

DYNAMIC ANALYSIS OF DARRIEUS VERTICAL  
AXIS WIND TURBINE ROTORS\*

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ABSTRACT

The dynamic response characteristics of the VAWT rotor are important factors governing the safety and fatigue life of VAWT systems. The principal problems are the determination of critical rotor speeds (resonances) and the assessment of forced vibration response amplitudes. The solution to these problems is complicated by centrifugal and Coriolis effects which can have substantial influence on rotor resonant frequencies and mode shapes. This paper will describe and discuss the primary tools now in use at Sandia National Laboratories for rotor analysis. These tools include a lumped spring-mass model (VAWTDYN) and also finite-element based approaches. The discussion will center on the accuracy and completeness of current capabilities and plans for future research.

INTRODUCTION

The primary goal at Sandia in the dynamic analysis of vertical axis wind turbines (VAWT) is to accurately predict vibratory and mean stress levels throughout the rotor system. In most VAWT designs to date, quasi-static analysis methods have been the primary tools utilized for dynamic analysis. This simple approach was motivated by the observation that the VAWT rotor is stiff relative to the excitation frequencies. However, experience has indicated that substantial resonances can and do occur for certain operating conditions in the VAWT rotor. There is clearly a need to construct relatively complete dynamic models to identify critical resonance conditions and, for near resonant operations, to predict dynamic amplification factors.

Techniques for predicting vibratory stress levels near resonance are hampered by uncertainties in the aerodynamic wind loading and structural damping. In the latter case, since the VAWTs encountered to date have all been very lightly damped (.1 to .5% of critical), near resonance, slight variations in the magnitude of the damping produce large variations in the vibratory stress levels.

Although techniques for predicting vibration amplitudes are still being pursued, the major effort is being expended on developing methods to identify critical resonances. With a knowledge of the natural frequencies of the turbine and the frequency content of the aerodynamic wind loading, both as a function of the turbine operating speed, one can identify possible turbine speeds which may produce resonance. Some of these critical speeds can be eliminated by considering the modal content

of the wind loading as compared with the mode of vibration in question.

This all seems straightforward enough, but it is considerably complicated by the fact that the turbine modes and frequencies as well as wind forcing functions must be obtained relative to the rotating frame. Due to this added complexity and a scarcity of intuition for the behavior of rotating structures, experimental data have been relied upon whenever possible to verify the mathematical models.

HISTORY

Originally at Sandia, finite element techniques which accounted only for the rotational effects of centrifugal stiffening were used to determine the spectral characteristics of VAWTs. A version of the SAP IV code, modified to include centrifugal stiffening, was utilized in this regard. It had also been determined, from symmetry arguments, that for two-bladed rotors, modes which involve axisymmetric motion about the turbine axis are driven only by even per rev excitations, whereas modes which involve lateral tower motion are driven by odd per rev excitations. A fan plot for the DOE/Alcoa low-cost 17 meter turbine which was generated with SAP IV is shown in Fig. 1.

The first revelation that this dynamic analysis technique was inadequate came when Alcoa's ALVAWT 6342 turbine was put into operation. A one per rev tower resonance, observed at the operating speed, was not predicted by the SAP IV analysis as all modes which involve tower motion crossed the one per rev excitation line well away from the operating speed. In an attempt to understand this apparent anomaly, closed form solutions of the whirling shaft problem were examined, whereupon the necessity of

\*This work supported by the U.S. Department of Energy under Contract No. DE-AC04-76DP00789.



including all of the rotating coordinate system effects was immediately realized.

To include these effects, a simple, seven-degree-of-freedom, spring-mass model of a two-bladed VAWT, which is displayed in Fig. 2, was developed. In this model, the tower is represented by two rigid links joined together with a "U" joint. Torsional springs are mounted across the joint to account for the tower bending stiffness. The blades, which are also assumed to be rigid, are attached at the top and bottom of the tower through ball joints with torsional springs representing the blade lead-lag stiffness. Linear springs, which model the cable stiffness, are attached at the top of the tower, tending to restore it to its upright position. Torsional springs represent the drive train stiffness and blade aerodynamic loads are determined using a single streamtube aerodynamic model. The equations for the model are developed in a frame which rotates with the turbine, taking into account all low order rotating coordinate system effects. Solutions are obtained using time marching techniques developed for initial value problems. This analysis package, which goes by the name VAWTDYN, is covered in detail in a Sandia National Laboratories report.\*

