LONG TERM FREQUENCY STABILITY ANALYSIS OF THE GPS NAVSTAR 6 CESIUM CLOCK

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ABSTRACT

Time domain measurements, taken between the NAVSTAR 6 Spacecraft Vehicle (SV) and the Vandenberg GPS Monitor Site (MS), by a pseudo random noise (PRN) receiver, have been collected over an extended period of time and analyzed to estimate the long term frequency stability of the NAVSTAR 6 onboard frequency standard, referenced to the Vandenberg MS frequency standard.

The technique employed separates the clock offset from the composite signal by first applying corrections for equipment delays, ionospheric delay, tropospheric delay, earth rotation and the relativistic effect. The data are edited and smoothed using the predicted SV ephemeris to calculate the geometric delay. Then all available passes from each of the four GPS monitor stations, are collected at 1-week intervals and used to calculate the NAVSTAR orbital elements. The procedure is then completed by subtracting the corrections and the geometric delay, using the final orbital elements, from the composite signal, thus leaving the clock offset and random error.

Frequency stability estimates of clock performance are then made using the clock offsets to calculate the Allan Variance, $\sigma_y(\tau)$, for the spacecraft oscillator, using sample times that vary from 1 to 10 days. The results indicate a combined clock/ephemeris frequency stability of 1.3×10^{-13}, or less, for sample times varying from one day up to ten days. Future work will include analysis of cesium standards on SV 5 and 6 as well as a rubidium standard on SV 5.

INTRODUCTION

As part of the Navy support to the NAVSTAR GPS Clock Development Program, the Naval Research Laboratory (NRL) has continued (1) research and development of precise time and frequency standards. This paper describes the on-orbit performance evaluation of the NAVSTAR 6 cesium clock.

The cesium clock in NAVSTAR 6 is the fifth one to be orbited in the GPS clock development program. This cesium clock was built under contract to the Navy by Frequency and Time Systems (FTS). The NAVSTAR 6 cesium clock was activated on 26 April 1980 (Day number 117, 1980) and has been in continuous operation for more than one year.

The FTS clock is a preproduction model (PPM) which is designated as PPM-11. The preproduction series of cesium beam frequency standards is an evolutionary development from the prototype model orbited in the Navigation Technology Satellite II (NTS-2). The preproduction cesium frequency standards built by FTS are scheduled to be placed in the NAVSTAR 5, 6, 7, and 8 spacecraft.

The NAVSTAR 6 spacecraft is also known as SV 9. The SV identification is given as part of the navigation message.

GPS SYSTEM DESCRIPTION

The NAVSTAR Global Positioning System is a Department of Defense (DOD) space-based system employing a constellation of satellites which broadcast signals that are synchronized in both time and frequency. Information is given in the ephemeris message which can be combined with measurements from four GPS satellites and used to calculate the user's instantaneous position and velocity in all three coordinates, as well as precise time and frequency. Signals from the GPS satellites can be rapidly acquired and processed independently of all other systems. The precise time and frequency information can be used to provide a common, worldwide time grid for referencing scientific measurements by laboratories all over the world.

The NAVSTAR GPS system is comprised of four segments:

(1) Control Segment

(2) Space Segment

(3) User Segment

(4) Engineering Segment

The Space, Control, and User segments of GPS will be discussed with emphasis on factors that are important to this paper.

The GPS Control Segment consists of a Master Control Station (MCS), located at Vandenberg, CA, and four remote Monitor Sites (MS), located at Vandenberg, Hawaii, Alaska, and Guam. These four stations track the GPS spacecraft vehicles (SV). Data from these sites are collected at the MCS and processed to determine SV health, orbits, and clock offsets.

The GPS Space Segment currently consists of six NAVSTAR SVs, with a total of 18 satellites scheduled to be operational in the 1986-87 time frame. These satellites are placed in (nominal) 12 hour near-circular orbits, with occasional orbit adjust maneuvers which maintain a repeating ground track for each SV. The configurations of the NAVSTAR have been under active study since the original recommended constellation (2). Recent studies (3) have produced configurations which result in a small improvement in the GPS coverage.

