

The UTRC Wind Energy Conversion System Performance Analysis
for Horizontal Axis Wind Turbines (WECSPER)

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ABSTRACT

The theory for the UTRC Wind Energy Conversion System Performance Analysis (WECSPER) for the prediction of horizontal axis wind turbine performance is presented. Major features of the analysis are the ability to: (1) treat the wind turbine blades as lifting lines with a prescribed wake model; (2) solve for the wake-induced inflow and blade circulation using real nonlinear airfoil data; and (3) iterate internally to obtain a compatible wake transport velocity and blade loading solution. This analysis also provides an approximate treatment of wake distortions due to tower shadow or wind shear profiles. Finally, selected results of internal UTRC application of the analysis to existing wind turbines and correlation with limited test data are described.

INTRODUCTION

The analytical capabilities required to accurately predict horizontal axis wind turbine rotor performance are varied and complex depending on the turbine design, operating conditions, and the desired computational accuracy. Such factors as rotor yaw angle, tower shadow, and wind shear create inflow profiles which are unsteady and nonuniform. Conditions where the turbine rotor wake is close to the rotor disk result in wake induced effects which can significantly affect performance predictions. Further complexities occur when rotor aeroelastic effects are considered. Many of these effects are generally neglected for wind turbine rotor performance predictions because the increased computational costs required to obtain the gain in predictive accuracy are not justifiable.

The UTRC Wind Energy Conversion System Performance Analysis (WECSPER) for horizontal axis wind turbine performance is capable of treating uniform wind conditions using rigid blade aerodynamics which include real airfoil section properties (lift and drag) for prescribed wake geometries. In addition, it can treat nonuniform inflow profiles for conditions which do not violate the assumptions of the analysis. The computer code is computationally very fast, highly modular, and well structured. This analysis is a logical extension and refinement of the UTRC Prescribed Wake Rotor Performance Method of Landgrebe (Refs. 1 and 2) for hovering helicopter rotors which has been adapted and applied to statically thrusting propellers (Ref. 3) and high speed propeller configurations (Ref. 4). This analysis (Ref. 2) and similar derivatives for helicopter forward flight applications have been expanded in capability and combined into a single comprehensive rotor inflow analysis, the UTRC Rotorcraft Wake Analysis (Ref. 5).

THEORY

General

Briefly, the method is derived utilizing blade-element lifting line theory and incorporates a prescribed wake model consisting of a finite number of trailing vortex filaments. The trajectories and positioning of these filaments are prescribed through internal equations or input coordinates. To reduce the computational time, the original analysis makes use of the fact that for zero yaw angle and steady uniform wind conditions the flow is steady with respect to the turbine rotor blades and has an axially symmetric wake. The analysis uses a cylindrical coordinate system axially aligned with the trajectory of the wake. All velocities and lengths are defined in the positive sense consistent with the right hand rule. Figure 1 illustrates this coordinate system. Figure 2 is an illustration of two types of prescribed wake models used for hovering helicopter rotors (similar representations are used for propellers). The classical model is the one generally used for wind turbine application and is used herein. This model is generally acceptable for wind turbine applications because the wake is transported rapidly away from the blades. A free wake analysis can be used to obtain a wake geometry which models the self-induced distortions; however, this is a costly computational procedure and is generally not warranted. Once the position of the wake is prescribed, a set of equations in terms of the unknown bound circulations is generated utilizing the Kutta-Joukowski and Biot-Savart relationships and the airfoil section lift characteristics. The solution for the blade bound circulation distributions is found and the corresponding induced velocity and section angle of attack distributions are calculated. With the use of the two-dimensional airfoil data the complete

blade loading distribution (lift and drag) and rotor performance (thrust and power) are then obtained.