The dynamic behavior predicted by VAWTDYN differed markedly from that of the SAP IV model. This is shown in Fig. 3, where a VAWTDYN analysis of the DOE/Alcoa 17 meter is summarized. The natural frequencies of the turbine, which previously only increased with increasing rpm due to centrifugal stiffening, now varied in either direction reminiscent of the whirling shaft behavior. In addition to this, mode shapes which had been independent of each other became coupled. For example, modes which contained motion either in the plane of the blades or out of it, as predicted by SAP IV, now possessed both types of motion.

For verification purposes, VAWTDYN results were compared to the limited amount of experimental data available. VAWTDYN accurately predicted the tower resonance of the ALVAWT 6342 and, in fact, was relatively successful in all these verification tests. As a result, even though it is a relatively crude model, capable of representing only 3 or 4 rotor modes, a fair amount of confidence was developed in the VAWTDYN package.

However, after the erection of the DOE/Alcoa low-cost 17 meter turbine, a signi-

\*D. W. Lobitz and W. N. Sullivan, "VAWTDYN - A Numerical Package for the Dynamic Analysis of Vertical Axis Wind Turbines," SAND80-0085, July 1980.

ficant three per rev blade resonance was observed which VAWTDYN did not predict. And, in fact, no reasonable amount of variation in the parameters of the VAWTDYN model would produce the resonance or the associated mode of vibration. Speculating that the difficulty was associated with the crudeness of the model, a much more refined model, described in the next section was developed.

#### CURRENT METHODS

The model which is now used at Sandia for dynamic analysis of VAWTs is derived using finite element techniques. A coordinate system is employed, fixed in space at the base of the tower, which rotates at a constant angular velocity. Turbine motions within this system are assumed to be small. Manifestations of this rotating system appear primarily in the inertia terms of the equations of motion.

Due to the structural nature of VAWTs, the finite element equations need only be developed for beam elements and concentrated masses. In the case of the beam elements, for developing the inertia matrices, the displacements and velocities are assumed to vary linearly along the length of the element. Using the following equation for the total velocity at a point,

$$\chi_t = \dot{\chi} + \dot{\Omega} \times (\chi + \mu), \quad (1)$$

where,

$\chi_t$  is the total velocity vector,  
 $\chi$ ,  $\mu$ , and  $\dot{\chi}$  are the original position, the displacement, and the velocity vectors, respectively, as observed in the rotating coordinate system, and  
 $\dot{\Omega}$  is the angular velocity vector of that system,

an expression for the kinetic energy of an elemental mass at that point can be developed. After substituting for  $\chi$ ,  $\mu$ , and  $\dot{\chi}$ , their linear functional forms and integrating the kinetic energy along the length of the element, the total kinetic energy is obtained. The appropriate finite element matrices can now be developed using Hamilton's Principle. For the special case of  $\dot{\Omega}$  constant and directed along the "z" axis of the rotating system, i.e.,

$$\dot{\Omega} = \begin{Bmatrix} 0 \\ 0 \\ \Omega_z \end{Bmatrix}, \quad (2)$$

the various inertia matrices and their corresponding vector multipliers, for an arbitrary element, are given below:



$$\text{Mass} \quad \rho \ell \begin{bmatrix} 1/3 & 0 & 0 & 1/6 & 0 & 0 \\ & 1/3 & 0 & 0 & 1/6 & 0 \\ & & 1/3 & 0 & 0 & 1/6 \\ & & \text{sym} & 1/3 & 0 & 0 \\ & & & & 1/3 & 0 \\ & & & & & 1/3 \end{bmatrix} \begin{Bmatrix} \ddot{u}_{x_1} \\ \ddot{u}_{y_1} \\ \ddot{u}_{z_1} \\ \ddot{u}_{x_2} \\ \ddot{u}_{y_2} \\ \ddot{u}_{z_2} \end{Bmatrix} \quad (3)$$

