PERFORMANCE OF WIND TURBINES IN A TURBULENT ATMOSPHERE

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ABSTRACT

The effect of atmospheric turbulence on the power fluctuations of large wind turbines is studied. The significance of spatial non-uniformities of the wind is emphasized. The turbulent wind with correlation in time and space is simulated on the computer by Shinozukas method. The wind turbulence is modelled according to the Davenport spectrum with an exponential spatial correlation function. The rotor aerodynamics is modelled by simple blade element theory. Comparison of the spectrum of power output signal between 1-D and 3-D turbulence, shows the significant power fluctuations centered around the blade passage frequency.

INTRODUCTION

One of the considerations associated with the integration of large wind turbines into an electrical network is the fluctuation of output power/torque due to turbulence in the atmospheric wind. Information on the statistics of shaft power/torque fluctuations is essential for the design of control systems for large wind turbines.

The effect of one dimensional turbulence (i.e. only temporal variations about a uniform mean wind over the entire rotor plane) has been studied by Frost [Ref. 1] and der Kinderen et al [Ref. 2]. The one dimensional analysis shows how the high frequency oscillations in power due to the wind fluctuations can be attenuated by large rotor inertia.

In this paper we emphasize the significance of spatial turbulence on large diameter rotors. [See Fig. 1.] Observing the wind energy spectrum of turbulence in the atmosphere [Ref. 26] we see that the predominant energy is at the low frequency end. This low frequency is associated with large size eddies on the order of hundreds of feet. When the rotor diameter approaches the size of these eddies, then the effect of the blade chopping through this spatial turbulence is to produce fluctuations in the power output at the rotor frequency and its multiples. The study is subdivided into two distinct tasks.

(i) a suitable model to describe the turbulence in the atmosphere and the simulation of wind velocities conforming to this model.

(ii) a suitable rotor model to produce the fluctuating power output signal with the above wind input.

WIND MODEL

The objective of the wind model is to simulate the turbulent wind components. The model takes into account the structure of the atmospheric turbulence, and generates a turbulent wind signal appropriate to the terrain, the scale of turbulence and the mean wind velocity.

A comprehensive survey of the subject of atmospheric turbulence [Ref. 16,17,18,19,24,25,26,27] gives a good understanding of the mechanics of turbulence in the atmosphere and also empirical equations to describe the structure of turbulence close to the ground [13,14].

Later we will show that the horizontal component of wind turbulence makes the largest contribution to the output power fluctuations. Hence our attention is focused on the modelling of the horizontal component alone.

The Davenport Model

According to Davenport [14] the spectral density function for horizontal component is given by:

$$\frac{n\ S(n)}{C_T \cdot V_{M,33}^2} = 4\ \frac{x^2}{(1+x^2)^{4/3}} \tag{1}$$

where: $x = 4000.\ \frac{n}{V_{M,33}}$

n = frequency (Hz)
$S(n)$ = Spectral density (m^2/s^2 per Hz)
$V_{M,33}$ = Mean Wind velocity at reference height of 10m(33') above ground (m/s)
C_T = Ground roughness factor (akin to roughness coeff. for pipe flow). Suggested values: 0.0005 for open sea, 0.050 for Urban area

The shape of this function [Fig. 2] conforms to observed general results of experimental measurements [Ref. 14]. The value of the constant C_T can be adjusted according to the terrain to give a model of the atmospheric turbulence at a particular site.

Davenports analysis of recorded data also shows that an exponential correlation between points in space agrees well with experimental results. The Spatial Correlation function is given by:

$$\text{Cor.} = \exp\left(\frac{-Cn\ell}{V}\right) \tag{2}$$

where: V = Mean Wind velocity
C = correlation constant

This function is plotted in Figure 3 for C=6. Taking the Fourier transform of this function with respect to ℓ, we get a spectral description in terms of the wave number $k = \frac{2\pi}{\ell}$ (rads/m)

we get therefore, for spatial correlation:

$$S(k) = \frac{\alpha|\omega|}{\pi(\alpha^2\omega^2+k^2)} \tag{3}$$

where: $\alpha = \frac{C}{2\pi V}$ (per m)