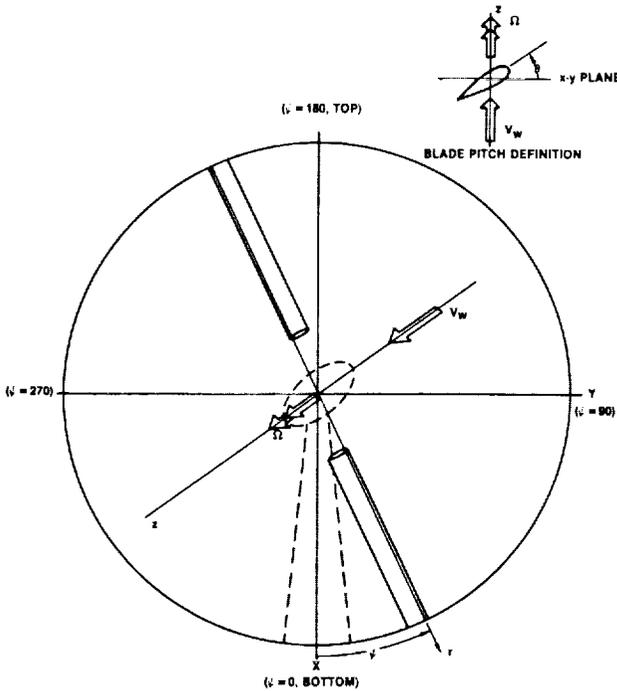


Figure 1. Coordinate System Used in the UTRC Horizontal Axis Wind Turbine Performance Analysis

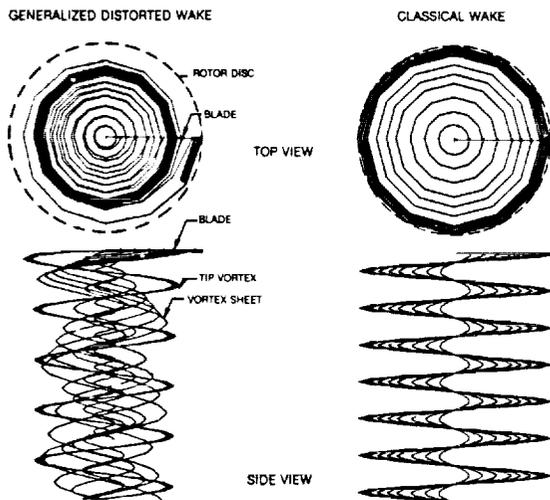


Figure 2. Computer Wake Representation for One Blade of a Hovering Rotor, Classical and Generalized Distorted Wake Models

Lifting Line - Wake Modeling

The concept of a prescribed wake, blade element, lifting line theory applied to wind turbine rotors assumes that each blade of the rotor is represented by a segmented bound vortex lifting line located along the rotor

blade quarter chordline with a spanwise varying concentrated circulation strength proportional to the local blade lift (Kutta-Joukowski Law). The wake is assumed to be modeled by a system of discrete segmented trailing vortices shed from the junction points of the bound vortex segments. The circulation strength of these trailing segments is a function of the spanwise blade bound circulation gradients. A finite wake whose trailing filament segmentation is defined by a specified azimuthal step size ($\Delta\psi$), is used which is of sufficient length to approximate an infinite wake. Figure 3 is an illustration of this modeling procedure.

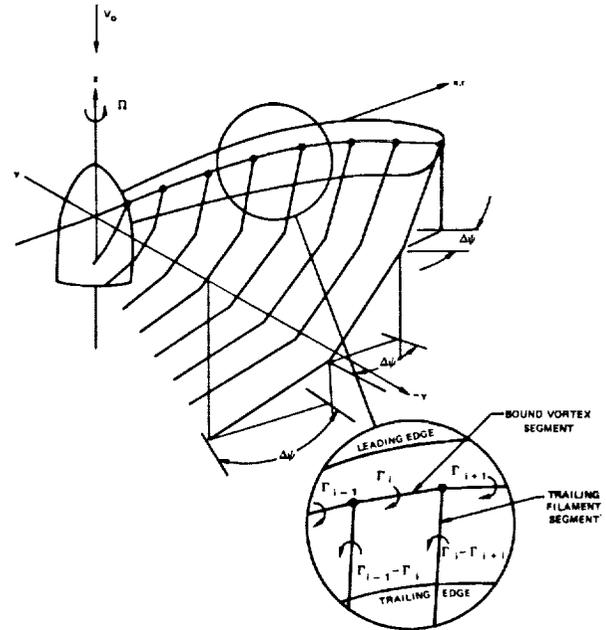


Figure 3. Representation of a Blade and Wake by Bound and Trailing Vortex Segments

