# Life and Reliability Models
# for Helicopter Transmissions
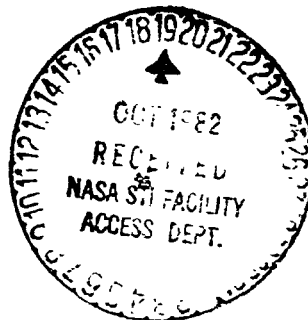
Michael Savage and Raymond James Knorr
*The University of Akron*
*Akron, Ohio*

and

John J. Coy
Propulsion Laboratory
AVRADCOM Research and Technology Laboratories
*Lewis Research Center*
*Cleveland, Ohio*

**NASA**

# LIFE AND RELIABILITY MODELS FOR HELICOPTER TRANSMISSIONS

Michael Savage
The University of Akron
Akron, Ohio

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Raymond James Knorr
The University of Akron
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John J. Coy
Propulsion Laboratory
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Lewis Research Center
Cleveland, Ohio

## Abstract

Life and reliability models are pre-
sented for planetary gear trains with a
fixed ring gear, input applied to the sun
gear, and output taken from the planet
arm. For this transmission the input and
output shafts are co-axial and the input
and output torques are assumed to be co-
axial with these shafts. Thrust and side
loading are neglected. The reliability
model is based on the Weibull distribu-
tions of the individual reliabilities of
the main transmission components. The
system model is also a Weibull distribu-
tion. The load versus life model for the
system is a power relationship as are the
models for the individual components.
The load-life exponent and basic dynamic
capacity are developed as functions of
the component capacities. The computer
models are used to compare three and four
planet, 150 kw (200 horsepower), 5:1
reduction transmissions with 1500 RPM
input speed to illustrate their use.

## Nomenclature

C   basic dynamic component capacity (N)
D   basic dynamic system capacity (N-m)
e   Weibull exponent
f   gear face width (mm)
F   force (N)
$\ell$   life in millions of component load
     cycles
L   life in millions of sun gear rotations
n   number of planets
N   number of gear teeth
p   load-life exponent
R   gear radius (mm)
S   probability of survival (reliability)

$T_i$   input torque (N-m)
V   stressed volume (mm³)
z   depth to maximum shear stress (mm)
$\zeta$   curvature sum at pitch point (1/mm)
$\theta$   angular rotation
$\tau$   maximum shear stress (pa)
$\phi$   pitch line pressure angle

## Subscripts

A   planet carrier or arm
a   first load
B   planet bearing
b   second load
c   third load
r   radial direction
P   planet gear
R   ring gear
S   sun gear
T   transmision
t   tangential direction
10   corresponding 90 percent probability of
     survival

## Introduction

Recently in the field of mechanical
design, the fixed-load, fixed-strength
factor of safety approach to design has
come under considerable doubt[1,2,3]*.
The value of this factor of safety comes
under serious question when one becomes
aware of the actual variations in service
loads and in device strengths which exist
in machinery today. Selecting the worst
load and the weakest unit and using a
factor of safety greater than unity leads
to heavy, overdesigned machinery. Using
nominal loads and strengths leads to the
possibility of a signigicant underdesign
for a large population of the in service

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* Numbers in superscript denote references
at the end of the paper.

devices due to the statistical variation in the service load conditions and actual device strengths.

A more realistic approach is offered by the methods of probabilistic mechanical design[2]. In probabilistic design, a proposed design is evaluated in terms of statistically varying load and strength characteristics which more nearly model the situation. A statistical or probabilistic approach requires knowledge of the nominal loads and strengths as well as the statistical variations in each. With this knowledge, the designer should be able to assess the reliability or probability of survival of the total mechanical system[2,3].

The utility of a probabilistic approach to design is apparent in the design of airborne power transmission systems. The requirements of low weight, high power densities, and high speeds must be balanced against requirements of reliability, maintainability, and long mean time between overhauls (MTBO's).

Currently, there is no system probabilistic design procedure for designing lightweight geared power transmissions such as those found in helicopter drive systems. It has been shown by Lundberg and Palmgren[4,5] and by Coy, Townsend and Zaretsky[6,7,8] that rolling element bearings and high strength steel gear teeth have finite lives under any level of applied stress. The statistical models for the lives and the capacities of these components follow the Weibull distribution[3,4,5,8,9,10]. The finite lives of these components are due to the nature of pitting fatigue to which both bearings and gears are subjected. Even in carefully designed gears and bearings where the lubrication is adequate and the service loads do not increase unexpectedly, pitting fatigue failures will eventually end the useful lives of both bearings and gears[4,5,8]. Thus, pitting fatigue is the mode of failure on which the reliability of each component is based.

Short cycle static overloading must also be considered in the design of any device to minimize startup and crash type failures. However, the static overload design consideration must be applied in addition to the reliability design, since these failures are not by pitting fatigue and thus are not considered in this system model.

In this study, the Weibull distribution is used to describe the failure spectrum of the units. This distribution of probability of failure versus life at a given load is commonly accepted for bearing life analyses[4,5,13] and is being applied for other components[6-10] due to its ability to describe failure distributions which are not normally distributed. It matches component failure test data more closely than the simpler bell curve normal distribution[9,10] due to its ability to model skewed distributions.

In addition it is assumed that the load life relationship is independent of this failure probability versus life at a given load relationship[4,5,9]. Since the load life relationship is an inverse power relationship, it can be used to obtain a weighted average load from the mission spectrum. It is this weighted average load which is called the nominal load or service load of the transmissi

In a prior paper[11], the derivations of the specific load and reliability equations for the system life and dynamic capacity of a planetary gear reduction with stepped planets are presented. In this paper, these relations are applied to the most common subset of this transmission - that with simple planet gears. The kinematic inversion of the planetary gear reduction studied is that of a single plane arrangement with a sun, several planets and a fixed ring gear. The sun is the input and its shaft is co-axial with the planet arm's shaft which is the output. Both input and output shafts are loaded with pure torques.