For a complete 3-dimensional simulation of the horizontal wind turbulence as a function of time and two spatial coordinates Y and Z in the plane of the rotor we use the composite spectral energy density function non-dimensionalized by mean wind velocity

$$\frac{S(n,k_y,k_z)}{V^2_{M,33}} = \frac{4.C_T}{\pi^2} \cdot \frac{x^2}{(1+x^2)^{4/3}}$$

$$\frac{\alpha|\omega|}{(\alpha^2\omega^2+k_z^2)} \cdot \frac{\beta|\omega|}{(\beta^2\omega^2+k_y^2)} \tag{4}$$

Where k_y, k_z are the wave numbers corresponding to the separation lengths in the y and z directions respectively. α and β are related to the correlation constant C. (Note: C could be different for correlation in y and z directions).

Simulation of Wind Velocities

A very comprehensive method of simulating a multi-dimensional multivariate process is described by Shinozuka [21]. A random process is simulated by a series of cosine waves at almost evenly spaced frequencies and with amplitudes weighted according to the spectral energy at the wave frequency.

For the simple case of a random function of time with spectral energy function $S(\omega)$ the process is simulated as:

$$F(t) = \sum_{j=1}^{N} [2.S(\omega_j)\Delta\omega]^{\frac{1}{2}}\cos(\omega_j t+\theta_j) \tag{5}$$

where: $\Delta\omega = \frac{\omega_u}{N}$

ω_u = Upper cutoff frequency, above which $S(\omega)$ is practically zero.

ω_j = $j\Delta\omega + \delta\omega, \delta\omega$ is randomly small quantity $<\Delta\omega/20$ is introduced to avoid a periodic repetition of the frequency ω_j

θ = Random phase angle $0<\theta<2\pi$

Shinozuka has shown that with as low as 50 frequencies one can obtain a fairly good representation of the spectrum. [Ref. 22] This procedure can be expanded to 3 dimensions as follows: [Ref. 21,22,23]

$$F(t,y,z) = \sqrt{2}\sum_{j=1}^{N_1}\sum_{k=1}^{N_2}\sum_{l=1}^{N_3}[S(\omega_j,k_{y_k},k_{z_l})\Delta\omega\Delta k_y\Delta k_z]^{\frac{1}{2}}$$

$$\cos(\omega_j t+k_{y_k}y+k_{z_l}z+\theta_{jkl}) \tag{6}$$

To obtain a good representation of the spectrum in the turbulent signal, the values of N_1,N_2,N_3 should be large. Direct evaluation of the above function turns out to be extremely time consuming computation.

By suitably rewriting the above equation the Fast Fourier Transform technique can be applied in place of the above triple summation. [Ref. 23]

For the present simulation the constants in the spectral density function are:

Mean Wind: $V_{M,33}$ = 32kmph(20 mph)

Terrain Roughness Factor = C_T = 0.005

Spatial Correlation
Constants $\alpha=\beta=$ 0.098 per m(.03 per ft)
Cutoff Frequency n_u = 0.5 Hz
Cutoff Wave Number = k_u = 2 rads/m(0.628 rads/ft)(chosen to give a grid spacing of 1.5m(5ft).

After the simulation the turbulent wind data is stored as tabulated information. The table consists of 512 time planes at 1 sec intervals. In each time plane the wind velocities are tabulated at grid points 1.5m(5ft) apart in both y and z directions and covering a total area of 98x98m(320 x320 ft) in the y-z plane. (64 points in each direction)

Instead of generating the turbulent signal as a function of t,y,z by evaluating the triple series a table look up is done in the tabulated wind data with simple linear interpolation.

In the present work we evaluate the turbulent wind component at different radial stations on the wind turbine blade. Interpolation has been restricted to between time planes only. For turbulence at a particular spatial position we choose the value at the grid point closest to this position.

Details of the wind model and computational techniques are given in Ref.30.

An example of the accuracy of the simulation method is shown in Fig. 4 where a comparison between the Davenport Spectrum and a spectrum obtained by taking the Fourier Transform of a one-dimensional turbulent signal generated using Eq. 5, shows excellent agreement.