Each GPS spacecraft broadcasts spread spectrum modulated signals which are precisely related to the on-board clock. The spacecraft navigation message (4) is also modulated onto the signal at precise epochs which aid in defining GPS time.

The "precise", or P-code modulation is generated at two frequencies in the L-band; these are designated as the L_1 and L_2 signals. The P-code signals are modulated at a rate of 10.23 Mbps (million bits per second). The P-code modulation provides the capability for high precision time difference measurements, and is resistant to electronic countermeasures and multipath interferences. The P-code employs a very long code that is reset once per week. The second code which is designated as the coarse/acquisition code, or (C/A) code, is modulated at 1.023 Mbps and repeats every millisecond. It provides a coarse signal that is a factor of ten less precise than the P-code. The C/A signal is biphase modulated with the P-code and may be rapidly acquired by all users. Each GPS SV has an atomic frequency standard which controls the broadcast frequency of each satellite to the same nominal value. The use of the spread spectrum modulation and separate codes for each GPS SV permits multiple access to any of the satellites that are above the user's horizon.

A GPS user would be required to have an appropriate antenna, receiver, processor, and output device to receive the precise time and time interval signals and perform a navigation solution. A fully operational GPS user would select four NAVSTARs from the 6 to 9 satellites that would be available in such a fashion as to minimize the Geometrical Dilution Of Precision (GDOP), a quantity [5,2] that relates to the navigation accuracy available from GPS. The user would acquire and lock the receiver to signals broadcast from four of the SVs, and then make simultaneous measurements of time difference (pseudo-range) and frequency difference (pseudo-range-rate) between each of the SV clocks and the receiver clock. The user would then use the four pseudo-range measurements to calculate clock offset, latitude, longitude, and height. The four pseudo-range-rate measurements would be used to calculate frequency offset and velocity in all three components.

LONG TERM FREQUENCY STABILITY

The GPS system is capable of providing instantaneous precise navigation because the satellite clocks are synchronized in time and frequency. Therefore a fundamental measure of system performance is given by the long term frequency stability of each of the SV clocks.

The technique that has been developed for analyzing frequency stability performance of orbiting clocks and frequency standards was developed (6) at the Naval Research Laboratory (NRL) in 1975.

This procedure has evolved into an analytical procedure depicted in Figure 1. Each major component of the long term frequency stability analyses will be described in the development of this paper.

FREQUENCY STABILITY MODEL

The Allan variance was adopted by the IEEE as the recommended measure of frequency stability. Reference 7 presents a theoretical development which results in a relationship between the expected value of the standard deviation of the frequency fluctuations for any finite number of data samples and the infinite time average of the standard deviation. Equation (1) presents the Allan variance expression for M frequency samples with sample period T equal to the sampling time, τ.

$$\text{Eq (1)} \quad \sigma^2_y(2,\tau) = \frac{1}{(M-1)} \sum_{k=1}^{M-1} \frac{(\bar{y}_{k+1} - \bar{y}_k)^2}{2}$$

The average frequency values y_k are calculated from pairs of clock offsets, Δt, separated by sample time, τ, is given by

$$\text{Eq (2)} \quad y_k = \frac{\Delta t_{k+1} - \Delta t_k}{\tau}$$

The clock offset is not directly observable from a pseudo-range measurement; other variables must be measured or estimated.

