Department of Geodetic Science and Surveying

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1. CURRENT TECHNICAL OBJECTIVES

1. Optimal Utilization of Laser and VLBI Observations for Reference Frames for Geodynamics (Grant NSG 5265)

2. Utilization of Range Difference Observations in Geodynamics (Contract NAS 5-25888)

3. Development of Models for Ice Sheet and Crustal Deformations (Grant NSG 5265)

2. ACTIVITIES

2.1 Effects of Adopting New Precession, Nutation and Equinox Corrections on the Terrestrial Reference Frame

A paper on this topic was presented at the XVII General Assembly of the International Astronomical Union, Patras, Greece, August 17-26, 1982, and appears in its entirety below. It will also appear in Bulletin Geodesique.

PREFACE

These projects are under the supervision of Professor Ivan I. Mueller, Department of Geodetic Science and Surveying, The Ohio State University. The Science Advisor of RF 711055 is Dr. David E. Smith, Code 921, Geodynamics Branch, and the Technical Officer is Mr. Jean Welker, Code 903, Technology Applications Center. The Technical Officer for RF 712407 is Mr. C. Stephanides, Code 942. The latter three are at NASA/GSFC, Greenbelt, Maryland 20771.

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EFFECTS OF ADOPTING NEW PRECESSION, NUTATION AND EQUINOX CORRECTIONS ON THE TERRESTRIAL REFERENCE FRAME [1]

Sheng-Yuan Zhu* and Ivan I. Mueller
Department of Geodetic Science and Surveying
The Ohio State University, Columbus, Ohio 43210 USA

ABSTRACT. First, the paper is devoted to the effects of adopting new definitive precession and equinox corrections on the terrestrial reference frame: The effect on polar motion is a diurnal periodic term with an amplitude increasing linearly in time; on UT1 it is a linear term. Second, general principles are given the use of which can determine the effects of small rotations (such as precession, nutation or equinox corrections) of the frame of a Conventional Inertial Reference System (CIS) on the frame of the Conventional Terrestrial Reference System (CTS). Next, seven CTS options are presented, one of which is necessary to accommodate such rotations (corrections). The last of these options requiring no changes in the origin of terrestrial longitudes and in UT1 is advocated; this option would be maintained by eventually referencing the Greenwich Mean Sidereal Time to a fixed point on the equator, instead of to the mean equinox of date, the current practice. Accomodating possible future changes in the astronomical nutation is discussed in the last section. The Appendix deals with the effects of differences which may exist between the various CTS's and CIS's (inherent in the various observational techniques) on earth rotation parameters (ERP) and how these differences can be determined. It is shown that the CTS differences can be determined from observations made at the same site, while the CIS differences by comparing the ERP's determined by the different techniques during the same time period.

INTRODUCTION

New general precession and equinox corrections are being introduced in the 1984 star catalogues and ephemerides. These corrections in turn will affect the earth rotation parameters (ERP), i.e., polar motion coordinates and UT1, and thus may change the frame of the Conventional Terrestrial Reference System

*On leave from Shanghai Observatory, China.
[1] Presented at XVIII General Assembly of the International Astronomical Union, Patras, Greece, August 17-26, 1982.

(CTS) [Mueller 1981]. Williams and Melbourne [1981] have already given a detailed discussion of these effects on UT1 and on the origin of terrestrial longitudes. In fact, it was this work which gave us motivation to expand the discussion to include the effects on all ERP's and offer additional options on how the necessary changes in the CTS could be accommodated. The approach is strictly geometric, i.e., we try to answer the question how definitive corrections to precession, nutation and the equinox affect the ERP's, and thus the CTS. Williams and Melbourne [1981] emphasize the point of how UT1 and the origin of longitudes will be affected in the future by the uncertainties in the newly adopted corrections or how these corrections can be improved in the future from ensembles of Very Long Baseline Interferometer (VLBI) or Lunar Laser Ranging (LLR) orbservations, with the desire that no or minimum additional changes result in the CTS. They assume that VLBI sources are observed randomly over the sky, while LLR observations are equally distributed only along the ecliptic, and therefore the resulting equations defining the changes of the origin of terrestrial longitudes and UT1 are technique dependent, whereas ours are not. (Putting it differently, they imply that if the analysis of future VLBI or LLR ensemble observations indicate necessary changes in UT1 and in the origin of terrestrial longitudes, such changes are due to the still existing imperfections in the newly adopted corrections to precession, equinox, etc., and when determined they will be biased with respect to each other because of the different sensitivities of the two ensembles of observations.) This difference in the results should not confuse the reader who recognizes the different purposes for which these papers were written.

1. EFFECTS OF ADOPTING NEW PRECESSION AND EQUINOX CORRECTIONS ON THE FRAME OF THE CONVENTIONAL TERRESTRIAL REFERENCE SYSTEM

1.1 Transformation Between Conventional Inertial (CIS) and Terrestrial Reference Frames (CTS)

The transformation at an epoch T between the CIS at some fundamental epoch (e.g., 1950.0) and the CTS is

$$[\underline{CTS}] = SNP(M)\ [\underline{CIS}] \tag{1}$$

(see [Mueller 1981]). Here

$$S = R_2(-x_p)\ R_1(-y_p)\ R_3(\theta)$$

is the earth rotation matrix, in which x_p and y_p are the polar motion components, and θ is the Greenwich Apparent Sidereal Time (corresponding to the epoch T) computed from

$$\theta = (GMST)_0 + \omega_e \, UT1 + Eq.\ E.$$

where $(GMST)_0$ is the Greenwich Mean Sidereal Time at 0^h UT1, ω_e is the conversion factor from mean time to mean sidereal time, and Eq. E. is the equation of the equinox (nutation in right ascension). The other matrices N, P, M in equation (1) are the nutation, precession, and proper motion matrices respectively [Mueller 1969, p. 123]. Parentheses around the M matrix indicate that proper motion is applied only in the case of a stellar CIS.

Let prime (') denote the case with the precession, nutation and equinox changes introduced. The transformation equation (1) also holds for the corrected case:

$$[\underline{CTS}]' = S'N'P'(M')\,[\underline{CIS}]' \tag{1'}$$

In this section only the precession and equinox changes are considered so that N' = N. From the definitions (or stipulations), one can determine directly or indirectly the relations between P' and P, M' and M, and [CIS]' and [CIS] at some epoch, leaving S' and [CTS]' to be solved for.

One cannot solve for both S' and [CTS]' simultaneously, hence some additional constraint is needed. There are several options for the constraint, and they will be discussed later in Section 2.2. For the time being we will conform with the IAU adopted constraint, namely: Let the new ERP's be the same as the old ones at some epoch T_u (in this paper T denotes the epoch, and t the time interval between T and some fundamental epoch, e.g., 1950.0); solve for [CTS]' at this time, then keep it time invariant and solve for the resulting time variations in the new ERP's.

1.2 The Effect in the Case of a Stellar CIS

The new (1976) corrections for lunisolar precession in longitude and planetary precession in right ascension are [Williams and Melbourne 1981]

$$\Delta p_1 = 1''\!.1/\text{cy}$$
$$\Delta \dot{\chi} = -0''\!.029/\text{cy}$$

The correction to the equinox is $E_0 + \dot{E}t$, where $E_0 = 0''.525$ is the offset at 1950.0, $\dot{E} = 1''.275/cy$, and t is the time elapsed from 1950.0 [Fricke 1981].

The new precession matrix P' can be written with sufficient approximation as

$$P' = R_2(\Delta n t)\, R_3(-\Delta m t)\, P \tag{2}$$

with

$$\Delta n = \Delta p_1 \sin\varepsilon$$
$$\Delta m = \Delta p_1 \cos\varepsilon - \Delta\dot{\chi}$$

where ε is the obliquity of the ecliptic, and Δn, Δm are the general precession changes in declination and in right ascension. Due to the equinox correction, the equation for the Greenwich Mean Sidereal Time is to change(without terms of higher order) to [Aoki et al. 1982]

$$(GMST)_0' = (GMST)_0 + E_0 + \dot{E}t \tag{3}$$

For the stellar (i.e., classical optical) CIS the change caused by the equinox correction at the fundamental epoch 1950.0 is

$$[\underline{CIS}]' = R_3(-E_0)\, [\underline{CIS}] \tag{4}$$

The new proper motion matrix is

$$M' = R_2(-\Delta n t)\, R_3[(\Delta m - \dot{E})t]\, M \tag{5}$$

The proper motion components in right ascension and declination are

$$(\mu_\alpha)' = (\mu_\alpha) + \dot{E} - \Delta m - \Delta n \sin\alpha \tan\delta$$
$$(\mu_\delta)' = (\mu_\delta) - \Delta n \cos\alpha$$

Substituting the above new values of P', M', $[\underline{CIS}]'$ and $(GMST)_0'$(i.e., eqs. (2) - (5)) into eq. (1'), one gets

$$[\underline{CTS}]' = R_2(-x_p')\, R_1(-y_p')\, R_3[(GMST)_0 + \omega_e\, UT1' + Eq.\ E.]\, R_3(E_0 + \dot{E}t)\, N \cdot$$
$$\cdot\, R_2(\Delta n t)\, R_3(-\Delta m t) P\, R_2(-\Delta n t)\, R_3[(\Delta m - \dot{E})t]\, M\, R_3(-E_0)\, [\underline{CIS}]$$

Except for $(GMST)_0$, all rotation angels are small; neglecting the second-order terms, approximately,

$$[\underline{CTS}]' = R_2(-x_p')\, R_1(-y_p')\, R_3[(GMST)_0 + \omega_e\, UT1' + Eq.\ E.]\, NPM\, [\underline{CIS}] \tag{6}$$

(For the above given Δp_1 and $\dot{E}$ values, neglecting the modulation of NP will cause an error of less than 0".0001 in t = 10 yr.) Combining the above equation with the mentioned constraints at epoch T_u: $x_p' = x_p$, $y_p' = y_p$, and UT1' = UT1, one obtains

$$\boxed{[\underline{CTS}]' = [\underline{CTS}]}$$

The well-known conclusion is that in the case of the stellar CIS, the CTS and ERP's are unaffected because changes in the proper motion compensate for the equinox and precession changes. This statement is naturally valid not only at the epoch T_u but at any time before or after.

1.3 The Effect in the Case of a Non-Stellar CIS

For any non-stellar (e.g., VLBI or LLR) CIS, the proper motion matrix is no longer taken into consideration; the P' and $(GMST)_0'$ are the same as in the stellar case (eq. (2) - (3)). The relationship between $[\underline{CIS}]'$ and $[\underline{CIS}]$ depends on the particular CIS under consideration. Generally,

$$[\underline{CIS}]' = E_I\ [\underline{CIS}]$$

If the considered CIS is aligned with the dynamic equator and equinox, then $E_I = I$, where I is a unit matrix.

If the non-stellar CIS is aligned with the stellar system equinox at some epoch T_0, then E_I will be a little complicated. At this time due to the equinox correction,

$$[\underline{CIS}]'_{T_0} = R_3(-E_0 - \dot{E}t_0)\ [\underline{CIS}]_{T_0} \tag{7}$$

(More exactly, a second-order term could be considered.) The precession effect on the CIS's for the time interval t_0 between the fundamental epoch 1950.0 and the alignment epoch T_0 is

$$[\underline{CIS}]_{T_0} = P(t_0)\ [\underline{CIS}]$$

$$[\underline{CIS}]'_{T_0} = P'(t_0)\ [\underline{CIS}]'$$

With equations (2) and (7) one gets at the fundamental epoch

$$[\underline{CIS}]' = R_2(-\Delta n t_0)\ R_3[(\Delta m - \dot{E})t_0]\ R_3(-E_0)\ [\underline{CIS}] = E_I\ [\underline{CIS}] \tag{8}$$

i.e.,

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$$E_I = R_2(-\Delta n t_0)\, R_3[(\Delta m - \dot{E})t_0 - E_0] \tag{9}$$

The corresponding corrections in right ascension ($\Delta\alpha_{E_I}$) and declination ($\Delta\delta_{E_I}$) are

$$\Delta\alpha_{E_I} = E_0 + (\dot{E} - \Delta m)t_0 - \Delta n t_0 \sin\alpha \tan\delta$$

$$\Delta\delta_{E_I} = -\Delta n t_0 \cos\alpha \ .$$

Now, substituting P', $(GMST)_0'$, and [CIS]' (i.e., eq. (2), (3) and (8)) into eq. (1'),

$$\underline{[CTS]}' = R_2(-x_p')\, R_1(-y_p')\, R_3[(GMST)_0 + E_0 + \dot{E}t + \omega_e\, UT1' + Eq.\ E]\, N \cdot$$
$$\cdot\, R_2(\Delta n t)\, R_3(-\Delta m t)\, P\, E_I\, \underline{[CIS]} \tag{1''}$$

As stated before, the ERP's are continuous, that is, at the alignment epoch T_u, $x_p' = x_p$, $y_p' = y_p$, UT1' = UT1. Thus

$$\underline{[CTS]}' = R_2(-x_p)\, R_1(-y_p)\, R_3[(GMST)_0 + \omega_e\, UT1 + Eq.\ E]\, R_3(E_0 + \dot{E}t_u)\, N \cdot$$
$$\cdot\, R_2(\Delta n t_u)\, R_3(-\Delta m t_u)\, P\, E_I\, \underline{[CIS]} =$$
$$\doteq SNP\, R_2(\Delta n t_u)\, R_3[E_0 + (\dot{E} - \Delta m)\, t_u]\, E_I\, \underline{[CIS]} \tag{10}$$

If the CIS is linked with the stellar system equinox at epoch T_0, i.e., E_I is expressed by eq. (9), then

$$\underline{[CTS]}' \doteq SNP\, R_2[\Delta n(t_u - t_o)]\, R_3[(\dot{E} - \Delta m)(t_u - t_o)]\underline{[CIS]}$$
$$\doteq S\, R_2[\Delta n(t_u - t_o)]\, R_3[(\dot{E} - \Delta m)(t_u - t_o)]\, NP\, \underline{[CIS]} \tag{10'}$$

As pointed out previously, the modulation of NP is negligible, but the modulation of $R_3(\theta)$, included in S, must be taken into consideration.

$$R_3(\theta)\, R_2[\Delta n(t_u - t_o)] = \{R_3(\theta)\, R_2[\Delta n(t_u - t_o)]\, R_3(-\theta)\}\, R_3(\theta)$$
$$= R_1[\Delta n(t_u - t_o)\sin\theta]\, R_2[\Delta n(t_u - t_o)\cos\theta]\, R_3(\theta)$$

Substituting this into equation (10'),

$$\underline{[CTS]}' \doteq R_1[\Delta n(t_u - t_o)\sin\theta]\, R_2[\Delta n(t_u - t_o)\cos\theta]\, R_3[(\dot{E} - \Delta m)(t_u - t_o)]\, SNP\, \underline{[CIS]}$$

Thus for the case of <u>CIS alignment with the stellar system</u>

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$$\boxed{[\underline{CTS}]' = R_1[\Delta n(t_u-t_o)\ \sin\theta]\ R_2[\Delta n(t_u-t_o)\ \cos\theta]\ R_3[(\dot{E}-\Delta m)(t_u-t_o)][\underline{CTS}]} \tag{11}$$

If the CIS is aligned with the dynamic equinox, that is, $E_I = I$, then

$$[\underline{CTS}]' = SNP\ R_2(\Delta n\ t_u)\ R_3[E_0 + (\dot{E}-\Delta m)\ t_u][\underline{CIS}]$$

Thus

$$\boxed{[\underline{CTS}]' = R_1(\Delta n\ t_u\ \sin\theta)\ R_2(\Delta n\ t_u\ \cos\theta)\ R_3[E_0 + (\dot{E}-\Delta m)\ t_u][\underline{CTS}]} \tag{12}$$

If the alignment is made over some time period (say, five days or so) T_u is the mean epoch of alignment, and the values $\sin\theta$ and $\cos\theta$ are the mean values within this time span and can be averaged to zero. In this case

$$[\underline{CTS}]' = R_3[(\dot{E}-\Delta m)(t_u-t_o)][\underline{CTS}] \tag{11'}$$

for the CIS linked with the stellar system equinox, and

$$[\underline{CTS}]' = R_3[E_0 + (\dot{E}-\Delta m)\ t_u][\underline{CTS}] \tag{12'}$$

when aligned with the dynamic equinox. Thus the relation between the new and old CTS's is a small rotation around the third axis. Expressed in longitude (positive to the East),

$$\delta\lambda = \lambda' - \lambda = (\Delta m-\dot{E})(t_u-t_o) \tag{11"}$$

for the CIS linked with the stellar system equinox, and

$$\delta\lambda = \lambda' - \lambda = (\Delta m-\dot{E})\ t_u - E_0 \tag{12"}$$

when aligned with the dynamic equinox.

For a CIS linked with the stellar system, if $t_u = t_o$, then $\delta\lambda = 0$; otherwise a shift in longitude is necessary. As for a CIS aligned with the dynamic equinox, the CTS longitude origin shift generally cannot be avoided.

1.4 The Effect of the Time-Invariant CTS on the ERP's

The new CTS' at the time of alignment T_u can then be determined as outlined in the previous section, i.e., in the stellar CIS case $[\underline{CTS}]' = [\underline{CTS}]$, and in the non-stellar cases as given by eqs. (11), (11') or (12), (12').

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The next step is to keep the new CTS time invariant and to find the resulting ERP's at any time other than T_u. Substituting eq. (11') for the left side of eq. (1"), and eq. (9) in the right-hand side, after some derivation and neglecting second-order terms, one gets

$$[\underline{CTS}] \doteq R_2(-x_p') \, R_1(-y_p') \, R_3[(GMST)_0 + \omega_e \, UT1' + Eq.\,E] \cdot$$
$$\cdot \, R_2[\Delta n(t-t_o)] \, R_3[(\dot{E}-\Delta m)(t-t_u)] \, NP \, [\underline{CIS}]$$

Comparing this equation with eq. (1),

$$R_2(-x_p') \, R_1(-y_p') \, R_3[(GMST)_0 + \omega_e \, UT1' + Eq.\,E] \, R_2[\Delta n(t-t_o)] \cdot$$
$$\cdot \, R_3[(\dot{E}-\Delta m)(t-t_u)] = S$$

or

$$R_2[-x_p' + \Delta n(t-t_o)\cos\theta] \, R_1[-y_p' + \Delta n(t-t_o)\sin\theta] \, R_3\{(GMST)_0 + Eq.\,E +$$
$$+ [\omega_e \, UT1' + (\dot{E}-\Delta m)(t-t_u)]\} =$$
$$= R_2(-x_p) \, R_1(-y_p) \, R_3[(GMST)_0 + \omega_e \, UT1 + Eq.\,E]$$

From the above it is obvious that over a limited time span (otherwise second-order terms must be added),

$$\begin{aligned} \Delta x_p &= x_p' - x_p = \Delta n(t-t_o) \cos\theta \\ \Delta y_p &= y_p' - y_p = \Delta n(t-t_o) \sin\theta \\ \Delta UT1 &= UT1' - UT1 = (\Delta m-\dot{E})(t-t_u)/\omega_e \end{aligned} \tag{13}$$

The above are in the case of a non-stellar CIS linked with the stellar system. For the dynamic equinox alignment, substitute eq. (12') for the left side of eq. (1") and let $E_I = I$. The results are

$$\begin{aligned} \Delta x_p &= \Delta nt \cos\theta \\ \Delta y_p &= \Delta nt \sin\theta \\ \Delta UT1 &= (\Delta m-\dot{E})(t-t_u)/\omega_e \end{aligned} \tag{14}$$

For both cases $\Delta UT1$ is the same; so is the rate of $\Delta UT1$:

$$\frac{d\Delta UT1}{dt} = (\Delta m-\dot{E})/\omega_e = -0.157 \text{ ms/yr} \tag{15}$$

In conclusion, in the case of a non-stellar CIS, changes in the precessional constant and the equinox will result in changes in both the CTS and the ERP's. The CTS change is a longitude origin shift. The ERP changes are diurnal terms in the polar motion components with amplitudes linearly increasing with time and a constant rate change in UT1. One point worth stressing is that these are the differences of the same system (technique) between the new and old cases, not the differences between different systems (techniques). Also the diurnal term which is evident in polar motion is not the diurnal true polar motion, but an artifact due to the time invariant CTS constraint applied.

2. GENERAL SOLUTION: SMALL CIS ROTATIONS AND THEIR EFFECT ON THE CTS

2.1 Changes in the Earth Rotation Parameters

In the general case, eq. (13) or (14) can be written in the following form

$$
\begin{aligned}
\Delta x_p &= -\alpha_1 \sin\theta + \alpha_2 \cos\theta \\
\Delta y_p &= \alpha_1 \cos\theta + \alpha_2 \sin\theta \qquad (16) \\
\Delta\theta &= -\alpha_3
\end{aligned}
$$

where the small angles α_i represent the changes in the sense

$$N'P'M' \, [\underline{CIS}]' = R_1(\alpha_1)\, R_2(\alpha_2)\, R_3(\alpha_3)\, NPM \, [\underline{CIS}]$$

Since

$$\theta = (GMST)_0 + \omega_e \, UT1 + Eq.\ E$$

there are several possibilities for changing θ. If the nutation is assumed to be unchanged, α_3 either may be absorbed into $(GMST)_0$, i.e., it becomes a change in the Greenwich Mean Sidereal Time; or, as before, it may go into UT1 ($\Delta UT1 = \alpha_3/\omega_e$); or it can be incorporated partially in $(GMST)_0$ and partially in UT1. When α_3 is placed (fully or partly) into UT1, then if UT1 is still to be continuous at the epoch T_u, the longitude λ has to absorb the one-time discontinuity as shown before. Finally, if α_3 is a nutation correction, than α_3 must be combined with Eq. E (see Section 3).

The corrections for precession, for the equinox, and for proper motion may be written in the following forms respectively

$$\Delta p = R_2(\Delta nt)\, R_3(-\Delta mt)$$
$$E_I = R_2(E_{I_2})\, R_3(E_{I_3})$$
$$\Delta M = R_2(\Delta M_2)\, R_3(\Delta M_3)$$

where, from comparisons with earlier results, in the case of the stellar CIS, $\Delta M_2 = -\Delta nt$, $\Delta M_3 = (\Delta m-\dot{E})t$, $E_{I_2} = 0$, $E_{I_3} = -E_0$; for the non-stellar CIS aligned with the dynamic equinox, $E_{I_2} = E_{I_3} = \Delta M_2 = \Delta M_3 = 0$; and in the case of the non-stellar CIS linked with the stellar system, $E_{I_2} = -\Delta nt_0$, $E_{I_3} = -E_0 + (\Delta m-\dot{E})t_0$, $\Delta M_2 = \Delta M_3 = 0$.

In any of the above cases

$$\begin{aligned} \alpha_1 &= 0 \\ \alpha_2 &= \Delta nt + E_{I_2} + \Delta M_2 \\ \alpha_3 &= -\Delta mt + E_{I_3} + \Delta M_3 \end{aligned} \tag{17}$$

Thus, for example, in the case of the stellar CIS

$$\begin{aligned} \alpha_1 &= \alpha_2 = 0 \\ \alpha_3 &= -E_0 - \dot{E}t \\ \Delta\theta &= E_0 + \dot{E}t \end{aligned} \tag{17'}$$

If we let $\Delta\theta$ be the $\Delta(\mathrm{GMST})_0$, as we did before, then eq. (17') is equivalent to eq. (3).

In the case where the non-stellar CIS is linked at T_0 with the stellar system,

$$\begin{aligned} \alpha_1 &= 0 \\ \alpha_2 &= \Delta n(t-t_0) \\ \alpha_3 &= -E_0 + (\Delta m-\dot{E})t_0 - \Delta mt \\ \Delta\theta &= E_0 + (\dot{E}-\Delta m)t_0 + \Delta mt \end{aligned} \tag{17''}$$

If we let $\Delta(\mathrm{GMST})_0 = E_0 + \dot{E}t$, and let the ERP's be continuous at T_u, then eq. (17") is equivalent to eqs. (11") and (13). The analogy can also be established for the case of the non-stellar CIS linked to the dynamic equinox (eq. (14)).

2.2 Options to Change the CTS (Due to $\Delta\theta$)

As shown, in the case of equinox and precession constant corrections,

$\Delta\theta = E_0 + \dot{E}t$ for the stellar system, and

$\Delta\theta = -E_{I_3} + \Delta mt$ for the non-stellar systems

$\Delta\theta$ can also be written (assuming no change in nutation and the one-time discontinuity in UT1 absorbed in the longitudes, $\delta\lambda$, mentioned earlier)

$$\Delta\theta = \Delta(GMST)_0 + \omega_e \, \Delta UT1 + \delta\lambda$$

Thus, as stated above, one can absorb $\Delta\theta$ either in $\Delta(GMST)_0$, or $\Delta UT1$, or $\delta\lambda$, or in some combinations of these. To get a definite (unique) solution, some constraint is needed. Mathematically, there are quite a number of possible choices for such a constraint, but practically only a few are meaningful. Below we deal with three sets of options. Which option is the best surely will be the subject of many discussions.

<u>Set A Options</u>. Here the basic requirements are: (i) no discontinuity in ERP's at the epoch T_u, (ii) the change in the Greenwich Mean Sidereal Time formula is the same for all CIS's, though different for each option.