The reliability model is based on the reliabilies of the individual gears and bearings and is Weibull in nature. The transmission reliability is presented as a system life for 90 percent probability of survival of the entire assembly based on corresponding lives for the individual components. The transmission's basic dynamic capacity is defined as the input torque which may be applied for one million rotations of the input sun gear with a 90 percent probability of survival. The variation of life with load for a given reliability is modeled with a power law relation. When plotted on log-log coordinates the relation becomes a straight line. The relationship is treated as being uncoupled from the Weibull relationship of reliability to life at a given load.[4,7,9]. The load-life exponent and basic dynamic capacity are developed as functions of the component capacities.

Numerical studies of a 150 kw (200 horsepower) 5:1 reduction transmission operating at 1500 revolutions per minute input speed are presented and discussed. Both three and four planet transmissions are considered.

## Force and Motion Analysis

The gear train under consideration is shown in Fig. 1 in a three planet arrangement. It is assumed in this model that all the planets share the load equally.
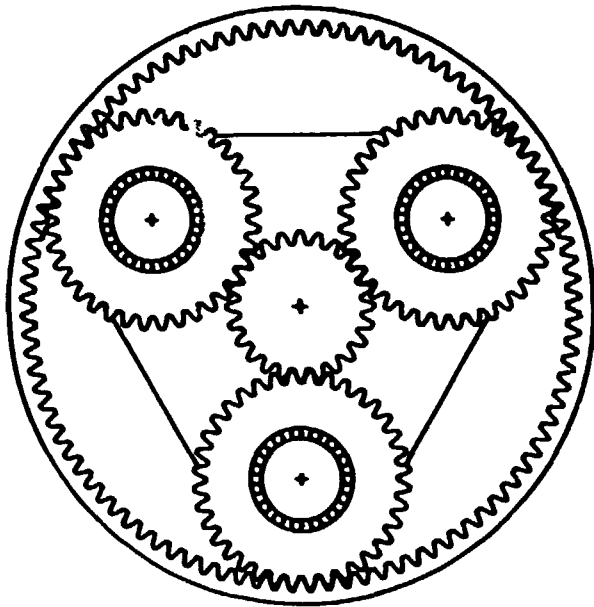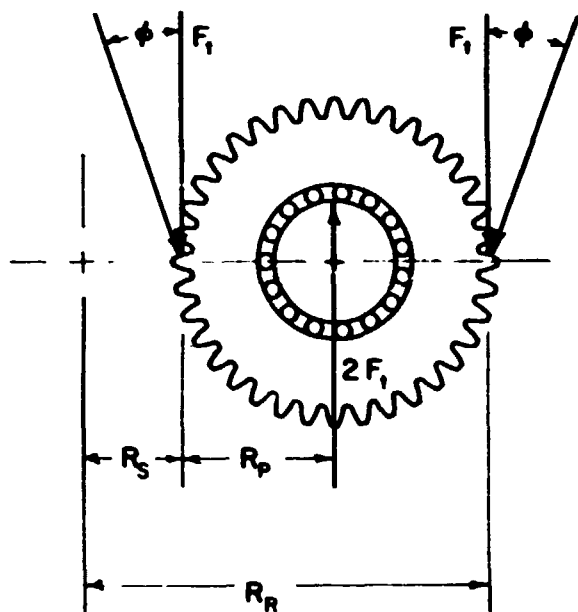


Fig. 1   Planetary gear reduction



Fig. 2   Planet gear forces

The major loads on the components are shown in Fig. 2 which is a force diagram for a single planet gear. In this figure are shown the gear radii as well as the forces acting on the planet gear. The force component acting tangent to the pitch circle, $F_t$, is the same for both contact with the sun gear and contact with the ring gear and is:

$$F_t = \frac{T_i}{nR_S} \tag{1}$$

where $T_i$ is the input torque and n is the number of planets in the transmission. The normal force on the teeth is this load divided by the cosine of the pressure angle. The total bearing load at each planet for this symmetric planet is twice the tangential tooth load:

$$F_B = 2F_t \tag{2}$$

A kinematic analysis of this planetary is also required to determine the relative number of load cycles that each component sees as the input sun rotates. This is needed for the fatigue life analysis. The kinematic analysis has been derived in reference 12. The results are presented in table 1, where the rotation of each component is given in terms of the rotation of the sun gear. All rotations are taken in the coordinate frame of the ring gear which is held fixed. Reading across in the table, for each of the components i, one obtains the terms for the following relative angular motion expression

$$\theta_i = \theta_{i/A} + \theta_A \tag{3}$$

where A represents the arm or spider.

The itemized angular rotations in this table can be used to relate the number of load cycles of the various components to the number of input sun rotations.

### Bearing Reliability and Capacity

The reliability and capacity of the planetary assembly is a function of the reliabilities and capacities of its components. These quantities have been well defined for the bearings[4,5,13]. The fatigue life model proposed in 1947 by Lundberg and Palmgren[4] is still the commonly accepted theory. The reliability of a single bearing can be expressed in terms of its probability of survival, S, for a life of $\ell$ rotations by the following relation

$$\log \frac{1}{S} \sim \tau^c \ell^{e_B} \frac{V}{z^h} \tag{4}$$

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where $\tau$ is the critical shearing stress beneath the surface, $z$ is the depth under the surface to the location of the critical stress, and $V$ is stressed volume. The exponents are determined from experimental life testing on groups of bearings run under identical conditions. The Weibull exponent $e_B$ is a measure of the scatter in the distribution of bearing lives.

Table 1. Angular motion of components in terms of angular motion of sun gear

| $i$ | $e_i$ | $=$ | $\theta_{i/A}$ | $+$ | $e_A$ |
|---|---|---|---|---|---|
| SUN | $e_S$ | | $\dfrac{e_S}{1 + \dfrac{R_S}{R_R}}$ | | $\dfrac{e_S}{1 + \dfrac{R_R}{R_S}}$ |
| PLANET | $\dfrac{(1 - \dfrac{R_R}{R_P})e_S}{1 + \dfrac{R_R}{R_S}}$ | | $\dfrac{-e_S}{\dfrac{R_P}{R_S} + \dfrac{R_P}{R_R}}$ | | $\dfrac{e_S}{1 + \dfrac{R_R}{R_S}}$ |
| RING | 0 | | $\dfrac{-e_S}{1 + \dfrac{R_R}{R_S}}$ | | $\dfrac{e_S}{1 + \dfrac{R_R}{R_S}}$ |
| SPIDER | $\dfrac{e_S}{1 + \dfrac{R_R}{R_S}}$ | | 0 | | $\dfrac{e_S}{1 + \dfrac{R_R}{R_S}}$ |

The above formula for probability of survival reflects the observed effects of stress, stress field, and stress cycles on reliability. Greater stress, $\tau$, decreases reliability. A more shallow stress field (smaller $z$) decreases reliability. This is true because it is expected that a microcrack beginning at a point of maximum stress under the surface requires some time to propagate to the surface for the more shallow stress field.