The influence (induced velocity) of the bound and trailing vortex segments at any field point is computed by using the Biot-Savart Law for finite length, straight line segments of constant strength. The induced velocity of a filament segment due to a unit strength is called the geometric influence coefficient. The calculation of these coefficients is the most time consuming portion of the analysis.

Within this analysis it is possible to prescribe internally several different wake geometries or to input the wake geometry from an external source. Only the classical wake model which is currently used for wind turbine applications is described. The classical wake model is defined by the addition of the wind speed and the momentum induced velocity for the condition being investigated.

$$V_T = V_W + v_{imom} \quad (1)$$

No radial wake expansion or contraction is used. The resulting wake shape is a helix for which the pitch rate depends on the wind speed and thrust level (Fig. 2).

Vortex core effects are not modeled in this analysis because it is assumed that conditions for which the vortex core influence should be considered will not occur (i.e., close blade-vortex interactions). The roll-up of the vortex sheet into a tip vortex is modeled by prescribing the wake roll-up for the tip region if desired. Our experience has shown that for most wind turbine operating conditions, wake roll-up modeling is unnecessary for performance predictions, even though flow visualization studies have clearly shown the existence of a tip vortex.

Blade Element Aerodynamics

The modeling of the wind turbine blade by the lifting line approach defines the inflow and the effective angle of attack at each blade segment. This aerodynamic model is shown conceptually in Fig. 4. Tabulated linearized airfoil data are used to relate the effective angle of attack at each blade element segment to the local section lift, thus inherently accounting for the chordwise vorticity distribution and the Kutta condition. For this discretized system, the section bound circulation (Γ) is related to the local velocity (U), chord (c), and lift coefficient (C_l) at a section through the Kutta-Joukowski Law.

$$\Gamma = \frac{1}{2} c C_l(\alpha) U \quad (2)$$

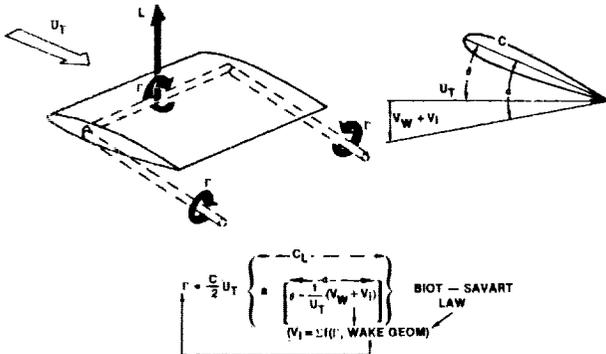


Figure 4. Lifting Line/Wake Aerodynamics - Linearized Model

The local velocity and effective angle of attack (α) are functions of the local tangential velocity (U_T), axial induced velocity (v_z), wind velocity (v_w), and blade pitch angle (θ).

$$U^2 = (U_T)^2 + (v_w + v_z)^2 \quad (3)$$

$$\alpha = \theta + \tan^{-1} [(v_w + v_z)/U_T] \quad (4)$$

The local axial induced velocity due to a given wake geometry is a function of the unknown bound circulation distribution and known geometric influence coefficients (GC) for the particular wake geometry.

$$v_z = \frac{1}{4\pi R} \sum_{j=1}^N GC_j \Gamma_j \quad (5)$$

These relationships (Eqs. 2-5) result in a system of simultaneous nonlinear equations in terms of the wake geometric influence coefficients (GC), the inflow and section properties at each blade element, the two dimensional lift coefficients, and the unknown blade circulation distribution.