Set A Options		Stellar CIS $\Delta\theta = E_0 + \dot{E}t$	Non-stellar CIS $\Delta\theta = -E_{I_3} + \Delta mt$
Option I	$\Delta(GMST)_0$	$E_0 + \dot{E}t$	$E_0 + \dot{E}t$
	$\omega_e \, \Delta UT1$	0	$(\Delta m-\dot{E})(t-t_u)$
	$\delta\lambda$	0	$(\Delta m-\dot{E})t_u - E_{I_3} - E_0$
Option II	$\Delta(GMST)_0$	0	0
	$\omega_e \, \Delta UT1$	$\dot{E}(t-t_u)$	$\Delta m(t-t_u)$
	$\delta\lambda$	$E_0 + \dot{E}t_u$	$-E_{I_3} + \Delta mt_u$
Option III	$\Delta(GMST)_0$	Δmt	Δmt
	$\omega_e \, \Delta UT1$	$(\dot{E}-\Delta m)(t-t_u)$	0
	$\delta\lambda$	$E_0 + (\dot{E}-\Delta m)t_u$	$-E_{I_3}$

Options I, II, and III above are similar to Tables 1, 2, and 3 respectively in [Williams and Melbourne 1981]. (The main difference appears to be that for the non-stellar cases the general precession in right ascension Δm is replaced by what they call "the average value over all observations of the effects of the precession corrections in right ascension" $\langle\dot{\alpha}_p\rangle$. For VLBI $\langle\dot{\alpha}_p\rangle = \Delta p_1 \cos\varepsilon - \Delta\dot{\chi} = \Delta m$, but for LLR $\langle\dot{\alpha}_p\rangle = \Delta p_1 - \Delta\dot{\chi} \neq \Delta m$.) We already elaborated on Option I in Section 1.1. Option 1 is the presently accepted approach for the new FK5 CIS. But, as pointed out by [Williams and Melbourne 1981], for future possible new improvement of the precession constant and equinox corrections, this option might not be the best. They favor Option III because the space techniques are becoming the dominant source of information about the transformation parameters between the CIS and CTS frames and because this option keeps UT1 invariant to improved values of the precession constant and the equinox position for the space techniques. The common geodetic disadvantage of Set A options is the required shift in the longitude origin (except in Option I for the stellar CIS case), the worst thing being that these shifts are different in the cases of stellar and non-stellar CIS's.

Set B Options. Here the basic requirements are: (i) no change in the CTS, i.e., $\delta\lambda = 0$, (ii) as before, the Greenwich Mean Sidereal Time formula change is the same in all CIS cases, but different for each option.

The major inconvenience of Set B options is the change in UT1, not only in the rate, but also in the necessary discontinuity. The value of the discontinuity would need to be added with opposite sign to the UT1 at the epoch when the changes (new constants) are introduced.

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Set B Options		Stellar CIS $\Delta\theta = E_0 + \dot{E}t$	Non-stellar CIS $\Delta\theta = -E_{I_3} + \Delta mt$
	$\delta\lambda$	0	0
Option IV	$\Delta(GMST)_0$	0	0
	$\omega_e\ \Delta UT1$	$E_0 + \dot{E}t$	$-E_{I_3} + \Delta mt$
	$\delta\lambda$	0	0
Option V	$\Delta(GMST)_0$	$E_0 + \dot{E}t$	$E_0 + \dot{E}t$
	$\omega_e\ \Delta UT1$	0	$-E_{I_3} - E_0 + (\Delta m-\dot{E})t$
	$\delta\lambda$	0	0
Option VI	$\Delta(GMST)_0$	$-E_{I_3} + \Delta mt$	$-E_{I_3} + \Delta mt$
	$\omega_e\ \Delta UT1$	$E_0 + E_{I_3} + (\dot{E}-\Delta m)t$	0

Set C Option. Here the basic requirements are: (i) no change in CTS, (ii) no change in UT1, i.e., $\Delta\theta$ is entirely absorbed in $\Delta(GMST)_0$.

Set C Option		Stellar CIS $\Delta\theta = E_0 + \dot{E}t$	Non-stellar CIS $\Delta\theta = -E_{I_3} + \Delta mt$
	$\delta\lambda$	0	0
Option VII	$\omega_e\ \Delta UT1$	0	0
	$\Delta(GMST)_0$	$E_0 + \dot{E}t$	$-E_{I_3} + \Delta mt$

Although this option is probably the preference of geodesists, it may seem to be unorthodox from the traditional astronomical point of view. How can the formulae for Greenwich Mean Sidereal Time for different CIS's be different? What will the astronomical meaning of $(GMST)_0$ be? However, one can view the formula for Greenwich Mean Sidereal Time as composed of two parts: The first part, $(GMST)_0$, has its original astronomical meaning, while the second part, $\Delta(GMST)_0$ is only a correction particular for a given CIS. It would make sense that since the changes in precession and the equinox affect different CIS's in different ways, this correction should also be different. From this point of view, Option VII seems plausible and even preferable for geodetic use.

It should also be noted that after the new equinox and precession changes are introduced (once) into $\Delta(GMST)_0$, this option could become the equivalent of referencing the GMST to a fixed point on the equator, instead of to the mean equinox of date, the current practice. As pointed out by a number of authors, the advantage of such a change would be overwhelming and would make the future CTS stable against changes in the precession constant, etc. [Guinot 1979, Murray 1979, Williams and Melbourne 1981, Mueller 1981].

3. EFFECT OF ASTRONOMICAL NUTATION CHANGES ON EARTH ROTATION PARAMETERS

According to the principle in Section 2.1, it is also easy to deal with any future changes in nutation. The nutation matrix N is [Mueller 1969]

$$\begin{aligned} N &= R_1(-\varepsilon - \Delta\varepsilon)\, R_3(-\Delta\psi)\, R_1(\varepsilon) \\ &\doteq R_1(-\Delta\varepsilon)\, R_2(\Delta\psi \sin\varepsilon)\, R_3(-\Delta\psi \cos\varepsilon) \end{aligned}$$

where $\Delta\psi$ and $\Delta\varepsilon$ are the nutation in longitude and obliquity respectively, and ε is the obliquity of the ecliptic. If $\delta\Delta\varepsilon$ and $\delta\Delta\psi$ are the respective corrections to $\Delta\varepsilon$ and $\Delta\psi$, then one can easily obtain the nutation correction matrix,

$$\Delta N \doteq R_1(-\delta\Delta\varepsilon)\, R_2(\delta\Delta\psi \sin\varepsilon)\, R_3(-\delta\Delta\psi \cos\varepsilon)$$

Thus in the notation of eq. (16),

$$\begin{aligned} -\delta\Delta\varepsilon &= \alpha_1 \\ \delta\Delta\psi \sin\varepsilon &= \alpha_2 \\ -\delta\Delta\psi \cos\varepsilon &= \alpha_3 \end{aligned}$$

and, therefore,

$$\begin{aligned} \Delta x_p &= \delta\Delta\varepsilon \sin\theta + \delta\Delta\psi \sin\varepsilon \cos\theta \\ \Delta y_p &= -\delta\Delta\varepsilon \cos\theta + \delta\Delta\psi \sin\varepsilon \sin\theta \\ \Delta\theta &= -\alpha_3 = \delta\Delta\psi \cos\varepsilon \end{aligned}$$

Thus, as expected, the effects on polar motion components are diurnal terms ($\delta\Delta\psi$ and $\delta\Delta\varepsilon$ are long periodic). Again, this is a diurnal artifact in polar motion due to the introduction of the new nutation and not diurnal true polar motion.

As far as the term $\Delta\theta = \delta\Delta\psi \cos\varepsilon$ is concerned, if it is incorporated into the Eq. E, neither the longitude origin nor the UT1 will be affected.

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APPENDIX

EFFECTS OF DIFFERENCES BETWEEN VARIOUS CTS'S AND CIS'S ON EARTH ROTATION PARAMETERS AND THE DETERMINATION OF SUCH DIFFERENCES

The two CIS's (and two CTS's) inherent in two different techniques (e.g., SLR and VLBI) are generally not exactly identical [Mueller 1981]. Suppose the relation between the two CIS's at any epoch is (common nutation (N) and precession (P) matrices are assumed to be used in both techniques)

$$[\underline{CIS}]^{II} = R_1(\alpha_1)\ R_2(\alpha_2)\ R_3(\alpha_3)\ [\underline{CIS}]^{I} \tag{A.1}$$

Similarly, the relation between the two CTS's is

$$[\underline{CTS}]^{II} = R_1(\beta_1)\ R_2(\beta_2)\ R_3(\beta_3)\ [\underline{CTS}]^{I} \tag{A.2}$$

where α_i and β_i are small rotation angles about the axes "i".

The transformation from CIS to CTS again is

$$[\underline{CTS}]^{I} = S^{I}\ N\ P\ [\underline{CIS}]^{I} \tag{A.3}$$

and

$$[\underline{CTS}]^{II} = S^{II}\ N\ P\ [\underline{CIS}]^{II} \tag{A.4}$$

Substituting eq. (A.1) for the last term of the right-hand side of eq. (A.4), and eq. (A.2) for the left-hand side,

$$R_1(\beta_1)\ R_2(\beta_2)\ R_3(\beta_3)[\underline{CTS}]^{I} \;=\; S^{II}\ N\ P\ R_1(\alpha_1)\ R_2(\alpha_2)\ R_3(\alpha_3)[\underline{CIS}]^{I}$$

After some reduction, neglecting second-order terms,

$$[\underline{CTS}]^{I} = R_1(-\beta_1 + \alpha_1 \cos\theta + \alpha_2 \sin\theta)\ R_2(-\beta_2 - \alpha_1 \sin\theta + \alpha_2 \cos\theta)\ \cdot$$
$$\cdot\ R_3(-\beta_3 + \alpha_3)\ S^{II}\ N\ P\ [\underline{CIS}]^{I}$$

Comparing the above equation with (A.3)

$$S^{I} = R_1(-\beta_1 + \alpha_1 \cos\theta + \alpha_2 \sin\theta)\ R_2(-\beta_2 - \alpha_1 \sin\theta + \alpha_2 \cos\theta)\ \cdot$$
$$\cdot\ R_3(-\beta_3 + \alpha_3)\ S^{II}$$

Or

$$-\Delta y_p = -(y_p^{I} - y_p^{II}) = -\beta_1 + \alpha_1 \cos\theta + \alpha_2 \sin\theta$$

$$-\Delta x_p = -(x_p^{I} - x_p^{II}) = -\beta_2 - \alpha_1 \sin\theta + \alpha_2 \cos\theta$$

$$\omega_e \, \Delta UT1 = \omega_e (UT1^{I} - UT1^{II}) = -\beta_3 + \alpha_3 \tag{A.5}$$

Thus the CTS differences (β angles) cause biases in all earth rotation parameters. Because of the modulation of the earth's diurnal rotation, the effect of CIS differences (α_1, α_2) on polar motion components are diurnal terms, while the effect of α_3 on UT1 is again a bias.

The direct way to determine all the β angles is the method of station collocation, i.e., to position two different types of techniques at the same location.

The "observation" equation is

$$\Delta \underline{x}_i = \underline{x}_i^{I} - \underline{x}_i^{II} = - \begin{vmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{vmatrix} + \begin{vmatrix} 0 & \beta_3 & -\beta_2 \\ -\beta_3 & 0 & \beta_1 \\ \beta_2 & -\beta_1 & 0 \end{vmatrix} \begin{vmatrix} x_i \\ y_i \\ z_i \end{vmatrix} + c \begin{vmatrix} x_i \\ y_i \\ z_i \end{vmatrix} + \underline{v}_i$$

where $\underline{x}_i^{I}$ and $\underline{x}_i^{II}$ are the determined coordinates of the same collocated station i in the two CTS's, $\underline{\delta}_i$ is the translation vector, and c is the scale difference. One must have at least three collocated stations if all seven unknowns are to be solved for.

For connecting the CIS's, there are a few methods such as the use of space astrometry to connect the stellar CIS and the radio source CIS, or using differential VLBI (which, for example, was used when the Viking Mars Orbiters and a quasar were near eclipsing) to connect the planetary and radio source CIS's (see [Kovalevsky and Mueller 1981]). These are direct approaches. One indirect method is via station collocation, i.e., using the earth as an intermediate body (see [Kovalevsky 1980]): First by station collocation one determines the CTS difference (β angles) as above, then through earth rotation parameter differences determined over the same time period one finds the CIS difference (α angles). Eq. (A.5) is the basis for connecting the two CIS's. More details on this subject may be found in [Mueller et al. 1982].

When considering the above method one should note that the diurnal polar motion difference terms in eq. (A.5) will show up as long as there are differences between the two CIS's (i.e., α_1 and α_2 exist). This may even be the case in situations when one (or both) of the techniques solve for rotations of its CIS, resulting in no (individual) diurnal polar motion. This, of course, would mean that the adopted precessional constant is discarded.

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2.2 Utilization of Range-Difference Observations in Geodynamics (Research Contract NAS5-25888)

2.21 Utilization of Simultaneous Lageos Range-Differences in Geodynamics

Introduction

The following is a summary of the research performed during the past six months under the Lageos project, dealing with the utilization of simultaneous laser range-differences (SRD) for the determination of earth orientation and baseline variations. Reported are some results from the Aug. 1980 Lageos data collected during the short MERIT campaign, and simulations for a possible station arrangement for the main campaign (to begin in 1983).

2.211 Simulations for a proposed MERIT83 laser network.

Based on an optimal global laser station distribution (likely to be realizable by mid-1983) proposed at a recent meeting of the study group (cf. GOTES proposal in last semiannual report), a simulation study for baseline recovery was performed. Except for the fact that different stations (seventeen total) are involved, this simulation was similar to the one previously reported for the MERIT80 network in the last report. The station locations and the data distribution are given in Table 1 [(a)-(b)]. Baseline estimates and their statistics were computed for both the range and the SRD adjustments. In order to assess the effect of orbital biases on the baseline recovery, the orbit used in the adjustments (range and SRD) was biased as follows :

Radial bias	:	2.00 m
Along track bias	:	0.60 m
Across track bias	:	-1.20 m

Two different adjustments were performed. In the first case the coordinates of all stations were obtained in a simultaneous adjustment based on the data collected from all baseline pairs. On the basis of this solution the baselines between all possible station combinations were obtained along with their formal accuracies and differences with respect to their "true" values. The results of this solution for the station coordinates are given in Table 2 for the range adjustment and in Table 3 for the SRD adjustment. The baseline results are shown in Table 4.

As it can be seen from the last table, in all cases except for two, the baseline lengths have been overestimated although the errors in the SRD case are about an order of magnitude smaller than the ones for the range adjustment. Since the radial bias results in an "expansion" of the network of satellite positions, this should come as no surprise. The stations have a global distribution and since the observations from all stations are adjusted simultaneously, their positions become interdependent and the aforementioned expansion affects all of them similarly.

The results of this first adjustment prompted us to test the recovery of baselines from individual adjustments. In this second case the data collected from each pair of stations are adjusted independently and the estimated baselines are only the ones defined by coobserving station pairs. The results of this second type solution are shown in Table 5. What is obvious again is that the SRD results are again superior to the range results for baselines and station coordinates alike. The quality of the results with respect to the latter is characterized by the norm $||X||$ of the six coordinate differences between the true and estimated positions of the stations defining each baseline.

The most interesting observation though in this solution is that on the basis of the same data, the range adjustment now underestimates the baselines and the recovery errors are all negative. For the SRD results, there seems to be no bias preference and those errors are rather randomly distributed and in almost all cases at the centimeter level. The three baselines for which the range adjustment has given better results than the SRD, all have lengths in excess of 7000 km and very few observations. As it has been previously reported the SRD mode is much more geometry dependent than the range mode, and as the results of this table show it admits of its limitations very eagerly (note the formal accuracies on those baselines !). Unlike the SRD mode, the formal accuracies for the range mode give no hint whatsoever as to the real accuracy of the results. Even though the recovery errors are of the order of a few decimeters in all cases, the reported σ's are hardly ever higher than 2 cm !

On the basis of these simulations one can conclude that the

Table 1 (a)

PARTICIPATING STATION COORDINATE LIST

STATION	#	LATITUDE	LONGITUDE	HEIGHT	X (M)	Y (M)	Z (M)	SIGMA (M)
FTDAVS	7086	30 38 0.0	256 3 0.0	1580.000	-1324510.442	-5332139.932	3231791.956	0.100
WETTZE	7914	49 9 0.0	12 53 0.0	0.0	4074613.305	931963.678	4801492.271	0.100
HAWAII	7120	20 42 0.0	203 44 0.0	0.0	-5464096.683	-2402363.153	2240358.273	0.100
STALAS	7063	39 1 0.0	283 10 0.0	0.0	1130304.818	-4831721.449	3993759.624	0.100
WESTFO	7091	42 36 46.518	288 30 22.720	67.400	1492212.742	-4458121.791	4296085.489	0.100
ONSALA	7095	57 0 0.0	13 0 0.0	0.0	3392750.872	783278.257	5325986.607	0.100
HERSTM	7911	51 0 0.0	0 0 0.0	0.0	4022035.768	0.0	4933550.635	0.100
GRAZ	7999	48 0 0.0	15 0 0.0	0.0	4130031.490	1106638.602	4716882.075	0.100
JAPAN	7935	35 0 0.0	142 0 0.0	0.0	-4121637.800	3220176.370	3637871.320	0.100
RICHMO	7069	25 36 47.130	279 37 4.200	100.000	961533.601	-5674186.968	2740519.741	0.100
QUINCY	7051	39 58 0.0	239 4 0.0	0.0	-2516274.896	-4198843.469	4075154.589	0.100
YARAGA	7090	-29 -3 0.0	115 21 0.0	0.0	-2389125.331	5042839.038	-3078750.728	0.100
CHILBO	7901	53 0 0.0	358 0 0.0	0.0	3844341.319	-134247.357	5070549.699	0.100
ORRORA	7943	-45 0 0.0	160 0 0.0	1000.000	-4245816.653	1545350.882	-4488060.975	0.100
DIONYS	7940	37 0 0.0	22 0 0.0	0.0	4728637.251	1910493.462	3817397.791	0.100
AREQUI	7907	-16 -28 0.0	288 30 0.0	0.0	1941330.115	-5802924.122	-1796312.986	0.100
GRASSE	7942	44 0 0.0	8 0 0.0	0.0	4550759.258	639567.505	4408696.973	0.100

(b) Distribution of Observations per Baseline

BASELINE

PASS#	B0608	B0815	B0602	B0510	B0416	B0116	B1016	B1001	B0411	B1101	B0311	B0301
1	3739	3310	3806	2248	512	497	858	2055	1972	2648	1475	925
TOTAL	3739	3310	3806	2248	512	497	858	2056	1972	2648	1475	925

BASELINE

PASS#	B1302	B0615	B1708	B0617	B0506	B0407	B1017	B0715	B1317	B1702	B0706	B1715
1	3601	2988	3730	3477	1297	1221	223	2942	3531	3734	3725	3248
TOTAL	3601	2988	3730	3477	1297	1221	223	2942	3531	3734	3725	3248

BASELINE

PASS#	B0104	B0316	B0501	B0314	B0309	B0914	B0912	B1214	B1203	B0911	*TOTAL*
1	2333	0	2174	48	671	0	183	1613	0	206	64991
TOTAL	2333	0	2174	48	671	0	183	1613	0	206	64991

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Table 2 Recovered Station Coordinates (Range Mode)

	X	Y	Z
STATION NO. : 7051	X	Y	Z
APRIORI ESTIMATE	-2516274.896042	-4198843.469479	4075154.588717
ADJUSTMENTS	-1.287283	-1.351438	1.822092
ADJUSTED POSITION	-2516276.183325	-4198844.820917	4075156.410809
STATION NO. : 7063	X	Y	Z
APRIORI ESTIMATE	1130304.817676	-4831721.449137	3993759.624496
ADJUSTMENTS	-0.267394	-1.937618	1.854899
ADJUSTED POSITION	1130304.550281	-4831723.386754	3993761.479395
STATION NO. : 7069	X	Y	Z
APRIORI ESTIMATE	961533.600910	-5674186.967561	2740519.740502
ADJUSTMENTS	-0.264610	-2.204586	1.526215
ADJUSTED POSITION	961533.336300	-5674189.172147	2740521.266717
STATION NO. : 7086	X	Y	Z
APRIORI ESTIMATE	-1324510.442373	-5332139.932091	3231791.055906
ADJUSTMENTS	-0.902349	-1.830874	1.691947
ADJUSTED POSITION	-1324511.344721	-5332141.762966	3231792.747852
STATION NO. : 7090	X	Y	Z
APRIORI ESTIMATE	-2389125.331291	5042839.037557	-3078750.728221
ADJUSTMENTS	-0.975459	1.621840	-1.359841
ADJUSTED POSITION	-2389126.306750	5042840.659397	-3078752.088062
STATION NO. : 7091	X	Y	Z
APRIORI ESTIMATE	1492212.741998	-4458121.790935	4296005.488571
ADJUSTMENTS	-0.037315	-1.836603	1.862885
ADJUSTED POSITION	1492212.704682	-4458123.627538	4296007.351456
STATION NO. : 7095	X	Y	Z
APRIORI ESTIMATE	3392750.871854	783278.256725	5325906.606633
ADJUSTMENTS	1.469007	-0.126506	2.043865
ADJUSTED POSITION	3392752.340862	783278.130219	5325908.650498
STATION NO. : 7120	X	Y	Z
APRIORI ESTIMATE	-5464096.682969	-2402363.153199	2240358.272655
ADJUSTMENTS	-2.035242	-0.395526	1.552528
ADJUSTED POSITION	-5464098.718211	-2402363.540725	2240359.825183
STATION NO. : 7901	X	Y	Z
APRIORI ESTIMATE	3844341.318863	-134247.357044	5070549.689834
ADJUSTMENTS	1.470786	-0.208830	1.992617
ADJUSTED POSITION	3844342.789649	-134247.565874	5070551.682451

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Table 2 (cont'd)

	X	Y	Z
STATION NO. : 7907			
APRIORI ESTIMATE	1941330.114913	-5802024.122161	-1796312.985770
ADJUSTMENTS	-0.233499	-2.273246	0.774573
ADJUSTED POSITION	1941329.881414	-5802026.395407	-1796312.211197
STATION NO. : 7911			
APRIORI ESTIMATE	4022035.767630	0.0	4933550.635358
ADJUSTMENTS	1.390632	-0.343076	2.103931
ADJUSTED POSITION	4022037.158262	-0.343076	4933552.739290
STATION NO. : 7914			
APRIORI ESTIMATE	4074613.304579	931963.678222	4801492.271034
ADJUSTMENTS	1.547344	-0.189697	2.059665
ADJUSTED POSITION	4074614.851923	931963.488525	4801494.330699
STATION NO. : 7935			
APRIORI ESTIMATE	-4121637.799587	3220176.370484	3637871.319704
ADJUSTMENTS	-1.710228	1.738149	1.791909
ADJUSTED POSITION	-4121639.509815	3220178.108633	3637873.111614
STATION NO. : 7940			
APRIORI ESTIMATE	4728637.250678	1910493.461735	3817397.791492
ADJUSTMENTS	1.733165	-0.080564	1.959695
ADJUSTED POSITION	4728638.983843	1910493.381171	3817399.751187
STATION NO. : 7942			
APRIORI ESTIMATE	4550759.258444	639567.504711	4408096.973499
ADJUSTMENTS	1.616610	-0.203136	2.022975
ADJUSTED POSITION	4550760.875054	639567.301575	4408098.996474
STATION NO. : 7943			
APRIORI ESTIMATE	-4245816.653287	1545350.881948	-4488060.975056
ADJUSTMENTS	-0.801772	2.071760	-1.331585
ADJUSTED POSITION	-4245817.455059	1545352.953708	-4488062.306640
STATION NO. : 7999			
APRIORI ESTIMATE	4130031.489874	1106638.602427	4716882.074958
ADJUSTMENTS	1.644699	-0.043508	1.998229
ADJUSTED POSITION	4130033.134572	1106638.558919	4716884.073187

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Table 3 Recovered Station Coordinates (SRD Mode)

	X	Y	Z
STATION NO. : 7051	X	Y	Z
APRIORI ESTIMATE	-2516274.896042	-4198843.469479	4075154.588717
ADJUSTMENTS	-0.562246	0.077679	0.552835
ADJUSTED POSITION	-2516275.458287	-4198843.391800	4075155.141552
STATION NO. : 7063	X	Y	Z
APRIORI ESTIMATE	1130304.817676	-4831721.449137	3993759.624496
ADJUSTMENTS	-0.412984	-0.339656	0.568841
ADJUSTED POSITION	1130304.404692	-4831721.788792	3993760.193337
STATION NO. : 7069	X	Y	Z
APRIORI ESTIMATE	961533.600910	-5674186.967561	2740519.740502
ADJUSTMENTS	-0.493799	-0.372649	0.504026
ADJUSTED POSITION	961533.107111	-5674187.340210	2740520.244528
STATION NO. : 7086	X	Y	Z
APRIORI ESTIMATE	-1324510.442373	-5332139.932091	3231791.055906
ADJUSTMENTS	-0.614059	-0.110826	0.512840
ADJUSTED POSITION	-1324511.056431	-5332140.042917	3231791.568746
STATION NO. : 7090	X	Y	Z
APRIORI ESTIMATE	-2389125.331291	5042839.037557	-3078750.728221
ADJUSTMENTS	0.287588	0.480930	0.222993
ADJUSTED POSITION	-2389125.043703	5042839.518487	-3078750.505229
STATION NO. : 7091	X	Y	Z
APRIORI ESTIMATE	1492212.741998	-4458121.790935	4296005.488571
ADJUSTMENTS	-0.348903	-0.347854	0.591585
ADJUSTED POSITION	1492212.393095	-4458122.138789	4296006.080157
STATION NO. : 7095	X	Y	Z
APRIORI ESTIMATE	3392750.871854	783278.256725	5325906.606633
ADJUSTMENTS	0.250456	-0.309695	0.704342
ADJUSTED POSITION	3392751.122310	783277.947029	5325907.310975
STATION NO. : 7120	X	Y	Z
APRIORI ESTIMATE	-5464096.682969	-2402363.153199	2240358.272655
ADJUSTMENTS	-0.589684	0.486807	0.474197
ADJUSTED POSITION	-5464097.272654	-2402362.666392	2240358.746853
STATION NO. : 7901	X	Y	Z
APRIORI ESTIMATE	3844341.318863	-134247.357044	5070549.689834
ADJUSTMENTS	0.175767	-0.397336	0.686721
ADJUSTED POSITION	3844341.494631	-134247.754380	5070550.376555

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Table 3 (cont'd)