AERODYNAMIC MODEL OF ROTOR

All of the theories available for analyzing propellers are applicable for analysis of windmill rotors [3,4,12,13,14]. Based on the methods developed for propellers, the performance of windmills can be analysed [5,6,8,9,10,11]. Specific method for analyzing the unsteady aerodynamics has also been developed [8,9]. The study by Barlow [3,4] presents a fairly comprehensive understanding of the forces on a propeller operating in a turbulent atmosphere. As pointed out earlier, this analysis can be equally adapted to the windmill rotor.

Simple Blade Element Theory

For the purpose of the present analysis a relatively simple model of the rotor is adequate. The Blade -element theory is adopted because of its simplicity and ease of computation. Instead of computing tip loss corrections, and induced velocities, we simply introduced an overall induced velocity factor.

The resulting expression for the power developed is very simple. Comparison of the results from this simple model with that of the Wilson-Lissaman program shows that the trend of the results is the same. By a suitable choice of the induced velocity factor one can attain a specific operating point defined by the tip speed ratio λ and the power coefficient C_p, for a particular pitch setting angle θ_p.

Although the model is very simple, it still retains all the nonlinear features of the wind input versus the power output.

In analyzing the effect of wind fluctuation on output power we are interested more in the fluctuating component of power, than in the exact reproduction of the actual power. If one is interested in the exact power at different operating points the more exact methods detailed above can be used.

Integration of the Wind Model & Rotor Model

The aerodynamic model of the rotor consists of calculating the forces and moments on each element of the rotor from the resultant velocity at that station. The free stream wind is modeled as the sum of the mean wind velocity including the effects of wind shear and the turbulence component as generated by the wind model. That is

$$\vec{V} = \vec{V}(z) + \vec{V}'(t,y,z) \qquad (7)$$

In terms of the freestream turbulent wind components u',v' and w' the fluctuating power coefficient is shown to be (Ref. 30)

$$C_p' = \int_0^1 \frac{C_{l\alpha}}{\pi} \frac{c(x)}{R} \frac{x^2}{(1+\lambda^2 x^2)}$$

$$\left\{ -\frac{v'}{U}\sin\theta - \frac{w'}{U}\cos\theta + \frac{u'}{U}\lambda x \right\} x\,dx \qquad (8)$$

$\theta = \Omega t$ = angular position of rotor

$\lambda = \frac{\Omega R}{U}$ = tip speed ratio

From this equation we see the dominance of the $\frac{u'}{U}$ term which is multiplied by λ the tip speed ratio.

In the present model we only use the horizontal component of wind turbulence and treating the free stream wind velocity to be only in the axial direction we can easily find the resultant fluctuating wind input at each point along the rotor. As the rotor goes around in space each point on the rotor is at different spatial points at different times. Knowing the spatial point coordinates and time, the wind model can generate the turbulent velocity which then is added to the mean wind velocity to generate the resulting local wind velocity at that rotor station.

It can be observed, that the output power would be fluctuating even if the input wind has a frozen turbulence. That is, there is correlation with respect to space only. An analysis with frozen turbulence for a simple element of the rotor is presented in [Ref. 30].

RESULTS AND DISCUSSIONS

The aim of the present study is to show the important contribution of the spatial non uniformities of atmospheric turbulence on the output power fluctuations of a large wind turbine.

Using the Davenport spectrum and the exponential correlation function, with Shinozukas method of generating a random signal a complete table of turbulent wind data has been generated.

The next step is to input this wind model to the rotor model and analyze the resulting power output. In order to emphasize our point about the contribution from the spatial turbulence effects, we first study the simple one dimensional wind turbulence input.

In Fig. 5 is shown the unsteady power output with a one dimensional wind input. It must be pointed out here that the power output obtained is an ideal one. This is the direct power obtained from the aerodynamic forces acting on the blade. The inertia of the rotor and the flexibility of the shaft have not been considered. For such an ideal case observe the exact reproduction of the wind fluctuations in the power fluctuation.