CLOCK DIFFERENCE MEASUREMENTS

Pseudo-range (PR) and accumulated delta pseudo-range (ADR) measurements are taken between the NAVSTAR SV clock and the MS clock using a spread spectrum receiver. The measurements are taken once every 6 seconds and then aggregated and smoothed once per 15 minutes. Figure 2 presents a plot of a typical pseudo-range signature obtained from a single NAVSTAR pass over a monitor station. Each measurement is corrected for equipment delay, ionospheric delay, tropospheric delay, earth rotation, and relativistic effects. Then the data are edited and smoothed using the predicted SV ephemeris to calculate the geometric delay. The clock offset at the mid point of the 15 minute data span is estimated, using both the pseudo-range and the pseudo-range rate measurements which are fitted to a cubic polynomial with epoch at time corresponding to the mid point of the data.

The pseudo-range measurements are resolved to (1/64) of a P-code chip, which corresponds to 1.5 nsec of time, or 46 cm in range. Nominal values for the pseudo-range noise levels are $\sigma_{PR} = 1.3$m for the L_1 measurements and $\sigma_{PR} = 2.0$m for the L_2 measurements. The L_1 and L_2 measurements are combined to correct for ionospheric refraction, which results in an increase to 4.53m for the corrected pseudo-range measurement. The accumulated delta pseudo-range measurement noise levels are 0.31 cm for L_1 and 0.56 cm for L_2. These measurements are also combined to correct for the accumulated pseudo-range measurements. The smoothing procedure uses the ADR measurements to aid the PR smoothing of each 15 minute segment of data. This process results in a smoothed pseudo-range measurement noise level of 18.5cm.

The equation which relates the pseudo-range measurements to the clock difference between the NAVSTAR SV and the monitor site (MS) is given by Equation (3).

$$\text{Eq (3)} \quad PR = R + c(t_{MS} - t_{SV}) + c\,\Delta t_A + \epsilon$$

where

PR = the measured pseudo-range

R = the slant, or geometric range, from the SV (at the time of transmission) to the MS (at the time of reception)

c = the speed of light

t_{MS} = the MS clock time

t_{SV} = the SV clock time

t_A = ionospheric, tropospheric, and relativistic delay, with corrections for antenna and equipment delays

ϵ = the measurement error

The clock difference, denoted by Δt_k for the k_{th} measurement is obtained by rearranging Eq (3) into

$$\text{Eq (4)} \quad \Delta t_k = (t_{SV} - t_{MS}) = R/c + \Delta t_A + \epsilon/c - PR/c$$

The particular evaluation of Eq (4) that will be used in this paper is obtained by designating the NAVSTAR 6 by SV 9 and by designating the Vandenberg Monitor Site as VMS. This particular case of Eq (4) is given by

$$\text{Eq (5)} \quad \Delta t_k = (SV9 - VMS)$$

SMOOTHED ORBIT ESTIMATION

All of the smoothed pseudo-range measurements are collected from the four GPS monitor sites for one week. The Naval Surface Weapons Center (NSWC) then estimates a smoothed NAVSTAR orbit using an orbit estimation program which extensively models the dynamics of the satellite motion, including solar radiation pressure, and orbit adjust maneuvers. The NSWC post-fit ephemeris calculations employ the highly redundant set of range-difference (5) values, which are calculated from the smoothed 15 minute pseudo-range measurements.

The purpose of the smoothed orbit estimation is to separate the clock and orbital components by modeling the clock as a constant, (but unknown frequency) during the one week span. The model incorporates the feature of inclusion of an (unknown) aging rate, which may be used for frequency standards that exhibit aging. The model also is capable of segmenting the clock bias solution to allow for frequency adjustments of the MS or SV clock.

The clock differences used for analyzing the spacecraft clock incorporate the smoothed orbit and the set of 15 minute pseudo-range measurements to calculate the clock difference at the time of each measurement according to Eq (4). The clock differences for each NAVSTAR pass are then used to estimate the clock differences at the time-of-closest-approach (TCA) of the NAVSTAR SV over the monitor site. This procedure results in either one or two points per day. The NAVSTAR orbit and the monitor site location determine whether one or two points per day will be available. For NAVSTAR 6, one pass per day is available from the Vandenberg Monitor Site (VMS).