STATION NO. : 7907	X	Y	Z
APRIORI ESTIMATE	1941330.114913	-5802024.122161	-1796312.985770
ADJUSTMENTS	-0.444374	-0.522985	0.256872
ADJUSTED POSITION	1941329.670539	-5802024.645146	-1796312.728899

STATION NO. : 7911	X	Y	Z
APRIORI ESTIMATE	4022035.767630	0.0	4933550.635358
ADJUSTMENTS	0.198181	-0.407657	0.673034
ADJUSTED POSITION	4022035.965812	-0.407657	4933551.308392

STATION NO. : 7914	X	Y	Z
APRIORI ESTIMATE	4074613.304579	931963.678222	4801492.271034
ADJUSTMENTS	0.301524	-0.373080	0.668115
ADJUSTED POSITION	4074613.606103	931963.305142	4801492.939149

STATION NO. : 7935	X	Y	Z
APRIORI ESTIMATE	-4121637.799587	3220176.370484	3637871.319704
ADJUSTMENTS	0.044690	0.580912	0.490394
ADJUSTED POSITION	-4121637.754897	3220176.951396	3637871.810098

STATION NO. : 7940	X	Y	Z
APRIORI ESTIMATE	4728637.250678	1910493.461735	3817397.791492
ADJUSTMENTS	0.444574	-0.387945	0.619332
ADJUSTED POSITION	4728637.695252	1910493.073790	3817398.410824

STATION NO. : 7942	X	Y	Z
APRIORI ESTIMATE	4550759.258444	639567.504711	4408096.973499
ADJUSTMENTS	0.296323	-0.433382	0.648499
ADJUSTED POSITION	4550759.554768	639567.071329	4408097.621998

STATION NO. : 7943	X	Y	Z
APRIORI ESTIMATE	-4245816.653287	1545350.881948	-4488060.975056
ADJUSTMENTS	-0.249487	0.494039	0.067337
ADJUSTED POSITION	-4245816.902774	1545351.375987	-4488060.907719

STATION NO. : 7999	X	Y	Z
APRIORI ESTIMATE	4130031.489874	1106638.602427	4716882.074958
ADJUSTMENTS	0.324886	-0.372003	0.668722
ADJUSTED POSITION	4130031.814759	1106638.230423	4716882.743680

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Table 4 Recovered Baselines and Associated Statistics

	Baseline for Stations	A Priori Length	Ranges			SRD's		
			Adjusted	Error	$\hat{\sigma}$	Adjusted	Error	$\hat{\sigma}$
1	7051 ==> 7063	3701986.397	3701987.501	1.104	0.014	3701986.615	0.218	0.009
2	7051 ==> 7069	4006624.584	4006625.884	1.300	0.014	4006624.825	0.241	0.010
3	7051 ==> 7086	1848222.236	1848222.838	0.602	0.012	1848222.336	0.100	0.006
4	7051 ==> 7090	11687738.184	11687742.486	4.302	0.025	11687738.714	0.530	0.076
5	7051 ==> 7091	4022930.960	4022932.239	1.279	0.014	4022931.203	0.242	0.010
6	7051 ==> 7095	7829591.498	7829594.393	2.895	0.010	7829591.889	0.391	0.018
7	7051 ==> 7120	3909408.182	3909409.312	1.130	0.020	3909408.428	0.246	0.011
8	7051 ==> 7901	7613750.166	7613753.103	2.936	0.013	7613750.547	0.380	0.017
9	7051 ==> 7907	7544174.051	7544175.685	1.634	0.037	7544174.479	0.428	0.022
10	7051 ==> 7911	7817713.002	7817715.815	2.812	0.012	7817713.391	0.389	0.017
11	7051 ==> 7914	8384065.582	8384068.542	2.960	0.011	8384065.995	0.413	0.018
12	7051 ==> 7935	7603305.998	7603309.103	3.106	0.037	7603306.364	0.366	0.029
13	7051 ==> 7940	9480463.508	9480466.631	3.123	0.012	9480463.975	0.468	0.019
14	7051 ==> 7942	8571116.796	8571119.846	3.050	0.011	8571117.219	0.423	0.018
15	7051 ==> 7943	10455416.980	10455421.363	4.383	0.014	10455417.555	0.575	0.078
16	7051 ==> 7999	8528384.534	8528387.646	3.112	0.011	8528384.954	0.420	0.018
17	7063 ==> 7069	1519487.440	1519487.859	0.419	0.012	1519487.521	0.081	0.006
18	7063 ==> 7086	2618612.747	2618613.369	0.622	0.013	2618612.908	0.161	0.007
19	7063 ==> 7090	12645700.300	12645705.074	4.774	0.023	12645700.939	0.639	0.076
20	7063 ==> 7091	601586.746	601586.951	0.205	0.013	601586.791	0.045	0.006
21	7063 ==> 7095	6198507.839	6198510.154	2.315	0.010	6198508.137	0.298	0.016
22	7063 ==> 7120	7243088.373	7243090.573	2.200	0.022	7243088.834	0.461	0.018
23	7063 ==> 7901	5530979.463	5530981.811	2.348	0.013	5530979.726	0.263	0.016
24	7063 ==> 7907	5926566.472	5926567.587	1.115	0.036	5926566.802	0.331	0.019
25	7063 ==> 7911	5708839.391	5708841.621	2.230	0.012	5708839.660	0.269	0.016
26	7063 ==> 7914	6522380.757	6522383.146	2.389	0.011	6522381.062	0.305	0.016
27	7063 ==> 7935	9619907.288	9619911.155	3.867	0.034	9619907.812	0.524	0.030
28	7063 ==> 7940	7644381.054	7644383.631	2.577	0.012	7644381.414	0.360	0.017
29	7063 ==> 7942	6465770.398	6465772.873	2.475	0.011	6465770.699	0.301	0.016
30	7063 ==> 7943	11895840.209	11895844.871	4.663	0.017	11895840.939	0.731	0.072
31	7063 ==> 7999	6692188.453	6692191.006	2.553	0.011	6692188.766	0.313	0.016
32	7069 ==> 7086	2363121.040	2363121.745	0.705	0.013	2363121.196	0.156	0.007
33	7069 ==> 7090	12646954.988	12646959.747	4.759	0.024	12646955.633	0.646	0.073
34	7069 ==> 7091	2044497.683	2044498.217	0.534	0.013	2044497.802	0.119	0.006
35	7069 ==> 7095	7368439.441	7368442.015	2.575	0.011	7368439.812	0.371	0.017
36	7069 ==> 7120	7227981.538	7227983.929	2.391	0.020	7227982.014	0.476	0.018
37	7069 ==> 7901	6665624.561	6665627.134	2.572	0.013	6665624.894	0.333	0.017
38	7069 ==> 7907	4643187.992	4643188.735	0.743	0.035	4643188.248	0.256	0.016
39	7069 ==> 7911	6809732.429	6809734.911	2.481	0.012	6809732.766	0.336	0.017
40	7069 ==> 7914	7588155.202	7588157.844	2.642	0.012	7588155.572	0.370	0.018
41	7069 ==> 7935	10283655.504	10283659.652	4.148	0.037	10283656.062	0.557	0.031
42	7069 ==> 7940	8536867.895	8536870.719	2.823	0.012	8536868.310	0.415	0.019
43	7069 ==> 7942	7451634.061	7451636.775	2.713	0.011	7451634.423	0.361	0.018
44	7069 ==> 7943	11466935.187	11466939.925	4.738	0.015	11466935.897	0.710	0.073
45	7069 ==> 7999	7741122.810	7741125.605	2.795	0.012	7741123.188	0.378	0.018
46	7086 ==> 7090	12190017.682	12190022.207	4.525	0.025	12190018.257	0.575	0.074
47	7086 ==> 7091	3135345.207	3135346.041	0.834	0.013	3135345.406	0.199	0.008
48	7086 ==> 7095	8002263.047	8002265.839	2.792	0.009	8002263.455	0.408	0.018
49	7086 ==> 7120	5167466.031	5167467.779	1.748	0.019	5167466.358	0.327	0.014
50	7086 ==> 7901	7557522.680	7557525.492	2.812	0.011	7557523.066	0.385	0.017

Table 4 (cont'd)

51	7086 ==> 7907	6014011.635	6014012.800	1.165	0.035	6014011.974	0.338	0.018
52	7086 ==> 7911	7740365.514	7740368.214	2.699	0.011	7740365.906	0.392	0.017
53	7086 ==> 7914	8417451.703	8417454.565	2.861	0.010	8417452.125	0.421	0.018
54	7086 ==> 7935	9007271.328	9007274.972	3.644	0.038	9007271.779	0.451	0.030
55	7086 ==> 7940	9457233.777	9457236.821	3.044	0.011	9457234.249	0.472	0.020
56	7086 ==> 7942	8459537.776	8459540.720	2.945	0.009	8459538.199	0.423	0.018
57	7086 ==> 7943	10743836.617	10743841.260	4.643	0.013	10743837.225	0.608	0.078
58	7086 ==> 7999	8568278.243	8568281.260	3.018	0.010	8568278.671	0.428	0.019
59	7090 ==> 7091	12638040.638	12638045.406	4.769	0.022	12638041.280	0.643	0.077
60	7090 ==> 7095	11054963.383	11054967.922	4.540	0.013	11054964.034	0.651	0.081
61	7090 ==> 7120	9652947.997	9652951.496	3.498	0.029	9652948.411	0.413	0.069
62	7090 ==> 7901	11492146.332	11492150.861	4.529	0.015	11492146.996	0.664	0.080
63	7090 ==> 7907	11747703.941	11747708.043	4.102	0.031	11747704.602	0.661	0.059
64	7090 ==> 7911	11433730.158	11433734.779	4.621	0.015	11433730.816	0.657	0.080
65	7090 ==> 7914	10989879.121	10989883.735	4.613	0.013	10989879.768	0.647	0.080
66	7090 ==> 7935	7171939.095	7171942.195	3.100	0.031	7171939.379	0.284	0.073
67	7090 ==> 7940	10393796.101	10393800.671	4.570	0.013	10393796.733	0.632	0.077
68	7090 ==> 7942	11117719.497	11117724.116	4.619	0.013	11117720.151	0.654	0.079
69	7090 ==> 7943	4203079.994	4203079.533	-0.461	0.042	4203080.272	0.279	0.039
70	7090 ==> 7999	10897934.223	10897938.794	4.571	0.013	10897934.872	0.649	0.080
71	7091 ==> 7095	5669657.481	5669659.600	2.119	0.011	5669657.738	0.257	0.015
72	7091 ==> 7120	7539368.002	7539370.323	2.321	0.022	7539368.484	0.482	0.019
73	7091 ==> 7901	4982802.192	4982804.336	2.145	0.013	4982802.411	0.220	0.015
74	7091 ==> 7907	6254928.000	6254929.140	1.140	0.036	6254928.357	0.357	0.019
75	7091 ==> 7911	5165396.234	5165398.253	2.018	0.012	5165396.461	0.226	0.014
76	7091 ==> 7914	5998110.612	5998112.791	2.179	0.012	5998110.876	0.264	0.015
77	7091 ==> 7935	9534396.737	9534400.606	3.869	0.032	9534397.260	0.523	0.030
78	7091 ==> 7940	7159802.313	7159804.669	2.356	0.012	7159802.634	0.321	0.016
79	7091 ==> 7942	5945898.372	5945900.626	2.254	0.011	5945898.632	0.260	0.015
80	7091 ==> 7943	12088278.997	12088283.622	4.625	0.017	12088279.749	0.752	0.070
81	7091 ==> 7999	6172664.180	6172666.525	2.345	0.012	6172664.452	0.271	0.016
82	7095 ==> 7120	9905183.912	9905187.285	3.373	0.020	9905184.478	0.567	0.026
83	7095 ==> 7901	1054037.162	1054037.247	0.085	0.010	1054037.210	0.049	0.005
84	7095 ==> 7907	9808100.813	9808103.428	2.615	0.034	9808101.384	0.571	0.025
85	7095 ==> 7911	1078641.514	1078641.604	0.090	0.010	1078641.566	0.052	0.004
86	7095 ==> 7914	872957.116	872957.157	0.041	0.010	872957.167	0.051	0.004
87	7095 ==> 7935	8077993.108	8077996.681	3.573	0.022	8077993.613	0.505	0.033
88	7095 ==> 7940	2308853.694	2308853.924	0.230	0.009	2308853.824	0.130	0.006
89	7095 ==> 7942	1484591.097	1484591.233	0.135	0.008	1484591.180	0.082	0.004
90	7095 ==> 7943	12459631.945	12459636.131	4.185	0.025	12459632.803	0.857	0.058
91	7095 ==> 7999	1009482.790	1009482.972	0.182	0.010	1009482.846	0.056	0.004
92	7120 ==> 7901	9990062.600	9990066.034	3.434	0.021	9990063.173	0.573	0.026
93	7120 ==> 7907	9093555.719	9093558.233	2.515	0.035	9093556.311	0.592	0.028
94	7120 ==> 7911	10149450.364	10149453.725	3.361	0.021	10149450.941	0.577	0.026
95	7120 ==> 7914	10424208.891	10424212.360	3.469	0.020	10424209.479	0.588	0.026
96	7120 ==> 7935	5947116.046	5947118.193	2.147	0.045	5947116.282	0.236	0.028
97	7120 ==> 7940	11179428.021	11179431.635	3.615	0.019	11179428.647	0.626	0.027
98	7120 ==> 7942	10688768.532	10688772.104	3.572	0.020	10688769.136	0.604	0.026
99	7120 ==> 7943	7895585.976	7895589.858	3.882	0.016	7895586.379	0.403	0.078
100	7120 ==> 7999	10511591.643	10511595.224	3.581	0.020	10511592.237	0.594	0.027

Table 4 (cont'd)

101	7901 ==>	7907	9104885.947	9104888.507	2.560	0.034	9104886.479	0.532	0.025
102	7901 ==>	7911	261469.713	261469.531	-0.182	0.013	261469.730	0.017	0.006
103	7901 ==>	7914	1123487.006	1123487.024	0.018	0.011	1123487.060	0.053	0.004
104	7901 ==>	7935	8761366.855	8761370.526	3.670	0.023	8761367.381	0.526	0.033
105	7901 ==>	7940	2556038.730	2556038.940	0.210	0.012	2556038.864	0.134	0.007
106	7901 ==>	7942	1239620.646	1239620.717	0.070	0.011	1239620.713	0.067	0.005
107	7901 ==>	7943	12634822.678	12634826.951	4.273	0.025	12634823.537	0.859	0.059
108	7901 ==>	7999	1321551.217	1321551.409	0.191	0.012	1321551.278	0.061	0.005
109	7907 ==>	7911	9126000.454	9126003.032	2.578	0.034	9126000.981	0.527	0.025
110	7907 ==>	7914	9665843.112	9665845.834	2.722	0.033	9665843.662	0.550	0.026
111	7907 ==>	7935	12152779.136	12152783.306	4.170	0.053	12152779.816	0.680	0.037
112	7907 ==>	7940	9938096.258	9938099.181	2.923	0.031	9938096.817	0.559	0.027
113	7907 ==>	7942	9316540.457	9316543.238	2.781	0.032	9316540.987	0.530	0.026
114	7907 ==>	7943	9975480.531	9975484.652	4.121	0.023	9975481.210	0.679	0.064
115	7907 ==>	7999	9743805.430	9743808.250	2.821	0.033	9743805.985	0.555	0.026
116	7911 ==>	7914	942740.742	942740.909	0.167	0.010	942740.783	0.041	0.005
117	7911 ==>	7935	8852555.558	8852559.214	3.655	0.023	8852556.086	0.528	0.033
118	7911 ==>	7940	2322728.588	2322728.978	0.389	0.011	2322728.705	0.117	0.006
119	7911 ==>	7942	982189.734	982189.990	0.256	0.010	982189.783	0.049	0.005
120	7911 ==>	7943	12629816.243	12629820.537	4.294	0.025	12629817.099	0.855	0.058
121	7911 ==>	7999	1132809.489	1132809.826	0.337	0.011	1132809.536	0.048	0.005
122	7914 ==>	7935	8588856.921	8588860.579	3.659	0.022	8588857.444	0.523	0.033
123	7914 ==>	7940	1534180.499	1534180.712	0.213	0.010	1534180.582	0.083	0.005
124	7914 ==>	7942	683352.290	683352.365	0.075	0.009	683352.323	0.033	0.003
125	7914 ==>	7943	12486056.145	12486060.345	4.200	0.026	12486057.002	0.857	0.056
126	7914 ==>	7999	201844.964	201845.143	0.179	0.010	201844.972	0.007	0.004
127	7935 ==>	7940	8948456.154	8948459.829	3.675	0.021	8948456.694	0.540	0.033
128	7935 ==>	7942	9080928.389	9080932.138	3.748	0.022	9080928.931	0.542	0.033
129	7935 ==>	7943	8297664.519	8297667.497	2.978	0.047	8297664.955	0.436	0.044
130	7935 ==>	7999	8586113.915	8586117.604	3.689	0.022	8586114.442	0.526	0.034
131	7940 ==>	7942	1412734.544	1412734.695	0.151	0.009	1412734.616	0.072	0.004
132	7940 ==>	7943	12233347.755	12233351.785	4.030	0.028	12233348.613	0.858	0.053
133	7940 ==>	7999	1346693.532	1346693.574	0.043	0.010	1346693.608	0.077	0.004
134	7942 ==>	7943	12543596.675	12543600.914	4.239	0.026	12543597.537	0.862	0.056
135	7942 ==>	7999	700368.121	700368.199	0.079	0.009	700368.153	0.033	0.003
136	7943 ==>	7999	12453042.887	12453047.068	4.181	0.026	12453043.748	0.861	0.056

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Table 5 Baseline Recovery from Individual Adjustments

BASELINE FOR STATIONS			APRIORI LENGTH	RANGE ADJUSTMENT				SRD ADJUSTMENT			
				#OBS	‖X‖	ERROR	$\hat{\sigma}$	#OBS	‖X‖	ERROR	$\hat{\sigma}$
7901	==>	7914	1123487.006	7202	3.572	-0.117	0.017	3601	1.959	-0.005	0.018
7095	==>	7940	2308853.694	5976	3.469	-0.362	0.019	2988	1.712	-0.047	0.026
7942	==>	7999	700366.121	7460	3.730	-0.096	0.019	3730	1.196	0.010	0.016
7095	==>	7942	1434591.097	6954	3.582	-0.228	0.020	3477	1.371	0.004	0.020
7091	==>	7095	5009057.481	2594	2.848	-0.973	0.027	1297	0.454	0.413	0.326
7063	==>	7911	5700639.391	2442	2.865	-1.076	0.033	1221	0.445	0.323	0.279
7069	==>	7942	7451634.061	446	2.605	-1.397	0.165	223	0.591	0.311	5.1[illegible]3
7911	==>	7940	2322728.588	5884	3.518	-0.382	0.020	2942	1.619	-0.038	0.030
7901	==>	7942	1239620.046	7062	3.631	-0.187	0.020	3531	1.471	-0.006	0.020
7942	==>	7914	683352.290	7468	3.728	-0.091	0.019	3734	1.030	-0.001	0.015
7911	==>	7095	1078641.514	7450	3.594	-0.153	0.020	3725	1.541	-0.009	0.017
7942	==>	7940	1412734.544	6496	3.684	-0.207	0.020	3249	1.576	-0.008	0.021
7095	==>	7999	1009482.790	7478	3.618	-0.158	0.020	3739	1.261	0.002	0.017
7999	==>	7940	1346093.532	6620	3.671	-0.194	0.019	3310	1.392	0.006	0.019
7095	==>	7914	672957.116	7612	3.629	-0.122	0.020	3806	1.074	0.025	0.016
7091	==>	7069	2044497.683	4496	3.544	-0.305	0.022	2248	1.336	0.010	0.029
7063	==>	7907	5926566.472	1024	2.809	-1.165	0.057	512	0.888	0.059	0.483
7086	==>	7907	6014011.635	994	2.798	-1.264	0.060	497	1.023	-0.085	0.027
7069	==>	7907	4643167.992	1716	3.052	-0.864	0.037	858	0.766	0.168	0.175
7069	==>	7086	2303121.040	4112	3.552	-0.375	0.025	2056	1.250	0.041	0.036
7063	==>	7051	3701936.397	3944	3.264	-0.006	0.025	1972	1.150	0.038	0.069
7051	==>	7086	1848222.236	5296	3.641	-0.388	0.023	2648	1.145	0.020	0.027
7120	==>	7051	3909463.182	2950	3.220	-0.724	0.030	1475	1.366	-0.035	0.065
7120	==>	7080	5167465.031	1850	2.968	-0.991	0.040	925	0.687	0.249	0.220
7086	==>	7063	2513612.747	4666	3.500	-0.452	0.025	2533	0.925	0.090	0.039
7091	==>	7086	3135345.207	4348	3.396	-0.569	0.026	2174	1.520	-0.041	0.050
7120	==>	7935	3947116.046	1342	2.836	-1.211	0.052	671	1.570	-0.302	0.443
7935	==>	7090	7171959.095	366	2.609	-1.008	0.155	183	7.275	4.760	3.401
7090	==>	7943	4203079.994	3226	3.160	-0.611	0.029	1613	1.428	-0.069	0.096
7935	==>	7051	7603305.998	412	2.582	-1.458	0.202	206	10.245	7.029	8.130

SRD mode will in all likelihood provide more meaningful results in the presence of unmodeled orbital biases than the range mode, and it will also give more reliable accuracy estimates for those results. Comparing the batch (global) solution to that of individual adjustments, the latter seems to be by far a better approach in the case of SRD observations, although the opposite is true for the range observations. Compare for instance the level of recovery errors between Tables 4 and 5.

2.212 Preliminary Results from Lageos Data Analysis.

Lageos ranging data collected from ten stations over the period August 14-29,1980 (during the short MERIT campaign) were used in GEOSPP81 for baseline recovery. A total of 24240 ranges were selected with effort to balance the distribution among stations whose observability performance shows wild variations (cf. station 7090 with over 60000 ranges during August, and station 7092 with hardly over 3000 in the same period of time). The summary of the data distribution per pass per station is given in Table 6 [(a)-(j)]. The ill-conditioning of the normal equations due to the lack of origin of longitudes definition is overcome by applying a small weight in all three coordinates of all stations, corresponding to a $\sigma=\pm 50$ m. This way the origin of longitudes does not depend on a single station but rather the ensemble of them. The separation of the X and Y coordinates is thus not as good as it would be if one longitude were fixed absolutely, but that has no effect on the baselines. This high correlation between X and Y is also reflected in the estimated

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formal accuracies for these coordinates, Table 7. The orbital model and the constants used in the solution are shown in Table 8. Baseline results of the adjustment are given in Table 9, and an analytical breakdown of the residuals after the adjustment are given in Table 10 [(a)-(j)]. Notice that the fact that station 7907 (ARELAS) is the one with the fewest observations (only 489) shows very clearly in the estimation of baselines which emanate from that station (Table 9).

Care should be taken in comparing these results with other solutions for the fact that these baselines are reckoned between the optical centers of the corresponding laser instruments and not the stations' validation points.

This investigation is now being completed, and the final report is in preparation by E. Pavlis, to appear in the report series of the Department of Geodetic Science and Surveying, The Ohio State University.

