AERODYNAMIC POTPOURRI

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ABSTRACT

Aerodynamic developments for vertical axis and horizontal axis wind turbines are given that relate to the performance and aerodynamic loading of these machines. Included are: (1) a fixed wake aerodynamic model of the Darrieus vertical axis wind turbine; (2) experimental results that suggest the existence of a laminar flow Darrieus vertical axis turbine; (3) a simple aerodynamic model for the turbulent windmill/vortex ring state of horizontal axis rotors; and (4) a yawing moment of a rigid hub horizontal axis wind turbine that is related to blade coning.

I. DARRIEUS FIXED WAKE THEORY

A fixed wake theory for the Darrieus Rotor has been developed and compared to test results (1). A comparison of theory and test results is shown below in Figure 1.

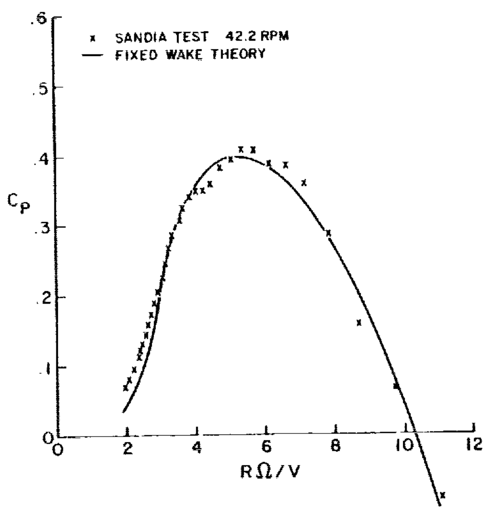

Figure 1. Fixed Wake Theory and Test Results for the Sandia 17 m Machine.

The induced velocities at the front and rear of the streamtube shown in Figure 2 are determined from the shed vorticity in the wake. For the front position (segment BC), a semi-infinite wake of strength directly proportional to the circulation at BC is developed. No effect from the rear position is felt along BC.

At the rear position, DA, the induced velocity is the result of the wake from the front blade which appears as an infinite wake and a semi-infinite wake of strength directly proportional to the circulation at DA. A point in the far wake is subject to infinite wakes from both the front and rear blade positions. If we denote the induced velocity by $\Delta U_F \equiv a_F V_\infty$ and $\Delta U_R \equiv a_R V_\infty$ where F stands for front and R for rear and we use Γ_F and Γ_R to represent the front and rear blade circulations, then we may write

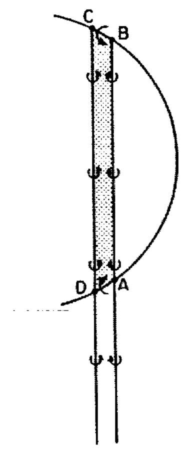

Figure 2. Fixed Wake Streamtube.

$$\Delta U_F = k|\Gamma_F| = a_F V_\infty$$

$$\Delta U_R = k[2|\Gamma_F| + |\Gamma_R|] = a_R V_\infty$$

$$\Delta U_{wake} = k[2|\Gamma_F| + 2|\Gamma_R|] = 2aV_\infty$$

Here the far-wake induced velocity is defined as 2a. Accounting for the sign of the circulation, these results may be put in the form

$$\frac{a_R}{a_F} = 2 - \frac{\Gamma_R}{\Gamma_F}\ , \ a_R - a_F = a$$

These relations are exactly the same as developed by Holme (2). Momentum considerations yield an expression for the dimensionless induced velocity

$$a(1-a) = \frac{BcX}{8\pi R}\left[\frac{C_L W}{V_\infty}\bigg|_F - \frac{C_L W}{V_\infty}\bigg|_R\right]$$

where B is the number of blades, c is the chord, X is the tip speed ratio, C_L is the lift coefficient and W is the velocity relative to the blade. Both the lift coefficient and the relative velocities are determined from the local induced velocities. When the circulation is expressed in terms of the lift coefficient, the above three equations can be used to determine a, a_R and a_F.

For idealized aerodynamics, $C_L = 2\pi m \sin\alpha$, the above relations can be solved in simple form. The results are:

$$a(1-a) = \frac{BcmX}{4R} \cos\gamma \sin\theta(1-a + \sqrt{1-2a})$$

$$a_F = \frac{1}{2}(1 - \sqrt{1-2a})$$

$$a_R = \frac{1}{2}(1+2a - \sqrt{1-2a})$$

where γ is the inclination of a blade element with respect to the vertical and θ is the blade angular position, 90° being directly upwind.

II. DARRIEUS DRAG COEFFICIENT MEASUREMENTS

During testing of the 5 meter and 17 meter Darrieus vertical axis turbines at Sandia Laboratories, data has been obtained at wind speeds below 1 mph (3). At such wind speeds, the tip speed ratio is 70 or above so that the blades are operating with very small angles of attack. In this mode of operation, power input is required to overcome blade drag, so that the power input, RPM and rotor geometry can be used to determine the blade drag coefficient C_{D_0}. Data from the Sandia Laboratories test site is reduced using the method of bins (4). The results for NACA 0015 blades are shown below. Each point represents over 400 samples using the bins method. Also shown are the results of a numerical prediction for the NACA 0015 airfoil. The computation method is described in Reference (5).