$$\text{Coriolis} \quad 2\rho \ell \Omega_z \begin{bmatrix} 0 & -1/3 & 0 & 0 & -1/6 & 0 \\ 1/3 & 0 & 0 & 1/6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1/6 & 0 & 0 & -1/3 & 0 \\ 1/6 & 0 & 0 & 1/3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{u}_{x_1} \\ \dot{u}_{y_1} \\ \dot{u}_{z_1} \\ \dot{u}_{x_2} \\ \dot{u}_{y_2} \\ \dot{u}_{z_2} \end{Bmatrix} \quad (4)$$

$$\text{Softening} \quad \rho \ell \Omega_z^2 \begin{bmatrix} 1/3 & 0 & 0 & 1/6 & 0 & 0 \\ & 1/3 & 0 & 0 & 1/6 & 0 \\ & & 0 & 0 & 0 & 0 \\ & & \text{sym} & 1/3 & 0 & 0 \\ & & & & 1/3 & 0 \\ & & & & & 0 \end{bmatrix} \begin{Bmatrix} u_{x_1} \\ u_{y_1} \\ u_{z_1} \\ u_{x_2} \\ u_{y_2} \\ u_{z_2} \end{Bmatrix} \quad (5)$$

$$\text{Centrifugal Force} \quad \rho \ell \Omega_z^2 \begin{bmatrix} 1/3 & 0 & 0 & 1/6 & 0 & 0 \\ & 1/3 & 0 & 0 & 1/6 & 0 \\ & & 0 & 0 & 0 & 0 \\ & & \text{sym} & 1/3 & 0 & 0 \\ & & & & 1/3 & 0 \\ & & & & & 0 \end{bmatrix} \begin{Bmatrix} r_{x_1} \\ r_{y_1} \\ r_{z_1} \\ r_{x_2} \\ r_{y_2} \\ r_{z_2} \end{Bmatrix} \quad (6)$$

The subscripts x, y, and z refer to the vector components along the rotating coordinate system axes and the "1's" and "2's" denote the value at either the first or second node point of the element. The quantity,  $\rho$ , is the mass/unit length and  $\ell$  is the length of the element.

Assembling the contributions from all the elements in the discretized model and denoting the total mass, Coriolis, and softening matrices by M, C, and S, respec-

tively, the resulting finite element equations are given by

$$\ddot{M}\dot{U} + C\dot{U} - SU + K\begin{Bmatrix} U \\ \theta \end{Bmatrix} = F_C + F_A \quad (7)$$

The load vectors,  $F_C$  and  $F_A$ , respectively, represent the centrifugal force loading, which results from the element contributions detailed in Eq. (6), and blade loads caused by aerodynamic forces.

The matrix, K, is the usual assembled stiffness matrix for the beam elements, and consequently contains terms associated with rotations in addition to displacements, as indicated. In general, due to the stretching of the neutral axis, K is a function of the displacement. This introduces a nonlinearity in Eq. (7) which results in solution procedures of much greater complexity. To avoid this additional complexity, the stiffness matrix is developed commensurate with the quasi-static displacement field associated with the time-independent centrifugal loading only, neglecting variations which result from the time-dependent aerodynamic loads. With this approximation, K is constant, which eliminates the nonlinearity, and the equations represent small vibrations about a centrifugally prestressed state.

A critical factor in the development of this method is that at no time was it required to make vector transformations between stationary (groundbased) and rotating coordinate systems. For most VAWTs, the physical connections of the rotor to the ground occur through the tiedown cables and the tower base connection. Since the base of the tower is stationary, zero displacements exist in both systems. Furthermore, if there are three or more equally spaced tiedown cables with mass small relative to the rotor, identical in length, cross-sectional area, pretension, and angle of inclination, the restoring forces depend only on the displacement of the top of the tower from the vertical. These forces are directed toward the undisturbed vertical position of the rotor. Consequently, the tiedowns can be represented by massless linear springs which rotate with the turbine and are connected between its top and the vertical. To date, all turbines analyzed by this method have possessed this type of tiedown system.

In certain rotor designs, the restoring forces may also depend on the azimuthal angle of the turbine relative to ground (as in the case of unequally spaced cables, for example). In these situations, appropriate transformation must be implemented and time dependent coefficients appear in the stiffness matrix. Although the equations retain their linearity, the existence of these coefficients would require the current solution procedure to be extensively modified.