The stressed volume $V$ is also an important factor. Pitting initiation occurs near any small stress raising imperfection in the material. The larger the stressed volume, the greater the likelihood of failure.

Relation 4 is built around the basic two parameter Weibull distribution relationship for reliability, S, as a function of life, $\ell$. Since this distribution is exponential in form, different slopes, $e_B$,

will represent distributions with different skews. Thus the bearing life distribution which has a Weibull slope of 1.2 is skewed to the high end of the life region. As a result more bearings fail at lives below the average life than above it. An increase in the Weibull slope, $e_B$, would reduce this skewness and produce a symmetric distribution. Even higher values of $e_B$ (above 3.57) produce a distribution with an opposite skew and more failures above the average life. The proportionality of relation 4 can be removed by dividing relation 4 by a similar relation with a specific reliability, S= .9, or probability of survival of 90 percent and the corresponding life, $\ell_{B10}$ for the identical condition of stress and geometry. The two parameters of the distribution are now the Weibull slope, $e_B$, and the life for 90 percent survival, $\ell_{B10}$, and the Weibull distribution becomes:

$$\log \frac{1}{S} = \log (\frac{1}{.9})(\frac{\ell_B}{\ell_{B10}})^{e_B} \qquad (5)$$

This Weibull distribution relates the bearing life, $\ell_B$, to the probability of survival at that life in terms of the two parameters $e_B$ and $\ell_{B10}$.

The relationship between the bearing life and its load for a 90 percent probability of survival is

$$\ell_{B10} = (\frac{C_B}{F_B})^{p_B} \qquad (6)$$

where $F_B$ is the nominal load on the bearing. $p_B$ is the load-life exponent and $C_B$ is the basic dynamic capacity of a single bearing. The basic dynamic capacity is defined as the load which may be endured by 90 percent of the bearings for one million inner race revolutions under certain operating conditions.

To facilitate the combination of lives and capacities of all the transmission components into a single life and capacity for the transmission, the lives and the dynamic capacities of each of the components will be expressed in terms of input sun gear rotations and input sun gear torque.

From Table 1, the bearing inner rotations are given in terms of sun rotations

$$\theta_{B/A} = \frac{1}{\frac{R_P}{R_S} + \frac{R_P}{R_R}} e_S \qquad (7)$$

4

Using lower case $\ell$'s to designate
component lives in terms of component
cycles and upper case L's to designate
component lives in terms of input sun gear
rotations, Eq. (7) transforms Eq. (5) into

$$\log \frac{1}{S_B} = \log \frac{1}{.9}\left(\frac{R_S R_R L_B}{R_P(R_S + R_R)\ \ell_{B10}}\right)^{e_B} \quad (8)$$

For a 90 percent survival rate for a
planet bearing, $S_B = .9$ and $L_B = L_{B10}$ sun
gear rotations. Substitution into
Eq. (8) yields:

$$L_{B10} = \frac{R_P(R_S + R_R)}{R_S R_S}\ \ell_{B10} \quad (9)$$

as expected from Eq. (7).

To obtain the load-life relation for
the bearing in terms of transmission in-
put parameters, one can substitute the ex-
pressions for bearing load in terms of
input torque as given by Eqs. (1,2) into
Eq. (6) and substitute all of this into
Eq. (9):

$$L_{B10} = \frac{R_P\ (R_S + R_R)}{R_S R_R}\left(\frac{n R_S C_B}{2 T_i}\right)^{P_B} \quad (10)$$

The dynamic capacity of a planet bearing
is now the input torque on the sun
shaft which may be applied with 90 percent
of the planetary bearings surviving for
one million sun shaft revolutions. From
Eq. (10) the planet bearing system
dynamic capacity of $T_i = D_B$ is obtained
when $L_{B10} = 1.0$. The result for the
dynamic capacity is

$$D_B = \left(\frac{R_S R_S}{R_P(R_S + R_R)}\right)^{1/P_B}\left(\frac{n R_S C_B}{2}\right) \quad (11)$$

The relationship between bearing life in
millions of sun rotations and applied sun
shaft torque for which 90 percent of the
bearings will endure is given by

$$L_{B10} = \left(\frac{D_B}{T_i}\right)^{P_B} \quad (12)$$

The fundamental quantities that describe
the reliability and life distribution for
single bearings and bearings treated as
transmission components have now been de-
termined. Finally, the probability dis-
tribution for the reliability of a planet
bearing is written as

$$\log \frac{1}{S_B} = \log \frac{1}{.9}\left(\frac{L_B}{L_{B10}}\right)^{e_B} \quad (13)$$

Where $L_B$ is the number of million sun ro-
tations for which the bearing has the
probability of survival, $S_B$.

### Gear Reliability and Capacity

Surface fatigue life and dynamic
capacity for a spur gear have been the
subjects of recent research[6,7,8]. This
research has applied the previously men-
tioned Lundberg-Palmgren reliability model
to spur gears.

Tests have shown that the pitting
fatigue life of gears follows this reli-
ability relationship, but with a different
Weibull exponent, $e_G$, than that for
bearings.

$$\log \frac{1}{S} = \log \frac{1}{.9}\left(\frac{\ell}{\ell_{10}}\right)^{e_G} \quad (14)$$

where S is the probability of survival
of a single gear tooth and $\ell$ is the number
of stress cycles imposed on the gear tooth
surface.

The load-life relationship for a
single tooth for a 90 percent probability
of survival is:

$$\ell_{10} = \left(\frac{C_\ell}{F}\right)^{P_G} \quad (15)$$

where F is the transmitted tangential
tooth load and $C_t$ is the basic dynamic
capacity of the tooth for one million load
cycles which has been developed in Refer-
ence 8,

$$C_t = B_1\ \frac{f}{\Sigma \rho} \quad (16)$$

where f is the active tooth face width,
$\Sigma \rho$ is the curvature sum at the pitch point,
and the constant $B_1$, measured in units of
stress (Pa), is based on experimental
results from gear life testing.[8] For case
hardened AISI 9310 Vacuum Arc Remelt
Steel gears, $B_1 = 144$ MPa (21,000 psi).

