TWO-DIMENSIONAL TURBULENCE MODELS

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ABSTRACT

Two-dimensional turbulence models described in the NASA technical paper 1359, Engineering Handbook on the Atmospheric and Environmental Guidelines for Use in Wind Turbine Generator Development, are compared with experimental measurements made using an array of instrumented towers. Discussion of the spatial correlation coefficient, the two-point spectrum or cross spectrum, and the coherence function is given. The prediction techniques in general agree reasonably well with the experimental results. Measurements of the integral length scale however, do not correlate well with the prediction model recommended in the design handbook.

Introduction

Information relative to two- and three-dimensional turbulence scales is very important to the design of WTG control systems, to the experimental verification of WTG power output, to the loading of the rotor blades resulting from nonuniform gusts over the span of the rotor, and many other such problems. Some two-dimensional turbulence effects are illustrated schematically in Figures 1 through 3. Figure 1 shows that if the control sensor is located on top of the nacelle, variation in wind speed in a longitudinal direction can result in the sensor monitoring a positive gust, whereas the rotor experiences a negative gust. Therefore, in locating the sensor and computing the appropriate control network, one must have an estimate of how well the wind measured at the sensor location is correlated with the wind which occurs at the rotor. Also, the lag time between the large gusts passing the sensor and impacting upon the rotor is of design interest. Figure 2 illustrates the same sort of problem associated with monitoring wind upstream of a wind turbine generator and attempting to correlate this with the wind monitored at the rotor and the fluctuating power output. Figure 3 schematically illustrates how the spatial variation in wind speed for turbulent gusts across the rotor can create antisymmetric loads and, consequently, large bending moments. These and other problems related to the analysis of turbulent effect on WTG's are related specifically to the spatial variation which occurs in gusts making up the turbulent field.

Spatial wind fields are normally determined either by assuming Taylor's hypothesis or by measuring the wind with an array of towers. Since wind speed is normally measured as a time signal at a single point in space, it is generally converted to a spatial distribution with Taylor's hypothesis, i.e.,

$$x = \overline{W}t$$

This is also referred to as the frozen turbulence concept. Figure 4 shows a time history of wind speed measured at the 24-m level of a tower. The mean wind speed at this level is 6.64 m s^{-1}. The figure illustrates the temporal variation and the spatial variation based on the frozen turbulence concept. Taylor's hypothesis assumes that the velocity profile illustrated would be distributed in space according to the horizontal x-axis.

Spatial variation at two or more points in space can also be measured with one or more towers. The spatial variation in the vertical direction can be measured with a single tower instrumented at different levels and in the horizontal or lateral direction with an array of towers. Figure 5 shows the former case of longitudinal wind speed measured at the 24-m, 12-m, 6-m, and 3-m levels. Many interesting features of two-dimensional turbulence are contained in this figure, and these will be discussed in subsequent sections.

Quantitative estimates of the effect of spatial variation in the wind fields are provided by the three statistical quantities:

1. Correlation coefficient

2. Cross spectra or cross correlation

3. Coherence function

Definitions of these terms are given in References 1, 2, and 3.

The purpose of this paper is to describe these quantities physically and to review the models recommended in the Engineering Handbook on the Atmospheric Environmental Guidelines for Use in Wind Turbine Development [1] for predicting their values. Recent experimental data measured with an array of towers is then compared with the recommended prediction techniques.

Tower Array

The tower array is illustrated schematically in Figure 6. Full details of this Atmospheric Boundary Layer Test Facility, Atmospheric Sciences Division, Space Sciences Laboratory, NASA Marshall Space Flight Center, are given in Reference 4. The original purpose of the data measurements reported herein were to investigate the influence of a simulated block building on the wind field surrounding the building. The wind fields at the top of the towers (20 m in height) and well downstream of the building, however, were found to be insensitive to the presence of the building. Therefore, the upper level wind measurements

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represent essentially undisturbed flow and provide meaningful results for WECS design criteria comparisons.