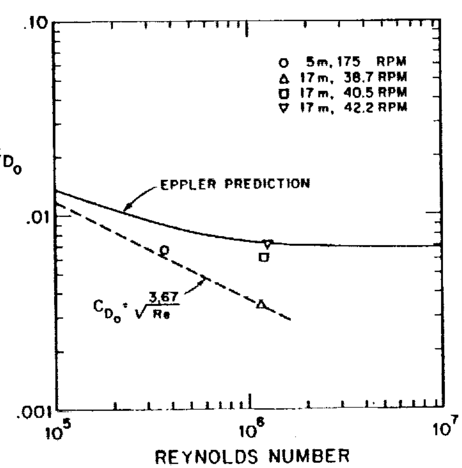

Figure 3. C_{D_0} versus Reynolds Number for the NACA 0015 Airfoil.

These few data points suggest that laminar flow may exist up to a chord Reynolds Number of $1.1 \cdot 10^6$ for the NACA 0015 extruded aluminum Darrieus vertical axis wind turbine blades. The test results for the 17 m at 38.7 RPM show a higher peak power coefficient (0.46) and lower peak power output ($K_{p_{max}} = 0.0080$) than achieved for operation at higher RPM. If laminar flow characteristics can be incorporated into the design of a vertical axis wind turbine, significant improvements can be made in the COE and reliability of a Darrieus Rotor.

III. TURBULENT WINDMILL/VORTEX RING STATE

Strip theory analysis of horizontal axis wind turbines determines local induced velocity by equating blade normal forces to the momentum change in a streamtube. Figure 4 below illustrates the operating states of a rotor expressed in terms of the thrust coefficient, C_T, and the induced velocity, a.

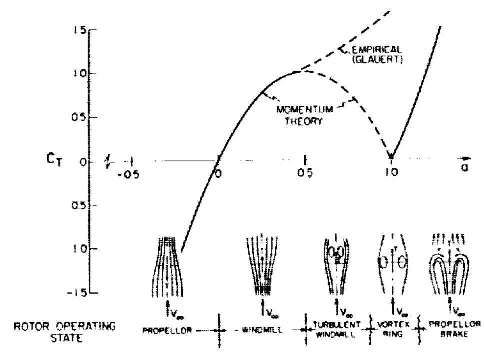

Figure 4. Rotor Operating States.

For operations at induced velocities greater than 0.5, it may be seen that there is a great difference in the value of C_T between the momentum and Glauert (6).

The aerodynamic loads and the performance predicted using the momentum and Glauert values of C_T show corresponding differences. The Glauert empirical curve is represented by an implicit algebraic equation, however, approximating the Glauert relation by a straight line gives a useful and accurate representation the local thrust coefficient for strip theory use. Using the expressions given below, strip theory calculations were made for a two-bladed rotor with NACA 0015 constant chord, untwisted blades. The local values of C_T used were

$$C_{T_L} = 4a(1-a) \qquad a < 0.38$$

$$C_{T_L} = 0.5776 + 0.96a \qquad a > 0.38$$

Test data (7) and calculations are illustrated in Figure 5 below. The measured value of $C_T)_{Rotor}$ increases with tip speed ratio for 0° and 2° blade pitch angle cases. The calculated flow for these cases are in the turbulent windmill state. Agreement between theory and test suggests the above relations can be used to predict the aerodynamic loads of horizontal axis wind turbines operating in the turbulent windmill state.

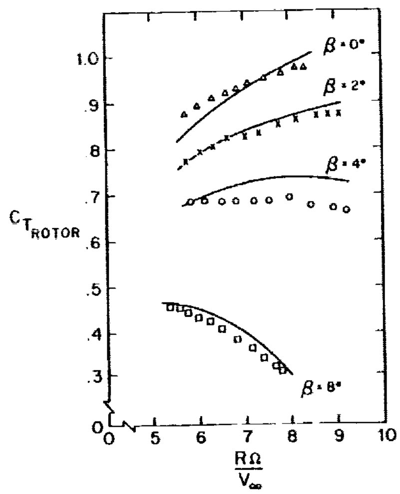

Figure 5. Measured and Predicted Rotor Thrust Coefficients at Various Blade Pitch Angles.

IV. YAW MOMENT FOR A HORIZONTAL AXIS WIND TURBINE

Downwind rigid-hub horizontal axis wind turbines derive their yaw stability from blade coning. The coning acts similar to dihedral in an airplane by increasing the velocity normal to the upwind blade. Yaw forces developed by rigid hub rotors are quite small so that the yaw moment controls yaw stability. The yaw moment can be expressed in terms of integrals over the blade length. Denoting the yaw angle as δ and the position of the ith blade by θ_i, as shown in Figure 6, the rotor yaw moment is the moment about the z axis. Considering the flow induced by rotor yaw, the yaw moment is obtained by the approach of Ribner (8) in which the yaw moment is determined from blade forces and momentum considerations.

