

SWECS TOWER DYNAMICS ANALYSIS METHODS AND RESULTS

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ABSTRACT

At the Rocky Flats Wind Systems Center, several different tower dynamics analysis methods and computer codes are used to determine the natural frequencies and mode shapes of both guyed and freestanding wind turbine towers. In this paper these analysis methods are described and the results for two types of towers: a guyed tower and a freestanding tower are shown. The advantages and disadvantages in the use of and the accuracy of each method are also described.

INTRODUCTION

The accurate prediction of tower vibration frequencies and mode shapes is important in avoiding unwanted vibration problems. At present, there is a variety of structural dynamic analyses covering a range of complexity and application. In this paper, some of the existing analyses and corresponding computer codes will be examined in order to determine those which can be of use to the SWECS industry.

A tower dynamics supporting research and technology project has been conducted at the Rocky Flats Small Wind Systems Center. The objective of this project has been to determine those analyses which are simple to use but give adequate results compared to test results. This paper will present some of the simpler tower dynamic analyses, their correct use and accuracy. The simpler analysis methods will be presented in order of increasing complexity and accuracy. The theory and use of the methods will first be described, as well as the accuracy of results for towers with various mass and stiffness distributions. The results from these analyses will then be compared to the test results for two types of towers at Rocky Flats: 1) a guyed tower with uniform mass and stiffness, and 2) a freestanding tower with uniform mass but tapered stiffness. It will be shown that accurate determination of the bending frequencies for towers with tapered stiffness is more difficult, using the simpler analyses. This will require the designer to use a more complex analysis, such as SAPIV. It should be emphasized that this paper deals specifically in determination of tower bending frequencies. The case of torsional frequencies or coupled bending torsional

frequencies have not been analyzed, using the methods of this paper. The results for these frequencies may be the subject of a later paper.

THEORY AND USE OF THE METHODS

The Rayleigh Quotient

The Rayleigh Quotient forms the basis for some approximate methods: the Rayleigh Method and the Rayleigh Ritz Method, both to be described here. The system to be analyzed is shown in Figure 1. The freestanding tower has variable mass and stiffness distributions $\rho A(x)$ and $EI(x)$, respectively. A concentrated mass, M , is attached at the tower top at a height, L , above the ground.

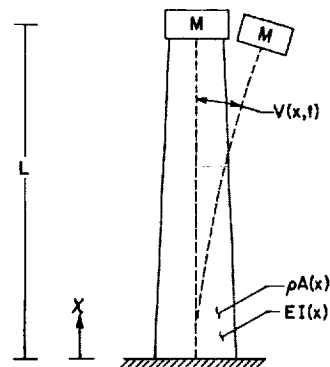


Figure 1: System to be Analyzed.

By assuming the deflection $v(x,t)$ in the form:

$$v(x,t) = \phi(x) \sin pt$$

and equating the tower potential energy at maximum displacement to the kinetic energy at minimum displacement, the Rayleigh Quotient can be expressed:

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$$p^2(\phi) = \frac{\int_0^L EI(x) [\phi''(x)]^2 dx}{\int_0^L \rho A(x) [\phi(x)]^2 dx + M[\phi(L)]^2} \quad (1)$$

When ϕ is an exact vibration mode of the tower, p has the value of the corresponding exact natural frequency; also, p^2 is stationary with respect to variations in ϕ at each of these points (Ref. 3). This expression can be expected to give a good approximation to the frequency if a good approximation to the mode shape is used to evaluate Equation 1. A simple analysis can be formed for the special case of a tower with uniform or linearly tapered mass and stiffness distributions, in order to find the first bending frequency.

A Simple Analysis

For a freestanding tower with linear mass and stiffness distributions of the form:

$$EI(x) = EI_0 (1 - \beta x/L)$$

$$\rho A(x) = \rho A_0 (1 - \gamma x/L)^*$$

the approximate shape:

$$\phi(x) = \left(\frac{x}{L}\right)^4 - 4\left(\frac{x}{L}\right)^3 + 6\left(\frac{x}{L}\right)^2 **$$

can be used to evaluate Equation 1 with the result:

$$p^2 = \frac{EI_0}{\rho A_0 L^4} \cdot \frac{28.800 - 4.800\beta}{(2.311 - 1.854\gamma) + 9.00\mu} \quad (2)$$

where μ is the ratio $M/\rho A_0 L$.