Spectral analysis of the input one dimensional wind turbulence and the output fluctuating power is shown in Fig. 6. As expected from our observation of the time signals the spectra are identical in shape. Comparison of the Davenport spectrum of Fig. 3 with the spectrum of the wind turbulence Fig. 6 shows that the envelope of the peaks follows the Davenport spectrum. Although the simulated spectrum shows the correct shape, due to the finite length of the generated time signal the spectrum is not smooth at the low frequencies. This could be remedied by generating a longer time signal.

To show the effect of spatial turbulence we considered three different diameter rotors 30m, 60m, and 75m (100 ft, 200 ft, and 300 ft).

The results for the three different sizes of rotors are shown in Figures 7 to 9.* For comparison in each case is shown the power output with a one dimensional wind input and above it the power signal with a three dimensional wind input. (By three dimensional we mean the turbulent wind component as a function of time t, and spatial coordinates y and z in the planes of the rotor).

* All cases are for 2-bladed wind machines at the same tip speed ratio $\frac{\Omega R}{U} = 4.5$

Observe the high frequency fluctuations caused by the rotor chopping through the spatially non-uniform wind. The power spectrum of the time signals adequately show the important contributions from the spatial turbulence effects. Comparing the spectrum of the three different size rotors, observe how with increasing rotor diameter, the fluctuations caused by spatial turbulence dominate over the original Davenport spectrum due to the temporal fluctuations of the wind.

The blade passage is also indicated in the figure to show that the spatial turbulence effects are concentrated around the blade passage frequency.

Conclusion

The study conducted in the present simulation has shown the importance of spatial turbulence on performance of large wind turbines. This aspect of the fluctuations is an important consideration in the design of pitch control systems.

The above study can be easily extended to incorporate a more sophisticated rotor model, like the Wilson-Lissaman program or the vortex lattice method. In calculating the power output at the shaft the flexibility of the shaft and the rotor inertia can be introduced by writing the dynamic equilibrium equation for the entire rotor system in the presence of the applied aerodynamic torque and the torque load on the shaft. Solution of this dynamic equation will then provide the necessary shaft torque or power as a function of time. This extended model can be used to study the influence of rotor inertia and the flexibility of the shaft.

This complete dynamic model can then be merged into an overall simulation of a wind turbine power generation system as is done in Ref. 28 and 29.

ACKNOWLEDGEMENT

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REFERENCES

1. Frost, Walter, "Analysis of the Effect of Turbulence on Wind Turbine Generator Rotational Fluctuations." University of Tennessee Space Institute, Atmospheric Science Division, Report ASD-1, April 1977.

2. der Kinderen, W.J.G.J., Von Meel, J.J.E.A., Smulders, P.T., "Effects of Wind FLuctuations on Windmill Behaviour." Wind Engineering Vol. 1, No. 2, 1977, pp. 126-140.

3. Barlow, J.B., "On the Forces and Moments on a Propeller Moving Axially Through Homogenous Turbulence." AIAA Atmospheric Flight Mechanics Conference. AIAA paper No. 70-549, May 1970.

4. Barlow, J.B., "Theory of Propeller Forces in a Turbulent Atmosphere." VTIAS Rep. No. 155, September 1970.

5. de Vries, O., "Fluid Dynamic Aspect of Wind Energy Conversion." AGARD-AG-243, July 1979.

6. Wilson, Robert, E., Lissaman, Peter, B.S., "Applied Aerodynamics of Wind Power Machines." PB238595, July 1974.

7. Preuss, R.D., Suiciu, E.O., Morino, L., "Two General Methods for the Unsteady Aerodynamic Analysis of Horizontal Axis Windmills." Proceedings of 12th IECE Conference.

8. Preuss, R.D., "Unsteady Potential Aerodynamics of Rotors with Application to Horizontal Axis Windmills." AIAA Journal, Vol. 18, No. 4, April 1980.

9. Third Wind Energy Workshop, Vol. 1, September 1977, CONF770921. (Sponsored by United States Dept. of Energy and Division of Solar Energy Technology).