As shown in Reference 11, this fund-
amental gear tooth reliability equation
for the dynamic capacity of a single
tooth on a gear can be combined with
Eq. (1) and the kinematic analysis of
Table 1 to produce relations for the
basic dynamic capacity of each gear in
the transmission. As for the bearings,

these basic dynamic capacities are the inputs torques on the sun gear shaft which may be applied for 90 percent survival of that component in one million sun shaft revolutions. This capacity is:

$$D_S = \left(\frac{1}{N_S}\right)\frac{1}{e_G p_G}\left[\frac{R_R + R_S}{n\,R_R}\right]^{\frac{1}{p_G}}\left(n\,R_S\,C_S\right) \quad (17)$$

for the sun gear, where $C_S$ is the result of applying Eq. (16) to the parameters of the sun gear and its mesh with a planet gear.

For a planet gear this dynamic capacity becomes:

$$D_P = \left(\frac{1}{N_P}\right)^{1/e_G p_G}\left[\frac{R_P(R_S + R_R)}{R_S R_R}\right]^{1/p_G} \quad x$$

$$\left(\frac{n\,R_S\,C_S\,C_R}{C_S + C_R}\right) \quad (18)$$

where $C_S$ is the dynamic capacity of a tooth on the planet gear due to its mesh with the sun gear and $C_R$ is the dynamic capacity of a tooth on the planet gear due to its mesh with the ring gear. Since the planet gear teeth mesh with the sun gear on one face and with the ring gear on the opposite face, these dynamic capacities are independent of each other.

Finally, for the ring gear, this dynamic capacity becomes:

$$D_R = \left(\frac{1}{N_R}\right)\frac{1}{e_G p_G}\left[\frac{R_S + R_R}{n\,R_S}\right]^{\frac{1}{p_G}}\left(n\,R_S C_R\right) \quad (19)$$

For all the gears, the component Weibull exponent, $e_G$, and load-life exponent, $p_G$, remain unchanged for the relationships in terms of sun gear torque and sun shaft rotations.

Thus the relationship for sun gear life and applied sun shaft torque for which 90 percent of the sun gears will survive is now given by:

$$L_{S10} = \left(\frac{D_S}{T_i}\right)^{p_G} \quad (20)$$

and the probability distribution for the reliability of the sun gear can be written as :

$$\log\frac{1}{S_S} = \log\frac{1}{.9}\left(\frac{L_S}{L_{S10}}\right)^{e_G} \quad (21)$$

where $L_S$ is the number of million sun rotations for which the sun gear has the probability of survival $S_S$.

Eqs. (20) and (21) are also valid for the planet and ring gears with the replacement of the subscript S by the subscript P and R respectively.

## System Reliability and Capacity

The product rule may now be used to express the probability of survival of the total system consisting of the planet bearings, the sun gear, the planet gears and the ring gear.

$$S_T = S_B^n\,S_S\,S_P^n\,S_R \quad (22)$$

The probability distribution for the survival of the total transmission can be obtained by substituting Eqs. (13) and (21) into the natural log of the reciprocal of Eq. (22).

$$\log\frac{1}{S_T} = \log\frac{1}{.9}\left\{n\left(\frac{L_T}{L_{B10}}\right)^{e_B} + \left(\frac{L_T}{L_{S10}}\right)^{e_G}\right.$$
$$\left. + n\left(\frac{L_T}{L_{P10}}\right)^{e_G} + \left(\frac{L_T}{L_{R10}}\right)^{e_G}\right\} \quad (23)$$

Since all the component lives are counted in the same units of sun rotations, this count is now identical for all the components and is thus labeled as $L_T$ in the expression for the probability of survival $S_T$ for the entire transmission.

Unfortunately, Eq. (23) is not a strict Weibull relationship between system life and system reliability. The equation would represent a true Weibull distribution only if $e_B = e_G$ which is not the case in general. The relationship of Eq. (23) can be plotted on Weibull coordinates as shown in Fig. 3.