The evaluation of the clock difference at the TCA point minimizes the effect of the NAVSTAR orbit estimation for along-the-satellite-track and out-of-plane errors. However this procedure does not reduce the effect of radial orbit errors. Hence the estimate of radial orbit error will be one of the factors that limit the accuracy of the long term frequency stabilty analysis. The effect of the radial error on the frequency stability is given by

$$\text{Eq (6)} \quad \sigma_y(\tau) = \frac{\sqrt{3}\sigma_{RR}}{c\tau}$$

where

σ_{RR} = standard deviation estimate of radial component of orbit error

c = the speed of light

τ = sample time

NAVSTAR 6 RESULTS

The clock differences between the NAVSTAR 6 cesium clock and the Vandenberg Monitor Site (VMS) clock are presented in Figure (3). Each "X" symbol corresponds to a single measurement obtained from the smoothed 15 minute pseudo-range measurement. A total of 23 points are plotted for this NAVSTAR 6 pass over the VMS. These

23 points are analyzed and a subset of these data are used to estimate the clock offset at the TCA point.

The clock difference, which is denoted by (SV9 - VMS), corresponds to starting a time interval measurement with the NAVSTAR 6 clock, and stopping with the VMS clock (with corrections for the orbit and other delays). The clock offset changed by approximately 30ns during the SV pass. The clock offset presented here represents a clock difference which may be further processed to produce "GPS" time. The slope of these measurements indicates a frequency offset of -1.08×10^{-12} between the NAVSTAR 6 clock and the VMS clock. The magnitude of this offset is normally what would be expected (8) after the correction for the relativistic clock effect.

The clock differences for one week are presented by Figure 4. In this figure, each "X" symbol denotes one clock difference obtained from the smoothed 15 minute pseudo-range measurements. There are seven groups of "X" symbols, each one corresponding to a single NAVSTAR 6 pass that was observed by the Vandenberg Monitor Site. The slope of the clock differences for this one-week segment is -1.56×10^{-12}.

The clock differences for the entire 100 day data span are presented in Figure 5. Each vertical mark corresponds to the clock difference evaluated at the TCA point of a NAVSTAR pass over the Vandenberg Monitor Site.

Reference to Figure 5 indicates a total change in clock difference of about 10 μs in 100 days, or approximately 0.1 μs/day (-1.16×10^{-12}). It also indicates that a small frequency change, on the order of 5×10^{-13}, occurred between days 120 and 150. This small frequency shift will be further analyzed before computing the frequency stability.

The frequency differences for sample times of one, three, and ten days are presented in Figures 6, 7, and 8. Analysis of the results indicates that the total frequency change between the two cesium standards during the 100 day span was on the order of 1×10^{-12}. Reference to Figure 7 indicates three small frequency changes on the order of 3×10^{-13} occurred, with the majority of the frequency differences on the order of a few parts in 10(14). Analysis of other GPS data (not included in this paper) indicates that the Vandenberg MS cesium clock was responsible for the largest frequency changes analyzed.

The frequency measurements were then used to calculate the Allan variance for sample times varying from one to ten days. An interleaving (also called overlapping) data processing technique was used in order to obtain maximal use of the data. For instance, with a sample time of one day and the set of clock differences $\{\Delta t_1, \Delta t_2, \Delta t_3, \Delta t_4\}$ two variances were calculated. The first variance used the subset $\{\Delta t_1, \Delta t_2, \Delta t_3\}$ and the second variance was calculated using the subset $\{\Delta t_2, \Delta t_3, \Delta t_4\}$. Thus, the two $\sigma_y(\tau)$ values have the subset $\{\Delta t_2, \Delta t_3\}$ in common.

Frequency stability estimates, for the combined clock and ephemeris, are presented in Figure 9. The results are summarized by Table 1.