Solution

The circulation solution is based on the linearization of the above relationships to form a system of linear equations whose solutions can be obtained and corrected for the actual nonlinearities of the problem using a lagged iteration procedure. For the linearized solution it is assumed that all angles are small, and that the local velocity (U) can be approximated by the local rotational velocity (U_T). The lift coefficient at each section is modeled by a linear lift curve slope (a) and effective angle of attack adjusted for the lift offset (α_o). With these assumptions and Eqs. (2-5), the section circulation can be expressed as:

$$\Gamma = \frac{1}{2} c U_T a \left(\theta - \alpha_o + \frac{v_w + v_z}{U_T} \right) + \frac{1}{2} c a C_f \quad (6)$$

where,

$$C_f = \frac{U C_l}{a} - U_T \left(\theta - \alpha_o + \frac{v_w + v_z}{U_T} \right) \quad (7)$$

is the correction to the linearized equations for the nonlinearities of the actual problem. Since the induced velocity is also a function of the circulation distribution, the equation at the i th blade section can be rewritten as

$$(D_i - GC_i) \Gamma_i - \sum_{j \neq i} GC_j \Gamma_j = 4\pi R (U_T \bar{\theta} + C_f)_i \quad (8)$$

where $\bar{\theta} = \theta - \alpha_o + \frac{v_w}{U_T}$ and $D = \frac{8\pi R}{ac}$

This equation can be written for each blade segment, resulting in a system of simultaneous linear equations if the correction term (C_F) is assumed known. This system of equations can be expressed in matrix form for the n th iteration as,

$$A \Gamma^n = b + C(\Gamma^{n-1}) \quad (9)$$

where the correction vector $C(\Gamma^{n-1})$ is calculated based on the circulation solution from the previous iteration. When the solution procedure converges, the resulting circulation satisfies the original nonlinear relationships. Using the corresponding angle of attack (Eq. 4), the section lift and drag coefficients are obtained from tables of airfoil data for each segment. The lift and drag forces are then calculated and transformed to axial and rotational forces at each segment. The appropriate integrations of these forces yield the rotor thrust and torque.

Nonuniform Wind Conditions

As noted earlier, the inclusion of wind shear, tower shadow, and yawed wind direction introduce additional complexities into the problem of predicting the rotor performance. These nonuniform inflows create aerodynamic environments at the rotor blades which vary azimuthally and radially, and distort the wake geometry in a nonsymmetric manner. This dissymmetry in the wake and the azimuthal variation in the rotor inflow represent aerodynamic conditions for which the general formulation of the method described above is no longer valid since the solution is no longer independent of azimuth position. Rigorous treatment of the problem requires the use of a more sophisticated analysis which involves the wake dissymmetry and nonuniform rotor inflow. Such an analysis exists (the UTRC Rotorcraft Wake Analysis, Ref. 5), but it requires a significant amount of computer time to obtain the solution. In order to make use of the high computational efficiency of the above formulation, the WECSPER analysis includes an approximate treatment of the wake dissymmetry and azimuthal variation in the wind inflow. This treatment is broken into two portions; the wake geometry dissymmetry, and the azimuthal dependency.

Wake Dissymmetry

Comparisons made at UIUC between the predicted results using a Goldstein analysis and the WECSPER Analysis have shown that for most wind turbine operating conditions the wake induced influence at the turbine blade is strongly characterized by the immediate shed wake from that same blade. For these conditions the local wake displacement angle dominates the wake influence. The wake of the preceding blade is

transported rapidly away from the rotor disk and does not have the strong influence on the following blade that is typically seen for hovering helicopters or statically thrusting propellers. Thus, to account for the wake dissymmetry due to the nonuniform inflow, a pseudo wake distortion method can be used.

To treat wake dissymmetry in a manner which makes use of the computationally efficient solution procedure for the symmetric wake problem the trailing wake filaments are regionalized in terms of their wake age into three regions; a near wake region, an intermediate wake region and a far wake region (Fig. 5). The near wake region is defined from the blade which shed the wake to one half of the blade azimuthal spacing behind the blade. The blade spacing is defined as the azimuthal spacing between blades. The extent of the intermediate region is from the end of the near wake region to one and one-half blade azimuthal spacings away from the blade.