This data is plotted for the case of a three planet transmission as drawn in Fig. 1. It is for an output speed of 300 RPM at 150 kw (200 horsepower). The gears are of case hardened AISI 9310 Vacuum Arc Remelt Steel. They have a width of 51 mm (2.0 inches) and a module of 4.23 mm (a diametral pitch of 6 inches$^{-1}$). The pressure angle is 20 degrees. The sun gear has 24 teeth and a 102 mm (4 inch) pitch diameter while the planet gears have 36 teeth each and pitch diameters of 153 mm ( 6 inches ) each. The ring has 96 teeth and a pitch diameter of 406 mm (16 inches). The Weibull exponent for the gears is 2.5 while the load-life factor for them is 4.3[8]. The planet bearings are
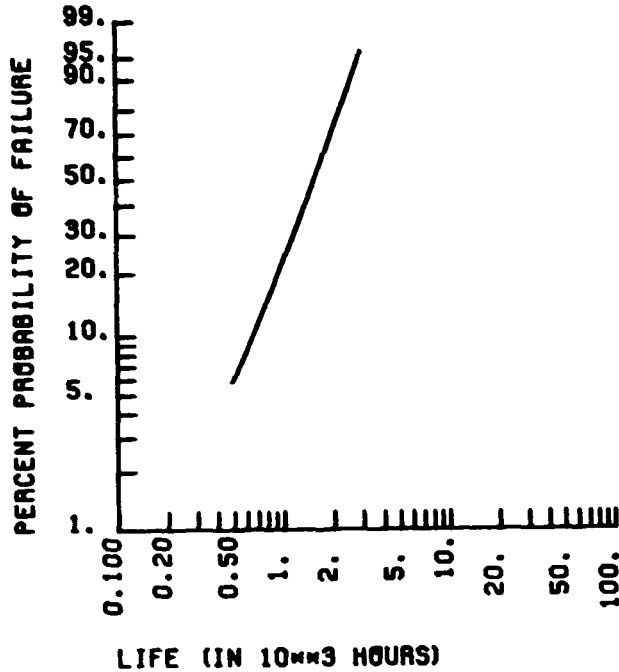
6
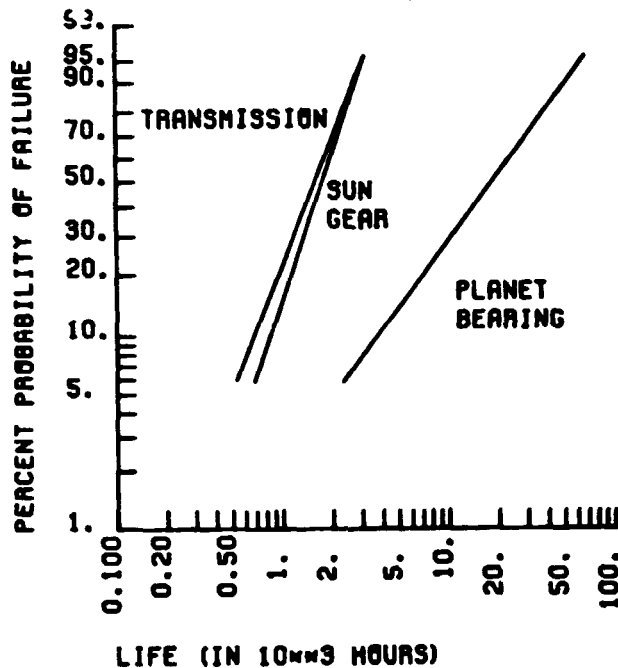
75-02 cylindrical roller bearings with a width of 25 mm (1 inch) and an outside diameter of 130 mm (5-1/8 inches). The basic dynamic capacity of the roller bearing is 81 kN (18,2000 pounds). Since the relative angular velocity of the bearing outer race with respect to the inner race is 800 RPM for this transmission, a composite life adjustment factor (including speed) of 1.5 is used in this analysis. Since the load is fixed relative to the inner race of the bearing, a load adjustment factor of 1.2 is used[13]. The bearings' Weibull exponent is 1.2 and their load-life factor is 3.33.

Although the plot of percent probability of failure versus transmission life in hours is not a straight line on Weibull coordinates, it can be approximated quite reasonably by a straight line relationship. This straight line approximation can be found using the least squared error approach over a range such as $0.5 \leq S_T \leq 0.95$. The slope of this straight line approximation is called the system Weibull slope $e_T$ and the system life of the straight line approximation at $S_T = 0.9$ is called the system 90 percent reliability life.

The exact $L_{T10}$ life can be calculated by setting $S_T = 0.9$ in Eq. (23) and iterating for $L_{T10}$ in the simplified equation:

$$1 = n\left(\frac{L_{T10}}{L_{B10}}\right)^{e_B} + \left(\frac{L_{T10}}{L_{S10}}\right)^{e_G} + n\left(\frac{L_{T10}}{L_{P10}}\right)^{e_G}$$

$$+ \left(\frac{L_{T10}}{L_{R10}}\right)^{e_G} \qquad (24)$$

For the cases studied in this research, the defined Weibull $L_{T10}$ life has not differed from the $L_{T10}$ life calculated from Eq. (24) by more than one percent. Since this error is considerably less than that between test data for the components and the resulting component Weibull lives, it is felt that the approximation is justified. When one component is weak relative to the rest of the transmission, the reliability model of the entire transmission and the least squares approximation will approach the Weibull model of the weak component.

For this straight line transmission Weibull curve, the reliability of the transmission is approximated by:

$$\log \frac{1}{S_T} = \log \frac{1}{.9}\left(\frac{L_T}{L_{T10}}\right)^{e_T} \qquad (25)$$

Fig. 3   Three planet transmission Weibull plot

Fig. 4   Three planet transmission and component Weibull plots

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In Fig. 4, this least squared error straight line Weibull curve is plotted for the three planet transmission. In addition, on the same set of coordinate axes is plotted the Weibull relationships for the two weakest components in the transmission. These components are the sun gear and a planet bearing. By comparing Figs. 3 and 4 one can see the asymptotic relationships between the sun gear Weibull distribution, the bearing Weibull distribution and the transmission's Weibull distribution. That these distributions are indeed different is graphically illustrated in the curves of Figs. 5 through 7. These are curves of the probability density functions for the three life distributions.

The bearing life distribution has the lowest Weibull slope. As a result its frequency distribution or percent probability of failure per $10^3$ hours is skewed to the low end of its curve with an $L_{10}$ life of 3870 hours relative to the distribution shown in Fig. 5. Since the gear Weibull slope is the highest in the transmission, the sun gear's life frequency distribution is significantly different from the bearings. As seen in Fig. 6, it is nearly a symmetric distribution. The $L_{10}$ life of the sun gear in this case is 810 hours. Even though this life is considerably less than the bearing's life, the fact that there are three bearings in the trans-
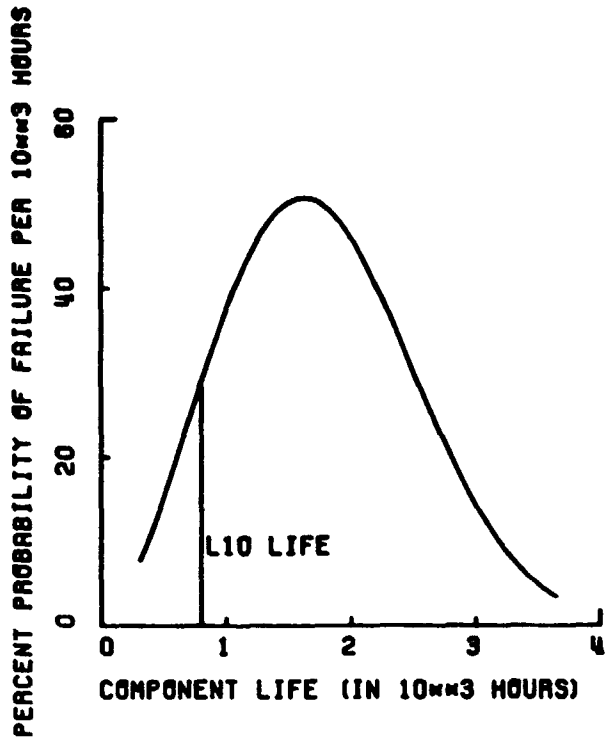
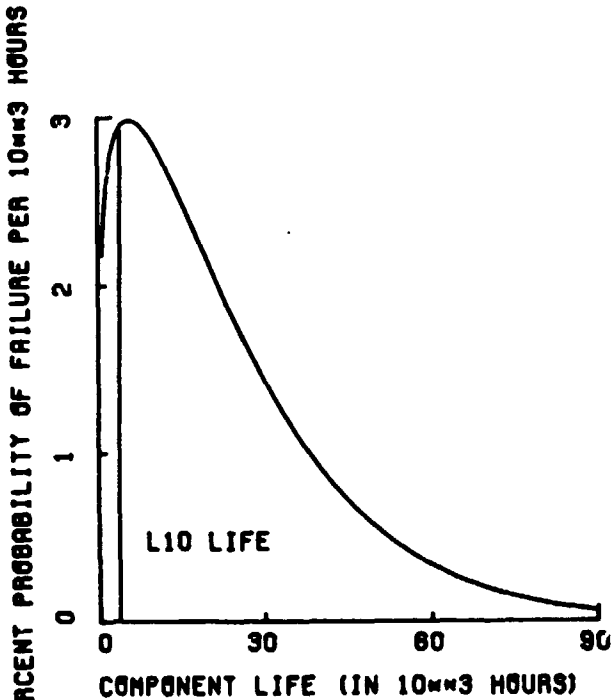Fig. 6. Sun gear life distribution for three planet transmission