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Table 6 (a)

O B S E R V A T I O N   S U M M A R Y

STATION IDENTIFICATION NO. : 7063   NUMBER OF PASSES TRACKED : 10   NUMBER OF OBSERVATIONS : 4284

PASS NO.	BEGINNING DATE YYMMDD HHMMSS.S		ENDING DATE YYMMDD HHMMSS.S		DURATION SECONDS	OBSERVATIONS	DENSITY LAG X (ONE POINT PER X SECS)
1	800814	1611 4.0	800814	163236.0	1292.0	4	323.00
2	800820	814 6.0	800820	84127.0	1641.0	477	3.44
3	800822	90040.0	800822	92534.0	1494.0	202	7.40
4	800822	122631.0	800822	125440.0	1689.0	6	281.50
5	800825	82148.0	800825	901 6.0	2358.0	859	2.75
6	800825	153110.0	800825	153110.0	0.0	1	0.0
7	800827	90532.0	800827	95222.0	2810.0	1550	1.81
8	800827	160010.0	800827	162513.0	1503.0	4	375.75
9	800827	192326.0	800827	200556.0	2550.0	14	182.14
10	800828	74317.0	800828	82441.0	2484.0	1167	2.13

(b)

O B S E R V A T I O N   S U M M A R Y

STATION IDENTIFICATION NO. : 7090   NUMBER OF PASSES TRACKED : 30   NUMBER OF OBSERVATIONS : 4143

PASS NO.	BEGINNING DATE YYMMDD HHMMSS.S		ENDING DATE YYMMDD HHMMSS.S		DURATION SECONDS	OBSERVATIONS	DENSITY LAG X (ONE POINT PER X SECS)
1	800814	73524.0	800814	81120.0	2156.0	97	22.23
2	800814	110144.0	800814	1138 6.0	2182.0	167	13.07
3	800814	1735 8.0	800814	1817 7.0	2519.0	182	13.84
4	800814	210657.0	800814	2148 8.0	2471.0	207	11.94
5	800815	93831.0	800815	102222.0	2631.0	196	13.42
6	800815	162235.0	800815	165245.0	1810.0	141	12.84
7	800815	1941 5.0	800815	202838.0	2853.0	263	10.85
8	800818	91445.0	800818	94842.0	2037.0	119	17.12
9	800818	124141.0	800818	1259 1.0	1040.0	67	15.52
10	800818	190634.0	800818	1953 1.0	2787.0	171	16.30
11	800819	111532.0	800819	114928.0	2036.0	136	14.97
12	800819	175015.0	800819	183136.0	2481.0	203	12.22
13	800819	212421.0	800819	213355.0	574.0	50	11.48
14	800820	629 6.0	800820	65033.0	1287.0	29	44.38
15	800820	200224.0	800820	204123.0	2339.0	136	17.20
16	800821	83132.0	800821	91537.0	2645.0	104	25.43
17	800822	72745.0	800822	74943.0	1318.0	55	23.96
18	800822	104041.0	800822	111838.0	2277.0	162	14.06
19	800822	171526.0	800822	175657.0	2491.0	173	14.40
20	800822	204756.0	800822	2126 5.0	2409.0	136	17.71
21	800826	84746.0	800826	928 7.0	2421.0	155	15.62
22	800826	122016.0	800826	123826.0	1090.0	41	26.59
23	800826	1534 1.0	800826	155759.0	1438.0	88	16.34
24	800826	1847 5.0	800826	193427.0	2842.0	233	12.20
25	800827	105652.0	800827	112919.0	1947.0	115	16.93
26	800827	143042.0	800827	143215.0	93.0	6	15.50
27	800827	172746.0	800827	181144.0	2638.0	189	13.96
28	800827	210429.0	800827	213938.0	2109.0	154	13.69
29	800828	161247.0	800828	164735.0	2088.0	154	13.56
30	800828	1937 5.0	800828	2022 5.0	2700.0	214	12.62

(c)

O B S E R V A T I O N   S U M M A R Y

STATION IDENTIFICATION NO. : 7091   NUMBER OF PASSES TRACKED : 4   NUMBER OF OBSERVATIONS : 1168

PASS NO.	BEGINNING DATE YYMMDD HHMMSS.S		ENDING DATE YYMMDD HHMMSS.S		DURATION SECONDS	OBSERVATIONS	DENSITY LAG X (ONE POINT PER X SECS)
1	800815	181925.0	800815	183830.0	1145.0	137	8.36
2	800818	72153.0	800818	75031.0	1718.0	352	4.88
3	800826	65525.0	800826	72644.0	1879.0	240	7.83
4	800826	103656.0	800826	110941.0	1965.0	439	4.48

(d)

O B S E R V A T I O N   S U M M A R Y

STATION IDENTIFICATION NO. : 7092   NUMBER OF PASSES TRACKED : 5   NUMBER OF OBSERVATIONS : 2253

PASS NO.	BEGINNING DATE YYMMDD HHMMSS.S		ENDING DATE YYMMDD HHMMSS.S		DURATION SECONDS	OBSERVATIONS	DENSITY LAG X (ONE POINT PER X SECS)
1	800823	153228.0	800823	160149.0	1761.0	322	5.47
2	800824	141353.0	800824	1433 6.0	1153.0	286	4.03
3	800824	173125.0	800824	1811 5.0	2380.0	1273	1.87
4	800825	1613 4.0	800825	165055.0	2271.0	363	6.26
5	800826	150956.0	800826	151250.0	174.0	9	19.33

Table 6 (cont'd)

(e)

OBSERVATION SUMMARY

STATION IDENTIFICATION NO. : 7096 NUMBER OF PASSES TRACKED : 6 NUMBER OF OBSERVATIONS : 2450

PASS NO.	BEGINNING DATE YYMMDD HHMMSS.S		ENDING DATE YYMMDD HHMMSS.S		DURATION SECONDS	OBSERVATIONS	DENSITY LAG X (ONE POINT PER X SECS)
1	800817	130510.0	800817	1345 7.0	2389.0	969	2.47
2	800818	150931.0	800818	1543 0.0	2009.0	461	4.36
3	800819	141549.0	800819	143418.0	1109.0	266	4.16
4	800820	62538.0	800820	63630.0	652.0	91	7.16
5	800827	35759.0	800827	41323.0	924.0	45	20.53
6	800827	135146.0	800827	141436.0	1366.0	616	2.22

(f)

OBSERVATION SUMMARY

STATION IDENTIFICATION NO. : 7114 NUMBER OF PASSES TRACKED : 11 NUMBER OF OBSERVATIONS : 1866

PASS NO.	BEGINNING DATE YYMMDD HHMMSS.S		ENDING DATE YYMMDD HHMMSS.S		DURATION SECONDS	OBSERVATIONS	DENSITY LAG X (ONE POINT PER X SECS)
1	800820	1148 6.0	800820	120748.0	1182.0	182	6.49
2	800820	2210 3.0	800820	222859.0	1136.0	17	66.82
3	800821	102434.0	800821	110649.0	2535.0	855	2.96
4	800821	205140.0	800821	211427.0	1367.0	9	151.89
5	800822	1234 9.0	800822	125059.0	1010.0	161	6.27
6	800825	115645.0	800825	123348.0	2103.0	390	5.39
7	800826	210644.0	800826	212131.0	887.0	6	147.83
8	800827	92627.0	800827	94352.0	1045.0	228	4.58
9	800827	125149.0	800827	1303 5.0	676.0	7	96.57
10	800827	194936.0	800827	200756.0	1080.0	7	154.29
11	800827	230754.0	800827	231434.0	400.0	4	100.00

(g)

OBSERVATION SUMMARY

STATION IDENTIFICATION NO. : 7115 NUMBER OF PASSES TRACKED : 12 NUMBER OF OBSERVATIONS : 2265

PASS NO.	BEGINNING DATE YYMMDD HHMMSS.S		ENDING DATE YYMMDD HHMMSS.S		DURATION SECONDS	OBSERVATIONS	DENSITY LAG X (ONE POINT PER X SECS)
1	800814	131518.0	800814	133542.0	1224.0	264	4.64
2	800814	195338.0	800814	200529.0	711.0	29	24.52
3	800815	1143 3.0	800815	121624.0	2001.0	384	5.21
4	800815	215545.0	800815	221246.0	1021.0	27	37.81
5	800818	110623.0	800818	114414.0	2271.0	500	4.54
6	800818	143855.0	800818	150048.0	1313.0	38	34.55
7	800818	211928.0	800818	214154.0	1346.0	172	7.83
8	800819	95759.0	800819	101248.0	889.0	63	14.11
9	800819	131239.0	800819	134432.0	1913.0	119	16.08
10	800820	114827.0	800820	123354.0	2727.0	568	4.80
11	800821	140436.0	800821	143124.0	1608.0	37	43.46
12	800822	1244 7.0	800822	125459.0	652.0	44	14.82

(h)

OBSERVATION SUMMARY

STATION IDENTIFICATION NO. : 7120 NUMBER OF PASSES TRACKED : 10 NUMBER OF OBSERVATIONS : 1904

PASS NO.	BEGINNING DATE YYMMDD HHMMSS.S		ENDING DATE YYMMDD HHMMSS.S		DURATION SECONDS	OBSERVATIONS	DENSITY LAG X (ONE POINT PER X SECS)
1	800814	131956.0	800814	135240.0	1964.0	225	8.73
2	800814	165838.0	800814	171227.0	829.0	44	18.84
3	800815	25247.0	800815	31424.0	1297.0	160	8.11
4	800815	121219.0	800815	122237.0	618.0	42	14.71
5	800815	152236.0	800815	160610.0	2014.0	187	13.96
6	800821	141436.0	800821	150035.0	2759.0	346	7.97
7	800822	125343.0	800822	133636.0	2573.0	401	6.42
8	800822	163825.0	800822	165250.0	865.0	50	17.30
9	800825	155656.0	800825	162431.0	1655.0	121	13.68
10	800828	1519 7.0	800828	155924.0	2417.0	328	7.37

Table 6 (cont'd)

(i) OBSERVATION SUMMARY

STATION IDENTIFICATION NO. : 7907   NUMBER OF PASSES TRACKED : 20   NUMBER OF OBSERVATIONS : 469

PASS NO.	BEGINNING YYMMDD	DATE HHMMSS.S	ENDING YYMMDD	DATE HHMMSS.S	DURATION SECONDS	OBSERVATIONS	DENSITY LAG X (ONE POINT PER X SECS)
1	800814	62230.1	800814	62722.5	292.4	12	24.37
2	800814	955 7.4	800814	1014 0.3	1133.0	41	27.63
3	800815	82930.0	800815	84352.5	862.5	51	16.91
4	800816	70052.5	800816	718 0.2	1027.8	19	54.09
5	800816	234237.7	800816	234837.8	360.2	5	72.03
6	800817	92152.3	800817	93645.2	892.9	52	17.17
7	800818	75452.3	800818	80552.6	660.3	24	27.51
8	800819	633 7.5	800819	64315.1	607.6	19	31.96
9	800819	101022.6	800819	102945.2	1162.6	35	33.22
10	800820	84452.5	800820	900 7.4	914.9	53	17.26
11	800821	71730.1	800821	73337.6	967.5	34	28.46
12	800822	60230.0	800822	60830.1	360.1	5	72.02
13	800822	93315.1	800822	95145.1	1110.0	24	46.25
14	800823	80922.6	800823	82052.6	690.0	17	40.59
15	800824	65237.6	800824	65937.6	420.0	8	52.50
16	800825	52522.5	800825	533 7.6	465.0	9	51.67
17	800825	90122.3	800825	91530.3	848.0	22	38.54
18	800827	95015.0	800827	1007 7.5	1012.5	28	36.16
19	800828	45537.5	800828	457 7.5	90.0	2	45.00
20	800828	82622.6	800828	83922.6	780.0	29	26.90

(j) OBSERVATION SUMMARY

STATION IDENTIFICATION NO. : 7943   NUMBER OF PASSES TRACKED : 32   NUMBER OF OBSERVATIONS : 3416

PASS NO.	BEGINNING YYMMDD	DATE HHMMSS.S	ENDING YYMMDD	DATE HHMMSS.S	DURATION SECONDS	OBSERVATIONS	DENSITY LAG X (ONE POINT PER X SECS)
1	800814	1101 0.2	800814	112015.2	1154.9	56	20.62
2	800815	92745.0	800815	100630.3	2325.3	122	19.06
3	800815	124959.9	800815	132145.0	1905.1	82	23.23
4	800815	1603 7.7	800815	1641 0.0	2272.3	98	23.19
5	800815	1945 7.6	800815	200952.4	1484.7	83	17.89
6	800816	81544.9	800816	84152.7	1567.8	85	18.44
7	800816	114022.8	800816	1204 7.8	1425.0	90	15.83
8	800816	144522.7	800816	1531 7.7	2745.0	212	12.95
9	800816	181515.0	800816	190345.0	2910.0	159	18.30
10	800817	101652.8	800817	1049 7.7	1934.9	47	41.17
11	800817	134030.4	800817	140545.0	1514.7	54	28.05
12	800817	170115.0	800817	1728 0.0	1605.0	75	21.40
13	800818	90422.7	800818	93445.2	1822.5	139	13.11
14	800818	1217 0.2	800818	124837.7	1897.5	100	18.97
15	800818	153015.1	800818	161930.1	2955.0	170	17.38
16	800818	191822.5	800818	194952.5	1890.0	106	17.83
17	800819	110015.2	800819	113215.1	1919.9	90	21.33
18	800820	125830.2	800820	133437.7	2167.5	44	49.26
19	800821	1507 7.7	800821	154515.2	2287.5	79	28.96
20	800821	183030.1	800821	191159.9	2489.8	178	13.99
21	800822	102622.7	800822	105115.2	1492.5	38	39.28
22	800822	134252.8	800822	141745.3	2092.5	88	23.78
23	800822	170915.1	800822	175822.4	2827.3	172	16.44
24	800823	92930.2	800823	940 0.1	629.9	73	8.63
25	800825	131730.2	800825	134430.1	1619.9	67	24.18
26	800825	163237.7	800825	165937.4	1619.7	96	16.87
27	800826	84152.6	800826	91352.5	1919.9	163	11.78
28	800826	115522.7	800826	122837.7	1995.0	191	10.44
29	800826	150922.7	800826	155922.5	2999.8	262	11.45
30	800828	92537.5	800828	958 0.2	1942.7	97	20.03
31	800828	1238 0.0	800828	131415.1	2175.1	61	35.66
32	800828	155722.7	800828	162230.1	1507.4	41	36.77

Table 7   A Priori Station Information and Final Solution Summary

APRIORI STATION INFORMATION :

STATION #	X (M)	Y (M)	Z (M)
7063	1130711.7	-4831371.3	3994088.7
7090	-2389002.3	5043332.5	-3078528.5
7091	1492450.9	-4457261.7	4296817.0
7092	-6143448.5	1364706.9	1034164.6
7096	-6100049.6	-996197.8	-1568978.3
7114	-2410426.2	-4477802.2	3838686.1
7115	-2350667.4	-4655546.1	3660999.2
7120	-5466003.7	-2404404.3	2242228.6
7907	1942786.1	-5804078.9	-1796938.6
7943	-4447545.7	2677137.8	-3694998.0

APRIORI WEIGHT MTX. :

```
STATION#  1                              STATION#  2                              STATION#  3
  0.40000-03                               0.40000-03                               0.40000-03
  0.0        0.40000-03                    0.0        0.40000-03                    0.0        0.40000-03
  0.0        0.0        0.40000-03         0.0        0.0        0.40000-03         0.0        0.0        0.40000-03

STATION#  4                              STATION#  5                              STATION#  6
  0.40000-03                               0.40000-03                               0.40000-03
  0.0        0.40000-03                    0.0        0.40000-03                    0.0        0.40000-03
  0.0        0.0        0.40000-03         0.0        0.0        0.40000-03         0.0        0.0        0.40000-03

STATION#  7                              STATION#  8                              STATION#  9
  0.40000-03                               0.40000-03                               0.40000-03
  0.0        0.40000-03                    0.0        0.40000-03                    0.0        0.40000-03
  0.0        0.0        0.40000-03         0.0        0.0        0.40000-03         0.0        0.0        0.40000-03

STATION#  10
  0.40000-03
  0.0        0.40000-03
  0.0        0.0        0.40000-03
```

ADJUSTMENT STATISTICS FOR ITERATION : 2

```
DEGREES OF FREEDOM FOR THIS ADJUSTMENT......................: 24234
PREVIOUS WEIGHTED SUM OF SQUARES OF THE RESIDUALS/D.F......:     0.1760
CURRENT  WEIGHTED SUM OF SQUARES OF THE RESIDUALS/D.F......:     0.1760
IMPROVEMENT IN PERCENT (NEG.SIGN INDICATES DETERIORATION)..:     0
CONTRIBUTION FROM STATION PARAMETER CONSTRAINTS............:  1745.D-04
CONTRIBUTION FROM POLAR MOTION PARAMETER CONSTRAINTS.......:  5508.D-12
CONTRIBUTION FROM ORBITAL PARAMETER CONSTRAINTS............:     0.
```

PASS BY PASS BREAKDOWN OF ADJUSTMENT STATISTICS FOR ITERATION : 2

PASS NO.	CONSTRAINTS FROM : STATIONS	P.M.STEPS	ORBIT	TOTAL NO.OF CONSTRAINTS	NUMBER OF OBSERVATIONS	NUMBER OF PARAMETERS	WEIGHTED SS. OF RESIDUALS	DEGREES OF FREEDOM	VARIANCE COMPONENT	WITHIN OBS.-6	RMS OF THE RESIDUALS	MEAN OF RESIDUALS
1	30	6	0	36	24240	42	0.42660D+04	24234	0.1761	0.18	0.4195	-0.0009

```
TOTAL NUMBER OF OBSERVATIONS..:   24240
ARITHMETIC MEAN OF RESIDUALS..: -0.0009
RMS OF ALL RESIDUALS..........:  0.4195
```

Table 7 (cont'd)

FINAL RESULTS FOR STATION # : 7063

	X (M)	Y (M)	Z (M)
APRIORI ESTIMATE :	1130708.42204	-4831371.26038	3994088.93494
CURRENT ADJUSTMENT :	0.02297	0.00509	-0.00009
TOTAL ADJUSTMENT :	-3.25499	0.03870	0.23484
ADJUSTED ESTIMATE :	1130708.44501	-4831371.26130	3994088.93484
STANDARD DEVIATION :	5.72794	1.34052	0.03958

FINAL RESULTS FOR STATION # : 7092

	X (M)	Y (M)	Z (M)
APRIORI ESTIMATE :	-6143446.15561	1364709.14351	1034165.06444
CURRENT ADJUSTMENT :	-0.00637	-0.02864	-0.00007
TOTAL ADJUSTMENT :	0.33802	2.21467	-1.73563
ADJUSTED ESTIMATE :	-6143446.16196	1364709.11487	1034165.06437
STANDARD DEVIATION :	1.61296	7.28384	0.64074

FINAL RESULTS FOR STATION # : 7090

	X (M)	Y (M)	Z (M)
APRIORI ESTIMATE :	-2388999.98444	5043334.84229	-3078527.61217
CURRENT ADJUSTMENT :	-0.02355	-0.01114	-0.00007
TOTAL ADJUSTMENT :	2.28901	1.34315	0.93176
ADJUSTED ESTIMATE :	-2389000.00799	5043334.83115	-3078527.61224
STANDARD DEVIATION :	5.97927	2.63255	0.03542

FINAL RESULTS FOR STATION # : 7096

	X (M)	Y (M)	Z (M)
APRIORI ESTIMATE :	-6100049.90984	-996195.17401	-1568977.31336
CURRENT ADJUSTMENT :	0.00456	-0.02844	-0.00006
TOTAL ADJUSTMENT :	-0.38128	2.62854	1.00356
ADJUSTED ESTIMATE :	-6100049.90528	-996195.20246	-1568977.31342
STANDARD DEVIATION :	1.18123	7.23222	0.04062

FINAL RESULTS FOR STATION # : 7091

	X (M)	Y (M)	Z (M)
APRIORI ESTIMATE :	1492448.19366	-4457282.67104	4296817.84539
CURRENT ADJUSTMENT :	0.02069	0.00677	-0.00015
TOTAL ADJUSTMENT :	-2.68505	-0.96427	0.84524
ADJUSTED ESTIMATE :	1492448.21435	-4457282.66427	4296817.84524
STANDARD DEVIATION :	5.28441	1.76962	0.04135

FINAL RESULTS FOR STATION # : 7114

	X (M)	Y (M)	Z (M)
APRIORI ESTIMATE :	-2410430.13262	-4477801.27485	3638688.97676
CURRENT ADJUSTMENT :	0.02092	-0.01121	-0.00004
TOTAL ADJUSTMENT :	-1.91570	0.93494	0.90572
ADJUSTED ESTIMATE :	-2410430.11170	-4477801.28606	3638688.97672
STANDARD DEVIATION :	5.30866	2.85773	0.03962

Table 7 (cont'd)

FINAL RESULTS FOR STATION # : 7115

	X (M)	Y (M)	Z (M)
APRIORI ESTIMATE :	-2350869.23703	-4655545.15656	3660999.85808
CURRENT ADJUSTMENT :	0.02109	-0.01091	0.00002
TOTAL ADJUSTMENT :	-1.85894	0.92454	0.83010
ADJUSTED ESTIMATE :	-2350869.21594	-4655545.16746	3660999.85810
STANDARD DEVIATION :	5.51947	2.76733	0.03718

FINAL RESULTS FOR STATION # : 7907

	X (M)	Y (M)	Z (M)
APRIORI ESTIMATE :	1942783.20050	-5804080.21407	-1796919.71300
CURRENT ADJUSTMENT :	0.02749	0.00906	-0.00004
TOTAL ADJUSTMENT :	-2.87201	-1.30502	10.88696
ADJUSTED ESTIMATE :	1942783.22799	-5804080.20502	-1796919.71304
STANDARD DEVIATION :	6.68123	2.30354	0.09142

FINAL RESULTS FOR STATION # : 7120

	X (M)	Y (M)	Z (M)
APRIORI ESTIMATE :	-5466004.68925	-2404402.05471	2242229.46133
CURRENT ADJUSTMENT :	0.01128	-0.02552	0.00007
TOTAL ADJUSTMENT :	-0.99198	2.22478	0.80840
ADJUSTED ESTIMATE :	-5466004.67798	-2404402.08022	2242229.46140
STANDARD DEVIATION :	2.85082	6.48051	0.03966

FINAL RESULTS FOR STATION # : 7943

	X (M)	Y (M)	Z (M)
APRIORI ESTIMATE :	-4447544.46005	2677140.24779	-3694997.07404
CURRENT ADJUSTMENT :	-0.01248	-0.02076	-0.00009
TOTAL ADJUSTMENT :	1.20847	2.41503	0.87687
ADJUSTED ESTIMATE :	-4447544.47253	2677140.22703	-3694997.07413
STANDARD DEVIATION :	3.17406	5.27300	0.03368

FINAL RESULTS FOR PASS # :7603901.01

	X (M)	Y (M)	Z (M)	XDOT (M/S)	YDOT (M/S)	ZDOT (M/S)
APRIORI ESTIMATE :	-5097881.44816	-5961549.88883	-9377290.43571	-4720.07387	-871.25436	3111.28230
CURRENT ADJUSTMENT :	0.02741	-0.02379	0.00014	0.00000	-0.01002	0.00003
TOTAL ADJUSTMENT :	-82.04308	22.97666	29.38775	0.00603	0.04008	0.02003
ADJUSTED ESTIMATE :	-5097881.42075	-5961549.91262	-9377290.43557	-4720.07387	-871.25438	3111.28230
T.D.R.D. ESTIMATE :	-5098243.47143	-5961677.20100	-9377012.67520	-4720.05663	-870.98060	3111.38509
STANDARD DEVIATION :	7.06781	6.04394	0.04245	0.00103	0.00560	0.00002

RMS POSITION : 5.36920

RMS VELOCITY : 0.00329

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Table 8 Constants and Force Model for GEOSPP

EARTH CONSTANTS USED BY GEOSPP80 :

SEMI-AXIS MAJOR...............:	0.637814411000000D+07	(M)	SPEED OF LIGHT.....:	0.299792458000000D+09	(M/S)
INVERSE FLATTENING............:	0.298255000000000D+03	---	ASTRONOMICAL UNIT..:	0.149597870950000D+12	(M)
GRAVITATIONAL CONSTANT (GM)...:	0.398600412500000D+15	(M3/S2)	SOLAR PRES AT 1AU..:	0.462576125869690D-05	(N/M2)
ROTATIONAL RATE...............:	0.729211585470000D-04	(RAD/S)	SINGLE RANGE ACC...:	0.100000000000000D+01	(M)

GEOPOTENTIAL MODEL USED IN THIS RUN : PGSL1 (12 X 12)

ZONALS

INDEX N	M	VALUE	INDEX N	M	VALUE	INDEX N	M	VALUE	INDEX N	M	VALUE	INDEX N	M	VALUE
2	0	-1.082630D-03	3	0	2.533890D-06	4	0	1.624210D-06	5	0	2.298210D-07	6	0	-5.432490D-07
7	0	3.573730D-07	8	0	2.105480D-07	9	0	1.212660D-07	10	0	2.444660D-07	11	0	-2.334080D-07
12	0	1.930420D-07												

SECTORIALS AND TESSERALS

INDEX N	M	VALUE C	S	INDEX N	M	VALUE C	S	INDEX N	M	VALUE C	S
2	1	-3.409330D-09	-3.454640D-09	2	2	1.573620D-06	-9.030690D-07	3	1	2.189960D-06	2.065240D-07
3	2	3.084020D-07	-2.097680D-07	3	3	1.005870D-07	1.973640D-07	4	1	-5.051410D-07	-4.440710D-07
4	2	7.930240D-08	1.477160D-07	4	3	5.939230D-08	-1.209680D-08	4	4	-4.040560D-09	6.456920D-09
5	1	-4.446730D-08	-7.980520D-08	5	2	1.047310D-07	-5.340600D-08	5	3	-1.507260D-08	-7.066600D-09
5	4	-2.255800D-09	3.880350D-10	5	5	4.356500D-10	-1.630330D-09	6	1	-6.153510D-08	1.474180D-08
6	2	6.195170D-09	-4.407070D-08	6	3	1.249020D-09	3.434250D-11	6	4	-3.893620D-10	-1.757040D-09
6	5	-2.132600D-10	-4.335200D-10	6	6	1.889560D-12	-5.525090D-11	7	1	1.954290D-07	6.706580D-08
7	2	3.341790D-08	1.073410D-08	7	3	3.491840D-09	-3.055330D-09	7	4	-5.993040D-10	-2.668410D-10
7	5	4.654430D-12	1.514140D-11	7	6	-2.503990D-11	1.055500D-11	7	7	1.792480D-13	5.633430D-13
8	1	1.849820D-08	4.080880D-08	8	2	6.160660D-09	4.160450D-09	8	3	-5.810170D-11	-8.892400D-10
8	4	-3.105830D-10	9.785540D-11	8	5	-2.723420D-12	1.467590D-11	8	6	-1.903880D-12	8.083050D-12
8	7	3.407770D-13	3.701740D-13	8	8	-1.568580D-13	1.642560D-13	9	1	1.025460D-07	1.250220D-08
9	2	1.540690D-09	-1.905360D-09	9	3	-1.212910D-09	-7.324920D-10	9	4	-8.514040D-12	7.978520D-12
9	5	2.489480D-13	-6.905710D-12	9	6	5.715720D-13	2.942020D-12	9	7	-1.915640D-13	-1.558640D-13
9	8	6.010060D-14	-4.857840D-15	9	9	-4.329410D-15	7.203170D-15	10	1	4.894940D-08	-8.250060D-08
10	2	-4.080740D-09	-8.272660D-10	10	3	-1.022380D-10	-9.860540D-10	10	4	-5.704760D-11	-3.833820D-11
10	5	-3.929630D-12	-2.454440D-12	10	6	-2.564670D-13	-6.034600D-13	10	7	2.165860D-15	7.610900D-15
10	8	4.564400D-15	-8.955630D-15	10	9	2.282340D-15	-9.243230D-16	10	10	4.525390D-16	-1.038020D-16
11	1	-5.620910D-10	-9.122240D-09	11	2	2.478020D-09	-4.871040D-09	11	3	-9.479060D-11	-6.372540D-10
11	4	-1.800820D-11	-2.393130D-11	11	5	1.906600D-12	3.264410D-12	11	6	8.332730D-15	1.773780D-13
11	7	5.610050D-15	-3.247200D-14	11	8	1.723160D-16	1.141720D-15	11	9	-1.839830D-16	2.003920D-16
11	10	-5.912810D-17	-4.834260D-18	11	11	6.338160D-18	-1.003840D-17	12	1	-3.547220D-08	-3.123210D-08
12	2	3.939780D-10	-3.280170D-10	12	3	2.509300D-10	1.128970D-10	12	4	-2.105490D-11	-2.031570D-12
12	5	1.172210D-12	3.610940D-13	12	6	2.154850D-14	8.434420D-14	12	7	-2.238300D-15	9.043440D-15
12	8	-5.563390D-16	5.276510D-16	12	9	1.049730D-16	3.202290D-17	12	10	2.002970D-18	1.433320D-17
12	11	7.217460D-19	-2.911490D-19	12	12	-6.815260D-20	-1.084510D-19				