10. AIAA, Collection of Technical Papers. AIAA/SERI, Wind Energy Conference, April 9-11, Boulder, Colorado.

11. Gaonkar, Gopal, Hohemsor, Kuh, H., "Flapping Response of Lifting Rotor Blades to Atmospheric Turbulence." Journal of Aircraft, Vol. 6, No. 6, November-December 1969, pp. 496-503.

12. Sullivan, J.P., "The Effect of Blade Sweep on Performance." AIAA Paper No. 77-716.

13. Chang, Li, Ko, "The Theoretical Performance of High Efficiency Propellers." Ph.d Thesis Purdue University, December 1980.

14. Davenport, A.G., "The Spectrum of Horizontal Gustiness Near the Ground in High Winds." Quarterly J. Roy Met. Soc.. Vol. 87, 1961, pp. 194-211.

15. Frost, Walter, Long, B.H., Turner, R.E., "Engineering Handbook on the Atmospheric Environmental Guidelines for use in Wind Turbine Generator Development." NASA TP-1359, December 1978.

16. Frost, Walter, Turner, RObert, E., "Summary of Atmospheric Wind Design Criteria for Wind Energy Conversion System Development." NASA TP-1389, 1979.

17. Fichtl, George H., Kaufman, John W., Vaugham, William W., "The Characteristics of Atmospheric Turbulence as Related to Wind Loads on Tall Structures." NASA Marshall Space-flight Center, Huntsville, Alabama.

18. Scroggins, J.R., "Wind Effects on Launch Vehicles." The Wind Field, AGARDOGRAPH 115.

19. Taylor, J., "Manual on Aircraft Loads." Chapter 2 Atmospheric Environment. AGARDOGRAPH 83.

20. Shinozuka M., Takeo Masaru, Vicaitis Rimas, "Parametric Study of Wind Loading on Structures." Journal of Structural Div. Proceedings of ASCE, March 1973.

21. Shinozuka, M., "Simulation of Multivariate Multi-Dimensional Random Process." Journal of Acoustical Soc. of America, Vol. 49, No. 1, part 2, 1971.

22. Shinozuka, M., "Digital Simulation of Random Processes and its Applications." Journal of Sound and Vibrations (1972) 25(1), pp. 111-128.

23. Shinozuka, M., "Digital Simulation of a Random Process with the aid of FFT Technique." Stochastic Problems in Mechanics. Univ. of Waterloo Press, 1976, pp. 227-286.

24. Lumley, John L., Panofsky, Hans, A., "The Structure of Atmospheric Turbulence." Monographs and Texts in Physics and Astronomy, Vol. Xii, 1964, Book.

25. Simiu, Emil, Scanlon, Robert H., "Wind Effects on Structures, An Introduction to Wind Engineering." Chapter 2, The Atmospheric Boundary Layer.

26. Sachs, Peter, "Wind FOrces in Engineering." 2nd Edition, Pergamon Press, 1978.

27. Etkin, Bernard, "Dynamics of Atmospheric Flight." Chapter 13, Flight in a Turbulent Atmosphere, Wiley, New York, 1972.

28. Krause, P.C., Man, D.T., "Transient Behaviour of a Class of Wind Turbine Generators During Electrical Disturbances." School of Electrical Engineering, Purdue University.

29. Wasuczuk, D., Man, D.T., Sullivan, J.P., "Dynamic Behaviour of a Class of Wind Turbines During Random Wind Fluctuations." School of Electrical Engineering, Purdue University.

30. Sundar, R.M., "Performance of Large Wind Turbines in Turbulent Atmosphere." M.S. Thesis. (To be published May 1981), School of Aeronautics & Astronautics, Purdue University.

