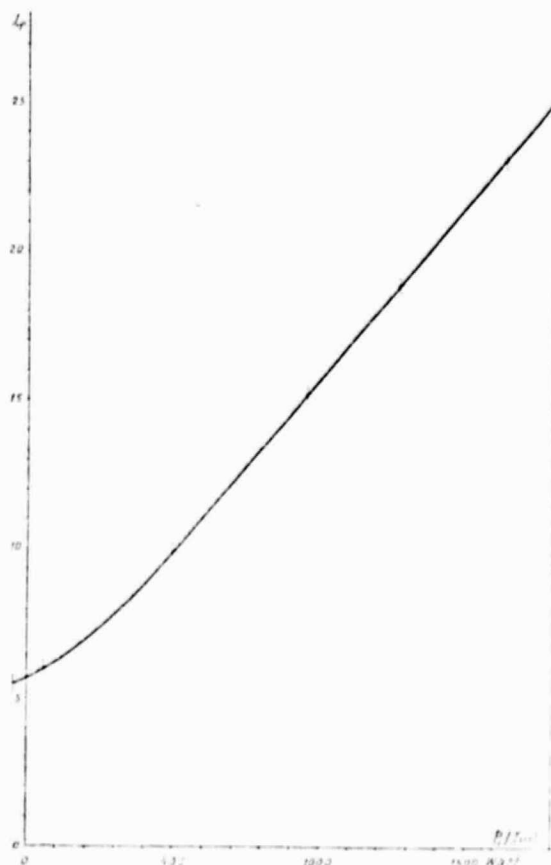
Calculations:

c_s Ohm	J_s Amp.	P_{i_s} Watt/Phase	$\cos\phi_s$ %	J_r Amp.	$\cos\phi_r$ %	P_{k_m} kW	P_{mech} kW	η %
/	3,06	54,1	9,45	5,95	8,43	0,243	0,986	24,95
93	3,285	271,5	52,6	7,55	43,3	1,513	2,290	66,10
44	4,11	498,4	77,7	9,90	61,0	2,797	3,651	76,65
28	5,23	728,2	89,0	12,55	69,6	4,056	5,030	80,60
20	6,495	959,0	93,6	15,25	74,4	5,286	6,415	82,40
14	8,26	1274,0	96,5	18,9	78,2	6,913	8,305	83,30
10	10,33	1639,0	97,8	25,1	80,3	8,725	10,495	83,15

The characteristic lines of the machine are drawn in Fig. 56
chapter (8.2.)) after this table.

116

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CURRICULUM VITAE

I was born in ████ on ████████████. I received my high-school diploma in May, 1939 and started to go to Fouad University, Cairo the same year. I received my engineering diploma in May, 1944. From October 1944 to Septmber 1946 I worked as an assistant in the deparment of electrical technology and also did research at the same university. Then, I came to Zurich as member of the Egyptian Mission in order to get my Ph.D. Under the supervision of Prof. E. Duenner I carried out the theoretical and experimental research of the present study at the Institute of Electrical Machines of the Swiss Technological University, Zurich.

Zurich, May 1948.

118