Correlation Coefficients

The correlation coefficient is defined as

$$R_{ij}(r) = \overline{w_i(x)w_j(x + r)}/\sigma_i\sigma_j$$

This quantity is a measure of how fluctuations in the wind speed component, w_i, measured at the position, x, correspond or correlate with fluctuations in the wind speed component, w_j, at x + r. Two common mathematical forms of correlation coefficients are the Dryden and the von Karman [1]. Figure 7 is a plot of both the longitudinal and transverse von Karman correlations. The insert defines longitudinal and transverse correlation coefficients. The integral length scale, L_p, appearing in the correlations is discussed in a latter section.

Figure 8 shows the longitudinal correlation with wind parallel to the tower array. The top row of correlation values is plotted as solid circles on Figure 7. The measured correlation in the boundary layer is slightly lower than the theoretical curve. However, the agreement is reasonable considering that near the surface the flow will not be isotropic.

Some interesting properties of the correlation coefficient are observed in Figure 8. It can be seen that directly behind the block building there is very little correlation with the upstream position. This indicates a breakdown in the flow structure between the undisturbed flow upstream and the wake region behind the building. Also, as would be expected, the correlations in the free stream decrease with an increase in the distance separating the two points from which measurements were made. It should be noted that a correlation of unity indicates that the exact signal is felt instantaneously at the two points in question, and it is obvious that a point correlated with itself gives a value of unity (i.e., at the 20-m level on tower 1). A negative correlation between two points suggests a structured reverse flow region where a longitudinal fluctuation in the positive direction at one point results in a negative fluctuation at another point. If the correlation is equal to -1, then the velocities are equal in magnitude but opposite in sign. More experimental values of spatial correlations are given in References 4 and 5.

Integral Length Scale

Although the preceding correlations are based on the assumption of isotropic turbulence,which is a reasonable assumption at high altitudes (h > 300 m (1000 ft)), a technique frequently employed to adapt these isotropic relationships to low altitudes is to permit the integral length scale to vary with height, with surface roughness, and to be different for the longitudinal direction from what they are for the lateral and vertical directions. Relationships for the integral length scales recommended by Counihan [6] are shown in Figure 9. The mathematical definition of length scale is given in the insert. Measured values of L_{w_x} are shown in Figure 10. The scatter in the length scale values is very large and the results do not compare well with the theoretical results. Typically good correlations of measured length scales are difficult to achieve [6].

In theory, however, scale lengths are an indication of the size of the eddies. Figure 10 shows that the higher level usually has the larger length scale for the longitudinal gust, as predicted by the theoretical results shown in Figure 9. The vertical component of wind speed for the measured data [5] gives smaller length scales at a higher level. Conclusions from Figure 10 must be drawn with care since the presence of the building may influence the value of the length scales shown.

Two-Point Spectrum

The correlation coefficient provides a relationship between the correlation of fluctuations in the wind averaged over all values of gust sizes. In many cases, however, we are interested in the correlation between fluctuation in wind speeds of a prescribed frequency. In this case, the statistical quantity known as the two-point spectrum is useful.

Correlation of velocity over different spatial separations can be expressed by the two-point spectrum which has two parts--a real part, co-spectrum, and an imaginary part, quadrature spectrum. Also, this spatial variation in velocities can be expressed in terms of the dimensionless two-point spectrum. The latter form seems more manageable for wind loading applications and also has real and imaginary components as expressed below:

$$\hat{\phi}\left(x,x';\frac{\hat{n}}{\overline{W}}\right) = \frac{Co\left(x,x';\frac{\hat{n}}{\overline{W}}\right) + iQu\left(x,x';\frac{\hat{n}}{\overline{W}}\right)}{\left\{\phi\left(x;\frac{\hat{n}}{\overline{W}}\right)\phi\left(x';\frac{\hat{n}}{\overline{W}}\right)\right\}^{1/2}}$$

where $\hat{n}/\overline{W}$ is the wave number, x and x' = x + r are two spatial coordinates, Qu is the quadrature spectrum (out-of-phase component) of the two-point spectrum, Co is the co-spectrum (in-phase component) of the two-point spectrum, and ϕ is the one-point spectrum.