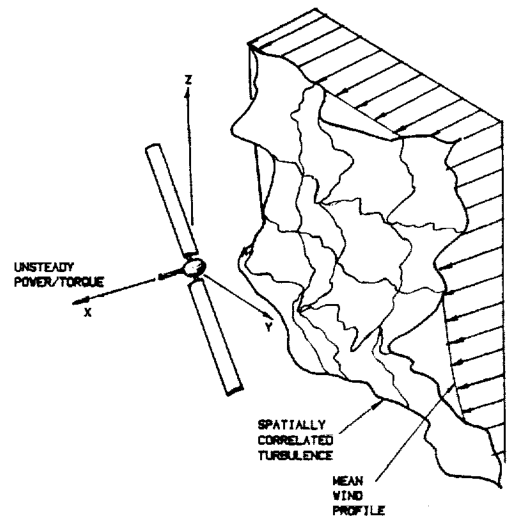

FIG. 1 THE PHYSICAL PROBLEM

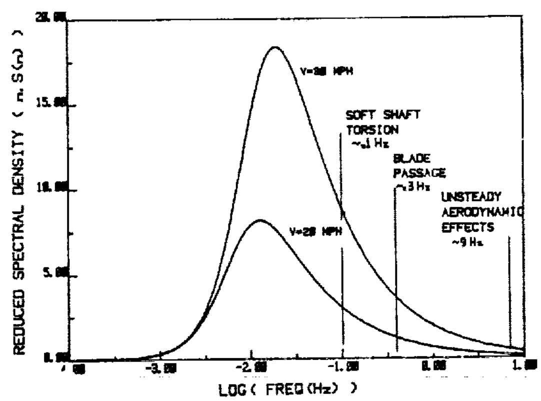

FIG. 2 DAVENPORT MODEL FOR SPECTRUM OF HORIZONTAL COMPONENT OF WIND TURBULENCE

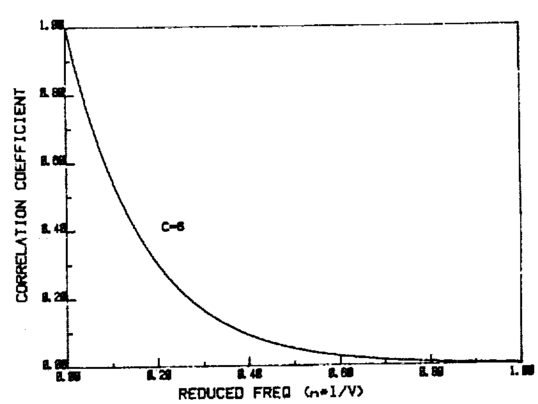

FIG. 3 CORRELATION FUNCTION FOR SPATIAL CORRELATION OF WIND TURBULENCE

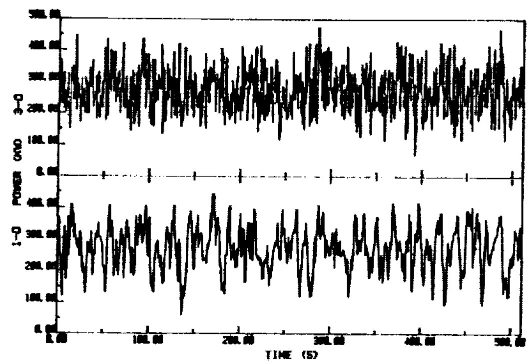

FIG. 8a UNSTEADY POWER OUTPUT WITH 1-D & 3-D TURBULENCE (D=200')

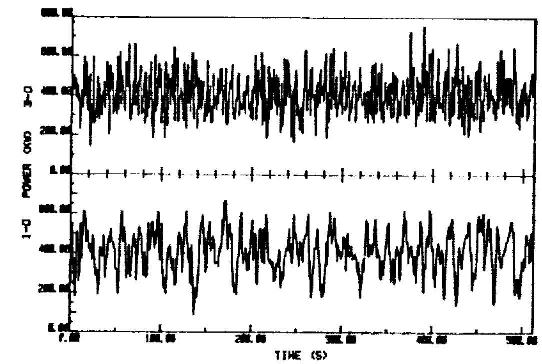

FIG. 9a UNSTEADY POWER OUTPUT WITH 1-D & 3-D TURBULENCE (D=300')