In those cases where exact frequencies for a cantilever beam with a tip mass have been calculated, the results from Equation 2 can be compared directly to give an indication of the accuracy. Table 1 shows the dimensionless frequency ratios:

$$W = \sqrt{\frac{\rho A_0 L^4}{EI_0}} p$$

obtained from Equation 1 compared to exact results calculated, using a power series method (Ref. 8).

* EI_0 and ρA_0 are the stiffness and mass distributions of the tower at the base. The case $\beta = 0$, $\gamma = 0$ is that of a uniform tower.

**The assumed mode shape should satisfy some or all of the cantilever tower boundary conditions. Here: $\phi(0) = 0$ and $\phi'(0) = 0$ are satisfied. The assumed mode shape should at least satisfy the base boundary conditions.

From Table 1 it can be seen that the accuracy of results from Equation 1 diminishes for towers with higher stiffness taper rates and larger tip masses because the assumed first mode shape $\phi(x)$, used in the above analysis, approximates the exact mode shape less accurately.

TABLE I: COMPARISON OF RESULTS OBTAINED FROM SIMPLE METHOD (W_s) TO EXACT RESULTS (W_e) FOR THE FIRST BENDING FREQUENCY RATIOS, FOR VARIOUS TAPERS AND TIP MASS RATIOS.

Uniform Stiffness, Uniform Mass: $\beta = 0.$, $\gamma = 0$

μ	W_e	W_s	% Error
1.0	1.56	1.60	2.6
2.0	1.16	1.19	2.6

Uniform Stiffness, Tapered Mass: $\beta = 0.$, $\gamma = .9$

μ	W_e	W_s	% Error
1.0	1.68	1.73	3.0
2.0	1.21	1.24	2.5

Tapered Stiffness, Uniform Mass: $\beta = .9$, $\gamma = 0$

μ	W_e	W_s	% Error
1.0	1.33	1.47	10.5
2.0	0.99	1.10	11.1

Limitations of the Method

This simple method is applicable to freestanding towers for mass and stiffness distributions varying approximately linearly. Only the first bending frequency can be found from Equation 2, although the second bending frequency might be estimated if the analyzer could evaluate Equation 1 with a good second mode shape approximation. Equation 2 is useful for obtaining a rough estimate of the first bending frequency of freestanding towers, without the use of a computer.

A Rayleigh Computer Code

A program developed at MIT, Program Rayleigh (Ref. 4), uses the Rayleigh procedure to evaluate the first bending mode for freestanding towers with nonlinear mass and stiffness distributions. The tower can be divided into N equal segments and constant values of mass and stiffness are input along each segment. Simpson's integration is then used to evaluate the integrals in the Rayleigh Quotient.

Both the Simple Method and the Rayleigh Program have the disadvantage that the user must input an assumed mode shape. The accuracy of the corresponding results depends on this one shape.

In a method to be described next, a linear combination of assumed shape functions is used to form the function $\phi(x)$ for use in the Rayleigh Quotient. This method is particularly useful for guyed towers, because the mode shapes are harder to approximate with a simple function, such as that used in the simple method above.

THE RAYLEIGH RITZ PROCEDURE

The Rayleigh Ritz method involves using a set of assumed shape functions $\psi_i(x)$ and combining them to form the mode shape:

$$\phi(x) = A_1 \psi_1(x) + A_2 \psi_2(x) + \dots + A_N \psi_N(x) \quad (3)$$

The A_i 's are constants to be determined and the ψ_i 's are a set of linearly independent functions, each satisfying some or all of the boundary conditions of the tower.*

For a guyed tower, Rayleigh's Quotient can be expressed:

$$p^2(\phi) = \frac{\int_0^L EI(x) [\phi''(x)]^2 dx + Kc[\phi(a)]^2}{\int_0^L \rho A(x) [\phi(x)]^2 dx + M[\phi(L)]^2} \quad (4)$$

where Kc is the guy wire stiffness coefficient and a is the guy wire attachment height, shown in Figure 2. For three guy wires spaced 120° apart, Kc can be shown to be (Ref. 6):

$$Kc = \frac{3}{2} \left(\frac{A'E'}{l} \right) \cos^2 \theta. \quad (5)$$

While for four guy wires spaced 90° apart, the factor $3/2$ in Equation 5 is replaced by a 2. These values of Kc are valid when the guy wires have been tensioned sufficiently so that there is no coupling between the guy wires and the tower (Ref. 1).