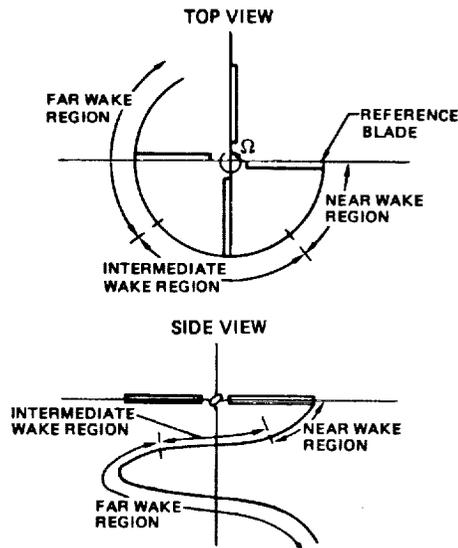


Figure 5. Wake Regionalization for a Tip Vortex Filament of a Four Bladed Rotor

The far wake region is the remaining portion of the wake. In the analysis, the geometric influence coefficients for all filaments are first calculated and stored according to the appropriate regions for the classical wake model. The near wake geometric influence coefficients of all filaments of a given azimuthal position are then scaled by the ratio of the cosines of the wake pitch angles defined by the uniform inflow definition and the local nonuniform inflow definition. The reason for scaling is that the local influence of the filament at the blade which shed the filament is characterized by the filament's orientation (Fig. 6). Geometric assumptions made to result in this simplified scaling factor have been investigated and the error introduced by the approximations were found to be insignificant for general wind

turbine applications. The intermediate wake geometry influence is computed based on corrections to the intermediate wake region geometric influence coefficients. These corrections are based on the change in axial location between the reference wake and the location of the displaced wake of this region (Fig. 6). In this region, changes in orientation are less significant than changes in axial location because of the strong inverse proportionality to displacement distance defined in the Biot-Savart Law.

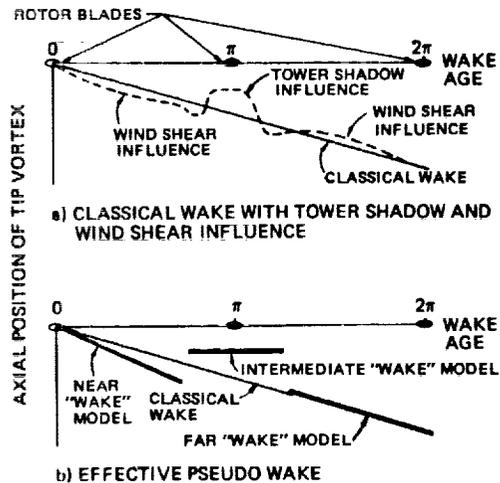


Figure 6. Pseudo Wake Distortion Model for a Tip Vortex of a Two Bladed Rotor

The far wake geometric influence coefficients are assumed to require no corrections since perturbations on the far wake geometry about a mean wake geometry tend to be felt as an average influence which is equal to the influence of the mean wake for small perturbations. With this scaling technique, the initial calculation of the classical wake geometries influence coefficients need only be done for one azimuthal position of the rotor.

Azimuthal Dependence

The performance solution for nonuniform inflow conditions such as wind shear and tower shadow is rotor azimuth dependent. The assumption of near wake dominance and quasi-steady aerodynamics allows for the uncoupling of the azimuthal dependence in the solution procedure. This in turn, allows for the original, computationally efficient, analysis solution procedure to be used.

The geometric influence coefficients are calculated by the pseudo-wake distortion procedure for the particular inflow condition at a specified rotor azimuth position and the circulation distribution and performance prediction are obtained. This is done for each azimuth position defining one revolution of

the rotor and the performance is integrated over this time period to calculate the time-averaged wind turbine performance.