To obtain the modes and frequencies of the turbine as observed in the rotating system, a complex eigenvalue extraction procedure must be employed. For this purpose, Eq. (7) is reduced to the following form:

$$M\ddot{U} + C\dot{U} - SU + K\begin{Bmatrix} U \\ \theta \end{Bmatrix} = 0, \quad (8)$$

with K corresponding to the pre-stressed state resulting from centrifugal loading, as discussed above. In the general case the damping matrix, C, produces complex eigenvalues and eigenvectors. However, in this case, where the C matrix represents Coriolis effects only and is consequently skew-symmetric, just the eigenvectors are complex.

Instead of developing a completely independent package for the eigensolution of Eq. (8), an existing code was modified. With this approach, duplication of such things as input, output, plotting, solution procedures, etc., is avoided. The MacNeal-Schwendler version of NASTRAN was selected here because the modification required was minimal and could be accomplished via DMAP programming, a feature which allows the NASTRAN user to modify the code without actually dealing with the FORTRAN coding. This version contains complex eigen-system solution procedures and also permits the stiffness, mass, and damping matrices to be modified through an input option. Thus, the special matrices required in Eq. (8), specifically the Coriolis (C) and softening (S) matrices, can be generated externally and read into NASTRAN as input. As the NASTRAN code handles non-symmetric as well as symmetric matrices, no special problems occur due to the skew-symmetry of the coriolis matrix. The mass (M) and stiffness (K) matrices are generated internally, complete with the effects of pre-stress in the stiffness matrix.

Although this method has been successfully tested with all available experimental data, the three per rev blade resonance which was observed in the DOE/Alcoa low-cost 17 meter turbine is of special significance, since it was not predicted by VAWTDYN. The fan plot shown in Fig. 4, which corresponds to the low-cost turbine, was developed using the current method. Note that the three per rev excitation line crosses the natural frequency line associated with the first in-plane mode very close to the 51.3 rpm operating speed, indicating the observed resonance. Moreover, at the operating speed, in addition to containing the observed three per rev blade edgewise motion, the first in-plane mode also contains the unusual tower motion which was measured. In this mode, even though the blade motion is predominantly out of the plane of the blades, reminiscent of flapping butterfly wings, the tower moves predominantly in the plane. This result,

which tends to defy intuition, is a result of the rotating coordinate system effects.

The other tests, which have been used to verify the method, have involved experimental data taken from the Alcoa ALVAWT 6342 turbine and Sandia's 17 meter research machine. The method has also been tested against existing closed form solutions such as that for the whirling shaft problem. Although verification will continue as new tests become available, to date, no failures have been experienced.

The primary strength of this method is that a general class of VAWTs can be analyzed with it relatively easily and in much detail using the various NASTRAN modeling features. Additional blades, struts, concentrated masses, etc. can be analyzed simply through the preparation of the appropriate NASTRAN input.

#### FUTURE ACTIVITIES

Two major activities are planned for extension of the finite element package described in the previous section. In order to predict vibratory stress levels during turbine operation, a capability for the analysis of forced vibration will be developed. Time marching as well as modal superposition methods will be pursued. Using this capability, an effort will be made to establish a general categorization procedure with regard to severity for the various crossings of the frequency and excitation lines on the fan plot.

The other activity involves the inclusion of aeroelastic effects in the finite element equations. To implement these effects, modifications will be made to the mass, damping, and stiffness matrices to incorporate the corresponding aerodynamic matrices. The aeroelastic effects will be used both in forced vibration and flutter instability analyses.

In addition to these extensions, a major experiment is planned to provide a relatively comprehensive test of accuracy of this method in predicting turbine modes and frequencies in the rotating system. The test will consist of measuring these spectral data for the Sandia 17 meter research turbine while rotating. By conducting tests at several rotational speeds, a fan plot such as the one shown in Fig. 4 can be experimentally developed for comparison with predictions.

#### CONCLUSIONS

The sophistication of dynamic analysis methods for VAWTs has undergone steady improvement at Sandia. The current method provides a means to straightforwardly predict the spectral characteristics

of rotating turbines which have a significant degree of structural complexity. Verification tests have shown the accuracy of the method to be quite satisfactory. After completion of the planned activities identified in the previous section, a strong capability for dynamic assessment of VAWTs should be available. This should be achieved within the current calendar year and will significantly improve the capability to structurally design advanced VAWT systems.