Substitution of the mode shape form in Equation 3 into Equation 4 gives:

$$p^2(\phi) = \frac{EI}{\rho A_0 L^4} \frac{\sum_j \sum_i A_i A_j d_{ij}}{\sum_j \sum_i A_i A_j b_{ij}} \quad (6)$$

Where:

$$d_{ij} = \int_0^1 f(\xi) \psi_i''(\xi) \psi_j''(\xi) d\xi + Kc \psi_i(\xi_a) \psi_j(\xi_a) \quad (7)$$

$$b_{ij} = \int_0^1 h(\xi) \psi_i(\xi) \psi_j(\xi) d\xi + \mu \psi_i(1) \psi_j(1)$$

$i, j = 1, \dots, N$

and ξ , $f(\xi)$, $h(\xi)$, ξ_a , Kc and μ are the dimensionless distance, stiffness, mass distribution, guy level attachment, guy stiffness coefficient, and tip mass ratio, respectively, as defined in the nomenclature.

*The ψ_i functions should satisfy at least the geometric boundary conditions (Ref. 3).

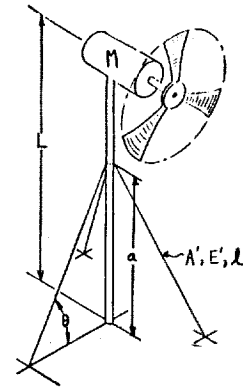


Figure 2: Guyed Tower to be Analyzed.

It is known that the Rayleigh Quotient is stationary with respect to variations in $\phi(x)$, when $\phi(x)$ is an exact mode shape for the tower. This can be shown to require that:

$$\text{Det} [d_{ij} - W^2 b_{ij}] = 0 \quad (\text{Ref. 3}).$$

The problem is thus reduced to finding the N values of W^2 which make the determinant of this $N \times N$ matrix equal to zero. The N values of W give estimates for the first N frequencies of the tower.

Simple Analysis

A simple two-mode analysis for freestanding towers can be performed by using two approximating functions of the form:

$$\begin{aligned} \psi_1(\xi) &= \xi^4 - 4\xi^3 + 6\xi^2 \\ \psi_2(\xi) &= 3\xi^5 - 10\xi^4 + 10\xi^3 \end{aligned} \quad (8)$$

The integrals d_{ij} and b_{ij} then are:

$$\begin{aligned} d_{11} &= 28.800 - 4.800\beta \\ d_{12} &= 24.000 - 6.857\beta \\ d_{22} &= 34.286 - 12.857\beta \\ b_{11} &= 2.3111 - 1.8540\gamma + 9.00\mu \\ b_{12} &= 2.0698 - 1.6997\gamma + 9.00\mu \\ b_{22} &= 1.8817 - 1.5732\gamma + 9.00\mu \end{aligned}$$

when $\text{det} [d_{ij} - W^2 b_{ij}]$ is evaluated, the polynomial $g(W^2)^2 - qW^2 + \lambda = 0$ must be solved where:

$$\begin{aligned} g &= b_{11}b_{22} - b_{12}^2 \\ q &= d_{11}b_{22} + b_{11}d_{22} - 2d_{12}b_{12} \\ \lambda &= d_{11}d_{22} - d_{12}^2 \end{aligned} \quad (9)$$

Table II shows the results for various values of tip mass ratio and taper rates. This shows that the simple two-mode method becomes inaccurate for the second bending frequency ratios of towers with large stiffness tapers. This method gives more accurate results for the first bending frequencies compared to the results of the previous sections.

TABLE II: EXACT (w_{1e}) AND APPROXIMATE (w_{1a}) FIRST AND SECOND BENDING FREQUENCY RATIOS FOR A FREESTANDING TOWER. TWO-MODE RAYLEIGH RITZ PROCEDURE, FOR VARIOUS TAPERS AND TIP MASS RATIOS.