Using all of the azimuthally varying bound circulation distributions obtained above, the induced velocity distribution at the rotor blades is recomputed by multiplying the pseudo-distorted wake influence coefficients for each rotor azimuth position by the appropriate time-dependent circulation values. Using these induced velocity distributions at each azimuth position, the resulting time averaged performance prediction is made. This essentially computes an approximately coupled azimuthally dependent solution. A measurement of the accuracy of the assumption of the near wake's dominance of the rotor performance is obtained by comparing the approximately azimuthally dependent and independent performance solutions. If there is a significant difference, the operating condition is sufficiently extreme to invalidate the assumptions used and requires the use of a more technically sophisticated analysis.

Rigorous treatment of yawed flow requires an analysis which computes the skewed unsymmetrical wake influence and the resulting azimuthally dependent circulation solution (Ref. 5). In the WECSPER Analysis, the assumption of local wake dominance is assumed to allow the influence of small yaw angles to be treated as an effective reduction in the uniform axial inflow profile and neglects the associated wake dissymmetry.

Wake Iteration

Once the rotor performance prediction is obtained, additional iterations may be required. The wake geometry is defined by both the non-induced and induced flow velocity field, and the induced field is unknown at the onset of the analytical procedure. The method used in the WECSPER Analysis is to calculate the classical wake model based on the predicted momentum-induced inflow from the previous solution iteration. The first iteration value is specified by the user. The complete performance solution is then repeated for each iteration until a converged momentum induced velocity solution is obtained. The complete prediction procedure is diagrammed in the flow chart shown in Fig. 7.

Inflow Profile Models

The analysis is capable of treating several types of inflow profiles with the assumptions noted earlier. There are currently three types of profiles available in the analysis, each shown pictorially in Fig. 8. The conventional mode of analysis uses a uniform wind profile model (upper portion of Fig. 8) for conditions without azimuthal or radial variation in the wind inflow velocity. A wind shear profile model can be used in the time dependent

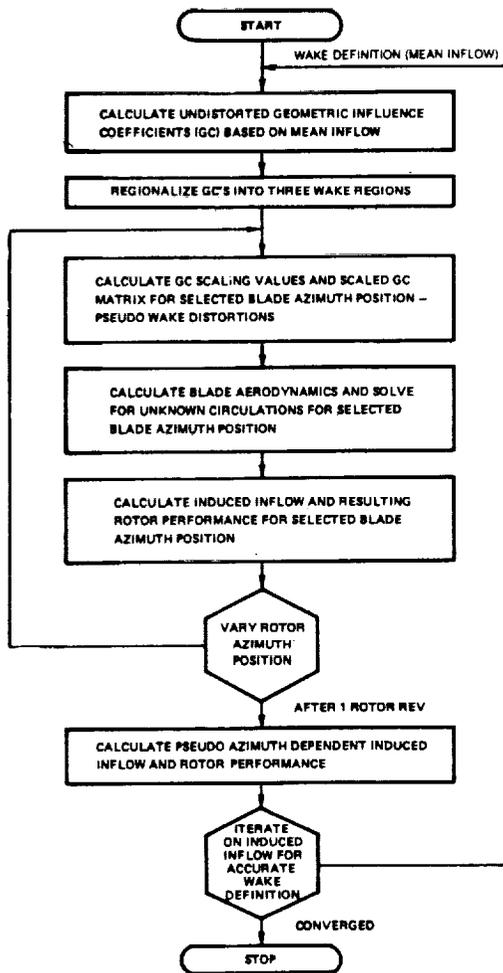


Figure 7. Simplified Flow Chart of Basic Structure for UTRC Wind Turbine Performance Analysis

mode of operation. This model is currently based on a user specified power law behavior (lower left portion of Fig. 8),

$$v_w = v_{ref} (h/h_{ref})^\alpha \quad (10)$$

The influence of a tower support structure on the wind turbine performance is modeled by assuming that the tower influence is represented as a constant velocity deficit from the uniform value over a selected region on the rotor disk, centered about the tower centerline. This region is defined by a tapered column of selected width and taper on the rotor disk (lower right portion of Fig. 8).