Fig. 5. Planet bearing life distribution for three planet transmission
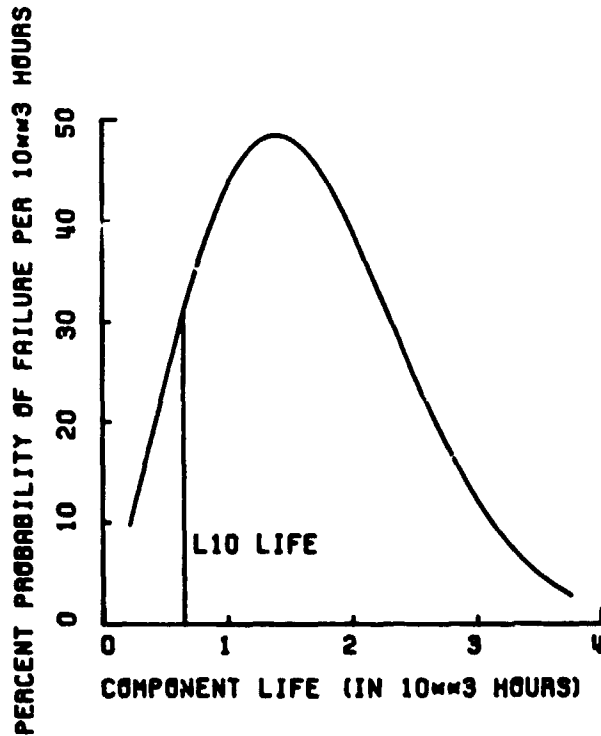
Fig. 7. Three planet transmission life distribution

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mission and only one sun gear helps the bearings to pull the system Weibull slope down to a value of 2.15 which produces the skewed system life distribution function shown in Fig. 7. The $L_{10}$ life of the three planet transmission at this power level is 650 hours according to the model presented in this paper.

The basic dynamic capacity for the transmission, $D_T$, is the sun input torque required to produce a system 90 percent reliablity life, $L_{T10}$, of one million sun rotations. By letting $S_T = 0.9$ in Eq. (23) and substituting Eqs. (12) and (20), one has for $L_{T10} = 1$;

$$1 = n\left(\frac{D_T}{D_B}\right)^{p_B e_B} + \left(\frac{D_T}{D_S}\right)^{p_G e_G} + n\left(\frac{D_T}{D_P}\right)^{p_G e_G}$$
$$+ \left(\frac{D_T}{D_R}\right)^{p_G e_G} \qquad (26)$$

The basic dynamic capacity of the transmission can be found by iterating this expression since the component exponents and capacities are known. It can also be found from Eq. (24) by determining a sequence of $L_{T10}$'s corresponding to a sequence of input sun torques, $T_i$'s, and plotting the natural log of $T_i$ versus the natural log of $L_{T10}$. The value of $T_i$ corresponding to $L_{T10} = 1$ million sun rotations is the transmission basic dynamic capacity. A plot of log $T_i$ versus log $L_{T10}$ is shown in Fig. 8 from the tranmsission example of Fig. 3. The slope of this curve is the negative of the load life exponent $P_T$ for the transmission. The case shown is that of nearly equal lives and capacities in which the deviation from a straight line relation is maximized. As for the transmission Weibull model, an approximate load-life curve is obtained by a least squares fit over a range of input torques (i.e , $0.1\ D_T \le T_i \le D_T$). With this approximation the load-life relation for the system is given by:

$$L_{T10} = \left(\frac{D_T}{T_i}\right)^{p_T} \qquad (27)$$

For the example plotted in Fig. 8, the transmission load life exponent, $p_T$, is 4.03 and the basic dynamic capacity is 2.56 kN-m (22,650 pound-inches).

As for the Weibull model, a weak component will dominate the transmission dynamic capacity and the system capacity and load life factor will approach that of the weakest component.
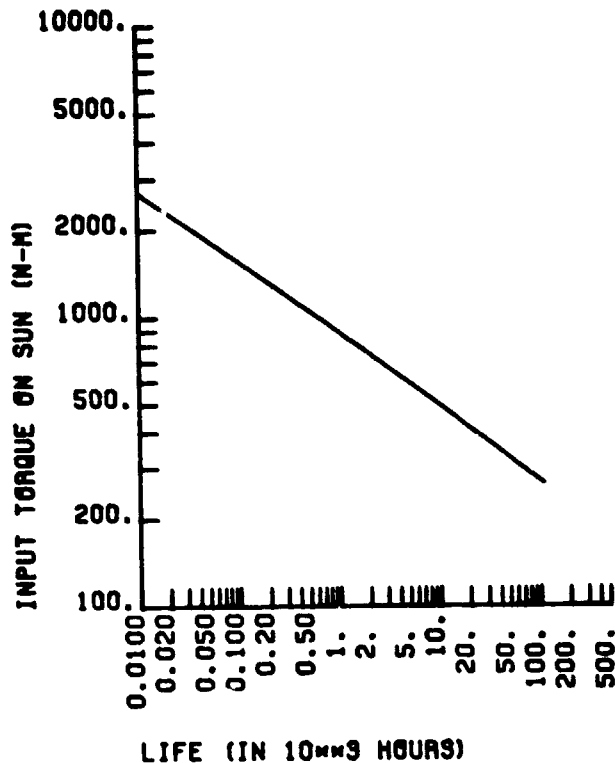