Table 8 (cont'd)

```
VARIABLE ORDER-VARIABLE STEPSIZE NUMERICAL INTEGRATION :
=========================================================

NOMINAL STEPSIZE.........:   140.0 SEC
MINIMUM STEPSIZE.........:     0.0 SEC
MAXIMUM STEPSIZE.........:   600.0 SEC
RELATIVE ACCURACY
EQUATIONS OF MOTION......: 1.0D-07
VARIATIONAL EQUATIONS....: 1.0D-04
INT.MESSAGE OUTPUT UNIT :       21


PERTURBATION MODEL :
====================

GEOPOTENTIAL...............................: (12,12)
FOR VARIATIONAL EQUATIONS..................: ( 4, 4)
MOON.......................................:   YES
SUN........................................:   YES
UNMODELED ACCELERATIONS ...................:   YES
     ALONG TRACK    = -0.3480D-11 (M/S2)
     CROSS TRACK    =  0.0        (M/S2)
     RADIAL         =  0.0        (M/S2)
SOLAR RADIATION PRESSURE...................:   YES
     SATELLITE AREA =    0.2827 (M2)
     SATELLITE MASS = 406.9650 (KG)
     SATELLITE REFLECTIVITY CK= 1.1729
SOLID EARTH TIDES..........................:   YES
     LOVE NO.       K2=0.2740
     PHASE ANGLE    E2=2.3300


INPUT INFORMATION FOR PASS : 7603901.01
===========================

                            YYMMDD HHMMSS.SSS
EPOCH OF ELEMENTS.......: 80 813 235930.028
OBSERVATIONS START AT...: 80 814  0 0 0.0
OBSERVATIONS END AT.....: 80 829  0 0 0.0
REFERENCE SYSTEM EPOCH..: 80 731


INERTIAL CARTESIAN ELEMENTS AT THE EPOCH :
==========================================
     X (M)            Y (M)            Z (M)
 -5098161.430     -5961700.182     -9377042.063

   XDOT (M/S)       YDOT (M/S)       ZDOT (M/S)
 -4720.0626632    -871.0200757     3111.3650676

APRIORI WEIGHT MATRIX :

0.0
0.0      0.0
0.0      0.0      0.0
0.0      0.0      0.0      0.0
0.0      0.0      0.0      0.0      0.0
0.0      0.0      0.0      0.0      0.0      0.0


LOVE NUMBER FOR RADIAL EXPANSION    H2 =  0.600
================================

SHIDA NUMBER FOR HORIZONTAL SHEAR   L2 =  0.075
=================================


10  STATIONS OBSERVING THIS PASS :
   7063   7090   7091   7092   7096   7114   7115   7120   7907   7943
```

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Table 9

BASELINE ESTIMATES AND RELATED STATISTICS :

BASELINE#	STATION#		STATION#	APRIORI EST.	ADJUSTED VAL.	DIFF.(A-E)	SIGMA	RELATIVE ACC.
1	7063	==>	7090	12645951.761	12645950.847	-0.914	0.018	3.34D-09
2	7063	==>	7091	602032.143	602032.169	0.026	0.036	1.44D-07
3	7063	==>	7092	10003296.515	10003295.833	-0.682	0.025	6.03D-09
4	7063	==>	7096	9896473.055	9896471.526	-1.528	0.022	5.19D-09
5	7063	==>	7114	3562138.713	3562137.442	-1.272	0.041	2.74D-08
6	7063	==>	7115	3501893.178	3501891.797	-1.381	0.037	2.52D-08
7	7063	==>	7120	7244020.742	7244019.261	-1.482	0.028	9.23D-09
8	7063	==>	7907	5928036.951	5928019.003	-17.948	0.085	3.41D-08
9	7063	==>	7943	12108539.054	12108538.064	-0.990	0.018	3.48D-09
10	7090	==>	7091	12638160.062	12638160.219	0.156	0.024	4.52D-09
11	7090	==>	7092	6674009.770	6674008.743	-1.027	0.024	8.61D-09
12	7090	==>	7096	7247520.432	7247520.743	0.311	0.023	7.68D-09
13	7090	==>	7114	11768618.014	11768618.337	0.323	0.018	3.07D-09
14	7090	==>	7115	11810628.856	11810629.014	0.158	0.016	3.20D-09
15	7090	==>	7120	9656458.579	9656458.910	0.331	0.021	5.12D-09
16	7090	==>	7907	11750456.119	11750458.620	2.500	0.034	6.92D-09
17	7090	==>	7943	3196328.733	3196328.646	-0.087	0.021	1.55D-08
18	7091	==>	7092	10141371.223	10141371.602	0.379	0.031	7.21D-09
19	7091	==>	7096	10199643.124	10199642.536	-0.587	0.025	5.84D-09
20	7091	==>	7114	3929728.803	3929728.019	-0.782	0.039	2.34D-08
21	7091	==>	7115	3900598.445	3900597.570	-0.876	0.034	2.07D-08
22	7091	==>	7120	7540273.824	7540273.123	-0.701	0.029	9.10D-09
23	7091	==>	7907	6257037.782	6257020.271	-17.511	0.088	3.35D-08
24	7091	==>	7943	12249596.212	12249596.272	0.059	0.022	4.38D-09
25	7092	==>	7096	3514556.686	3514554.371	-2.316	0.027	1.84D-08
26	7092	==>	7114	7479017.596	7479018.461	0.865	0.027	8.50D-09
27	7092	==>	7115	7584680.410	7584681.155	0.745	0.028	8.76D-09
28	7092	==>	7120	4015538.430	4015538.979	0.550	0.028	1.68D-08
29	7092	==>	7907	11171115.715	11171110.424	-5.291	0.041	8.74D-09
30	7092	==>	7943	5192643.026	5192640.982	-2.044	0.024	1.12D-08
31	7096	==>	7114	7414696.951	7414696.912	-0.040	0.023	7.48D-09
32	7096	==>	7115	7402692.901	7402692.731	-0.170	0.025	8.03D-09
33	7096	==>	7120	4112220.542	4112220.461	-0.081	0.025	1.44D-08
34	7096	==>	7907	9373094.052	9373093.497	-0.554	0.044	1.11D-08
35	7096	==>	7943	4554571.701	4554572.165	0.464	0.024	1.24D-08
36	7114	==>	7115	258289.958	258290.167	0.210	0.036	3.33D-07
37	7114	==>	7120	4022959.527	4022959.505	-0.022	0.031	1.81D-08
38	7114	==>	7907	7243602.178	7243588.024	-14.154	0.076	2.49D-08
39	7114	==>	7943	10587702.281	10587702.701	0.420	0.018	4.05D-09
40	7115	==>	7120	4096904.174	4096904.146	-0.027	0.031	1.79D-08
41	7115	==>	7907	7038726.657	7038712.246	-14.411	0.074	2.51D-08
42	7115	==>	7943	10595990.172	10595990.425	0.253	0.018	4.07D-09
43	7120	==>	7907	9097407.601	9097399.389	-8.212	0.052	1.36D-08
44	7120	==>	7943	7880988.899	7880989.300	0.401	0.022	6.75D-09
45	7907	==>	7943	10787493.058	10787496.735	3.676	0.041	8.98D-09

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Table 10 Residual Summaries by Station

(a)

CONSOLIDATED STATISTICS FOR STATION : 7063

PASS	OBSERV	RESID MEAN	RMS	DEVIATION	LENGTH	MIN RESD	MAX RESD	MEAN CLOS
1	4	-0.9288	5.383	6.123	1292.00	-8.493	5.007	-0.93
2	477	0.1070	0.237	0.212	1641.00	-2.866	0.398	0.11
3	202	0.1237	0.643	0.632	1494.00	-6.840	5.480	0.12
4	6	-2.0976	3.459	3.012	1689.00	-5.903	1.408	-2.10
5	859	0.1242	0.225	0.187	2358.00	-2.436	0.473	0.12
6	1	0.0458	0.046	0.0	0.0	0.046	0.046	0.05
7	1550	0.0139	0.322	0.321	2810.00	-4.383	8.987	0.01
8	4	-4.4022	5.625	4.043	1503.00	-9.652	-1.045	-4.40
9	14	-0.4982	2.473	2.514	2550.00	-4.545	5.946	-0.50
10	1167	-0.1706	0.464	0.432	2484.00	-6.694	7.124	-0.17

(b)

CONSOLIDATED STATISTICS FOR STATION : 7090

PASS	OBSERV	RESID MEAN	RMS	DEVIATION	LENGTH	MIN RESD	MAX RESD	MEAN CLOS
1	97	0.0882	0.130	0.095	2156.00	-0.221	0.344	0.09
2	167	-0.0325	0.104	0.099	2182.00	-0.337	0.177	-0.03
3	182	-0.0892	0.131	0.096	2519.00	-0.513	0.131	-0.09
4	207	0.0322	0.109	0.105	2471.00	-0.282	0.264	0.03
5	196	0.0335	0.140	0.137	2631.00	-0.433	0.448	0.03
6	141	-0.0832	0.142	0.116	1810.00	-0.386	0.192	-0.08
7	263	-0.0764	0.119	0.091	2853.00	-0.427	0.177	-0.08
8	119	0.0891	0.138	0.106	2037.01	-0.153	0.384	0.09
9	67	-0.0531	0.093	0.078	1040.00	-0.235	0.145	-0.05
10	171	-0.0780	0.122	0.094	2787.00	-0.471	0.181	-0.08
11	136	-0.0981	0.138	0.097	2036.00	-0.451	0.101	-0.10
12	203	-0.0640	0.112	0.092	2481.00	-0.526	0.145	-0.06
13	50	0.0123	0.080	0.079	574.00	-0.157	0.216	0.01
14	29	0.0674	0.143	0.128	1287.00	-0.219	0.286	0.07
15	136	-0.0940	0.124	0.081	2339.00	-0.347	0.092	-0.09
16	104	0.1106	0.435	0.422	2645.00	-3.875	0.378	0.11
17	55	0.1305	0.233	0.194	1318.01	-0.282	0.690	0.13
18	162	-0.1606	0.204	0.126	2277.00	-0.472	0.071	-0.16
19	173	0.0735	0.136	0.114	2491.00	-0.272	0.326	0.07
20	136	-0.1598	0.212	0.140	2409.00	-0.567	0.111	-0.16
21	155	0.0943	0.185	0.159	2421.00	-0.323	0.378	0.09
22	41	0.0313	0.069	0.062	1090.00	-0.113	0.143	0.03
23	88	0.1992	0.218	0.088	1438.00	-0.102	0.348	0.20
24	233	-0.0433	0.112	0.104	2842.00	-0.597	0.195	-0.04
25	115	0.1264	0.156	0.092	1947.00	-0.168	0.345	0.13
26	6	0.1585	0.172	0.073	93.00	0.071	0.280	0.16
27	189	0.0197	0.128	0.126	2638.00	-0.289	0.294	0.02
28	154	0.0887	0.123	0.085	2109.00	-0.164	0.328	0.09
29	154	-0.0249	0.223	0.223	2088.00	-0.567	0.398	-0.02
30	214	0.0598	0.128	0.114	2700.00	-0.379	0.292	0.06

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Table 10 (cont'd)

(c)

CONSOLIDATED STATISTICS FOR STATION : 7091

PASS	OBSERV	RESID MEAN	RMS	DEVIATION	LENGTH	MIN RESD	MAX RESD	MEAN CLOS
1	137	0.0824	0.271	0.259	1145.00	-0.750	0.683	0.08
2	352	-0.0646	0.182	0.171	1718.00	-1.040	0.346	-0.06
3	240	0.0505	0.169	0.162	1879.00	-0.545	0.450	0.05
4	439	-0.0202	0.314	0.313	1965.01	-1.142	4.742	-0.02

(d)

CONSOLIDATED STATISTICS FOR STATION : 7092

PASS	OBSERV	RESID MEAN	RMS	DEVIATION	LENGTH	MIN RESD	MAX RESD	MEAN CLOS
1	322	0.2185	0.268	0.156	1761.01	-0.258	0.527	0.22
2	286	-0.1712	0.268	0.206	1153.00	-0.845	0.666	-0.17
3	1273	-0.0004	0.239	0.239	2380.00	-1.234	0.986	-0.00
4	363	-0.0324	0.304	0.303	2271.00	-1.293	0.734	-0.03
5	9	0.0926	0.331	0.337	174.00	-0.367	0.[illegible]26	0.09

(e)

CONSOLIDATED STATISTICS FOR STATION : 7096

PASS	OBSERV	RESID MEAN	RMS	DEVIATION	LENGTH	MIN RESD	MAX RESD	MEAN CLOS
1	969	0.0078	0.189	0.189	2389.00	-0.583	0.546	0.01
2	461	0.0359	0.150	0.146	2008.99	-0.731	0.331	0.04
3	268	-0.1355	0.257	0.219	1109.01	-0.931	0.313	-0.14
4	91	-0.3075	0.391	0.244	652.00	-0.953	0.122	-0.31
5	45	0.0547	0.166	0.158	924.00	-0.451	0.356	0.06
6	616	0.0351	0.213	0.210	1368.01	-1.019	0.531	0.04

(f)

CONSOLIDATED STATISTICS FOR STATION : 7114

PASS	OBSERV	RESID MEAN	RMS	DEVIATION	LENGTH	MIN RESD	MAX RESD	MEAN CLOS
1	182	-0.0405	0.176	0.172	1181.99	-0.458	1.003	-0.04
2	17	-0.1346	1.475	1.514	1136.00	-4.979	2.632	-0.13
3	855	0.0155	0.263	0.262	2535.00	-3.965	1.966	0.02
4	9	1.1201	2.490	2.358	1367.00	-1.729	5.392	1.12
5	161	0.0939	0.155	0.124	1009.99	-0.310	0.465	0.09
6	390	-0.0036	0.129	0.129	2102.99	-0.387	0.968	-0.00
7	6	-0.7358	3.655	3.922	887.00	-4.838	6.072	-0.74
8	228	-0.0292	0.350	0.349	1045.00	-0.384	4.296	-0.03
9	7	0.0771	0.111	0.086	676.00	0.001	0.236	0.08
10	7	0.4243	0.703	0.606	1080.00	0.099	1.779	0.42
11	4	-4.5052	5.795	4.208	400.00	-8.096	0.651	-4.51

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Table 10 (cont'd)

(g) CONSOLIDATED STATISTICS FOR STATION : 7115

PASS	OBSERV	RESID MEAN	RMS	DEVIATION	LENGTH	MIN RESD	MAX RESD	MEAN CLOS
1	264	0.0850	0.131	0.099	1224.01	-0.178	0.379	0.08
2	29	0.2660	0.280	0.088	711.00	0.077	0.410	0.27
3	384	-0.0730	1.057	1.056	2001.00	-6.960	0.595	-0.07
4	27	-6.7929	6.793	0.090	1021.00	-6.969	-6.617	-6.79
5	500	0.0934	0.151	0.119	2271.00	-0.422	1.167	0.09
6	38	0.3169	0.328	0.088	1313.00	0.120	0.488	0.32
7	172	0.3589	0.511	0.364	1346.00	-0.525	4.824	0.36
8	63	-0.0468	0.145	0.138	889.00	-0.346	0.257	-0.05
9	119	-0.2538	0.312	0.181	1913.00	-0.739	0.504	-0.25
10	588	0.1708	0.213	0.128	2727.00	-0.255	1.811	0.17
11	37	0.1305	0.165	0.102	1608.00	-0.075	0.363	0.13
12	44	0.2436	0.286	0.151	652.00	-0.040	0.949	0.24

(h) CONSOLIDATED STATISTICS FOR STATION : 7120

PASS	OBSERV	RESID MEAN	RMS	DEVIATION	LENGTH	MIN RESD	MAX RESD	MEAN CLOS
1	225	-0.1213	0.142	0.073	1964.00	-0.380	0.120	-0.12
2	44	-0.0037	0.157	0.159	829.00	-0.772	0.180	-0.00
3	160	0.0996	0.140	0.098	1297.00	-0.225	0.313	0.10
4	42	-0.0689	0.108	0.084	618.00	-0.218	0.098	-0.07
5	187	0.0268	0.133	0.131	2614.00	-0.337	0.857	0.03
6	346	-0.0766	0.114	0.085	2759.00	-0.348	0.247	-0.08
7	401	0.0931	0.138	0.102	2573.00	-0.259	0.298	0.09
8	50	-0.2583	0.294	0.141	865.00	-0.511	0.034	-0.26
9	121	-0.1879	0.216	0.107	1655.00	-0.380	0.104	-0.19
10	328	0.1102	0.163	0.120	2417.00	-0.221	0.409	0.11

(i) CONSOLIDATED STATISTICS FOR STATION : 7907

PASS	OBSERV	RESID MEAN	RMS	DEVIATION	LENGTH	MIN RESD	MAX RESD	MEAN CLOS
1	12	0.1231	0.366	0.360	292.40	-0.408	0.939	0.12
2	41	-0.0135	0.604	0.611	1132.95	-1.520	0.761	-0.01
3	51	0.0917	0.509	0.505	862.51	-1.291	0.803	0.09
4	19	-0.0486	0.335	0.341	1027.76	-0.767	0.561	-0.05
5	5	-0.0084	0.596	0.666	360.15	-1.164	0.446	-0.01
6	52	0.1667	0.521	0.498	892.90	-0.951	2.032	0.17
7	24	0.0601	0.401	0.405	660.27	-0.925	0.724	0.06
8	19	-0.2028	0.393	0.346	607.64	-0.868	0.360	-0.20
9	35	0.0493	0.288	0.287	1162.58	-0.648	0.640	0.05
10	53	-0.0616	0.408	0.408	914.95	-1.008	0.881	-0.06
11	34	-0.0226	0.398	0.403	967.53	-1.199	0.697	-0.02
12	5	0.2501	0.257	0.065	360.10	0.169	0.303	0.25
13	24	-0.5918	1.005	0.830	1110.02	-2.043	0.962	-0.59
14	17	0.0004	0.813	0.838	689.98	-1.536	1.569	0.00
15	8	0.2717	0.528	0.484	420.00	-0.434	1.126	0.27
16	9	-0.1124	0.568	0.590	465.05	-1.312	0.452	-0.11
17	22	-0.1859	0.744	0.737	847.96	-1.970	1.863	-0.19
18	28	0.2387	0.405	0.333	1012.50	-0.646	0.796	0.24
19	2	-0.3987	0.425	0.207	90.00	-0.545	-0.252	-0.40
20	29	0.0272	0.411	0.417	779.97	-0.855	0.698	0.03

Table 10 (cont'd)

(J) CONSOLIDATED STATISTICS FOR STATION : 7943

PASS	OBSERV	RESID MEAN	RMS	DEVIATION	LENGTH	MIN RESD	MAX RESD	MEAN CLUS
1	56	0.0012	0.433	0.437	1154.92	-1.039	0.959	0.00
2	122	0.0426	0.338	0.336	2325.29	-1.432	0.815	0.04
3	82	-0.0668	0.439	0.437	1905.10	-1.252	1.074	-0.07
4	98	-0.0301	0.408	0.409	2272.34	-0.970	0.715	-0.03
5	83	0.0633	0.426	0.424	1484.75	-1.229	1.222	0.06
6	85	0.0516	0.337	0.335	1567.79	-0.917	0.872	0.05
7	90	0.1375	0.292	0.259	1424.99	-0.489	0.991	0.14
8	212	-0.0796	0.318	0.309	2744.95	-0.864	0.679	-0.08
9	159	0.0983	0.344	0.330	2910.00	-0.720	0.861	0.10
10	47	-0.0243	0.511	0.516	1934.86	-1.052	1.198	-0.02
11	54	0.0337	0.429	0.432	1514.65	-1.193	1.353	0.03
12	75	-0.1024	0.552	0.546	1605.00	-1.306	0.974	-0.10
13	139	0.0414	0.239	0.236	1822.48	-0.681	0.594	0.04
14	100	-0.0034	0.375	0.377	1897.49	-0.853	1.041	-0.00
15	170	-0.0420	0.414	0.413	2955.00	-1.097	1.295	-0.04
16	106	-0.0837	0.347	0.338	1890.00	-1.137	0.715	-0.08
17	90	-0.0132	0.280	0.281	1919.92	-0.878	0.702	-0.01
18	44	-0.0889	0.470	0.467	2167.47	-1.129	0.643	-0.09
19	79	0.1667	0.424	0.392	2287.55	-0.943	0.899	0.17
20	178	0.0212	0.282	0.282	2489.80	-0.607	1.253	0.02
21	38	-0.1376	0.625	0.617	1492.55	-1.638	1.460	-0.14
22	88	-0.0665	0.485	0.483	2092.54	-1.147	1.249	-0.07
23	172	-0.0355	0.263	0.261	2827.32	-0.671	0.611	-0.04
24	73	-0.3053	0.399	0.259	629.94	-0.896	0.119	-0.31
25	67	0.0351	0.448	0.450	1619.87	-1.124	0.798	0.04
26	96	-0.0945	0.596	0.592	1619.69	-1.605	1.349	-0.09
27	163	-0.1419	0.279	0.241	1919.90	-0.771	0.511	-0.14
28	191	0.1105	0.330	0.312	1994.97	-0.727	0.898	0.11
29	262	0.1799	0.365	0.318	2999.78	-0.819	0.837	0.18
30	97	0.1417	0.282	0.245	1942.65	-0.673	0.703	0.14
31	61	0.1398	0.466	0.448	2175.10	-0.953	0.875	0.14
32	41	-0.0845	0.485	0.483	1507.44	-1.351	0.607	-0.09

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2.22 Doppler Experiments

2.221 Geometric Adjustment of Simultaneous Doppler-Derived Range Differences

The results of work on this topic are described in a paper presented at the Third Internationl Symposium on the Use of Artificial Satellites for Geodesy and Geodynamics, Ermioni, Greece, September 20-25, 1982. It appears on the following pages and will be published in the proceedings of the symposium obtainable from the National Technical University, Athens.

Third International Symposium on the Use of Artificial Satellites for Geodesy and Geodynamics, Ermioni, Greece, September 20-25, 1982

GEOMETRIC ADJUSTMENT OF SIMULTANEOUS DOPPLER-DERIVED RANGE DIFFERENCES

Chengze Zhang* and Ivan I. Mueller
Department of Geodetic Science and Surveying
The Ohio State University, Columbus, Ohio 43210 USA

ABSTRACT. A mathematical model for the use of simultaneous Doppler-derived correlated ranges in the geometric mode is presented. The model is tested with data taken during the EDOC-2 campaign with different integration intervals. The results of this adjustment are compared with the EDOC-2 adopted solution and those from an uncorrelated model [Schneeberger et al. 1982] used earlier to provide more economical calculations.

The analysis of the comparison shows that the correlated mode is superior to the uncorrelated one when the optimum integration interval of 23 seconds is used.

1. INTRODUCTION

The geometric purpose of satellite geodesy is to tie remote stations together in the same geometric system. Its ultimate aim is to determine the coordinates of unknown ground stations [Mueller 1984].

Satellite geodesy with Doppler techniques is based on the principle that a frequency transmitted from a satellite-borne transmitter moving relative to a ground receiver is observed shifted by the Doppler effect. The observations are Doppler counts which are measures of the range change between the satellite and the receiver during the integration interval [Wells 1974].

In the geometric mode for Doppler observations, the satellite is regarded as a benchmark in space and its coordinates at the observation instants are unknowns which are solved in an adjustment with the unknown coordinates of ground stations. Such solutions are based on geometric rather than dynamic principles; therefore the calculations are relatively simple and do not require extensive computer programs.

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*On leave from the Institute of Geodesy and Geophysics, Chinese Academy of Sciences. Wuhan. People's Republic of China.

In a previous study [Schneeberger et al. 1982], the Doppler-derived ranges were regarded as uncorrelated pseudo-observations as a further simplification (to save computer time). In fact, since the Doppler-derived ranges are calculated from Doppler counts, it is obvious that there exist correlations between them in a given pass. The purpose of this study is to investigate the use of Doppler-derived correlated ranges in the geometric mode.

This method is then tested against a data set from which a dynamic solution is available. The results are compared with both the dynamic solutions and the uncorrelated geometric one.

2. SUMMARY OF THE PREVIOUS STUDY BASED ON UNCORRELATED OBSERVATIONS [SCHNEEBERGER ET AL. 1982]

2.1 Definitions

The *coordinate system* in which the computations are performed is an earth-fixed Cartesian system. It is defined by the assigned six coordinates distributed among at least three ground stations. A *satellite point* is the position of a satellite at a certain epoch. An *event* is the set of all observations to a satellite point. A *pass* is a set of satellite points between two epochs which are observed without interruption from at least six ground stations. A *Doppler-derived range* is a pseudo-observation derived by adding the range differences computed from Doppler counts to an estimated initial range.

2.2 Doppler-Derived Ranges

The basic equation which related the ratio between the received frequency f and the transmitted frequency f_0 to the range rate between transmitter and receiver ($\dot{r}$) is accredited to Doppler (1803-1853):

$$\frac{f}{f_0} = \left(\frac{c}{c + \dot{r}}\right) \approx \left(1 - \frac{\dot{r}}{c}\right)$$

where c is the velocity of propagation for electromagnetic waves in a vacuum. This equation has to be integrated to find a relation between the shifted frequency and the range difference during a time interval t. A detailed derivation can be found in [Brown and Trotter 1969] resulting in

$$r_j - r_{j-1} = \lambda_0(N_j - \Delta f_{00}\, t_j) + S \tag{1}$$

where

r_j = range between receiver and transmitter at epoch T_j

r_{j-1} = range at epoch T_{j-1}

N_j = the integrated Doppler shift over time interval $t_j = T_j - T_{j-1}$ (referred to as the Doppler count)

Δf_{00} = the difference between the transmitted frequency and the reference frequency generated in the receiver

$\lambda_0 = \frac{f_0}{c}$ = wavelength corresponding to the frequency of transmission f_0

S = correction term representing all systematic errors such as bias in the difference between the adopted transmitted and reference frequencies, and/or the drift rates of transmitter and receiver frequencies.