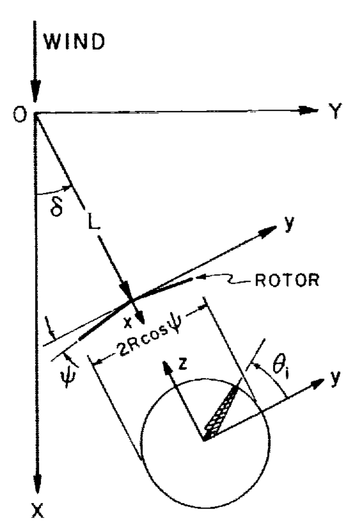

Figure 6. Wind Turbine in Yaw.

The results for a rotor with B blades is

$$C_{M_{Yaw}} \equiv \frac{M_z}{\frac{1}{2}\rho V_\infty^2 \pi R^3} = -\frac{\delta}{\pi}\sum_{1}^{B} \cos^2\theta_i \, \frac{I_1 I_{10}}{I_1+I_4}$$

where

$$I_1 = \int_0^1 \left(\frac{\partial C_{T_L}}{\partial a}\right) \eta^2 d\eta$$

$$I_4 = \frac{B}{2\pi}\int_0^1 \eta^2 \cos\psi \, F_1 d\eta$$

$$I_{10} = \int_0^1 \eta \sin\psi \, F_1 d\eta$$

and $F_1 = \left(\frac{c}{R}\right)[2C_n(1-a) + C_{n_\phi}\eta X]$

and $C_n = C_L\cos\phi + C_D\sin\phi$

$$C_{n_\phi} = C_{L_\alpha}\cos\phi + C_{D_\alpha}\sin\phi - C_t$$

$$C_t = C_L\sin\phi - C_D\cos\phi$$

Here the local blade coning angle is ψ, the blade chord is c and ϕ is the angle between the local relative wind and the plane of rotation. The variable η is the radial position r divided by the rotor radius R. For a rotor with 3 or more blades, the yaw moment becomes

$$C_{M_z} = -\frac{B\delta}{2\pi}\frac{I_1 I_{10}}{I_1+I_4}$$

The above relation has not been verified by comparison to test data, however, downwind free-yaw rigid rotors are observed to be stable in yaw and the above term has been found to be, by far, the largest stabilizing moment.

CONCLUSIONS

The material presented above should be of use in the design and design analysis of wind turbines. Three of the four developments have been compared with test data.

REFERENCES

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3. Worstell, M.J., Unpublished data.

4. Akins, R.E., Performance Evaluation of Wind Energy Conversion Systems Using the Method of Bins-Current Status, SAND77-1375, Sandia Laboratories, Albuquerque, NM, March 1978.

5. Eppler, R., Turbulent Airfoils for General Aviation, AIAA Journal of Aircraft, Vol. 15, No. 2, February 1978.

6. Glauert, H., The Analysis of Experimental Results in the Windmill Brake and Vortex Ring States of an Airscrew, British ARC R&M 1026, 1926.

7. Dugundji, J., Larrabee, E.E., and Bauer, P.H., Experimental Investigation of a Horizontal Axis Wind Turbine, Volume V, Wind Energy Conversion ASRL TR-184-11, MIT Department of Aeronautics and Astronautics, Cambridge, MA, 1978.

8. Ribner, N.S., Propellors in Yaw, NACA Report 820, Washington, D.C., 1948.

QUESTIONS AND ANSWERS

R.E. Wilson

From: T. Currin

Q: Effects of articulation on yaw forces and motions? Order of magnitude with coning?

A: *Professor Miller's paper, "On the Weathervaning of Wind Turbines," (Journal of Energy, Vol. 3, No. 5, Sept-Oct 1979, pp. 319-320), develops expressions for teetering and articulated rotors. His paper was the first to note the dihedral like effect of coning.*

From: J. Glasgow

Q: How did you get measurements for the power coefficient on Sandia machine?

A: *Data was obtained using the method of Bins (4) by Sandia (3).*

From: F. Perkins

Q: When will a wind turbine enter the vortex ring state, and what will the loads look like (steady state or not, large or small)?

A: *The blade normal loads as measured (7) are large. The time histories of these loads appeared to be steady.*

From: I. Paraschivoiu

Q: Did you calculate the loads distribution as a function of the azimuthal angle θ for a vertical-axis turbine?

A: *Yes, loads were calculated as a function of θ. As would be expected, the upwind blade positions experience the largest loads.*

From: L. Mirandy

Q: Vortex ring formula implies C_T can reach 1.5. I have never seen any data mech. over 1.0, have you?

A: *The data of Glauert (British Aeronautical Research Committee Reports & Memoranda No. 1026, Feb. 1926) has C_T up to 1.5 and above.*