Fig. 8.  Load-life curve for three planet transmission

In all of this work, a nominal load, $T_i$, has been imposed on the transmission and a total resultant life has been calculated for the transmission for a 90 percent probability of survival $L_{T10}$. In actual service, the transmission is not operated at a constant load. For situations like this, where a mission spectrum of loads and speeds is present, a method of properly summing the fatigue damage done at each load must be used to adequately predict the transmission life. The best method for summing this damage in cases where the sequence of loading is random, as in a helicopter mission spectrum, is often called Miner's Rule although it was originally presented by Palmgren in 1924[9]. This linear cumulative damage rule states that the fraction of fatigue life consumed at a given load is the ratio of the number of cycles at that load to the number of cycles to failure at that load. In terms of $\ell_{10}$ lives this rule can be stated is:

$$\frac{L_T}{L_{T10}} = \frac{L_a}{L_{a10}} + \frac{L_b}{L_{b10}} + \frac{L_c}{L_{c10}} + \ldots \qquad (28)$$

where $L_a$, $L_b$, and $L_c$ are the service lives at loads $T_{ia}$, $T_{ib}$, and $T_{ic}$ and the $\ell_{10}$

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lives $L_{a10}$, $L_{b10}$ and $L_{c10}$ are the lives for a 90 percent probability of survival at those same loads. The life $L_T$ is the total mission life:

$$L_T = L_a + L_b + L_c + \ldots \qquad (29)$$

and the $\ell_{10}$ life $L_{T10}$ is the averaged mission life for a 90 percent probability of survival for the mission spectrum. This averaged mission life corresponds to the nominal mission load, $T_i$, in Eq. (27). Using Eq. (27) for each of the service loads, and substituting into Eq. (28) yields:

$$\left(\frac{T_i}{D_T}\right)^{P_T} (L_a + L_b + L_c + \ldots) = \left(\frac{T_{ia}}{D_T}\right)^{P_T} L_a$$

$$+ \left(\frac{T_{ib}}{D_T}\right)^{P_T} L_b + \left(\frac{T_{ic}}{D_T}\right)^{P_T} L_c + \ldots$$

$$(30)$$

Solving Eq. (30) for the nominal mission load, gives the weighted average load, $T_i$:

$$T_i = \left[\frac{(T_{ia})^{P_T} L_a + (T_{ib})^{P_T} L_b + (T_{ic})^{P_T} L_c + \ldots}{L_a + L_b + L_c + \ldots}\right]^{1/P_T}$$

$$(31)$$

By using this value of $T_i$ in Eq. (27), a measure of the total mission life can be obtained in terms of the actual mission spectrum.

### Transmission Comparison

The three planet transmission of Fig. 3 was changed to a four planet transmission with the same power loads and components to study the relative life and reliability of the two transmissions. The four planet transmission is shown in Fig. 9 and its life distribution curve at 150 kw (200 horsepower) is shown in Fig. 10. For this version of the transmission the Weibull slope dropped from 2.15 to 2.05 while the $L_{10}$ life of the transmission increased from 650 hours to 1610 hours. As noted in Figs. 5 through 7, the effect of dropping the component loading was to make the sun gear less critical in the transmission's system life and the life of the transmission greater. This is due to the lower Weibull slope of the bearing life reliability relationship with a greater
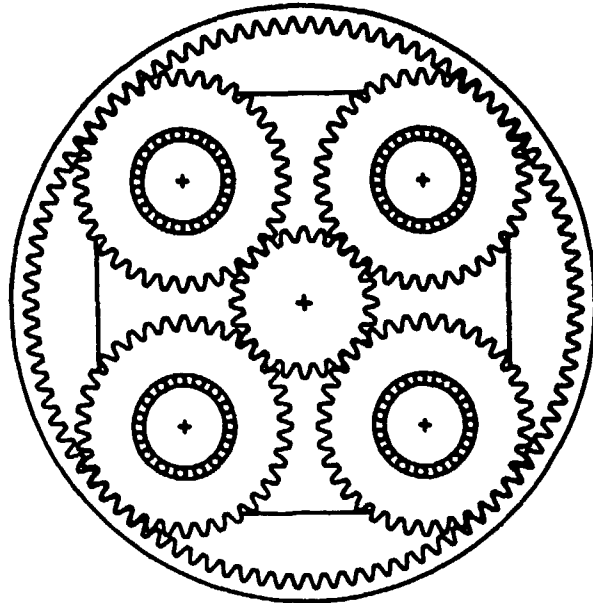
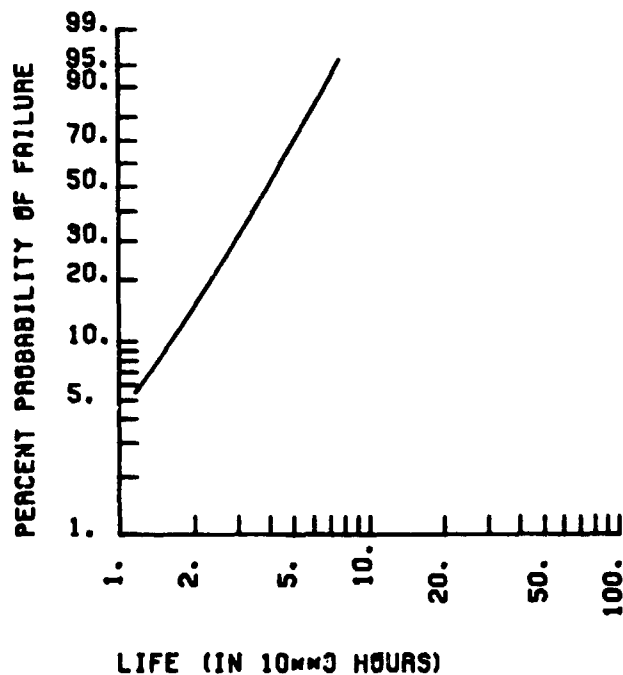Fig. 9  Four planet transmission

LIFE (IN 10**3 HOURS)

Fig. 10  Four planet transmission Weibull plot

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output shafts are assumed to be coaxial with the applied torques and each other; no side loading is considered.