Substituting the range difference computed from the Doppler count

$$\Delta r_j = \lambda_0 (N_j + \Delta f_{00}\, t_j)$$

into equation (1), the range at epoch T_j is

$$r_j = r_{j-1} + \Delta r_j + S_j$$

If the range r_0 at an initial epoch T_0 is known, the range for an epoch T_k can then be calculated from (taking into account that most instruments reset the Doppler count for each interval)

$$r_k = r_0 + \sum_{j=1}^{k} \Delta r_j + S_k \qquad (2)$$

This equation is correct only in a vacuum. Since the signal is passing through the ionosphere and the troposphere, the range has to be corrected for refractive effects. The ionospheric refraction is automatically compensated (to first order) by measuring the Doppler shift of the two different frequencies (400 and 150 MHz) [Krakiwsky and Wells 1971]. Each range has to be corrected therefore only for the tropospheric refraction ΔT_r. The tropospheric refraction model used in this study is the one outlined in [Brown and Trotter, 1973], using the Smith-Weintraub model for the index of refraction [Jordan et al. 1966].

Since the initial range in equation (2) is not known, we must use an approximate initial range r_0' and add a correction term a_0 to be estimated from the adjustment,

$$r_0 = r_0' + a_0$$

a_0 is considered part of the systematic error term S_k in equation (2). The modelling of the other systematic effects in S_k is given in great detail in [Brown and Trotter 1969; Kouba and Boal 1976]. In this study only two major terms are used: a + bt. The main cause of the constant term a is the possible bias in the adopted frequency f_0, and the initial range error a_0 above. The time dependent term bt is caused mainly by the difference in the adopted values for the transmitter and receiver frequencies (frequency offset) Δf_{00}.

Other terms in the systematic error model mentioned by Brown and Trotter [1969] but not considered in this study are range dependency, a function of the second power of time, and a function of the elevation angle (for residual refraction errors). An explanation of why only the above two terms are used here may be found in [Schneeberger 1982].

Substituting all terms for S and the correction for tropospheric refraction equation (2) can be written as

$$r_{ik} = r_0 + \sum_{j=1}^{k} \Delta r_j + \Delta T_r + a_i + b_i t_k$$

where the subscript i refers to ground station i. Defining the *Doppler-derived range* as

$$r_{D_k} = r_0 + \sum_{j=1}^{k} \Delta r_j + \Delta T_r \, , \tag{2'}$$

and recalling that

$$r_{ik} = \sqrt{(X_k - X_i)^2 + (Y_k - Y_i)^2 + (Z_k - Z_i)^2}$$

and changing the signs of a and b, we arrive at the mathematical model

$$r_{D_{ik}} = \sqrt{(X_k - X_i)^2 + (Y_k - Y_i)^2 + (Z_k - Z_i)^2} + a_{i\ell} + b_{i\ell} t_k \tag{3}$$

where $r_{D_{ik}}$ is the Doppler-derived pseudo-range (derived from the measured Doppler counts and corrected for tropospheric refraction), and the unknown parameters to be solved for in a least squares adjustment are

X_i, Y_i, Z_i — the unknown station (i) coordinates

X_k, Y_k, Z_k — the unknown satellite (k) coordinates

$a_{i\ell}, b_{i\ell}$ — the unknown coefficients used to model systematic errors for each station (i) and pass (ℓ)

t_k is the time elapsed from the epoch of the initial range r_0.

2.3 Least Squares Adjustment

The mathematical model developed above has the form of an observation equation:

$$L_a = F(X_a) \tag{4}$$

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where L_a is the adjusted Doppler-derived range, and X_a is the vector of the unknown parameters which can be divided into three subvectors:

$XG_a = XG_0 + XG$ containing the coordinates of the ground stations
$XC_a = XC_0 + XC$ containing the error coefficients a, b
$XS_a = XS_0 + XS$ containing the satellite coordinates

Equation (3) can be written in linearized form

$$r_{Dik\ell} + v_{ik\ell} = F^0_{ik\ell} + \frac{\partial F}{\partial XG}\bigg|_{X_0} \cdot XG_i + \frac{\partial F}{\partial XC}\bigg|_{X_0} \cdot XC_{i\ell} + \frac{\partial F}{\partial XS}\bigg|_{X_0} \cdot XS_k + \ldots \tag{5}$$

or, after neglecting higher-order terms

$$v_{ik\ell} = A_{ik\ell} \cdot XG_i + C_{ik\ell} \cdot XC_{i\ell} + S_{ik\ell} \cdot XS_k - W_{ik\ell}$$

where

$$A_{ik\ell} = \frac{\partial F}{\partial XG}\bigg|_{XG^0_i\ XC^0_{i\ell}\ XS^0_k} = \left(-\frac{X^0_k - X^0_i}{r_{0ik\ell}},\ -\frac{Y^0_k - Y^0_i}{r_{0ik\ell}},\ -\frac{Z^0_k - Z^0_i}{r_{0ik\ell}}\right)$$

$$C_{ik\ell} = \frac{\partial F}{\partial XC}\bigg|_{XG^0_i\ XC^0_{i\ell}\ XS^0_k} = (1,\ t_k)$$

$$S_{ik\ell} = \frac{\partial F}{\partial XS}\bigg|_{XG^0_i\ XC^0_{i\ell}\ XS^0_k} = -A_{ik\ell}$$

$$W_{ik\ell} = r_{Dik\ell} - (r_{0ik\ell} + a_{0i\ell} + b_{0i\ell} t_k)$$

$$r_{0ik} = \sqrt{(X^0_k - X^0_i)^2 + (Y^0_k - Y^0_i)^2 + (Z^0_k - Z^0_i)^2}$$

In this study all pseudo-range observations are assumed to have equal weight. For reason of convenience in programming, the a priori variance of unit weight is chosen to be equal to the variance of a range observation

$$\sigma_0^2 = \sigma_{DR}^2$$

Therefore all observations have the weight one. Further details of this least squares adjustment as used in the Geometric Doppler (GEODOR) computer program may be found in [Schneeberger 1982].

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3. ADJUSTMENT WITH CORRELATIONS CONSIDERED

3.1 Mathematical Model

The correlation existing in the Doppler-derived ranges are considered in this study by assuming that the range differences (computed from the Doppler counts) are independent observations.

Under this consideration, substituting eq. (2') into (3) and moving all the terms to the left side, we obtain

$$\sqrt{(X_k-X_i)^2 + (Y_k-Y_i)^2 + (Z_k-Z_i)^2} + a_{i\ell} + b_{i\ell}t_k - r_0 - \sum_{j=1}^{k} \Delta r_j - \Delta T_r = 0 \tag{6}$$

Thus the model becomes the form of a condition equation with parameters:

$$F(L_a, X_a) = 0 \tag{7}$$

Eq. (6) can be written in a linearized form, using the same notation as before,

$$A_{ik\ell}X_i + C_{ik}\, XC_{ik\ell} + S_{ik\ell}\, XS_k + \sum_{j=1}^{k} B_{ik\ell}\, v_j - W_{ik\ell} = 0 \tag{8}$$

where $B_{ik\ell}$ stands for the derivatives of F with respect to Δr_j, i.e.,

$$B_{ik\ell} = \frac{\partial F}{\partial \Delta r_j} = \begin{cases} -1 & \text{if } j \leq k \\ 0 & \text{if } j > k \end{cases} \tag{9}$$

All the observations are assumed to have equal weight. For convenience in programming, the a priori variance of unit weight is chosen to be equal to the variance of a range difference observation

$$\sigma_0^2 = \sigma_{\Delta r}^2$$

Therefore, all the observations have unit weight. For the detail of the derivations of the mathematical model and the method of solving this problem, see [Zhang 1982].

3.2 Construction of Normal Equations

The solution of the normal equation system for the least squares model of condition equations with parameters has the following form [Uotila 1976]:

$$X = -(A^T M^{-1} A)^{-1} A^T M^{-1} W \tag{10}$$

where

$$M^{-1} = (B\, P^{-1}\, B^T)^{-1}$$

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Therefore, before constructing the normal equation system, M^{-1} has to be found first. Fortunately, the matrix B has a regular configuration, and so does the matrix M^{-1} [Ashkenazi et al. 1980]. For the sake of simplicity, we investigate a matrix B for one station and one pass. From eqs. (8) and (9) it is evident that the matrix B has the form

$$B = \begin{vmatrix} -1 & 0 & 0 & 0 & \cdots & 0 \\ -1 & -1 & 0 & 0 & \cdots & 0 \\ -1 & -1 & -1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -1 & -1 & -1 & -1 & \cdots & -1 \end{vmatrix} \tag{12}$$

If one assumes uniform weight and no correlations between the range differences, and chooses the variance of unit weight equal to the variance of range difference observation, the matrix P will become an identity matrix. Then the matrix M can be written as

$$M = B\, P^{-1} B^T = \begin{vmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & 2 & 2 & 2 & \cdots & 2 \\ 1 & 2 & 3 & 3 & \cdots & 3 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 2 & 3 & 4 & \cdots & n \end{vmatrix} \tag{13}$$

where n is the number of observations in this pass. M^{-1} is found by inverting M:

$$M^{-1} = \begin{vmatrix} 2 & -1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & \cdots & 0 & -1 & 1 \end{vmatrix} \tag{14}$$

Since M^{-1} is a regular diagonal matrix, it will not invite much difficulty when constructing normal equations. For the case of more than one station and more than one pass, matrices B and M^{-1} can easily be found by using the same method [Zhang 1982].

After the matrix M^{-1} is found, all the coefficients of the normal equation system can be calculated. Since this normal equation system is still of the sparsity pattern, a method called second-order partitioned regression can be used to eliminate the unknowns to save storage and computing time [Brown and Trotter 1969].

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4. NUMERICAL TEST

4.1 Solutions and Their Comparisons

The data taken during EDOC-2 was used for testing the uncorrelated and correlated modes. Fig. 1 shows the network used which is chosen from EDOC-2. There were many solutions for each mode, but only the best

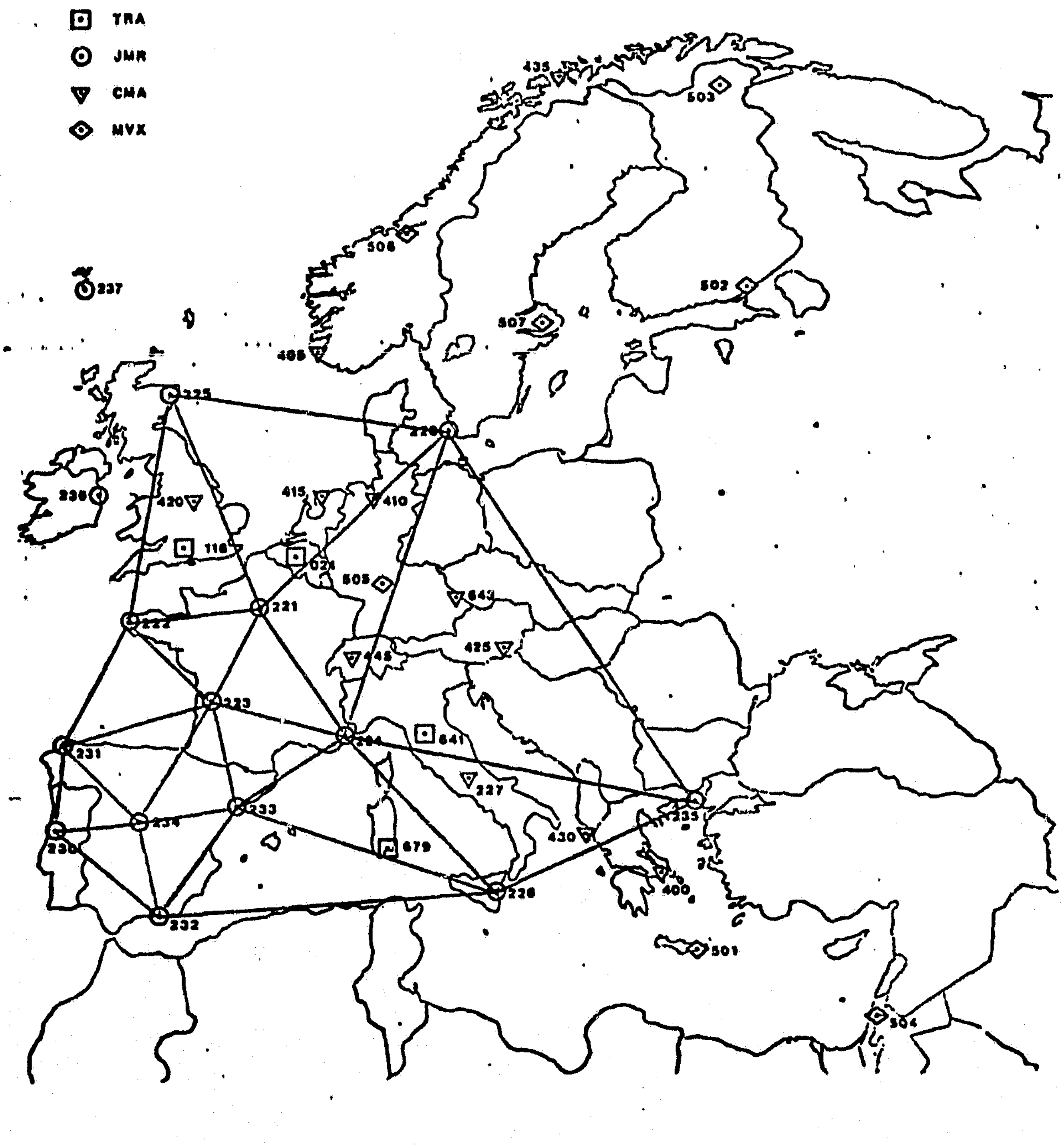

Fig. 1 EDOC-2 network [Boucher et al. 1981]

one of each mode can be presented here. Table 1 is a summary of these two solutions. Solution F4-5 is in the uncorrelated mode; solution C-5 in the correlated mode. The integration intervals of both solutions are 5 x 4.6 = 23 seconds.

The information of solutions using different integration intervals from the correlated mode is collected in Table 2. In the designation C-i, i indicates the integration intervals used, e.g., in case of i = 2, the range change is over 2 x 4.6 = 9.2 s. Fig. 2 gives a visual comparison of these solutions. It is obvious that the solution with i = 5 is the best.

4.2 Test of the Systematic Error Model

In this study as in the earlier one only the two major terms are used for modeling the systematic effects: a + bt. The residuals of the observations of randomly selected passes from the total of 193 passes were plotted for each station. Fig. 3 is one example. Investigating the distributions of the residuals of the observations at each station, no significant remaining systematic effect is found, which indicates that the two major terms used for modeling the systematic effects are reasonable.

4.3 Test of the Residuals

From Table 1 we can find that the correlated mode is superior to the uncorrelated one. In spite of that, there are still significant differences between solutions C-5 and EDOC-2. In order to find the reason, the residuals of all observations were investigated. Table 3 lists the statistics of the residuals of the observations over the ten worst passes.

Checking this table, one can see that the maximum residual is as large as 160 m, and the ratio of the number of the observations whose absolute residuals are larger than three times the standard deviation, to the total number of the observations for each one of the worst passes is high. The worst one is as high as 11.2%. This indicates that there may be blunders in the data set.

4.4 Problem of Weights

As stated earlier, all observations are assumed to have equal weight and the a priori variance of unit weight is chosen to be equal to the variance of a range difference observation

$$\sigma_0^2 = \sigma_{\Delta r}^2 = 1.0$$

In Table 2 one can see that the a posteriori standard deviations of unit weight for all the solutions are much larger than the chosen a priori one. For instance the a posteriori standard deviation of unit weight of the best solution, C-5, is as large as 3.4.

Table 1 Summary of Solutions F4-5 and C-5

Solution No.:	F4-5			C-5		
Total No. of Passes Processed	193			193		
Total No. of Events	3,430			3,430		
No. of Unknowns						
Station Coordinates	30			30		
Error Coefficients	3,312			3,312		
Satellite Coordinates	10,290			10,290		
Total No. of Unknowns	13,632			13,632		
Total No. of Observations	27,531			27,531		
Degrees of Freedom	13,899			13,899		
A Priori Weight Information:						
Range (or Range Difference)	1 m			1 m		
Error Coefficient σ_a	50 m			50 m		
Error Coefficient σ_b	38 m / 2 min			38 m / 2 min		
Fixed Station Coordinates σ_X, σ_Y, σ_Z	1 mm			1 mm		
Other Station Coordinates σ_X, σ_Y, σ_Z	100 m			100 m		
3 Satellite Events/Pass σ_X, σ_Y, σ_Z	10 m			10 m		
A Posteriori Standard Deviation of Unit Weight	3.5 m			3.4 m		
Coordinate Differences with Respect to EDOC-2 Solution (all units in m)	$\Delta\phi$	$\Delta\lambda$	ΔH	$\Delta\phi$	$\Delta\lambda$	ΔH
Station No. (* indicates fixed station) 220*	0.0	0.0	0.0	0.0	0.0	0.0
221	3.5	16.2	-4.4	3.0	7.0	-4.6
222	3.4	-4.3	-1.2	1.9	-3.0	-1.4
223	1.5	9.8	-7.1	1.7	5.2	-4.2
224	5.6	27.1	-12.6	5.3	13.2	13.1
225	-4.8	-10.9	6.7	-0.2	18.4	3.0
226*	0.0	0.0	0.0	0.0	0.0	0.0
230	1.0	-4.5	13.6	4.6	3.2	9.9
231*	0.0	0.0	0.0	0.0	0.0	0.0
232	4.5	-3.1	15.7	2.1	-4.6	20.5
233	0.8	8.7	-2.0	1.6	2.0	-0.9
234	0.1	-1.7	5.3	-1.0	1.1	7.5
235	1.0	44.2	70.0	8.2	-0.4	1.9
Average absolute difference (m) (10 stations)	2.6 ±2.8	13.0 ±5.2	14.0 ±4.6	2.2 ±4.7	5.8 ±8.9	6.7 ±7.9
Average absolute difference in position (m)	20.8 ±21.7			10.8 ±6.6		
Average absolute station-to-station chord distance difference (m)	10.2 ±10.5			5.5 ±4.5		

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Table 2 Comparison of the Different Integration Intervals Used in Adjustment

Name of Solution:	C-2	C-5	C-10	C-15
Integration interval (seconds)	9.2	23	46	69
Computing time (minutes)*	25.20	8.83	5.96	2.79
A posteriori standard deviation of unit weight	2.4	3.4	4.5	5.9
Average total absolute difference in position (m) (10 stations)	10.9 ±7.3	10.9 ±6.6	22.0 ±19.3	33.8 ±37.1
Average absolute station-to-station chord distance difference (m)	6.4 ±5.6	5.5 ±4.4	8.9 ±8.2	17.1 ±19.2

*using an Amdahl 470

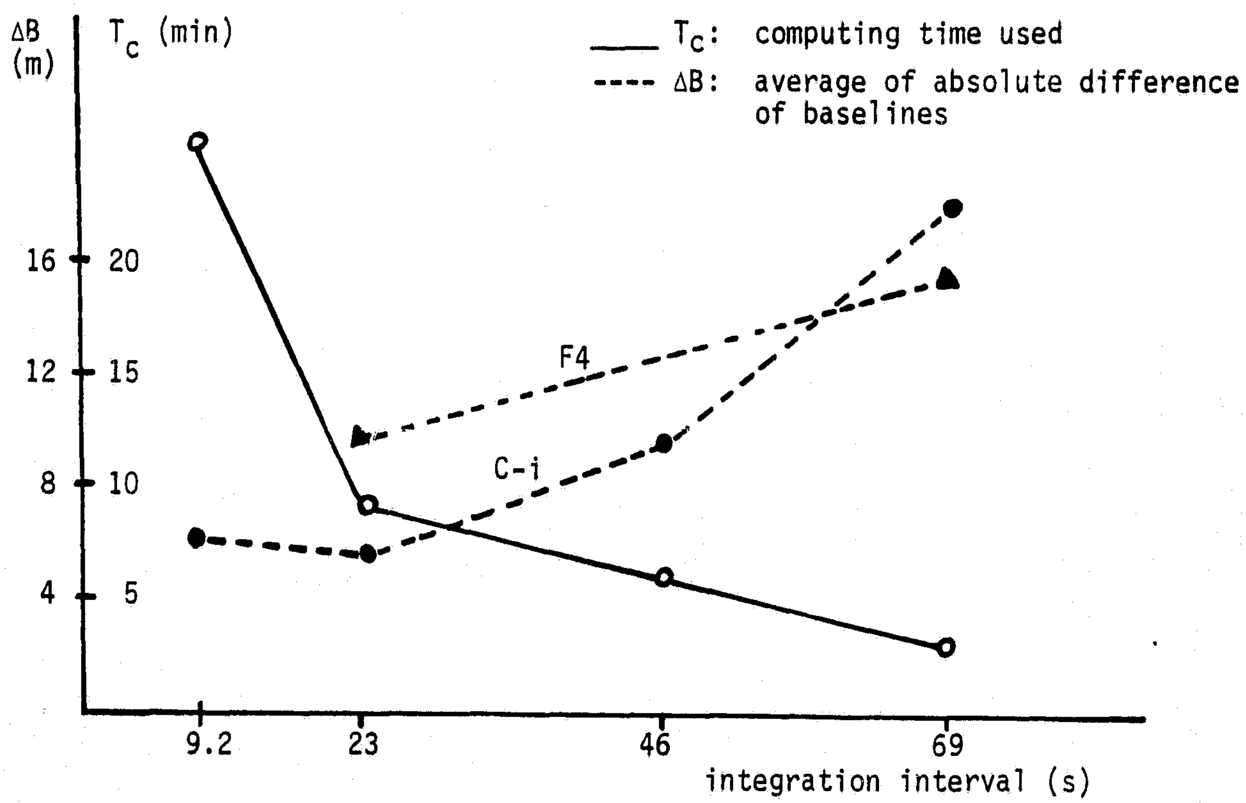

Fig. 2 Computing time used and average of absolute differences of baselines plotted against the length of integration interval

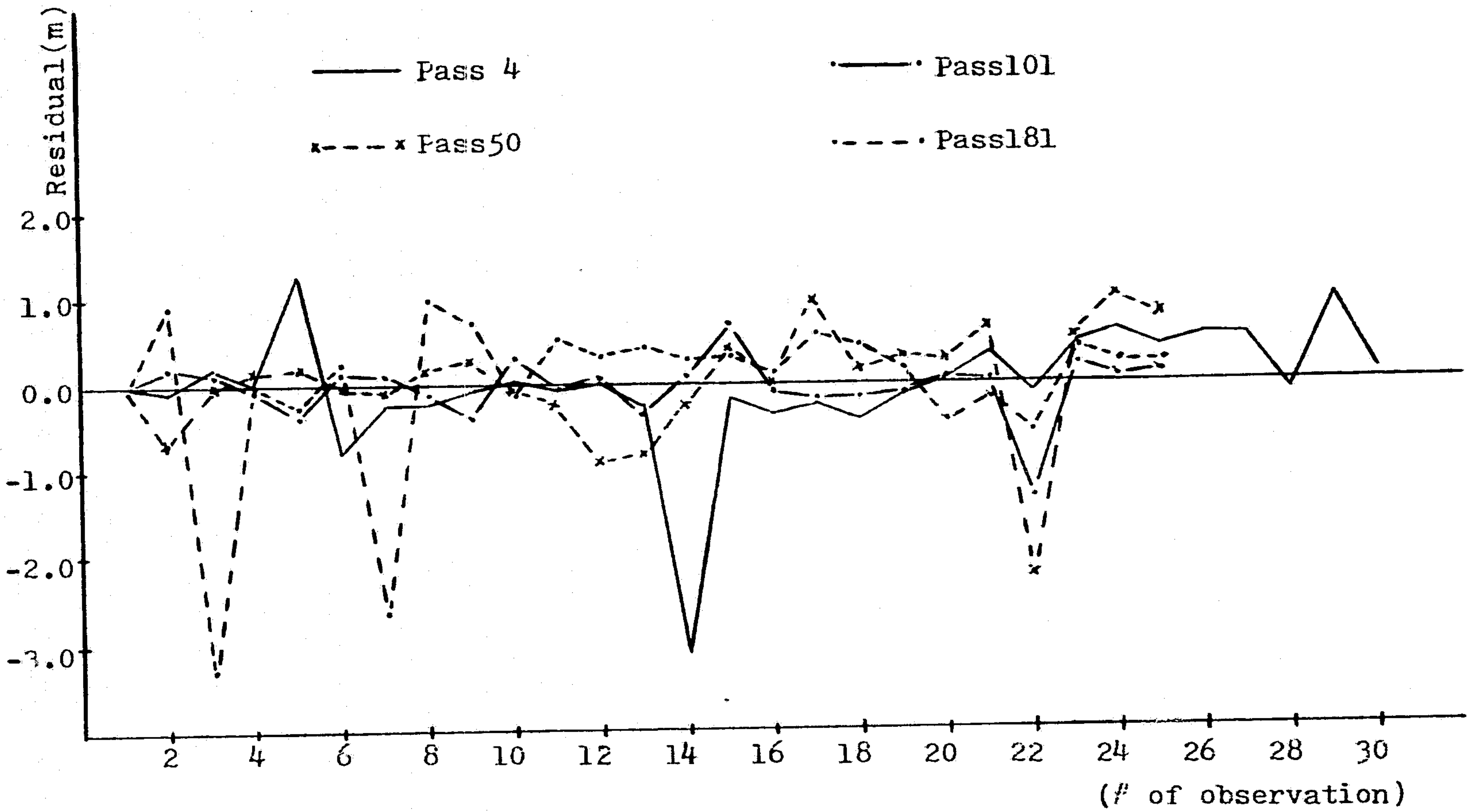

Fig. 3 Distribution of residuals at station 223 in passes 4, 50, 101 and 181.