Uniform Stiffness, Uniform Mass: $\beta = 0, \gamma = 0$

μ	w_{1e}	w_{1a}	% Error	w_{2e}	w_{2a}	% Error
1.0	1.56	1.56	0.0	16.25	17.61	8.4
2.0	1.16	1.16	0.0	15.86	17.29	9.0

Uniform Stiffness, Tapered Mass: $\beta = 0, \gamma = .9$

μ	w_{1e}	w_{1a}	% Error	w_{2e}	w_{2a}	% Error
1.0	1.68	1.70	0.6	22.37	23.56	5.3
2.0	1.21	1.21	0.0	22.17	23.00	3.7

Tapered Stiffness, Uniform Mass: $\beta = .9, \gamma = 0$

μ	w_{1e}	w_{1a}	% Error	w_{2e}	w_{2a}	% Error
1.0	1.33	1.37	3.0	11.81	15.36	30.1
2.0	0.98	1.02	4.1	11.53	15.06	31.0

This simple hand analysis can also be used for guyed towers, using these two approximating functions and including the guy wire stiffness effects in the terms d_{ij} . As an example, a single guyed tower with the dimensionless stiffness $K_c = 200$ and guy level $\xi_a = .8$ was examined for the first two bending frequencies. It was found that $w_1 \approx 20.16$ and $w_2 \approx 23.43$. These same results were calculated with a computer program, to be described in the next section, using a four-mode Rayleigh Ritz procedure with the results: $w_1 \approx 19.82$ and $w_2 \approx 22.45$. These results show that the simple two-mode Rayleigh Ritz procedure gives fair results for a guyed tower also.

As will now be shown, the use of more approximating ψ_i functions will improve the results of both the first and second bending mode frequencies and will also give results for the higher mode frequencies.

The Rayleigh Ritz Computer Program

The Rayleigh Ritz method can be used to determine any number of modal frequencies; however, the computational complexity increases greatly after the first two or three modes. For this reason, a computer code has been developed at Rocky Flats using four approximating functions of the form:

$$\psi_1(\xi) = \xi^4 - 4\xi^3 + 6\xi^2$$

$$\psi_2(\xi) = 3\xi^5 - 10\xi^4 + 10\xi^3$$

$$\psi_3(\xi) = 2\xi^6 - 6\xi^5 + 5\xi^4$$

$$\psi_4(\xi) = \xi^{10} - 2.5\xi^9 + 1.607\xi^8.$$

The program can be used for linear or nonlinear mass and stiffness distributions by dividing the tower into N sections (not necessarily equal) and inputting constant values of stiffness and mass along each section. Numerical integration techniques are then used to evaluate the integrals in the Rayleigh Quotient. Table III shows the improvement for the first and second bending frequency results, compared to the simple two-mode method of Table II.

TABLE III: EXACT (w_{1e}) AND APPROXIMATE (w_{1a}) FIRST AND SECOND BENDING FREQUENCY RATIOS FOR A FREESTANDING TOWER. FOUR-MODE RAYLEIGH RITZ PROCEDURE FOR VARIOUS TAPERS AND TIP MASS RATIOS.

Uniform Stiffness, Uniform Mass: $\beta = 0, \gamma = 0$

μ	w_{1e}	w_{1a}	% Error	w_{2e}	w_{2a}	% Error
1.0	1.56	1.56	0.0	16.25	16.26	0.1
2.0	1.16	1.16	0.0	15.86	15.88	0.1

Uniform Stiffness, Tapered Mass: $\beta = 0, \gamma = .9$

μ	w_{1e}	w_{1a}	% Error	w_{2e}	w_{2a}	% Error
1.0	1.68	1.70	0.6	22.37	22.49	0.5
2.0	1.21	1.21	0.0	22.17	22.30	0.6

Tapered Stiffness, Uniform Mass: $\beta = .9, \gamma = 0$

μ	w_{1e}	w_{1a}	% Error	w_{2e}	w_{2a}	% Error
1.0	1.33	1.34	1.0	11.81	12.75	8.0
2.0	0.98	.99	1.0	11.53	12.45	8.0

The program can also be run for guyed towers, when the guy wire tension is large enough so that no coupling between the guy wire first mode and tower first mode occurs (Ref. 1).