For reasons of symmetry, the quadrature spectrum between similar velocity components is usually zero for points in the same horizontal plane. For vertical separations, however, Qu is nonzero, although usually not as significant as Co. The existence of the quadrature component can be taken to indicate a preferred orientation of eddies and therefore only occurs when there is asymmetry present in the flow. For example, there is no significant quadrature component in the horizontal direction crosswind spectrum between like components of velocity; however, in the vertical direction where there is strong asymmetry, the quadrature component is significant, and the maximum correlation in the horizontal wind speed at two different heights occurs not simultaneously but when the signal from the lower station is delayed by time roughly equal to $\Delta h/\overline{W}$. It is interesting to inspect Figure 5 in this regard where it has been illustrated that the delay time between eddies of the 24-m and 12-m level is approximately $\Delta t = 1.7$ s whereas the calculated value of $\Delta h/\overline{W} = 1.9$ s. This means that in the vertical direction a signal hits the top of the tower before it hits the bottom because eddies lean into the wind as a result of wind shear.

Houbolt and Sen [7] have theoretically computed two-point spectra for vertical and longitudinal gusts in isotropic turbulence based on a von Karman correlation model. Their results for vertical fluctuations are plotted in Figure 11. This figure was developed for two-point spectra of nonuniform spanwise gusts on airfoils but has direct application to nonuniform spanwise gusts on WTG rotors.

Figure 12 illustrates the agreement of Houbolt and Sen's [7] theoretical model with experimental data. The model has a steeper slope at high frequencies and predicts less energy at low frequencies than the data show. Physically, this suggests that wind disturbances are more strongly correlated for small values of Δy but not as strongly correlated for intermediate values of Δy as predicted by the recommended design equations. In view of all the variables involved and of the fact that boundary-layer turbulence is not isotropic, however, the agreement with the prediction techniques is reasonably good.

Coherence

The coherence is defined as the absolute value of the two-point spectrum

$$coh = |\phi(x,x';n/\overline{W})|$$

and serves as a more useable form of the two-point spectrum.

Coherence is expressed by the relationship:

$$coh = e^{-a\hat{n}\Delta x/\overline{W}_{h=10m}}$$

where Δx is the spatial separation between the two points at which the wind speed is measured. The decay coefficient, a, is approximately equal to 7.5 for vertical separation and 4.5 for horizontal separation. These values represent an average of the decay coefficients reported in References 8, 9, and 10. The lateral decay coefficient is approximately equal to the vertical decay coefficient; hence,

$$a_x \simeq 4.5; \; a_y = a_z \simeq 7.5$$

The reader is cautioned that the values of a quoted are current state-of-the-art values and much research remains to be done before their value is confirmed. Moreover, it is known that a is dependent on terrain roughness, atmospheric stability, and spatial separation.

Measured lateral coherence functions are plotted as a function of reduced frequency, $\eta = \hat{n}\Delta y/\overline{W}_{h=10m}$ in Figure 13. The measured value of $a_y = 10.8$ is compared with the recommended value of $a_y = 7.5$.

The coherence function is a measure of the correlation of velocity fluctuations of frequency, $\hat{n}$, between spatial points separated by Δy.

CONCLUDING REMARKS

Comparisons of experimental data with the prediction models for two-dimensional turbulence design criteria given in [1] show that the models in predict the general trends in the data. The exception is length scales which show a very wide scatter in measured values.

The uncertainty in length scale however, does not impact the magnitude of the predicted statistical properties appreciable, and in general the guidelines recommended in [1] can be used for design with an appropriate safety factor.