APPLICATION

An application of the analysis to actual test conditions to validate the analysis is currently being funded by a DOE sponsored contract through Rockwell International. Results are currently not yet available; however, selected results of internal UTRC application of the analysis

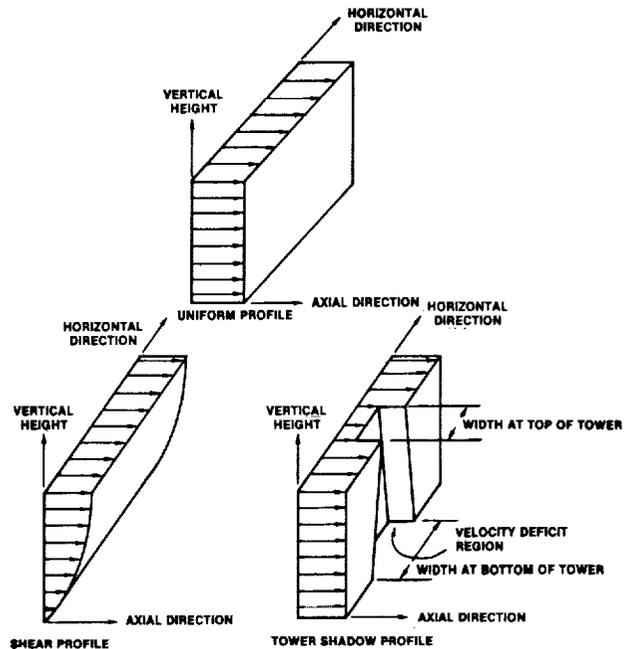


Figure 8. Wind Inflow Profile Models at Rotor Disk

are presented. Figure 9 presents a comparison of measured and predicted rotor performance at one blade angle in terms of power ratio versus velocity ratio for a 1/30 scale model of a Hamilton Standard 3.5 megawatt wind turbine tested in the UTRC main wind tunnel (Ref. 6). The low speed airfoil data used in this analysis was adjusted to reflect Reynolds number effects on the minimum drag coefficient and stall characteristics. These test results, presented for two different tip speeds, show some noticeable differences in measured results near the peak power ratios. These differences with tip speed could be attributable to both Reynold's number and compressibility effects.

The predicted results show fair to good correlation for the lower tip speed results except at the higher velocity ratios. The difference between measured and predicted results could be due at least in part to the Reynold's number corrections used on the airfoil data and/or the accuracy of the test measurements and data reduction procedures used for these low power output operating regimes (high velocity ratio). In addition, the lower tip speed results have a measured power ratio data point at a velocity ratio of 7 which appears to be slightly inconsistent with the other results. In general, when one considers the accuracy of the corrected airfoil data used in the analysis, the correlation as presented is good.

The results of another application of this analysis to the UTRC 8 kW wind turbine (Ref. 7) are shown in Fig. 10. This figure presents measured and theoretical power output versus wind speed for data taken onsite on several different test dates. The scatter in this data

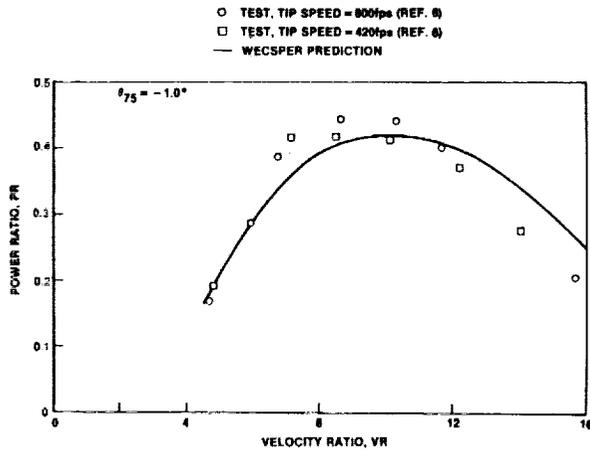


Figure 9. Comparison Between Measured Vs. Predicted Wind Turbine Performance For Rotor of Ref. 6

reflects the uncertainty in the measuring techniques associated with the fact that the actual site wind conditions are unsteady and that the wind shear profile is unknown. The theoretical results obtained were based on the uniform inflow model. The correlation is seen to be good except at the high wind speeds where the test data shows a distinct variation dependent on the particular test day.