Table 1

Sample Time (days)	Frequency Stability $\sigma_y(\tau)$ Parts in 10(13)
1	1.1
2	1.0
3	.9
4	.9
5	.9
6	1.0
7	1.1
8	1.2
9	1.3
10	1.3

The $\sigma_y(\tau)$ may be characterized by four segments, using a model given by

$$\text{Eq (7)} \quad \sigma_y(\tau) = a\,\tau^{\mu}$$

The coefficients and sample times for the combined clock and ephemeris frequency stability are summarized in Table 2.

Table 2

Sample Time (days)	Coefficient "a" PP 10(13)	Exponent "μ"
$1 \leq \tau \leq 3$	1.10	−0.18
$3 < \tau \leq 5$	.90	0.00
$5 < \tau \leq 9$	.33	.63
$9 < \tau \leq 10$	1.30	0.00

Comparison of the exponents with the type of noise process identifiable in reference (7) in atomic frequency standards indicate that the segment for sample times from three to five days is classified as flicker noise frequency modulation (FM). The segment for sample times of nine to ten days may also be classified as flicker noise FM. The other two segments can not readily be classified, however the first segment has an exponent of −0.18, which is close to flicker noise FM, and the third segment has a slope of 0.633, which is close to that expected from a random walk in frequency (with an exponent of 1.0).

The error sources that are believed to be most significant in limiting this analysis are

- o the use of a single monitor station frequency standard
- o the radial component of orbit error

The first factor can be reduced by incorporating into the analysis multiple frequency standards at a single monitor station, or by analyzing the spacecraft clock performance using multiple monitor sites. For example, the use of the other three GPS monitor sites will permit the identification of frequency changes such as evidenced in Figure 7. A time scale could then be formed in a manner similar to that described in reference (8). The effect of the radial orbit error can be estimated by using equation (6). For the best fit ephemerides used in this analysis, the average radial error was 2.1m, which corresponds to 1.4×10^{-13} for a sample time of one day. Additional orbit smoothing could produce better estimates for the orbit; however, the exponent of −1 (from Equation (6)) is unchanged by additional smoothing. Ultimately, the 18cm noise level obtained from the smoothed fifteen minute measurements yields a lower limit of 1.2×10^{-14} for a one day sample time.

CONCLUSIONS

- The frequency stability performance of the NAVSTAR 6 cesium clock, to date, is acceptable and within specifications.
- The combined (spacecraft clock, single monitor station clock, and ephemeris) frequency stability is equal to, or less than, 1.3×10^{-13} for sample times of one to ten days.