ACKNOWLEDGMENTS

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NOMENCLATURE

h	Height above the surface
$\hat{L}$	Integral turbulence length scale
L_p	Longitudinal isotropic turbulence integral scale
L_{w_x}	Characteristic length scale of longitudinal wind fluctuation
L_{w_y}	Characteristic length scale of lateral wind fluctuations
$\hat{n}$	Frequency in cycles per second
R_{w_x}	Lateral correlation
r	Separation distance in turbulence correlations
t	Time
W_h	Wind speed at height h
W_x	Longitudinal wind speed
w_x	Longitudinal wind speed fluctuation about the mean
W_y	Lateral wind speed
w_y	Lateral wind speed fluctuation about the mean
W_z	Vertical wind speed
w_z	Vertical wind speed fluctuation about the mean
z_0	Surface roughness
$\overline{n}$	Reduced frequency, $nh/\overline{W}$
$\overline{\zeta}$	Dimensionless two-point spectrum, $2\phi(\sigma,\overline{n})_{w_\alpha}/\sigma^2_{w_\alpha}$
σ_{w_α}	Standard deviations of the turbulence fluctuations where α represents x, y, or z, respectively
$\phi_{ij}(\hat{n})$	Two-point spectrum
$\phi_{w_\alpha}(\hat{n})$	Spectral density function for turbulence kinetic energy

Over Symbols

$(\overline{\ })$	10 min average or greater

Subscripts

α	Designates one of three wind vector component directions, x, y, or z

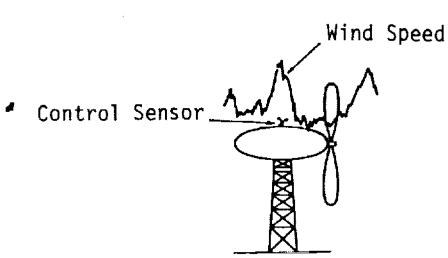

Figure 1 Control system requires correlation between wind speed at sensor and at rotor.

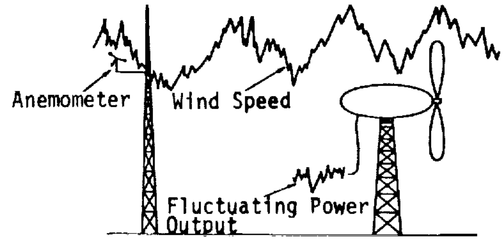

Figure 2 Correlation of wind speed at measuring tower with wind speed at rotor is needed for experimental verification.

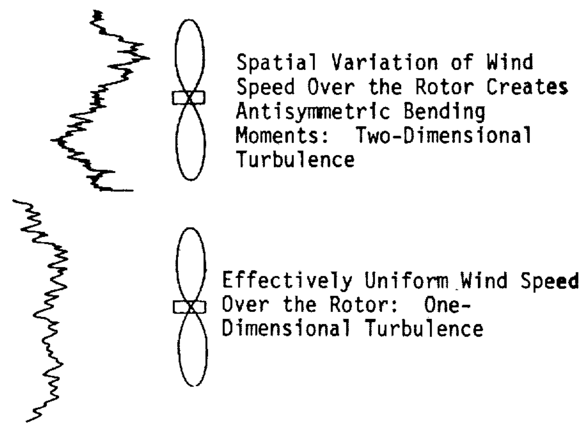

Figure 3 Wind speed fluctuation over the rotor.