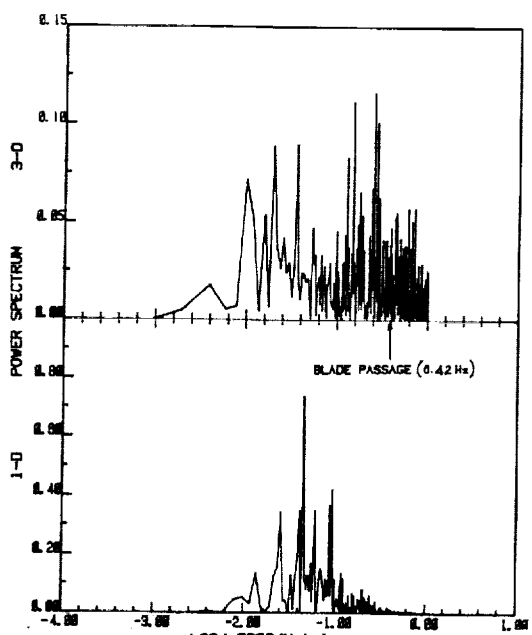

FIG. 8b SPECTRUM OF POWER OUTPUT WITH 1-D & 3-D TURBULENCE (D=200')

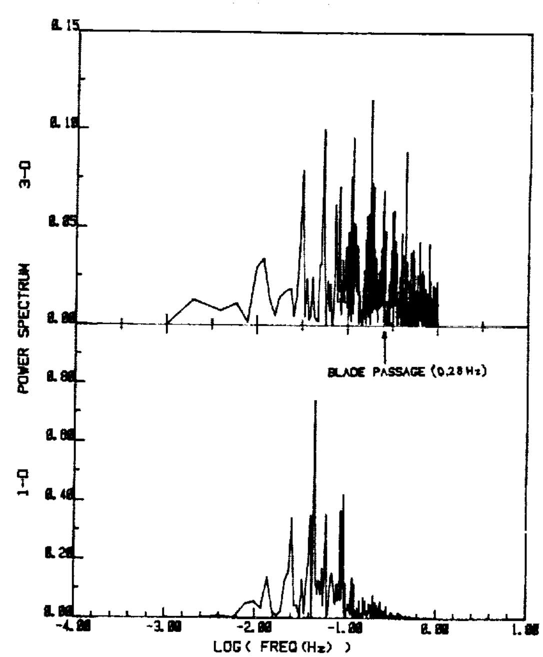

FIG. 9b SPECTRUM OF POWER OUTPUT WITH 1-D & 3-D TURBULENCE (D=300')