Limitations of the Rayleigh Ritz Method

Both the simple hand method and the four-mode Rayleigh Ritz program can be used to calculate first and second mode bending frequencies for freestanding and guyed towers. The simple method gives poor results for the second bending frequencies of towers with high stiffness tapers. The more complex four-mode method can be used for towers having more complex nonlinear mass and stiffness distributions, and gives better results than the two-mode method, for highly tapered towers. The disadvantage of this method is that a computer must be used.

A code for a TI-59 programmable calculator has been developed to handle towers with nonlinear mass and stiffness distributions.

Program TUSF (Turbine System Frequencies)

The two-mode Rayleigh Ritz procedure covered above can only be used for towers which have approximately linear mass and stiffness distributions. For towers with nonlinear distributions, this method would be hard to use, because the integrals in the Rayleigh Ritz Method would be too hard to evaluate.

To circumvent this difficulty, a method similar to the Rayleigh Ritz procedure has been developed for towers with nonlinear mass and stiffness (Ref. 6). The tower can be divided into N sections and constant values of mass and stiffness are input along each segment. Numerical integration techniques are then used to find the natural frequencies (Ref. 7).

In the techniques described previously, the nacelle and rotor were modeled as a single lumped mass at the tower top. This program includes the effects of rotor moments of inertia and nacelle-rotor C.G. location on the system frequencies. In addition, the effects of rotor spin rate on the natural frequencies are taken into account.

The program treats the tower as a flexible member and the nacelle and rotor as rigid bodies. It will be shown in the section on comparisons of analytical results to test results that the accuracy of results from this program are similar to the accuracy of results from the Rayleigh Ritz procedure, i.e., the results for the second bending frequencies are less accurate than the first bending frequencies.

In the approximate methods discussed above, an infinite degree of freedom structure is modeled as a beam having finite degrees of freedom. It can be shown that this causes the frequency estimates from these methods to be higher than the exact values (Ref. 6). A method using more degrees of freedom (such as SAPIV) gives more accurate frequency estimates.

In the next section, the test procedures and results for two types of towers will be presented. In the section on comparisons of analytical results to test results these simple methods, for these towers, will be compared. The results from more accurate structural codes (such as SAPIV) will also be shown. A recommendation as to which tower types can be analyzed, using the simple method, will also be given.

TEST PROCEDURES AND RESULTS

Objectives

This section deals specifically with the test methods and results for a Rohn 25G single guyed tower and a Rohn SSV freestanding tower. Impact tests were performed on each tower to determine the predominant modes of vibration and to compare the test results with various analytical methods.

Test Description and Theory

A technique currently being used at the Rocky Flats Test Center, for dynamic testing, is commonly known as impact testing. The structure can be excited with an impulse. This can be accomplished with the use of a hammer with a load cell attached, thereby exciting the structure with a known input, as shown in Figure 3.

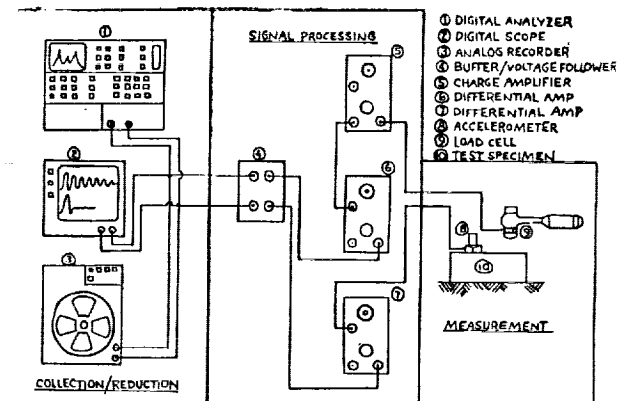


Figure 3.

With the load cell, the input force can be accurately measured; the response of the structure can be measured with the use of an accelerometer. Provided the input and response signals are Fourier transformable (Ref. 2), the frequency response function can be computed.

In practice, better results are obtained by computing frequency response functions as a ratio of the cross spectrum between the input and output, to the power spectrum of the input (Ref. 2). This is useful in eliminating the effects of noise on the input and output signal measurements. If the input and output noise is noncoherent, the effects on the cross spectrum, involving the noise, will yield zero.