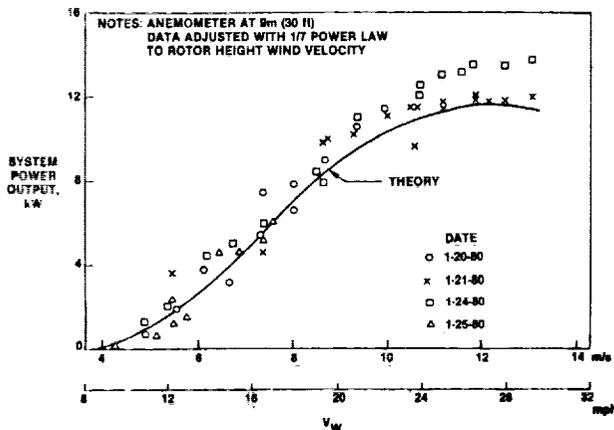


Figure 10. Power Output for UTRC 8 kW Rotor System

CONCLUDING REMARKS

The theory for the UTRC WECSPER Analysis has been presented, and selected results of the application of this analysis to existing UTC wind turbines, model and full scale, have been shown. These preliminary results indicate generally good agreement with measured test results. Discrepancies noted between test and theory may be related to the accuracy of the airfoil data and/or the accuracy of the test measurements. However, extensive validation of the analysis will require more data and comparisons to be made, such as that provided by the DOE funded validation activity noted above.

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QUESTIONS AND ANSWERS

T.A. Egolf

From: W.E. Holley

Q: Is it feasible to compute time varying induced velocity for turbulence inputs?

A: *Not with this analysis. The program assumes a quasi-steady flow aerodynamic to obtain solutions under "slowly" varying inflow conditions. Turbulence time scales are probably too small for application of this analysis.*

From: Anonymous

Q: Have you attempted to compare your code with other rotor prediction codes such as the efficient induction factor method of propeller theory?

A: *The analysis has been correlated in a limited manner with the Goldstein strip theory for W.T. applications. For helicopter applications (from which this analysis was originally derived) induction methods which do not recognize a wake model with significant wake distortion effects will not yield good performance prediction. For wind turbine applications this has not been shown to be the case, although neglecting the wake influence at high velocity ratio ($\Omega R/V$) may be a dangerous assumption.*

From: W.C. Walton

Q: Who funds this code development?

A: *The code development and the applications presented were funded internally by UTRC. There is a current validation activity being funded by DOE, through Rockwell International (Rocky Flats).*

From: F.W. Perkins

Q: Is the accuracy of your analysis significant with respect to uncertainties in the yaw response of wind turbines?

A: *The intent of this analysis is to predict integrated rotor performance (C_{p0} , C_T) within the operating conditions for which the assumptions used are valid. The accuracy of the analysis with respect to small variations in yaw angles is probably quite good. The analysis treats the effect of your angle as a reduction in the wind inflow. Large yaw angles invalidate the assumptions of the analysis and require a more sophisticated analysis (currently available at UTRC).*

From: T.E. Base

Q: How do you justify using potential flow theories in a shear flow?

A: *The treatment of a lifting surface with potential flow models is well justified in the open literature for fixed wing, propeller and helicopter applications. The viscous effects in the flow field can be neglected for most of these applications. On the lifting surface, the specification of a Kutta condition artificially replaces the actual viscous phenomenon. In the UTRC WECSPER analysis the Kutta condition is handled inherently through the use of actual airfoil data. Other viscous phenomena such as close blade/vortex interactions do not occur for general HAWT application.*

From: J. Tangler

Q: On the Hamilton Standard correlation, what was the rotor's Re number and blade geometry? Is a model like this adequate for twisted, tapered, blades using a constant pitch wake?

A: *The HSD correlation was for tapered, nonlinearly twisted blades using a constant pitch wake model. The tip Reynolds number was approximately 500,000. The analysis will handle most reasonable combinations of twist, taper and variable airfoil section types.*