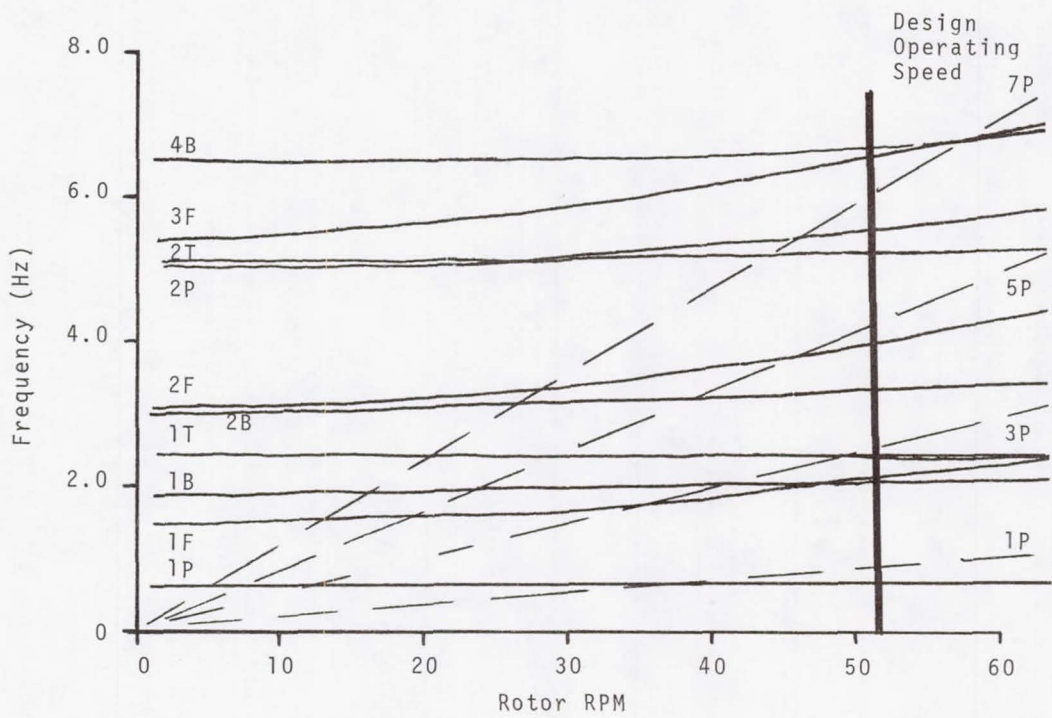


FIGURE 1: SAP IV Resonant Frequency Predictions for the DOE/Alcoa 17-m Rotor, Including Centrifugal Stiffening