Test Specimens and Test Results

A Rohn 25G Single Guyed Tower

Figure 4 shows a Rohn 25G 40-ft guyed tower. The tower has constant mass and stiffness distributions as given in Table IV. The tower stiffness distribution was determined by first finding the moment of inertia of a tower cross section about the centroid, as shown in Figure 5. Because the distance between the tower legs remains constant, for various heights, the moment of inertia and thus the stiffness distribution remains constant. The mass distribution also is constant, since the weight and length of each tower section is the same. The effects of the cross braces have been neglected in the stiffness determination, but have been included in the mass distribution.

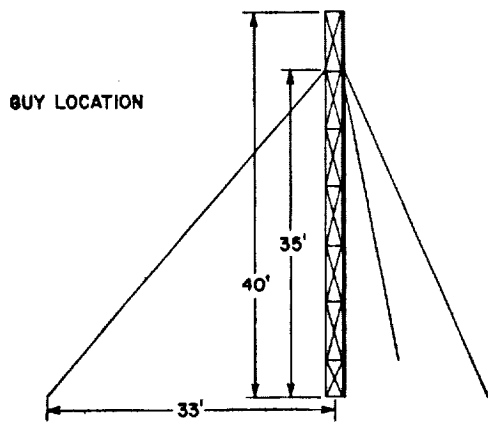


Figure 4.

TABLE IV: ROHN 25G PARAMETERS

Tower Height (L)	40.0 ft
Stiffness Distribution (EI)	$3.14 \times 10^6 \text{ lb-ft}^2$
Mass Distribution (ρA)	$0.124 \text{ lb-s}^2/\text{ft}^2$
Tower Top Mass (M)	$10.87 \text{ lb-s}^2/\text{ft}^2$
Guy Wire Attachment Level (a)	35.0 ft
Guy Wire Cross Sectional Area (A')	0.00015 ft^2
Guy Wire Elastic Modulus (E')	$2.88 \times 10^9 \text{ lb/ft}^2$
Guy Wire Length (ℓ)	48.45 ft
Guy Wire Stiffness Coefficient (Kc)	6365.76 lb/ft

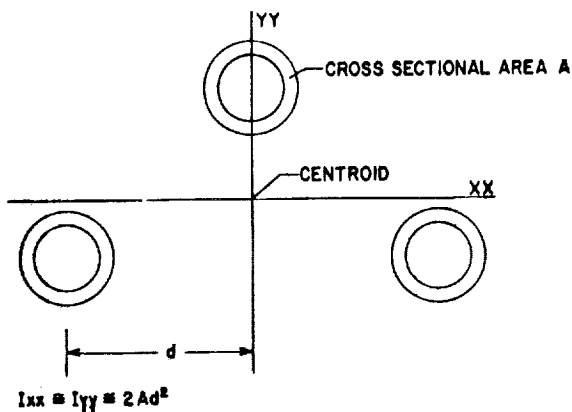


Figure 5.

The Rohn 25G was tested without a machine on top, but with the addition of 380 lb on the tower top. Table V shows the test results, using the impact testing methods. In this case, the three guy wires were tensioned sufficiently so that each had nearly the same fundamental frequency. As can be seen from Table V, the guy fundamental frequency was well separated from the tower first bending frequency, so that resonance between the guy wires and tower was not a problem (Ref. 1).

As will be shown in the next section, the simple methods can be used to get good estimates for the first bending frequencies for this tower.

TABLE V: ROHN 25G TEST RESULTS

First Mode Bending	2.6 Hz
First Mode Torsional	5.7 Hz
Second Mode Bending	7.8 Hz
Guy Wire Fundamental	6.9 Hz

A Rohn SSV Freestanding Tower

Figure 6 shows a 60-ft Rohn SSV freestanding tower. Unlike the Rohn 25G, the distance between tower legs decreases with increasing tower height. The stiffness distribution thus decreases from base to height. Figure 6 shows a plot of the mass and stiffness distribution for this tower. The mass distribution is nearly uniform, but the stiffness distribution tapers to 0.023 of the value at the tower base.