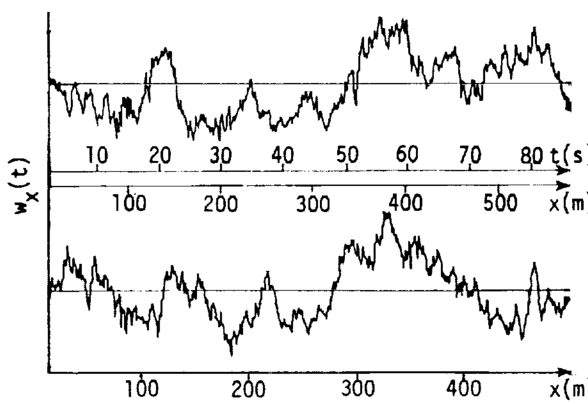

Figure 4 Illustrates Taylor's hypothesis (frozen turbulence) with data measured at 24-m and 12-m level.

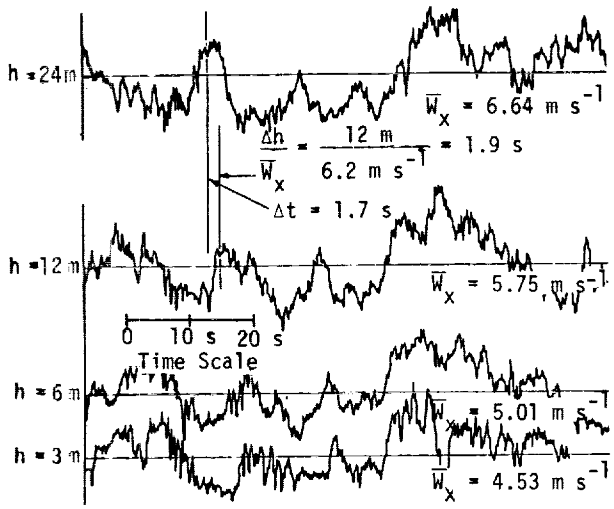

Figure 5 Features of two-dimensional turbulence measured with a single tower at 24-m, 12-m, 6-m, and 3-m level.

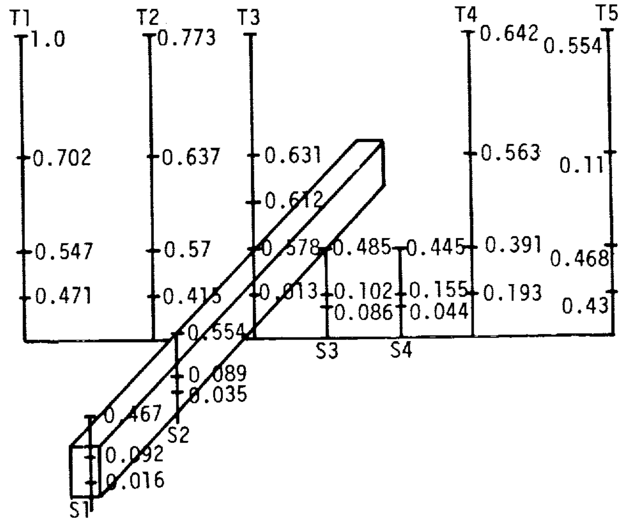

Figure 8 Spatial correlations of the longitudinal velocity component measured at the 20-m level on tower 1 with all other measuring stations [4].

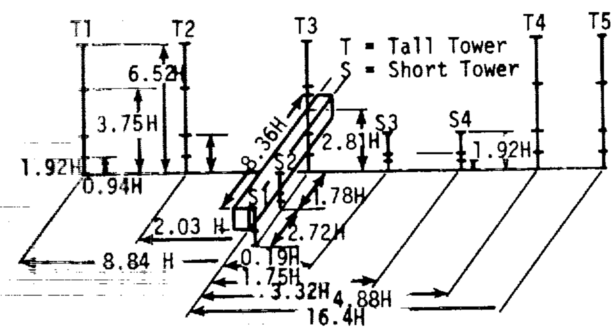

Figure 6 Schematic of the current configuration of the NASA Marshall Space Flight Center's Boundary Layer Facility. (Not to scale, H = 3.2 m.)