Table 3 Statistics of the Residuals of the Observations of the Ten Worst Passes

No.	Pass No.	Number of Observations							$\|v\|_{max}$ (m)
		Total	$\|v\| > 3\sigma$		$\|v\| > 2\sigma$		$\|v\| > 10.0$ m		
			Number	%	Number	%	Number	%	
1	49	207	20	9.7	27	13.0	17	8.2	160.8
2	46	187	18	9.6	38	20.3	13	7.0	126.1
3	187	143	16	11.2	27	18.9	12	8.4	41.4
4	21	262	19	7.3	32	11.8	10	3.8	53.0
5	43	181	9	5.0	17	9.4	8	4.4	39.5
6	180	142	9	6.3	15	10.6	8	5.6	25.6
7	51	221	10	4.5	15	6.8	7	3.2	17.2
8	26	186	10	5.4	12	6.5	5	2.7	26.0
9	25	105	6	5.7	7	6.7	5	4.8	31.6
10	16	195	7	3.6	10	5.1	4	2.1	37.4

Table 4 presents the comparison of the weights of each station calculated from the residuals over all passes. The weights of the stations differ from each other for solutions C-5; the largest one is ninefold as large as the smallest one. When the ten worst passes are taken out, the weights are close to each other, and the a posteriori standard deviation of unit weight is decreased from 3.4 to 2.0. It is seen that the existence of blunders is probably the most important detrimental factor in the solution.

Unfortunately, neither taking out the ten worst passes nor repeating the computation with the different weights for each station improved the result. It is likely that although taking out the ten worst passes removed the major blunders, it also resulted in losing many useful observations.

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Table 4 Comparisons of the Weights of Each Station

Station No.	All Passes Used			W/o 10 Worst Passes		
	No. of Obs.	$\hat{\sigma}$	p	No. of Obs.	$\hat{\sigma}$	p
220	1579	2.08	2.7	1473	1.73	1.3
221	1668	1.72	4.0	1606	1.56	1.6
222	2912	3.30	1.1	2711	1.33	2.2
223	1862	1.30	6.9	1777	1.27	2.4
224	2821	1.77	3.7	2631	1.49	1.7
225	1893	2.19	2.4	1757	1.33	2.2
226	845	2.56	1.8	763	1.26	2.4
230	2505	2.26	2.3	2435	1.21	2.7
231	2789	3.85	0.8	2609	1.87	1.1
232	2391	2.42	2.0	2205	1.30	2.3
233	2760	1.67	4.2	2575	1.30	2.3
234	2641	2.36	2.1	2444	0.99	4.0
235	865	1.25	7.5	809	1.01	3.8
Degree of Freedom	13,899			12,910		
$\hat{\sigma}_0$	3.4			2.0		

5. CONCLUSIONS

On the basis of the comparisons, the folloiwng conclusions can be drawn:

(1) The geometric mode of solving the problem of simultaneous Doppler-derived ranges without considering the correlation is a weak one.

(2) The correlated geometric mode leads to better results. Comparing with the uncorrelated solution, the correlated mode reduced the average total absolute differences (with respect to EDOC-2) in position from 20.8 ±21.7 m to 10.9 ±6.6 m; and the average absolute station-to-station chord distance differences from 10.2 ±10.5 m to 5.5 ±4.5 m.

(3) The choice of the optimum integration interval is very important for the use of simultaneous Doppler-derived ranges in the geometric mode. The examples of this study demonstrate that the optimum integration interval is 23 s, which agrees with that suggested by [Ashkenazi et al. 1980].

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2.222 Doppler Intercomparison Experiment

In the previous semi-annual report, preliminary results of the 1979 CSU comparison test of Doppler receivers are given. Since that time a final report on the comparison has been completed [Archinal, 1982] as a Master's thesis (and soon as a report of the Department of Geodetic Science and Surveying).

In this report, some of the results presented in the previous report are revised, and some additional final results are presented as well. For a more detailed discussion of the following, refer to [Archinal, 1982], and [Archinal and Mueller, 1982].

FINAL RESULTS OF DATA REDUCTION

As mentioned above, some of the results presented here are slightly different than those given in the last report. This is primarily due to:

a) The determination and use of receiver time delays in the GEODOP processing.

b) The modification of GEODOP to allow the input of a "common station noise" estimate, and use of this option, along with the use of a variance estimation process in GEODOP as well.

Therefore revised versions of the tracking statistics and chord difference results are given here, along with new information concerning the estimation of the receivers' observational (range rate) error and oscillator stability.

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Tracking Statistics

The statistics on the number of passes tracked, used in PREDOP, and two types of GEODOP runs are presented in table 1. Although several numbers have changed substantially from those given in the last report, most of the results given there are still valid. In addition to those results, it should be noted that if these statistics are broken down by antenna setup (as in [Archinal, 1982, pp. 61-70]), it becomes clear that:

a) The CMA-751 and the JMR-1As generally tracked about the same number of passes, and slightly more than the MX1502 (when operating correctly and tracking continuously).

b) There is no bias due to antenna location, at least when PREDOP rejections are considered. The relative percentages of rejections stayed fairly constant for all setups for the JMR-1A #2 and MX1502. No conclusions can be drawn for the CMA-751 due to its faulty antenna cable (on all but one site), or for the JMR-1A #1, since it only occupied one site.

The GEODOP statistics (for a multi-station broadcast ephemeris solution and a single station precise ephemeris solution) show a fairly consistant observations/pass value for all instruments, except for the MX1502, which has a higher value in both solutions. This higher value is due to the fact that the MX1502 was recording only (the better) passes which reached over 15 degrees altitude on the first setup, which strongly affects the grand totals shown here. The observations/pass for the CMA-751 are not representative here either, since it was operating properly only during the first and last setup.

Chord Difference Results

Table 2 shows absolute differences obtained in the chord distance between all pairs of instruments for each antenna setup for multi-station and precise ephemeris solutions. Many of these values are different from those given in the previous report, with generally smaller standard deviations and chord differences than previously reported. This is probably due to the changes in weighting and the better determined delays respectively, and points out the value of the

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TABLE 1 SUMMARY OF TRACKING AND PASS/DOPPLER COUNT ACCEPTANCE
1979 $274^D\ 14^H$ - $317^D\ 16^H$ ($43^D\ 02^H$ TOTAL)

INSTRUMENT	NO. PASSES TRACKED	NO. PASSES TRACKED PER DAY	NO. PASSES AFTER PREDOP		NO. PASSES (DOPPLER COUNT/PASS) AFTER GEODOP SOLUTION				
					MULTI-STA. - B.E.			POINT PS. - P.E.	
CMA-751 [1]	827	19.2	680	82% [4]	594	(17.6)	72% [4]	290	(17.5)
JMR-1A #1 [2]	231	20.6	197	85%	185	(16.5)	80%	77	(17.6)
JMR-1A #2	919	21.3	770	84%	642	(16.5)	70%	317	(16.5)
MX-1502 [3]	805	18.7	483	60%	429	(17.9)	53%	190	(18.4)

[1] CMA-751 LOST PASSES DUE TO FAULTY ANTENNA CABLE, INTERMITTENTLY BETWEEN 286^D AND 302^D.

[2] JMR-1A #1 OBSERVED ONLY DURING $274^D\ 14^H$ - $285^D\ 19^H$.

[3] MX-1502 WAS NOT TRACKING CONTINUOUSLY UNTIL $291^D\ 17^H$ AND HAD BREAKDOWNS INTERMITTENTLY BETWEEN $289^D\ 18^H$ AND $310^D\ 04^H$. THE LARGE DIFFERENCE BETWEEN NUMBER OF PASSES TRACKED AND AVAILABLE AFTER PREDOP IS DUE MAINLY TO 206 PASSES (26% OF THOSE TRACKED) WHICH THE MX-1502 WAS UNABLE TO MAJORITY VOTE.

[4] %'s ARE WITH RESPECT TO NUMBER OF PASSES TRACKED.

TABLE 2 ABSOLUTE DIFFERENCES IN CM OVER CHORD DISTANCES RANGING FROM 7.2 M TO 22.2 M IN LENGTH

ANTENNA SETUP #	MULTI-STATION SOLUTIONS BROADCAST EPHEMERIS - 5 SATELLITES						MULTI-STATION SOLUTIONS PRECISE EPHEMERIS - 2 SATELLITES					
	CMA-751 TO JMR-1A #2		CMA-751 TO MX-1502		JMR-1A #2 TO MX-1502		CMA-751 TO JMR-1A #2		CMA-751 TO MX-1502		JMR-1A #2 TO MX-1502	
	DIFF	σ	DIFF	σ	DIFF	σ	DIFF	σ	DIFF	σ	DIFF	σ
1	-17	6	-13	7	-20	13	1	10	-15	10	18	22
2	-60	11	3	5	-34	26	-30	16	63	27	-47	54
3	-25	25	- 5	22	-55	29	- 7	36	58	36	49	56
4	11	10	6	12	1	20	18	21	-12	20	21	36
5	6	10	-33	12	- 9	21	- 7	16	-83	17	73	35
WEIGHTED MEAN	21	4	14	5	20	8	17	7	33	7	33	15
	CMA-751 TO JMR-1A #1		JMR-1A #1 TO JMR-1A #2		JMR-1A #1 TO MX-1502		CMA-751 TO JMR-1A #1		JMR-1A #1 TO JMR-1A #2		JMR-1A #1 TO MX-1502	
1	3	12	13	7	11	10	28	20	13	10	16	15

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more rigorus weighting and better determination of delay for these solutions.

Even with the differences, the previous result still holds, that most of the differences (all execept two) lie within their three sigma value. A new result is that the single baseline determined between instruments of the same type (between the two JMR-1As on setup #1) did not show significantly better results than pairings between any other instrument combination. In conclusion, it appears that there is no evidence that any of these instruments are biased against one another for chord determinations (over short distances).

Another additional result shown by this table is that the precise ephemeris two satellite solutions for chord distances do not appear to have necessarily higher accuracy or precision than the corresponding broadcast ephemeris five satellite solutions, and in fact the precision of the broadcast ephemeris solution is better in all cases. This simply indicates that the greater number of observations in the broadcast ephemeris solution improves the results more than the corresponding increase in ephemeris accuracy of the precise ephemeris solution. This would imply that if only chord distances were needed from Doppler observations, then generally broadcast ephemeris solutions would be preferable to precise ephemeris solutions, since the former usually have more data available.

Range Rate Measurement Errors

Using procedures described in detail in [Archinal, 1982, pp. 70-79], estimates of the common station noise and each instrument's range rate standard deviation were made for each setup and precise ephemeris satellite. The results are shown in table 3 and discussed here.

First of all, the common station noise was estimated by processing only simultaneous observatins and precise ephemeris orbits. The common station (or "interstation" or "satellite" noise) estimates were made using the common station estimated variance-covariance matrix output by GEODOP to obtain the values shown in column three of table 3. The results vary with satellite and time during the entire test, with an amount between 3.4 and 7.5 cm/30 seconds. The overall average value (weighted mean of all observation pairs) is 4.9 cm/30 seconds. Since the range was not too great,

TABLE 3 ESTIMATION OF COMMON STATION NOISE AND INSTRUMENT RANGE RATE STANDARD DEVIATION

SETUP	SATELLITE NO.	NO. OF OBSERVATIONS (1)	COMMON STATION NOISE (2)	CMA-751 (3)	JMR-1A #1 (3)	JMR-1A #2 (3)	MX-1502 (3)
1	14	263	4.8	8.8	12.0	9.2	12.3
	19	277	3.4	7.9	11.8	9.5	11.4
2	14	75	3.4	--		--	--
	19	123	6.5	--		--	--
3	14	26	--	--		--	--
	19	52	4.0	--		--	--
4	14	161	7.5	12.8		12.4	14.5
	19	157	3.5	--		--	--
5	14	160	5.5	6.4		6.5	13.5
	19	276	5.6	11.3		11.4	12.8
ALL	14		5.4	9.6	12.0	9.9	13.0
	19		4.4	9.7	11.8	10.6	12.1
	BOTH		4.9	9.7	11.9	10.4	12.5

(1) NUMBER OF "30" SECOND COUNTS OBTAINED SIMULTANEOUSLY BY ALL INSTRUMENTS AND USED IN ESTIMATE. NO ESTIMATE DONE IF LESS THAN 10 OBS. PER INTERVAL.

(2) WEIGHTED MEAN OF RANGE RATE INTERSTATION NOISE FOR EACH 30-SECOND INTERVAL.(CM).

(3) WEIGHTED MEAN OF RESIDUALS (NOISE) FOR EACH 30-SECOND INTERVAL (CM).

and rather than change the value for each setup/satellite (perhaps based on too few observations), the GEODOP default of 5.0 cm/30 seconds was then used for all subsequent processing.

Secondly, the estimated receiver range rate standard deviations were computed for each setup and satellite, using the weighted mean of the diagonal elements of the estimated variance-covariance matrix of the residuals. The results for all three instrument types are shown in the last four columns of table 3. These values were obtained from GEODOP single station precise ephemeris solutions, in which the observations were approximately (the rejections due to statistical testing cause some exceptions) simultaneous. Although some of the individual solutions did not have enough data to be considered significant, using at least 600 observations in each case (but only 350 for the JMR-1A #1), the estimated range rate standard deviations were found to be 9.7, 11.9, 10.4, and 12.5 cm/30 seconds for the CMA-751, JMR #1, JMR-#2, and the MX1502 respectively, over the entire period of the test. The variations shown during the test may be partially instrument related, but they are probably due mostly to the satellite noise just discussed. The relative precision of the three instruments continuously observing also stays approximately the same for all time periods and satellites, which also indicates that the variations are non-receiver related. It is also significant that the variation between instruments is usually less than 3 cm/30 seconds, showing that these instruments are generally very similar, and that the variation in the common station noise is generally greater than this. The conclusion can therefore be drawn that the variation of the measurement precision between these instruments is <u>not significant</u>. The even more important conclusion which can be drawn is that the range rate accuracy obtainable depends in many cases more on the time and the satellite than it does on the receiver itself.

Lastly, to obtain the best possible estimates of the final variance-covariance matrices in GEODOP, the GEODOP option was used to allow an internal estimate of the range rate standard deviation and adjacent observation correlation for each pass to be made, with the previously estimated range rate standard deviation value (given in the last paragraph) used as an input approximate value. Although increasing the computational time by over 50% (all of the passes are processed twice), this method takes into account the variation of the satellite noise and possible variations in the receiver noise during the period under consideration. It is felt that this procedure, in conjunction with the

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first two above would result in the most rigorus processing of the data, to provide the best solutions.

Frequency Drift Results

The frequency drift of an instrument's oscillator is an important quantity which can be determined to fairly high accuracy during data reduction. In general, the more stable an oscillator (over the period of a satellite pass) the better the timing and Doppler count measurements can be made. If a drift is occurring, and remains fairly constant in time, it can be taken into account in the adjustment of the data (as in GEODOP), although it must still be assumed to be linear over a pass, and should not be very large in magnitude. If the drift is erratic, either changing during a pass or over just a few passes, the data will be very noisy due to these unmodeled changes in the oscillator. It is therefore important to check the frequency drift variations of these instruments. Ideally, one would like to check the short term drift which corresponds to the length of a satellite pass (over 100 seconds to about 15 minutes), but this is generally not possible unless an atomic standard is available for comparison. Instead, the long term drift of these instruments can be checked for variations (which may provide an indication of the short term stability), or at least checked against the manufacturer's specifications.

In the case of the data collected here, the frequency drift for each instrument for each setup and precise ephemeris satellite has been determined. The values have been obtained from the difference between the first and last (reasonable) frequency offsets computed for each instrument during a setup. The frequency offsets were determined from two satellite (one satellite at a time) precise ephemeris, single station solutions, and the antenna setup periods which ranged from about five to fifteen days in length. Note that to obtain the per day values given here, the assumption has been made that the frequency drift is constant during each setup. Examination of the GEODOP frequency plots supports this assumption.

The results for frequency drift are shown in table 4, and have been graphed in figure 1. They can be summarized as follows:

a) The CMA-751 had a fairly uniform value for frequency drift, using either satellite, and easily met its spe-

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TABLE 4 LONG-TERM OSCILLATOR FREQUENCY DRIFT [1]

SETUP NO.	SATELLITE NO.	CMA-751	JMR-1A #1	JMR-1A #2	MX-1502
1	14	0.45	1.65	3.15	0.41 [2]
	19	0.23	1.78	3.22	0.11 [2]
2	14	0.14		2.50 [2]	-- [3]
	19	-0.32		2.66 [2]	-- [3]
3	14	0.50		2.57 [2]	-2.88
	19	0.33		2.87 [2]	-2.72 [2]
4	14	0.39		2.27 [2]	0.47
	19	0.10		2.68 [2]	1.11
5	14	0.27		2.06	0.78 [2]
	19	0.50		2.47	0.74 [2]
SPECIFICATION:					
	/DAY	±1.00	±0.50	±0.50	?
	/100 s	±0.01	±0.05	±0.05	±0.08

[1] 10^{-10} PARTS PER DAY. DETERMINED FROM FREQUENCY OFFSET OF FIRST AND LAST PASS OF SINGLE STATION, PRECISE EPHEMERIS SOLUTION.

[2] SOLUTION SHOWS FREQUENCY JUMP AFTER FIRST OR SECOND PASS.

[3] TOO FEW PASSES IN SOLUTION, WITH TWO FREQUENCY JUMPS (OSCILLATOR DISTURBED DUE TO MAINTENANCE)

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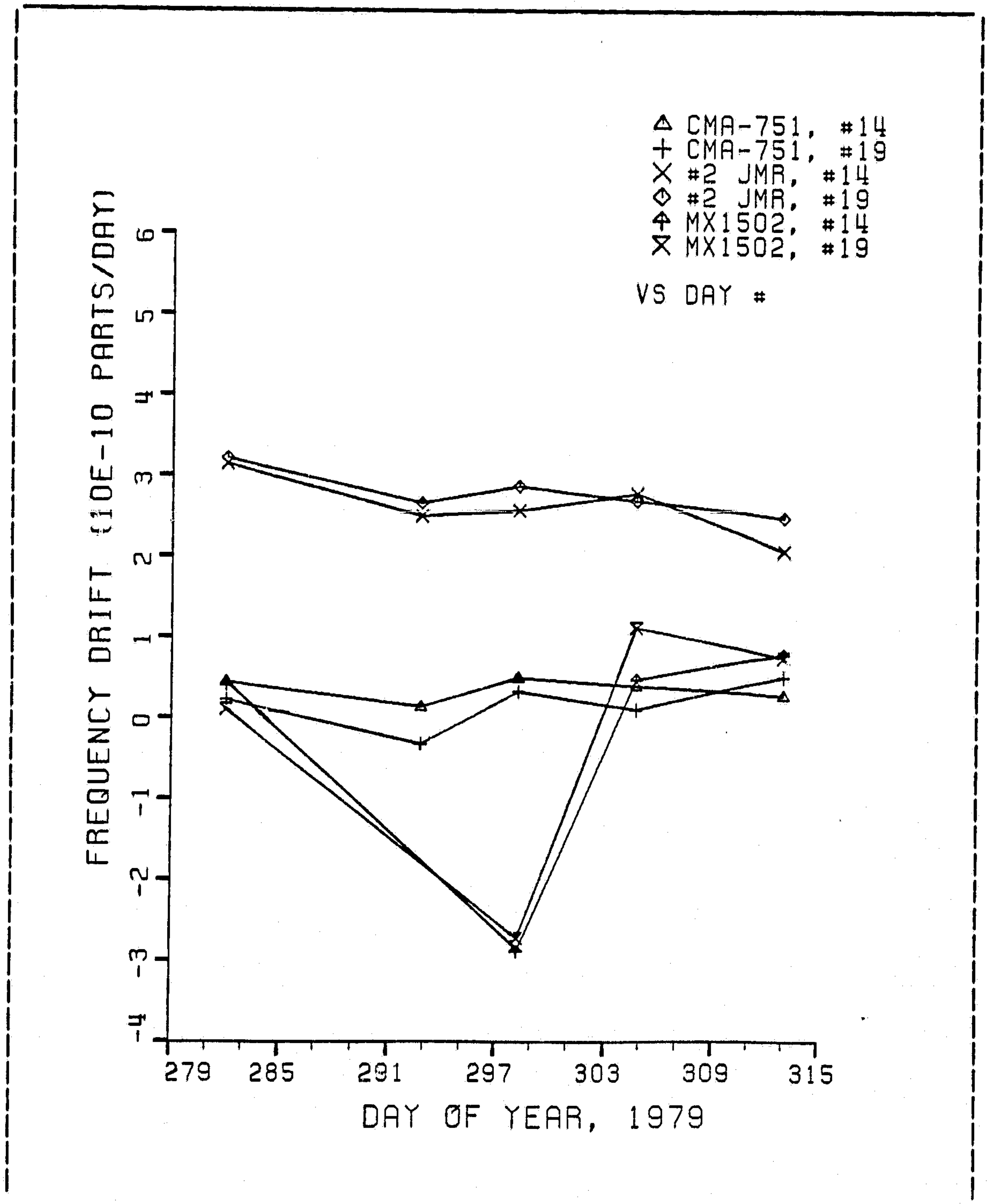

Fig. 4 Oscillator frequency drift versus time.

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cified 10^{-10} parts/day precision. The frequency drift was usually from one half to one tenth of that value, and even approached its 100 second specification.

b) The JMR-1A #2 also had a fairly uniform value for frequency drift, using either satellite. However, both it and the JMR-1A #1 failed to meet their 0.5×10^{-10} parts/day specified precision. (Note that this specification is actually for the JMR-1. It is assumed that the JMR-1A would have the same or a better specification.)

c) The MX1502 did not have a consistent value for its frequency drift, which shows oscillations during the second through fourth setups. Since the values for the first, fourth and last setups are at least similar, one would suspect that the frequency drift changes are mostly due to the various times that the instrument was opened (and its oscillator turned off) for repairs. No specification for the MX1502 drift per day is available for comparison purposes.

FINAL COMMENTS

The results presented here should be considered as the final ones of this comparison, although if time permits, some additional material will possibly be added to the report version of [Archinal, 1982] and the final version of [Archinal and Mueller, 1982]. Work is also continuing on the documentation and further testing of the IBM version of the GEODOP Program System.

As to the further use of the data obtained, the recommendation is made here that the data from both this comparison and the Ottawa comparison be finally processed together in multi-station solutions, to provide a comparison of how well the various possible instrument pairs can measure the long Columbus-Ottawa baselines involved. Further, it is also suggested that a similar reduction be made (if the data can be obtained) using the "Quebec" data described in [Moreau, 1981], which was also obtained during the operational phase of this comparison.

Other investigations are also possible, including extending the results given above by making further comparisons of the chords, comparing the vertical and horizontal positions of the stations through their coordinates, and comparing the computed coordinates with the available control coordinates. These items were not done in this study mainly because they are considered to be of lesser importance than the other results presented, and due to a general lack of time for these lengthy investigations. Other work concerning program options or comparisons of programs could also be done with this data.

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2.3 Earth Deformation Considerations for the Maintenance of a Conventional Terrestrial Reference System

The role of deformation analysis in the maintenance of a new Conventional Terrestrial Reference Frame has been outlined in previous semiannual reports and in [Bock and Zhu, 1982]. Basically, a set of fundamental coordinates X_0 of a global network of stations adopted at an initial epoch define the reference frame. The initial size and shape of this network is defined by the corresponding baseline lengths, D_0. By comparing the estimated baseline lengths at a later epoch to D_0, the deformations of the network can be estimated. This information is then used to improve the global estimates of variations in polar motion and earth rotation, with respect to the conventional axes defined by X_0.

Mathematical Model and Preliminary Estimation Model

The mathematical model for the deformation analysis is simply the chord length of baseline i-j

$$D_{ij} = [(x_j-x_i)^2 + (y_j-y_i)^2 + (z_j-z_i)^2]^{\frac{1}{2}}$$

This model is linearized about X_0 to yield

$$L = AX + V$$

where the observation vector L for the k^{th} baseline is

$$L_k = (D_{ij} - D_{ij_0})_k$$

and the parameter vector X represents the deformations, i.e., the change in coordinates between the initial epoch and a later one. V denotes the noise vector.

Since the design matrix A is rank deficient by 6, we are restricted to a Generalized Gauss-Markoff (GGM) model $(L, AX, \sigma_0^2P^{-1})$ where

$$E(L) = AX$$
$$D(L) = \sigma_0^2P^{-1}$$

If there is no a priori information for the deformations, the minimum bias P-least squares estimate for X is given by

$$\hat{X} = N^{+}U = A^{+}_{PI} \; L \; ; \qquad N = A^{T}PA, \quad U = A^{T}PL$$

using the notation of [Rao and Mitra, 1971], where P is the weight matrix of the observations. This estimate can be shown to be equivalent to that obtained from augmenting the normal matrix N by a set of constraints C such that [Blaha, 1971]

$$AC^{T} = 0$$
$$CX = 0$$

and

$$\hat{X} = (N + C^{T}C)^{-1} - C^{T}(CC^{T}CC^{T})^{-1} \, C$$

This means that we constrain the origin and orientation defined by the coordinates at some later epoch t to be equivalent to that defined by X_0.

Extended Models for A Priori Deformation Information

In the case of the availability of a priori information on the deformations of the network, e.g., as provided by absolute plate motion models, four possible estimators have been outlined and analyzed in [Bock, in preparation]. We briefly outline here the corresponding estimates and their respective properties.