The reliability model is based on reliability models of the bearing and gear mesh components which are two dimensional Weibull distributions of reliability as a function of life. The transmission's 90 percent reliability life and basic dynamic capacity are presented in terms of input sun rotations and torque. Due to the different Weibull distributions for the bearing and gearing components, the Weibull model for the planetary transmission is an approximate model. In this model, the transmission's 90 percent reliability life, Weibull exponent, basic dynamic capacity and load life exponent are presented.

Numerical examples are presented for two separate 150 kw (200 horsepower) transmissions to illustrate the use of the model. This power level is for a nominal transmission input torque and input speed. An equation is presented to obtain this weighted average torque from the mission spectrum torques and their relative durations. Both three and fo planet versions of a 5:1 reduction tra mission are considered.

The results show that the effect of adding a fourth planet more than doubles the life of the transmission. In addition, it is shown that due to the nature of the component life distributions, reducing the loading in the transmission makes the bearings more important in the life characteristics of the transmission, while increasing the loading makes the sun gear life more important in the overall life distribution of the transmission.
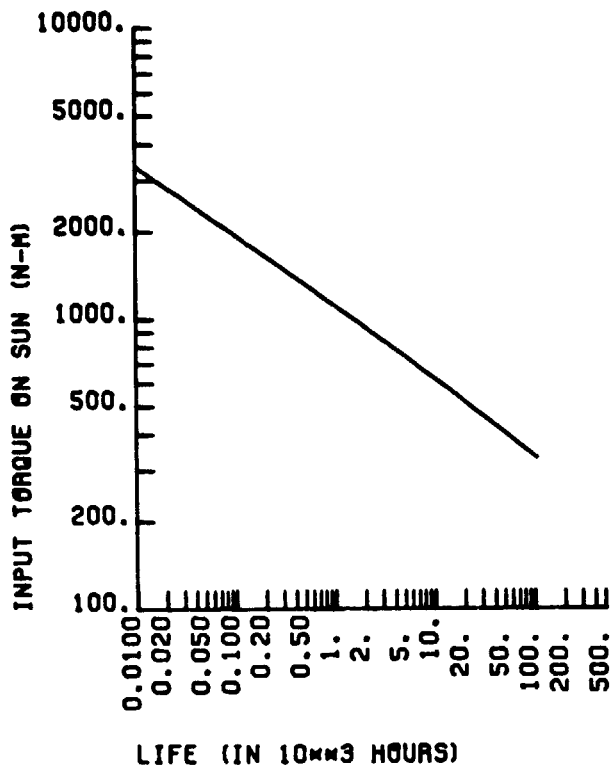


**LIFE (IN 10\*\*3 HOURS)**

Fig. 11  Load-life curve for four planet transmission

percentage of bearing failures in the lower life region in this skewed failure distribution relationship. Dropping the power level to the three planet transmission has the same dual effect of increasing the transmission life and reducing the Weibull slope to that of a more bearing dominated system.

Fig. 11 is the load-life curve for the four planet transmissions. The basic dynamic capacity of this transmission is 3.2 kN-m (28,300 pound-inches) as opposed to the 2.56 kN-m (22,650 pound-inches) for the three planet transmission while the load life factor only changed to 4.02 from 4.03. Thus the effect of increasing the number of planets on the load-life relationship for this transmission in only to increase the capacity by 25 percent. No significant change in the load-life factor occurred.

## Summary

A reliability model for the planetary gear train has been derived for use in the probabilistic design of this type of transmission. This gear train has the ring gear fixed, the sun gear as input and the planet carrier as output. The input and

### References

1. Bury, K. V., "On Probabilistic Design", Journal of Engineering for Industry, V96, N4, November, 1974, pp. 1291-1295.
2. Haugen, E. B., Probabilistic Mechanical Design, J. Wiley and Sons, 1980.
3. Smith, C.O., Introduction to Reliability in Design, McGraw-Hill, 1976.
4. Lundberg, G. and Palmgren, A., "Dynamic Capacity of Rolling Bearings", ACTA Polytechnica, Mechanical Engineering Series, V1, N3, 1947.
5. Lundberg, G. and Palmgren, A., "Dynamic Capacity of Roller Bearings", ACTA Polytechnica, Mechanical Engineering Series, V2, N4, 1952.
6. Coy, J. J., Townsend, D. P. and Zaretsky, E. V., "Analysis of Dynamic Capacity of Low Contact Ratio Spur Gears using Lundberg-Palmgren Theory", NASA TN D-8029, August, 1975.

7. Coy, J. J., Townsend, D. P. and Zaretsky, E. V., "Dynamic Capacity and Surface Fatigue Life for Spur and Helical Gears", Journal of Lubrication Technology, V98, N2, April, 1976, pp. 267-276.

8. Coy, J. J., Townsend, D. P. and Zaretsky, E. V., "An Update on the Life Analysis of Spur Gears", NASA CP 2210, 1982.

9. Juvinall, R. C., Engineering Considerations of Stress, Strain and Strength, McGraw-Hill, 1967, pp. 342-369.

10. Lipson, C. and Seth, N. J., Statistical Design and Analysis of Engineering Experiments, McGraw-Hill, 1973, pp. 36-44.

11. Savage, M., Paridon, C. A., and Coy, J. J., "Reliability Model for Planetary Gear Trains", ASME paper 82-DET-81 to be published, Journal of Mechanical Design.

12. Mabie, H. H. and Ocvirk, F. W., Mechanisms and Dynamics of Machinery, John Wiley and Sons, Third Edition, 1975, pp. 183-200.

13. Bamberger, E. N., et al., Life Adjustment Factors for Ball and Roller Bearings, ASME Engineering Design Guide, 1971.