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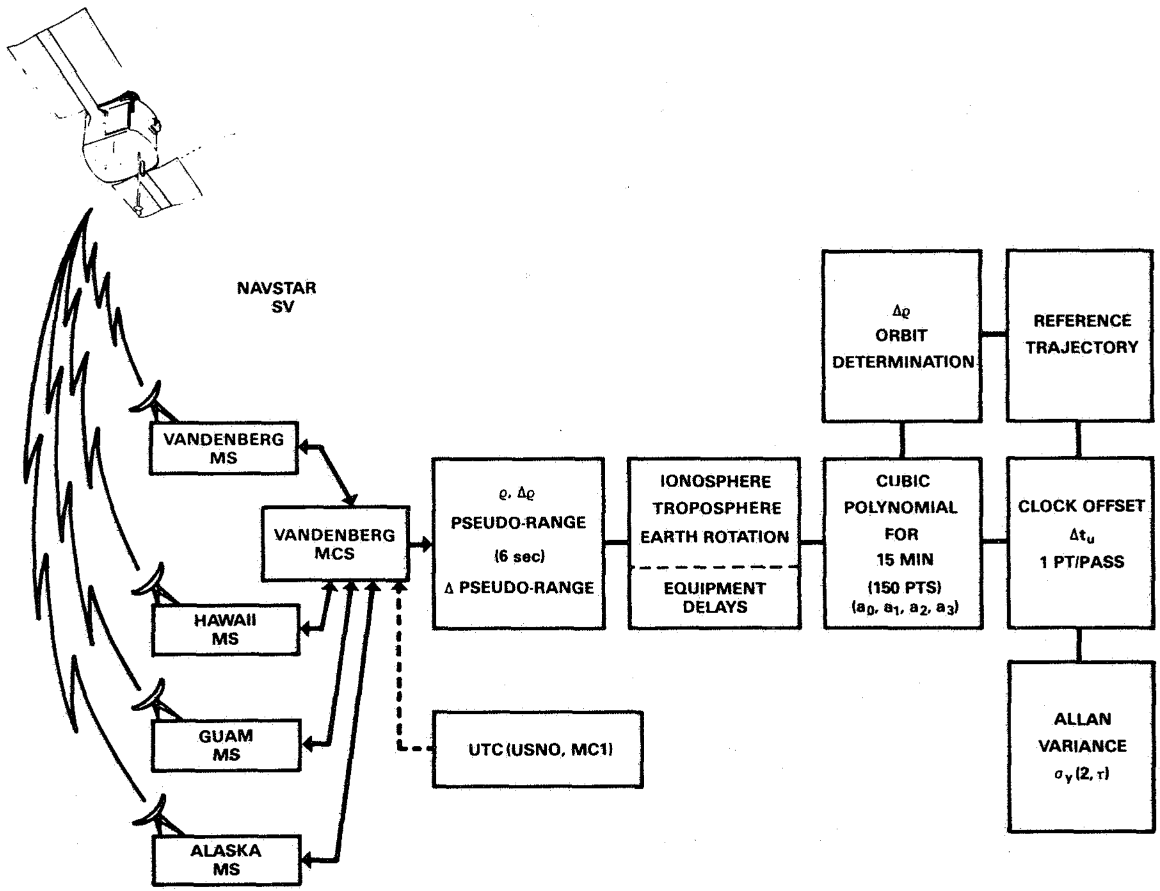

Figure 1 — GPS Frequency stability analysis procedure for analyzing on-orbit clock performance

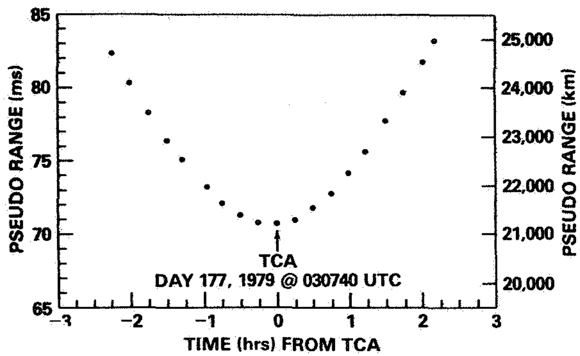

Figure 2—Typical NAVSTAR pseudo-range signature, plotted every 15 minutes

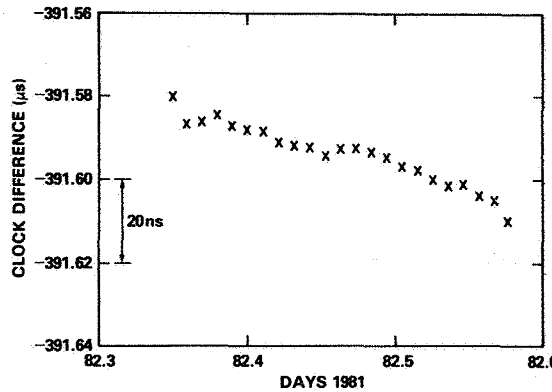

Figure 3—Single Pass Clock difference between NAVSTAR-6 (SV9) and the GPS Vandenberg Monitor Site (VMS)