Consider an expanded GGM model $(L, AX, Q_V, Q_{\overline{X}})$ where

$$E(L) = AX$$
$$D(L) = Q_V = E\{VV^{T}\} = \sigma_0{}^2P^{-1}$$
$$E(\overline{X}\,\overline{X}^{T}) = Q_{\overline{X}}$$
$$= \Sigma_{\overline{X}} + \mu_{\overline{X}}\,\mu_{\overline{X}}{}^{T} \qquad (\mu_{\overline{X}} = E\{\overline{X}\} = X)$$

where $\overline{X}$ is an independent estimate of the parameter vector. The resulting minimum M-norm P-least squares minimum variance estimate for X

$$\hat{X}_1 = Q_X N(NQ_X N)^{+} \, U$$
$$= M^{-1} \, N(NM^{-1} \, N)^{+} \, U$$

where $M = Q_X^{-1}$ (positive definite). $\hat{X}_1$ has the property of minimum bias. Therefore, this estimate is termed the BLIMBE (Best Linear Minimum Bias Estimate). In this case, it can be shown that this estimate is equivalent to that obtained from augmenting the normal equations by CM such that

$$AC^T = 0$$
$$CM\hat{X}_1 = 0$$

and

$$\hat{X}_1 = [(N + MC^T CM)^{-1} - C^T(CMC^T\ CMC^T)^{-1}\ C]^{-1}\ U$$

Therefore, we can say that the reference frame is maintained in a minimum M-norm P-least squares sense by a specified number of CTS stations.

For positive semidefinite Q_X, which would be the case for any plate model

$$\hat{X}_1 = (N + M)^{-1}\ N[N(N + M)^{-1}\ N]^+\ U$$

with $M = Q_X^+$. In this case, the estimate is minimum M-seminorm P-least squares but is no longer minimum bias.

For the BLIMBE we assume that the parameter vector X is deterministic and define a weighted norm in the parameter space on the basis of a priori information on X. Another possible biased estimator can be obtained by considering X as a random variable. Our estimation model is $(L, A\overline{X}, Q_V, Q_X)$ where

$$E\{X\} = \overline{X}$$
$$D[X] = E\{(X-\overline{X})(X-\overline{X})^T\} = \Sigma_X$$

which gives

$$Q_X = E\{XX^T\} = \Sigma_X + \overline{X}\ \overline{X}^T$$

The vector X includes the deformations computed from, say, a plate motion model and Σ_X is its covariance matrix. The distribution of L is given by

$$E\{L\} = A\overline{X}$$
$$D[L] = A\ \Sigma_X\ A^T + \sigma_0{}^2P^{-1}$$

from which

$$Q_L = E\{LL^T\} = A\ Q_X\ A^T + \sigma_0{}^2P^{-1}$$

In addition,

$$Q_{XL} = E\{XL^T\} = Q_X A^T$$

and we assume

$$Q_{XV} = 0$$

By the Gauss-Markoff theorem [Liebelt, 1967]

$$\begin{aligned}\hat{X}_2 &= Q_{XL} Q_L^{-1} L \\ &= Q_X A^T(A Q_X A^T + \sigma_0{}^2P^{-1})^{-1} L\end{aligned}$$

Which, for positive definite Q_X,

$$\hat{X}_2 = (N + M)^{-1} U : \qquad M = Q_X{}^{-1}$$

This estimate has been referred to as the Best (or Bayes) Linear Estimate or BLE for short [Rao, 1973, 1976]. While the BLIMBE has the minimum bias property, the BLE has minimum mean square error, i.e., it minimizes the sum of covariance and biased squares

$$MSE(\hat{X}) = \Sigma_{\hat{X}} + [X - E(\hat{X})][X - E(\hat{X})]^T$$

in the class of biased estimators. Note that the BLE requires some knowledge of the deformations in order to compute Q_X. Furthermore, while the BLIMBE reference system is maintained through the constraints $CM\hat{X}_1 = 0$, the deformations estimated by the BLE are with respect to an underlying reference frame of the deformation model from which Q_X is computed.

The previous two estimates are drawn from the class of biased estimators. If Q_X can be constructed, that is, if there exists a priori deformation information, then the origin and orientation singularities are essentially eliminated. We then are led to investigate whether an unbiased estimate exists and we find the Bayesian estimate. Consider the estimation model $(L, AX, Q_V, \overline{X}, \Sigma_{\overline{X}})$ where X is deterministic, $\overline{X}$ random and the set of observation equations

$$\begin{bmatrix} L \\ L_X \end{bmatrix} = \begin{bmatrix} A \\ I \end{bmatrix} X + \begin{bmatrix} V \\ V_X \end{bmatrix} ; \qquad L_X = \overline{X}$$

such that

$$E\{\overline{X}\} = X$$
$$D[\overline{X}] = E\{(\overline{X} - X)(X - X)^T\} = \Sigma_{\overline{X}}$$
$$E\{L\} = AX$$
$$D[L] = A\,\Sigma_{\overline{X}}\,A^T + \sigma_0^2 P^{-1} = \Sigma_L$$

The least squares solution for this model yields

$$\hat{X}_3 = \Sigma_{\overline{X}}\,A^T(A\,\Sigma_{\overline{X}}\,A^T + \Sigma_L)^{-1}\,L + [I - \Sigma_{\overline{X}}\,A^T(A\,\Sigma_{\overline{X}}\,A^T + \Sigma_L)^{-1}\,A]\,\overline{X}$$

for $\Sigma_{\overline{X}}$ positive semidefinite. For positive definite $\Sigma_{\overline{X}}$, this reduces to

$$\hat{X}_3 = (N+M)^{-1}\,U + [I - (N+M)^{-1}\,N]\overline{X}\ ; \qquad M = \Sigma_{\overline{X}}^{-1}$$
$$= \overline{X} + (N+M)^{-1}\,A^T P(L - A\overline{X})$$
$$= (N+M)^{-1}\,(U + M\overline{X})$$

It is easiliy seen that given this estimation model, particularly $E\{\overline{X}\} = X$, $E\{\hat{X}_3\} = X$, so that X is unbiased. This estimate has the minimum mean square error property which implies minimum variance since the bias is equal to zero. Note that in the BLE, the a priori information is incorporated into the moment matrix Q_X, while for $\hat{X}_3$, $\overline{X}$ is applied directly, and a residual deformation is estimated. Thus, we can consider the BLE ($\hat{X}_2$) as a "weak" Bayesian estimate and $\hat{X}_3$ a "strong" Bayesian estimate.

Assume again that some a priori deformations are available. In this case, the model may indicate that $CX = L_X$ where $L_X \neq 0$ which leads to an alternative approach to the constraints $CM\hat{X}_1 = 0$ of BLIMBE. Consider the following set of observation equations

$$\begin{bmatrix} L \\ L_X \end{bmatrix} = \begin{bmatrix} A \\ C \end{bmatrix} X + \begin{bmatrix} V \\ V_X \end{bmatrix}, \qquad L_X = C\overline{X}$$

We assume the estimation model $(L, AX \mid CX = C\overline{X},\ \Sigma_{\overline{X}},\ Q_V)$ where

$$E\{C\overline{X}\} = CX$$
$$D[C\overline{X}] = C\,\Sigma_{\overline{X}}\,C^T$$
$$E\{L\} = AX$$
$$D[L] = A\,\Sigma_{\overline{X}}\,A^T + \sigma_0^2 P^{-1}$$

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For this model, the least squares estimate is

$$\hat{X}_4 = [N + C^T P_X C]^{-1} U + C^T P_X C \overline{X}$$

where

$$P_X = (C \Sigma_{\overline{X}} C^T)^{-1}$$

From [Chipman, 1964]

$$A^+_{PI} = [N + C^T P_X C]^{-1} A^T P$$

$$C^+_{P_X I} = [N + C^T P_X C]^{-1} C^T P_X$$

so that

$$\hat{X}_4 = A_{PI}{}^+ L + C^+_{P_X I} \overline{X}$$

$$= N^+ U + C^+_{P_X I} \overline{X}$$

Therefore, $\hat{X}_4$ can be viewed as a correction term to the minimum I-norm P-least squares estimate $\hat{X}_1$, or a combination of the BLIMBE and Bayesian approaches.

The properties of the four estimators are summarized in Table 1.

Addition and Temporary Deletion of CTS Stations

The reference frame is defined by a particular number of CTS stations. It is quite possible that from time to time one or more of the stations will not be able to participate in a particular deformation analysis observing session which should involve all stations. Furthermore, it must be anticipated that new stations will be added to the frame periodically. Both of these occurrences must be dealt with in order to maintain continuity and avoid ambiguity in the reference frame definition. For the addition of CTS stations we use the filtering and estimation capabilities of least squares collocation. The model becomes

$$L = AX + BS + V$$

Where X is deterministic and represents the coordinates of the new stations to be estimated. The vector S, the signal, is random and includes the

Table 1 Properties of Deformation Estimators

Property \ Estimate	BLIMBE	BLE	Bayesian	BLICUE
Uniqueness	Yes	Yes	Yes	Yes
P-least squares	$\hat{V}^T P \hat{V} = \min$	$\hat{V}^T P \hat{V} + \hat{X}^T Q_X^{-1} \hat{X} = \min$	$\hat{V}^T P \hat{V} + \hat{V}_X^T \Sigma_{\overline{X}}^{-1} \hat{V}_X = \min$	$\hat{V}^T P \hat{V} = \min$
Minimum M-norm	In the class of P-least squares	Yes	No	No
Biasedness	Minimum bias*	Biased	Unbiased assuming $E(\overline{X}) = X$	Unbiased conditional on $E(C\overline{X}) = CX$
Minimum Variance	In the class of minimum bias estimators	In the class of biased estimators	Yes	Conditional
Minimum Mean Square Error	No	In the class of biased estimators	Yes	No
Estimation Model	$(L, AX, Q_V, Q_{\overline{X}})$	$(L, A\overline{X}, Q_V, Q_X)$	$(L, AX, Q_V, \overline{X}, \Sigma_{\overline{X}})$	$(L, AX \mid CX = C\overline{X}, \Sigma_{\overline{X}}, Q_V)$

*Only for positive definite $Q_{\overline{X}}$

filtered deformations. The L and V vectors are as before. From [Moritz, 1980),

$$\hat{X} = [A^T(BQ_SB^T + Q_V)^{-1}A]^+ A(BQ_SB^T + Q_V)^{-1} L$$

$$\hat{S} = Q_SB^T(BQ_SB^T + Q_V)^{-1} (L - A\hat{X})$$

where Q_S is the same as the previous Q_X.

If a station cannot observe, we can use the prediction capabilities of least squares collocation to predict the deformation via

$$\hat{S} = Q_SB^T(BQ_tB^T + Q_V)^{-1} L$$

where

$$S = \begin{bmatrix} t \\ u \end{bmatrix},$$

t includes the deformation of the observing station, and u the predicted deformations of the missing stations.

Conclusions

In order to test the properties of the four estimators and their suitability in estimating deformations, a series of simulations were run as described in [Bock, in preparation]. A 20-station, 8-plate network was chosen for the simulations as depicted in Fig. 1 and Table 2. The AM1-2 absolute plate motion model of [Minster and Jordan, 1978] was "adopted" (see Table 3).

The following conclusions were arrived at based on the simulations. Assuming that the absolute motion models available today are good to within their stated noise levels (this is reasonable considering that [Bender, 1981] indicates that their predicted deformations differ at the centimeter level), it is found that it is advisable to adopt a deformation model than not at all. This was seen from comparing the deformation estimates obtained with a deformation model and those obtained when M = I (no model) is assumed. If a model is adopted, then the BLE appears to be the best candidate for deformation analysis. This conclusion follows from several considerations.

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AM1-2 ABSOLUTE MOTIONS

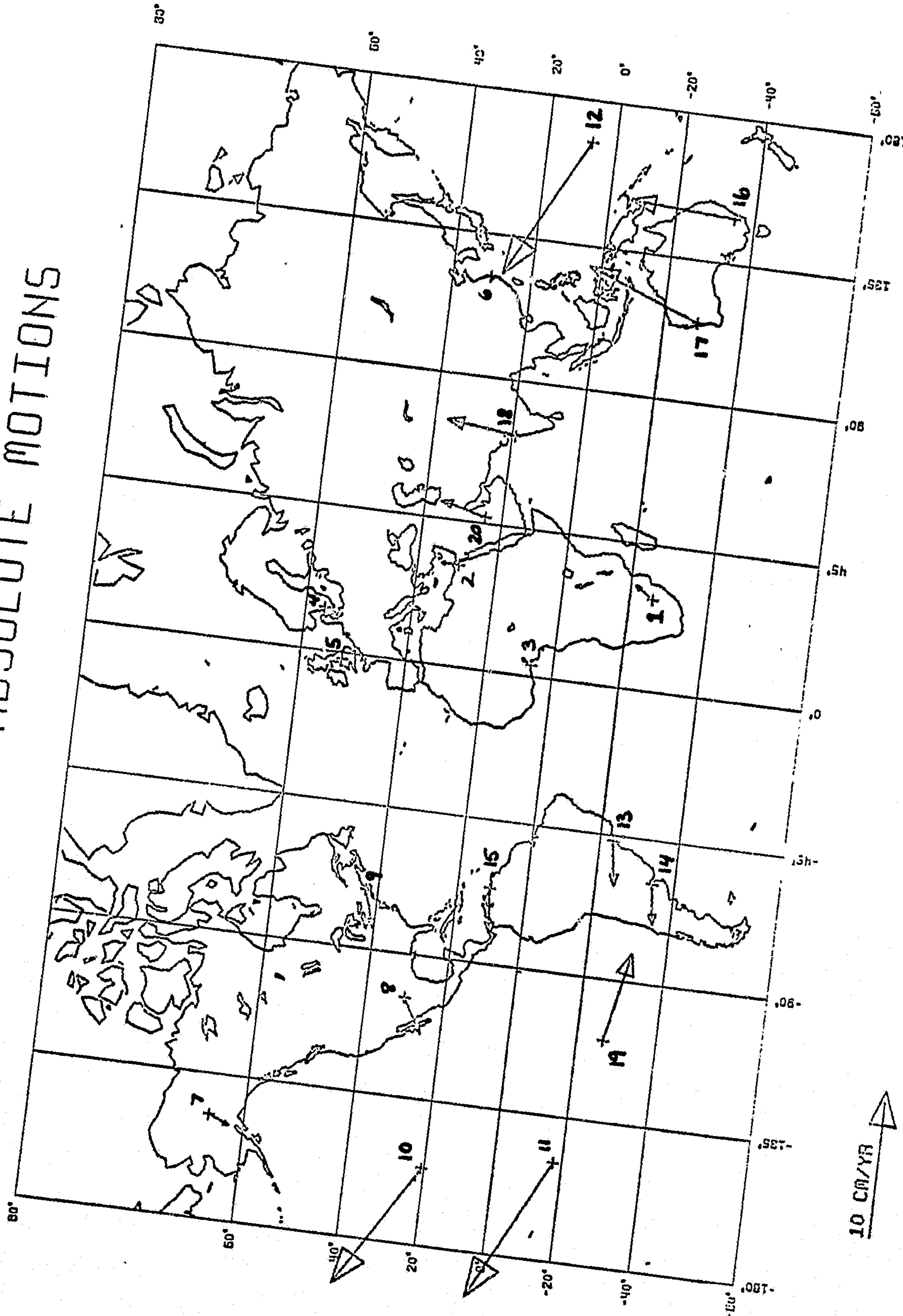

Fig. 1

Table 2 20-Station, 8-Plate Simulation Network and AM1-2 Velocities

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No.	Station	Latitude D	Latitude M	Longitude D	Longitude M	Plate	Velocity (cm/yr) X	Velocity (cm/yr) Y	Velocity (cm/yr) Z
1	Johannesberg*	-29	10	28	02	AFRC	0.03	0.99	1.00
2	Cairo	30	3	31	15	AFRC	-0.49	-0.31	1.01
3	Lagos	6	27	3	28	AFRC	-0.09	0.34	0.62
4	Onsala*	57	0	13	0	EURA	-0.14	-0.32	0.14
5	Jodrell Bank	53	0	358	0	EURA	-0.13	-0.31	0.09
6	Shanghai	30	45	121	45	EURA	-0.09	-0.20	0.21
7	Fairbanks	64	50	212	10	NOAM	-1.38	-0.15	-0.59
8	Ft. Davis*	30	38	256	3	NOAM	-2.43	-0.07	-1.11
9	Westford	42	36	288	30	NOAM	-2.27	-1.29	-0.54
10	Maui*	20	30	203	45	PCFC	-1.80	8.35	4.59
11	Tahiti	-17	30	210	30	PCFC	-6.11	7.59	4.48
12	Marshall Isles	7	0	167	0	PCFC	2.73	9.24	4.76
13	Sao Paulo*	-23	33	313	21	SOAM	-2.26	-1.94	-0.33
14	Buenos Aires	-34	37	301	36	SOAM	-2.44	-1.30	-0.25
15	Caracas	10	35	293	40	SOAM	-2.75	-1.26	-0.26
16	Orroral*	-35	18	149	8	INDI	-3.63	1.70	5.63
17	Yaragadee	-29	3	115	21	INDI	-4.20	1.99	6.47
18	Bombay	18	56	72	51	INDI	-0.95	-1.25	4.30
19	Easter Isle*	-27	5	250	39	NAZC	6.04	-1.56	-1.04
20	Arabia*	24	39	46	46	ARAB	-1.42	-0.36	2.70

* 8-Station 8-Plate Network

Table 3 AM1-2 Absolute Motion Plate Model (Adapted from [Minster and Jordan, 1981], Table 7)

		Absolute Rotation Vector					
	Plate	Deg (N)		Deg (E)		Deg/M.Y.*	
1.	African	18.76	33.93	338.24	42.20	0.139	0.055
2.	Eurasian	0.70	124.35	336.81	146.67	0.038	0.057
3.	North American	-58.31	16.21	319.33	39.62	0.247	0.080
4.	Pacific	-61.66	5.11	97.19	7.71	0.967	0.085
5.	South American	-82.28	19.27	75.67	85.88	0.285	0.084
6.	Indian-Australian	19.23	6.96	35.64	6.57	0.716	0.076
7.	Nazca	47.99	9.36	266.19	8.14	0.585	0.097
8.	Arabian	27.29	12.40	356.06	18.22	0.388	0.067
9.	Antarctic**	21.85	91.81	75.55	63.20	0.054	0.091
10.	Carribean**	-42.80	39.20	66.75	40.98	0.129	0.[illegible]04
11.	Cocos**	21.89	3.08	244.29	2.81	1.422	0.119

* Million Years
** Not used in the Simulations

First, the BLE provides the best estimates (in the sense of minimizing the root mean square error between true and estimated deformations) at the same level as the Bayesian estimate, in the case when the deformation model is correct (and then the deformation is just being filtered from the baseline noise). Second, and most important, it is markedly less sensitive to errors in the adopted deformation model. This is particularly apparent in the case that in reality there is no deformation but we assume some deformation model. These results are due to the minimum mean square error and minimum norm properties of the BLE and its "weak" Bayesian interpretation.

Finally, we should stress that the reference system is dependent on the choice of estimation models including the choice of M (as well as P, but to a lesser extent). This leads to the need for investigations concerning how sensitive the reference system is to changes in M and P. For example, what measures should be taken as M and P improve with time.

The algorithms presented here are general enough to incorporate geophysical as well as geodetic evidence of deformations. In [Bock, in preparation] only models for deformations of interplate type have been considered, to be monitored by periodic re-observations of the baseline lengths. Other aspects to be considered include intraplate and local motions (the site stability problem). Local effects can possibly be modeled on the basis of on-site observations such as by tidal gravimeters and local geodetic nets. It is necessary to investigate how to incorporate these and other types of observations (and their corresponding reference frames) into CTS operations.

This investigation is now being completed, and the final report is in preparation by Y. Bock, to appear in the report series of the Department of Geodetic Science and Surveying, The Ohio State University.

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2.4 Development of Models for Studying Ice Sheet and Crustal Deformations

The observed locations of survey markers change with time. When random and systematic errors are accounted for, what remains is actual movement. The movements of a network of stations can be described as the translation and rotation of the stations as a group and the deformation occurring within the network. Thus when a network of stations is resurveyed, it should be possible to obtain the geophysical parameters of velocity, rotation rate and strain rate [Dermanis, 1981; Livieratos, 1980; Reilly, 1979]. If the same network is resurveyed more than once, either the derivatives of these quantities or averaged values may be calculated.

As most stations are on the surface of the earth, it is natural to assume that all movements and deformations are two-dimensional. This may be adequate in many cases. However, vertical movement and deformation may occur because of irregularities in the surface, faulting, or from being buried under new material. Also, for networks covering relatively large areas, the surface of the earth cannot be well approximated by a plane. In this case, it may be better to determine the movements in an arbitrary (earth-centered) coordinate system and then transform these results to a latitude, longitude and elevation coordinate system.

A model is being developed to determine these geophysical parameters from the coordinates of a network that has been resurveyed at least once. Several methods have been proposed for obtaining sufficiently accurate coordinates [Brunner et al., 1981, Niemeier, 1979]. One technique that has been proposed for studying tectonic deformation is to use positions determined by Doppler satellite receivers [Malyevac and Anderle, 1979]. The precision of the receivers used individually (point positioning) is meters to tens of meters. But by using translocation between two or more receivers, the relative positions can be determined to within decimeters [Brown, 1976]. However, the movements and deformations of the crust are slow even in tectonically active areas [Savage, 1978; Minster and Jordan, 1978]; thus the time span between resurveying must be of the order of decades. Because the time period between reobservations is so long, it may be difficult to guarantee that the coordinate systems are identical.

For example, the coordinate system defining the broadcast ephemeris of the Navy Navigational Satellite System slowly varies with time. This problem could be overcome by using relative rather than absolute coordinates. Thus the velocities and rotation rates would be relative to some "fixed" stations. However, the deformation within the network can still be obtained by calculating the strains from the changes in the chord lengths between the stations. The only assumption needed for this is that the scale of the coordinate system has not changed. Because the strains obtained this way are theoretically identical to the strains obtained from coordinate differences, any differences can be attributed to rotations and/or translations of the coordinate system.

For the purposes of testing the model, the data set being used is from survey stations placed on the Greenland ice sheet. Seven Magnavox 1502 satellite receivers were used during the summers of 1980 and 1981 to obtain the movement of 22 stations on the ice sheet of Greenland. Using the data reduction program GEODOP [Kouba and Boal, 1976], the coordinates of the stations have been obtained relative to the positions of two stationary stations (which were located on the west coast of Greenland). The formal accuracy of the coordinates is under 20 cm. These stations are moving at velocities of up to 45 m per year, and the magnitude of the maximum strain rates are over 100 ppm.

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3. PERSONNEL

Ivan I. Mueller, Project Supervisor, part time

Brent Archinal, Graduate Research Associate, part time

Yehuda Bock, Graduate Research Associate, part time

George Dedes, Graduate Research Associate, part time

Alice J. Drew, Graduate Research Associate, part time

Erricos C. Pavlis, Graduate Research Associate, part time

Irene B. Tesfai, Secretary, part time from 6/1/82

Zhu Sheng-Yuan, Visiting Scholar, part time through 8/31/82

4. TRAVEL

Ivan I. Mueller

Patras, Greece — August 17-20, 1982
Attended XVIII General Assembly of the International Astronomical Union. Presented the paper which appears on pp. 2-18 and a report on progress in planning for the new Conventional Terrestrial Reference System to Commissions 4, 19 and 31. Chaired meetings of the IAG/IAU Working Group COTES.

Budapest, Hungary — August 20-26, 1982
Attended 3rd Symposium on the Study of Movements in Engineering Surveys. Presented a paper on the Greenland Ice Movement Study (see p. 87).

5. REPORTS PUBLISHED TO DATE

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262 The Observability of the Celestial Pole and Its Nutations
by Alfred Leick
June, 1978

263 Earth Orientation from Lunar Laser Range-Differencing
by Alfred Leick
June, 1978

284 Estimability and Simple Dynamical Analyses of Range (Range-Rate and Range-Difference) Observations to Artificial Satellites
by Boudewijn H.W. van Gelder
December, 1978

289 Investigations on the Hierarchy of Reference Frames in Geodesy and Geodynamics
by Erik W. Grafarend, Ivan I. Mueller, Haim B. Papo, Burghard Richter
August, 1979

290 Error Analysis for a Spaceborne Laser Ranging System
by Erricos C. Pavlis
September, 1979

298 A VLBI Variance-Covariance Analysis Interactive Computer Program
by Yehuda Bock
May, 1980

299 Geodetic Positioning Using a Global Positioning System of Satellites
by Patrick J. Fell
June, 1980

302 Reference Coordinate Systems for Earth Dynamics: A Preview
by Ivan I. Mueller
August, 1980

320 Prediction of Earth Rotation and Polar Motion
by Sheng-Yuan Zhu
September, 1981

329 Reference Frame Requirements and the MERIT Campaign
by Ivan I. Mueller, Sheng-Yuan Zhu and Yehuda Bock
June, 1982

Estimation of Earth Deformations for the Maintenance of a New Conventional Terrestrial Reference System
by Yehuda Bock
November, 1982 (in preparation)

On the Geodetic Applications of Simultaneous Range-Differencing to LAGEOS
by Erricos C. Pavlis
December, 1982 (in preparation)

The following papers were presented at various professional meetings and/or published:

"Concept for Reference Frames in Geodesy and Geodynamics"
AGU Spring Meeting, Miami Beach, Florida, April 17-21, 1978
IAU Symposium No. 82, Cadiz, Spain, May 8-12, 1978
7th Symposium on Mathematical Geodesy, Assisi, Italy, June 8-10, 1978
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"What Have We Learned from Satellite Geodesy?
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"Parameter Estimation from VLBI and Laser Ranging"
IAG Special Study Group 4.45 Meeting on Structure of the Gravity Field
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"Estimable Parameters from Spaceborne Laser Ranging"
SGRS Workshop, Austin, Texas, July 18-23, 1978

"Defining the Celestial Pole," manuscripta geodaetica, 4 (1979), No. 2 pp. 149-183.

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