The Rohn SSV was tested without a machine on top but with a top plate of approximately 30 lb. As will be shown in the next section, accurate estimation of the first and second bending frequencies for this tower is difficult, using

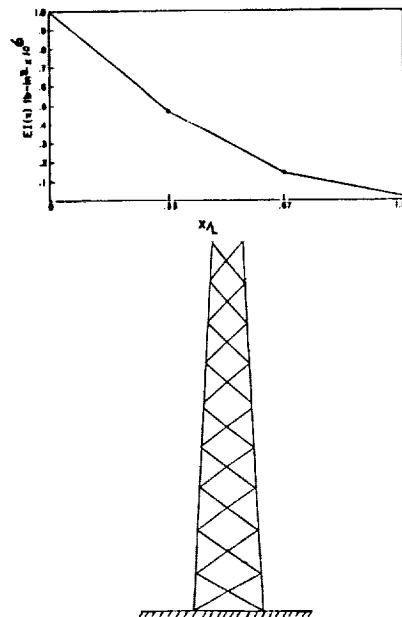


Figure 6.

the simple methods, because of the large stiffness taper. Also, the effects of the cross-members were neglected in the moment of inertia calculations, which poses further difficulties. Table VI shows the test results for the first two bending frequencies.

TABLE VI: ROHN SSV TEST RESULTS

First Mode Bending	3.3 Hz
Second Mode Bending	12.7 Hz

COMPARISONS OF ANALYTICAL RESULTS TO TEST RESULTS

In this section the results from the simple analysis methods and results from complex methods such as SAPIV (Ref. 5) will be compared to test results for the two towers presented in the previous section. A recommendation of the use and accuracy of the simple methods will be made.

Rohn 25G Results Summary

Table VII shows the results for the Rohn 25G. The simple two-mode Rayleigh Ritz method, a hand method, gives fair results for the first frequency (p_1), but poor results for the second frequency (p_2). The utility of this method is that it can be used to obtain a rough estimate, by hand calculations, for the first bending frequency of guyed or freestanding towers. This method gives less accurate results for towers with large stiffness tapers, however.

TABLE VII: ROHN 25G RESULTS SUMMARY

Method	p_1	Error	p_2	Error
Test	2.6	0.0%	7.8	0%
R-R-2 Mode	2.8	7.7%	9.0	16%
Program TUSF	2.6	0.0%	11.2	44%
R-R-4 Mode	2.6	0.0%	8.6	10%
MITGUY	2.6	0.0%	8.6	10%

Program TUSF, a hand calculator code, gives much more accurate results for the first mode. This code can also be used to determine the first bending frequency of towers with nonlinear mass and stiffness distributions, without use of a large computer.

The Rayleigh Ritz four-mode procedure, a small computer code developed at Rocky Flats, gives excellent results for the first mode and fair results for the second mode, while Program MITGUY (Ref. 4), a program utilizing matrix iteration methods, gives about the same results (to one decimal place).

The 10 percent inaccuracy in the second mode results is thought to be caused because of neglect of the cross-members in determination of the tower bending stiffness. This difficulty will be emphasized in the comparison of results for the Rohn SSV, to be shown next, in which use of the complex code SAPIV, which models the tower cross-members, is necessary.

Rohn SSV Results Summary

Table VIII shows the results for the Rohn SSV tower. The analysis of the Rohn SSV with the simple hand methods of the Rayleigh Method and Two-Mode Rayleigh Ritz Method is difficult, because the stiffness and mass distributions are nonlinear. For these analyses, the mass distribution was assumed constant, with a value equal to the base value. The stiffness distribution was approximated as linear, with a value of the taper rate β equal to 0.98. The resulting frequency estimates are too high, because the actual stiffness of the tower is less than what is given by this straight line approximation.

TABLE VIII: ROHN SSV RESULTS SUMMARY

Method	p_1	Error	p_2	Error
Test	3.3	0%	12.7	0%
Rayleigh	3.8	19%	--	--
R-R-2 Mode	3.6	12%	22.3	76%
Program TUSF	3.5	9%	16.3	28%
R-R-4 Mode	3.4	6%	15.5	22%
MITGUY	3.4	6%	14.8	17%
SAPIV	3.1	3%	13.0	2.4%

For towers with nonlinear mass and stiffness distributions, the program TUSF and the four-mode Rayleigh Ritz procedure can be used to find more accurate values of the first bending frequencies. The tower can be divided into N sections and constant values of mass and stiffness input along each segment. It was found that about 20 segments were needed in order to model the large stiffness taper correctly.