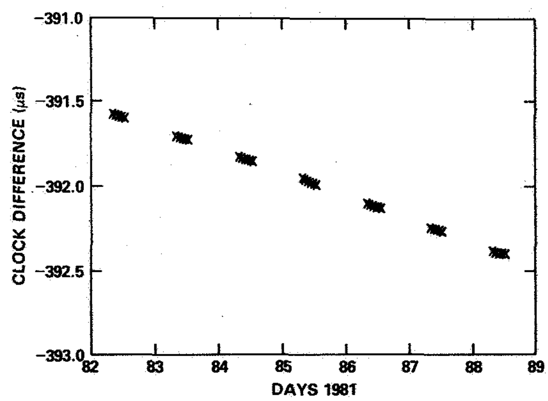

Figure 4—One week of clock differences between NAVSTAR-6 (SV9) and the GPS Vandenberg Monitor Site (VMS)

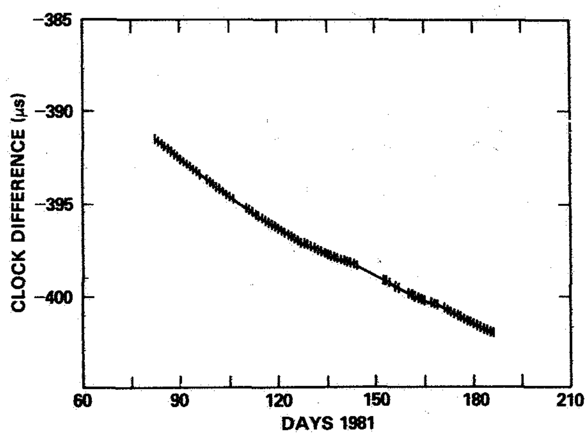

Figure 5—100 days of clock differences between NAVSTAR-6 (SV9) and the GPS Vandenberg Monitor Site (VMS)

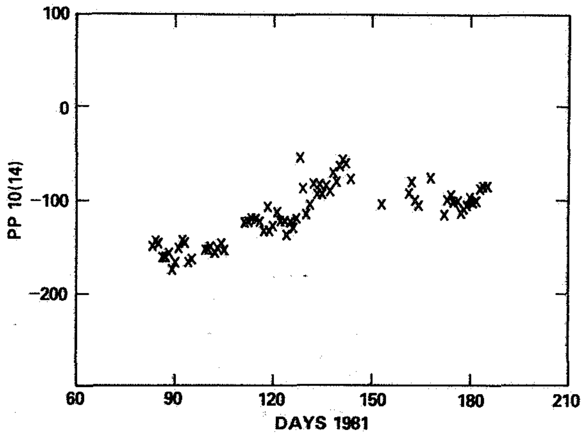

Figure 6 — 100 days of frequency differences between NAVSTAR-6 (SV9) and the GPS Vandenberg Monitor Site (VMS) for a one day sample time

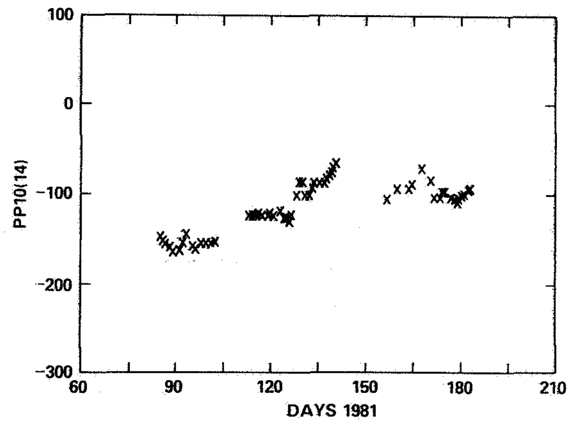

Figure 7 — 100 days of frequency differences between NAVSTAR-6 (SV9) and the GPS Vandenberg Monitor Site (VMS) for a 3 day sample time