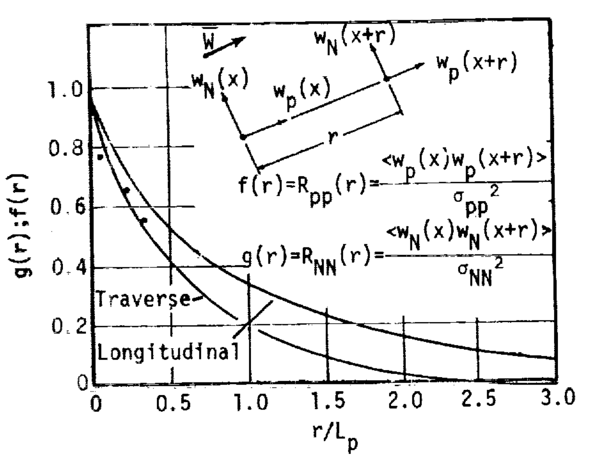

Figure 7 von Karman longitudinal and transverse fundamental correlation function for isotropic turbulence.

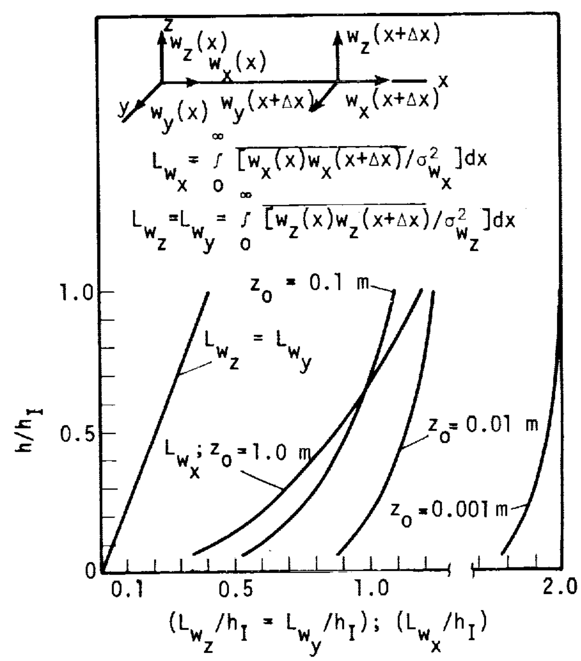

Figure 9 Integral scale length h_I = 250 m.

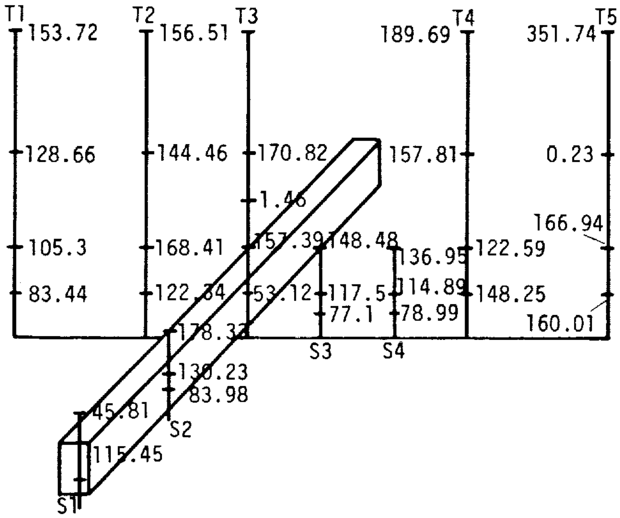

Figure 10 Integral length scale in meters for longitudinal component, L_{w_x}, wind perpendicular to array.