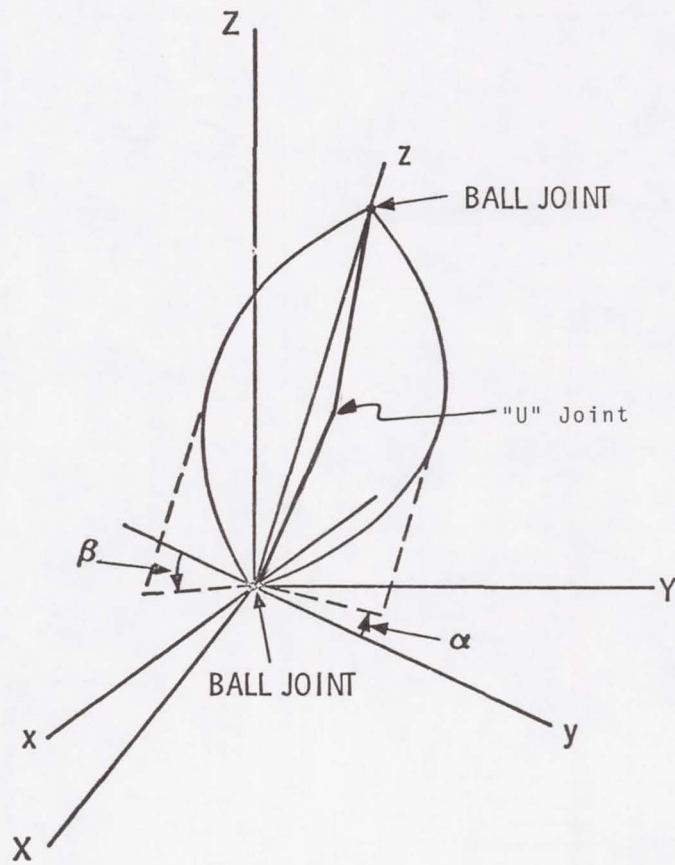


FIGURE 2: Blade and Tower Motions Included in the VAWTDYN Model

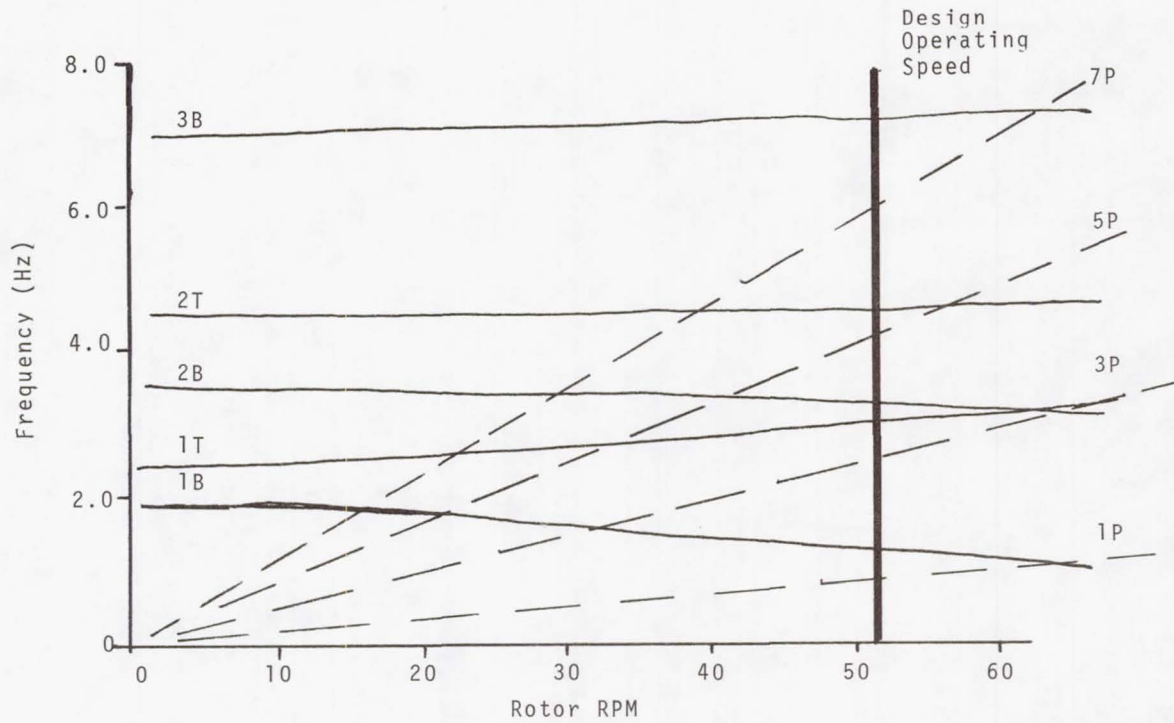


FIGURE 3: VAWTDYN Predictions of Resonant Frequencies for the DOE/Alcoa 17-m Rotor