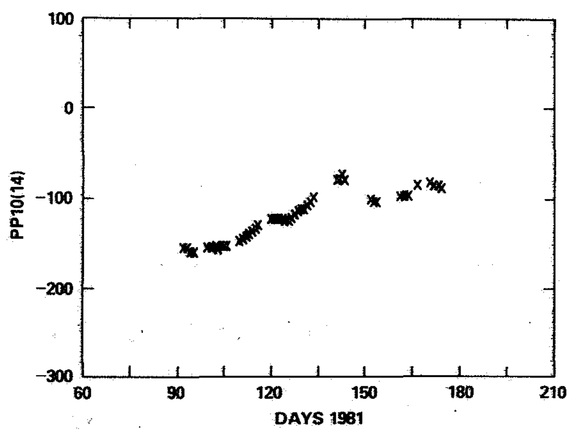

Figure 8 — 100 days of frequency differences between NAVSTAR-6 (SV9) and the GPS Vandenberg Monitor Site (VMS) for a 10 day sample time

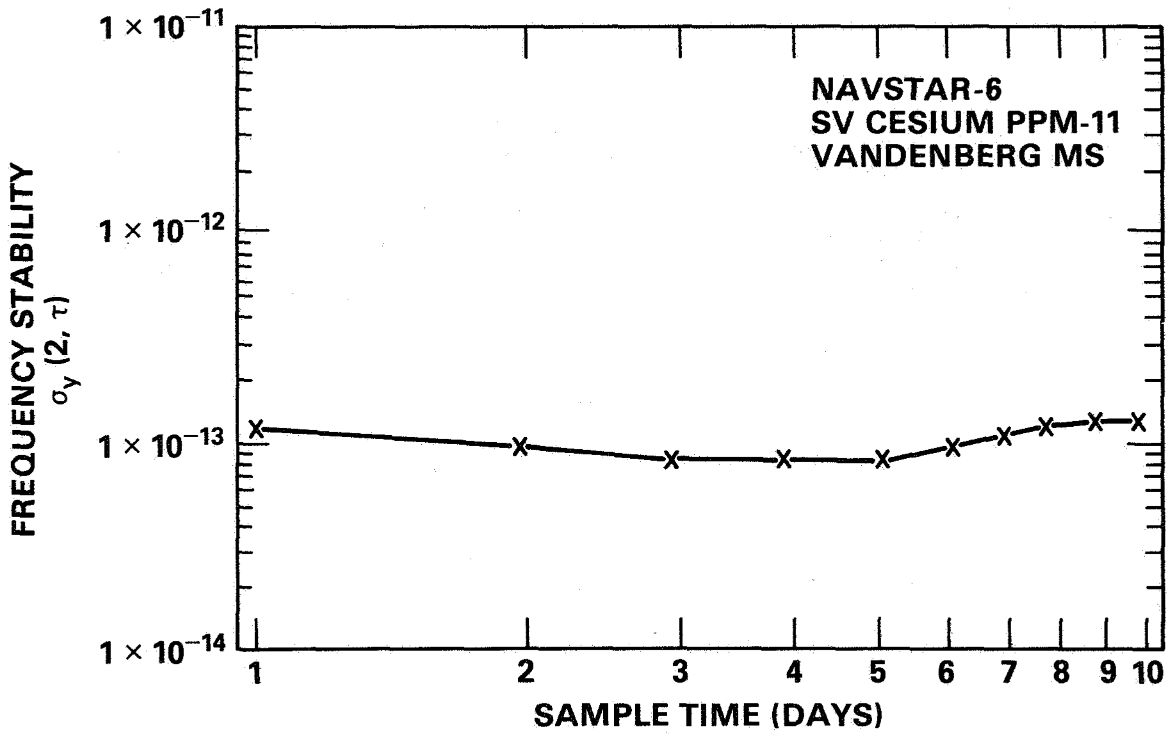

Figure 9 — NAVSTAR-6 Cesium Clock (PPM-11) on-orbit clock/ephemeris frequency stability for 1 to 10 day sample times