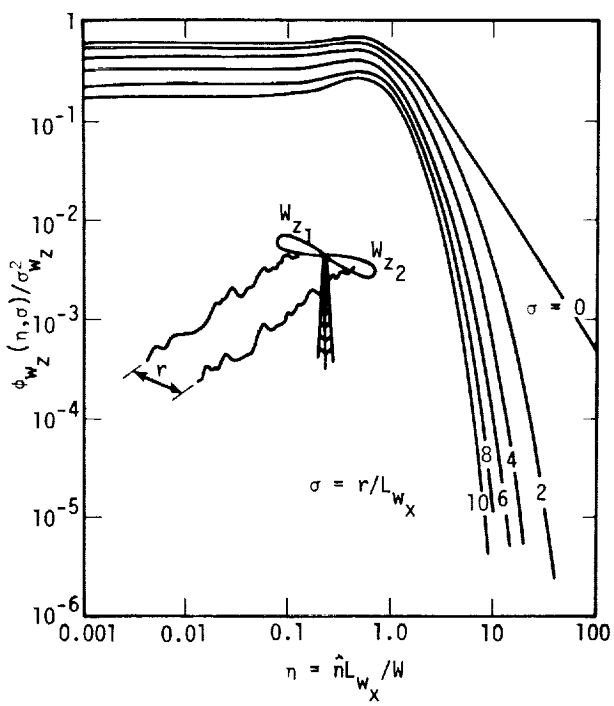

Figure 11 Theoretical two-point spectra for treatment of nonuniform spanwise gust.

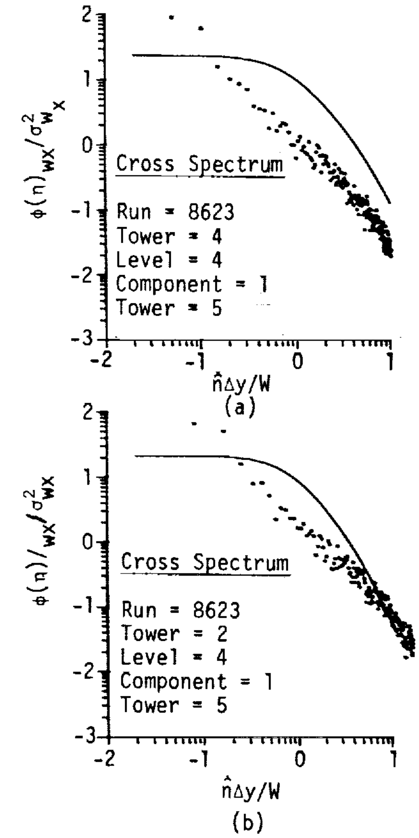

Figure 12 Comparison of theoretical two-point spectra with measured data.

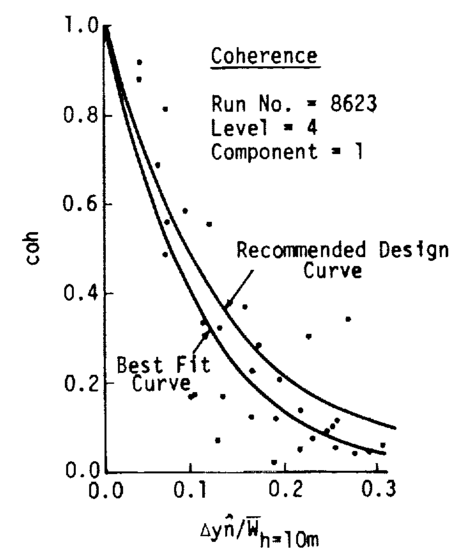

Figure 13 Experimental coherence function. Solid line is the best fit curve of $e^{-a\Delta yn/\overline{W}}$ where a = 10.8.

QUESTIONS AND ANSWERS

W. Frost

From: W.E. Holley

Q: Does the coherency = 1 at low frequency?

A: *Our data indicates that it does and, of course, the computation procedure forces it to unity when* Δy *goes to zero.*

From: D.C. Powell

Q: Why do you use length scale to calculate reduced frequency when you show that length scale is not a good parameter?

A: *Nearly all current prediction models use length scales as a scaling parameter. Personally I prefer the approach given in reference 1 of the paper which utilizes a characteristic reducing frequency and scales length with height above ground.*