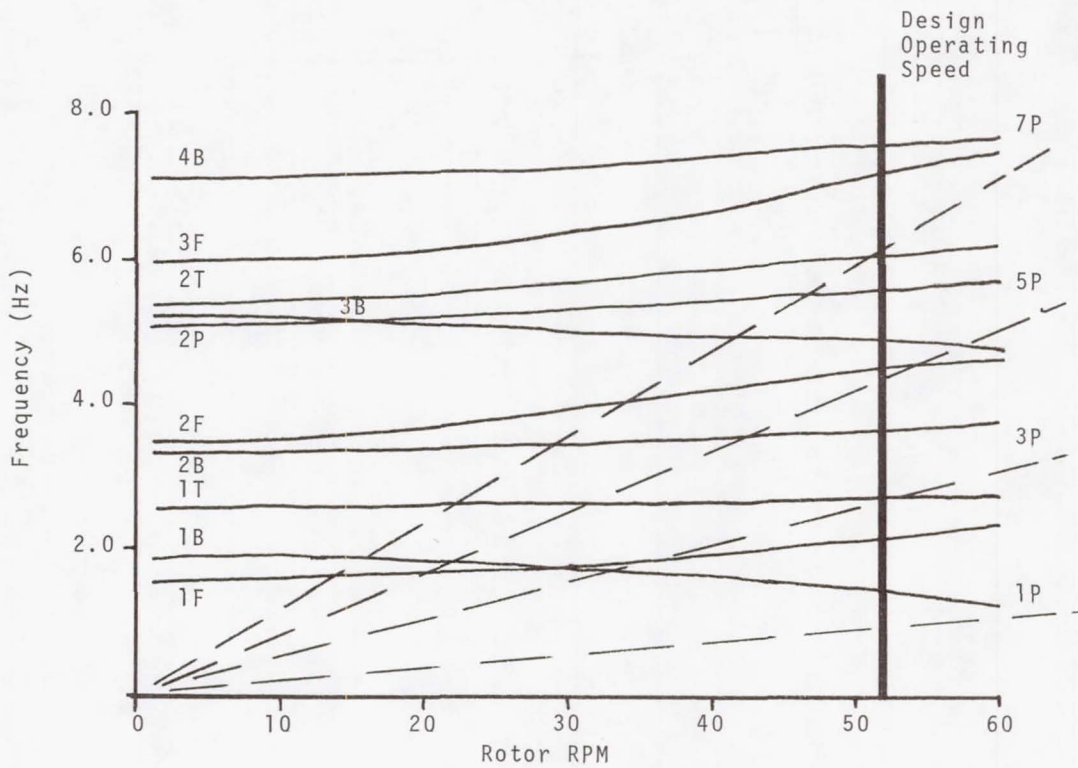


FIGURE 4: NASTRAN Resonant Frequency Predictions for the DOE/Alcoa 17-m Rotor, Including Rotating Coordinate System Effects



QUESTIONS AND ANSWERS

D.W. Lobitz

From: S.A. Shipley

Q: Is the Nastran modification applicable to HAWTS with N blades (including N = 2)?

A: *This modification is applicable to HAWT's of any number of blades under certain restrictions. These restrictions involve neglecting the inertia of the tower and modeling its stiffness by linear and rotational springs in the rotating system. These springs must be independent of the azimuthal position of that system. Of course, as with the VAWT's, the angular velocity of the rotating frame must be constant and motions within it, assumed small. For three or more bladed systems the multiblade coordinate transformation might yield an alternate NASTRAN approach. However, some method for affecting the equation manipulations would have to be developed. This approach would not be as restricted as the present one is.*

From: A. Wright

Q: How were centrifugal stiffening effects included in SAP-IV?

A: *Centrifugal stiffening effects are included in SAP-IV by internal modification of the stiffness matrix. These modifications are developed on the basis of a linear solution at 1 rpm. The alterations for other rpms is obtained by multiplication of the 1 rpm modification by the square of the desired rpm. Tests which involve a number of nonlinear iterations have shown this simple method to be quite accurate.*

From: G. Beaulieu

Q: How do you simulate the effects of the guy wires on the rotor into your NASTRAN model? Constant stiffness?

A: *The guy wires are modeled by a linear spring of constant stiffness which rotates with the turbine and tends to restore the turbine to its vertical position. As most of the cable systems we have dealt with are highly tensional, the sag contribution to the stiffness is minimal and the constant stiffness assumption is reasonably accurate. If the inertia of the guy wires was considered important and/or the spring constant was a function of the azimuthal position of the turbine, this NASTRAN solution procedure would be invalidated due to the introduction of non-constant coefficients. Fortunately, this has not been the case in the VAWT's encountered so far.*

From: Bill Wentz

Q: Please describe the "butterfly" mode.

A: *In a two-bladed VAWT system "butterfly" modes are characterized by lead-lag blade motions wherein the blades move simultaneously and equally in the same linear direction. The label "butterfly" is appropriate because when the turbine is viewed from the top, the motion is reminiscent of flapping butterfly wings. Three or more bladed systems also possess "butterfly" modes but of considerably greater complexity.*

From: W.C. Walton

Comment: Congratulations on the introduction of rotating effects in NASTRAN. We have also successfully used NASTRAN this way, though in a less complete and systematic way.