From Table VIII it can be seen that the second bending frequency results from: 1) the simple methods, and 2) the program MITGUY are in error by more than 17%. For this reason, the complex code SAPIV (Ref. 5) was used to model this tower, taking into account the effects of each cross-member. The improvement in the second bending frequency is very evident.

In the simple methods, as well as MITGUY, this tower was modeled as a beam, neglecting the effects of the cross-members in the bending stiffness. The cross-section moment of inertia was calculated, by taking into account the three legs only, as shown in Figure 5. The actual stiffness distribution of this tower is more complicated than this approximation because of the cross-members.

In the simple methods section it was shown that they give less accurate results for a beam with large stiffness tapers. For this truss-type of tower, this error is increased because the actual tower stiffness is more complicated than that of a beam in which the cross-members have been neglected.

CONCLUSION AND RECOMMENDATIONS

The simple methods can be used to determine good estimates of the first bending frequencies and rough estimates of the second frequencies of towers with uniform mass and stiffness distributions. Guyed towers can also be analyzed, using these methods, if the guy wire fundamental frequency is well separated from the tower first bending frequency so that guy wire-tower interaction will not occur.

For towers with high stiffness tapers, or stiffness distributions which are hard to determine accurately, more complex codes such as SAPIV may be needed, especially for accurate determination of the second and higher bending frequency.

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NOMENCLATURE

a	:	Guy wire attachment level.
A'	:	Guy cable cross-sectional area.
E'	:	Guy cable elastic modulus.
EI(x)	:	Tower stiffness at station x.
EI ₀	:	Tower stiffness at the base.

f(ξ)	:	EI(Lξ)/EI ₀
h(ξ)	:	ρA(Lξ)/ρA ₀
K _c	:	Guy wire stiffness coefficient.
K̄ _c	:	Dimensionless form of K _c : $\bar{K}_c = \frac{K_c L^3}{EI_0}$
L	:	Tower height.
l	:	Guy cable length.
M	:	Lumped mass at tower top.
p	:	Bending frequency (rad/sec) or (Hz).
t	:	Time (sec.).
v(x,t)	:	Tower deflection.
W	:	Dimensionless frequency.
A _i	:	Coefficients in approximate mode shape.
ψ _i	:	Functions for approximating mode shapes.
β	:	Rate of decrease of linear stiffness.
γ	:	Rate of decrease of linear mass distribution.
ξ	:	X/L.
ξ _a	:	a/L.
θ	:	Angle guy cable makes with ground.
μ	:	Tip mass ratio: M/ρA ₀ L.
ρA(x)	:	Mass distribution of tower at section x.
ρA ₀	:	Mass distribution of tower at tower base.
φ(x)	:	Tower mode shape.

QUESTIONS AND ANSWERS

A.D. Wright

From: G. Beaulieu

- Q: 1) Don't you believe that direct solution of 4th order differential equation would give more accurate mode shape and frequencies?
- 2) Did you take into account the rotary inertia of the rather large top mass?

- A: 1) *The direct solution of the beam governing equation would probably give more accurate results. The main emphasis of this paper, however, has been the use of some simple methods, such as the Rayleigh-Ritz method as a hand calculator code, such as program TUSF.*
- 2) *For the two towers presented in the paper, a Rohr 55V and a Rohr 25G, the rotary inertia effects were neglected in the analysis results. These effects could be easily taken into account, in the simple methods, by adding the appropriate term to the kinetic energy.*

From: F.W. Perkins

- Q: How do you determine a priori the location of nodes for higher mode analysis using Rayleigh's method?

- A: *For towers with complex mass and stiffness distributions, the precise mode shape, a priori, is difficult if not impossible to determine. The nodal points are therefore unknown. In the Rayleigh or Rayleigh Ritz procedure we choose a shape which we hope approximates the true mode shape. If the resulting frequency estimate, using this assumed shape, is inaccurate compared to test results, a better mode shape approximation is needed.*