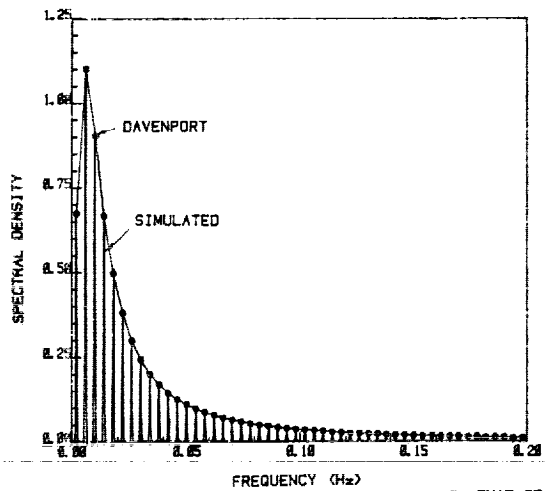

FIG. 4 COMPARISON OF DAVENPORT SPECTRUM WITH THAT OF 1-D TURB SIGNAL SIMULATED WITH 50 FREQUENCIES

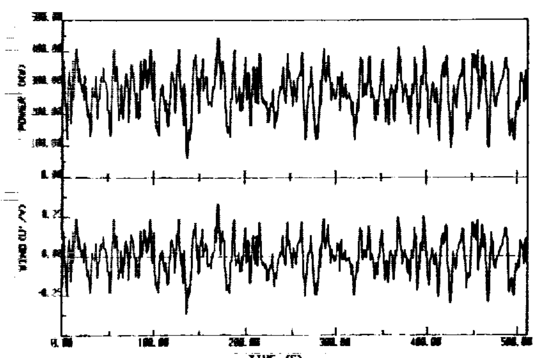

FIG. 5 UNSTEADY POWER OUTPUT WITH 1-D TURBULENCE

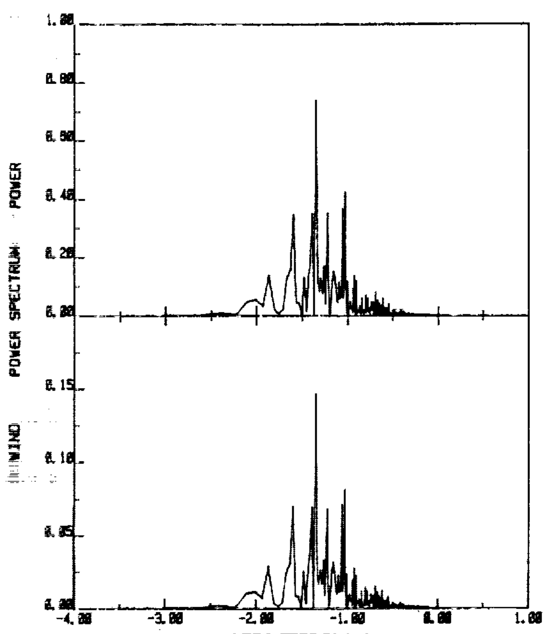

FIG. 6 SPECTRUM OF POWER AND 1-D TURBULENT WIND

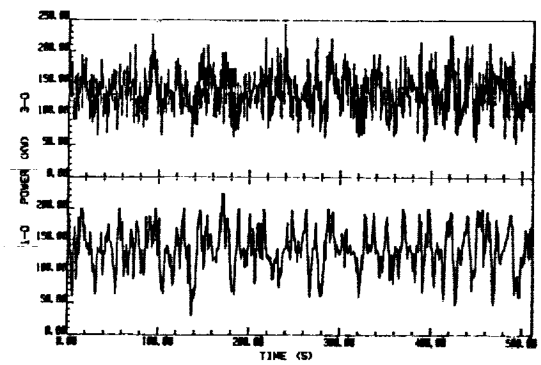

FIG. 7a UNSTEADY POWER OUTPUT WITH 1-D & 3-D TURBULENCE (D=100')

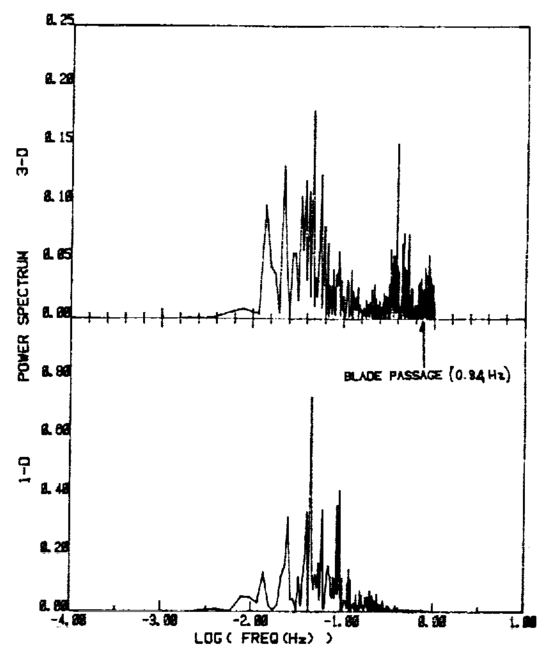

FIG. 7b SPECTRUM OF POWER OUTPUT WITH 1-D & 3-D TURBULENCE (D=100')

QUESTIONS AND ANSWERS

J.P. Sullivan

From: W.N. Sullivan

Q: Please comment on the effect of very soft drive trains on the output power spectra.

A: *A simulation of this problem is planned for the near future at Purdue.*

From: Walter Frost

Q: How is the coherence between frequencies of the wind simulated?

A: *With the exponential correlation function* $exp\ [-\frac{cn\ell}{V}]\ c = 6$.

From: F.W. Perkins

Q: Why does the turbulence seem to drift in a preferred direction, low right to high left?

A: *I don't know, but it may be just a figment of color image processing.*