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A DSN Optimal Spacecraft Scheduling Model

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A computer model is described which uses mixed-integer linear programming to provide optimal DSN spacecraft schedules given a mission set and specified scheduling requirements. A solution technique is proposed which uses Benders' Method and a heuristic starting algorithm.

I. Introduction

The DSN optimal scheduling problem refers to the problem of assigning tracking times to a set of spacecraft over a specified period of time, given the view periods of the spacecraft and a set of restrictive tracking requirements. For a typical DSN tracking situation, this is a complex decision-making problem with many variables and constraints.

A mixed-integer linear mathematical programming model is presented which may serve as a basis for both sophisticated constraint checking and automated optimal scheduling. A solution technique is proposed which uses a combination of Benders' Method and a heuristic algorithm. The proposed Benders' algorithm uses standard linear programming for the continuous variable subproblem and specialized 0,1 integer programming for the discrete variable subproblem. The proposed model will offer great facility for altering or updating the constraint set (tracking requirements) and is amenable to modern interactive computer graphics technology.

A prototype computer model was developed using the mixed-integer linear model and a standard mixed-integer programming subroutine. An experimental study was conducted using this model. The results of this study demonstrated the need for a decomposition technique such as Benders' Method and development of a specialized heuristic starting algorithm for the 0, 1 integer subproblem.

II. Problem Description

A. Problem Statement

The DSN spacecraft (S/C) scheduling problem is stated as: given the station rise and set times of a number of S/C at each of three locations (Goldstone, Australia, and Spain) and specified tracking requirements and constraints for each S/C, determine a tracking schedule which maximizes the total tracking time over all S/C and stations.

B. 24-Hour and *N*-Day Scheduling Models

The problem was formulated and explored in two ways:

(1) Develop an optimization model to determine a 24-hour tracking schedule which reflects tracking requirements.

(2) Develop an optimization model to determine a detailed N-day tracking schedule which reflects tracking requirements.

The first approach, that of determining a 24-hour schedule, is used to formulate and solve for an optimal tracking schedule using linear programming. The 24-hour tracking schedule model may be used to study the allocation of resources portion of the DSN scheduling problem and to develop a basic facility for the problem.

The second approach, that of determining a N-day schedule is more difficult. The N-day model may be used for determining real-time or actual DSN schedules. The N-day scheduling problem is nonlinear as it includes determining the sequence or order of S/C tracking as well as the linear resource allocation problem.

C. Possible Constraints for the Scheduling Model

In both the 24-hour and N-day models, the objective function chosen is the maximization of the total tracking time over all S/C and all stations. This form is a good candidate for a representation of DSN utilization. Several types of requirements or constraint possibilities were considered and are stated as follows:

(1) Minimum and maximum tracking pass length.

(2) Number of tracking passes per scheduling period.

(3) Minimum and maximum station-to-station overlap of tracking passes.

(4) Tracking on consecutive days or stations.

(5) Minimum elevation angle for tracking.

(6) Station handover or S/C switching time.

(7) Antenna selection at a station complex.

(8) S/C tracking time priority.

(9) Minimum and maximum acquisition of signal (AOS) and loss of signal (LOS) times.

(10) Pre- and post-calibration periods.

(11) Station downtime or maintenance periods.

Several of these constraint types were considered in developing the 24-hour model, and all were investigated in developing the N-day model.

III. 24-Hour Spacecraft Scheduling Model

A. Problem Formulation and Definitions

Given a subnet of DSN 64-meter Deep Space Stations at Goldstone, Australia, and Spain, and a view period schedule for n S/C at each station, the problem is to maximize the total daily tracking time subject to specified constraints for the missions.

For a given set of missions and corresponding daily view period schedule, a tracking schedule is defined as follows: for each view period, a station either does not track the S/C at all or tracks the S/C for a single subinterval of the view period. A station may track only one S/C at a time and each S/C is only tracked by one station at a time. Each tracking period must satisfy the minimum duration parameter δ. Also, in the usual case, weighting constraints on the relative total daily tracking time between missions are imposed. These weighting constraints are specified in the form of either equality or inequality constraints.

For each station, it is assumed that the n tracking periods are either continuous or disjoint subintervals of the cyclic 24-hour day. It is assumed that the n tracking periods occur in the same cyclic order as the rise times of the Goldstone view periods. Although the view periods for a S/C generally overlap stations in the east-west cyclic order of the stations (e.g., Goldstone, Australia, Spain, Goldstone), the view period configuration does not change much from station to station, on a given day, and this choice almost always provides an optimum schedule.

The problem may now be formulated as a linear program whose variables provide the start and end times of the tracking periods for each S/C at each station required to provide maximum tracking time (given the above assumptions). Let the view period of S/C i and station j have rise and set times r_{ij} and s_{ij}. The set times may be shifted by 24 hours so that $0 \leqslant r_{ij} < 24$, $r_{ij} < s_{ij} < r_{ij} + 24$. The tracking period for S/C i and station j is $(r_{ij} + z_{ij}, s_{ij} - w_{ij})$, where z_{ij} and w_{ij} are the non-negative variables of the linear program. Figure 1 illustrates these variables. Table 1 provides a list of variables and parameters used in the problem formulation.

B. Objective Function

The objective function to be maximized is the total weighted tracking time for n S/C, over stations j, $j = 1, 2, 3$.

Let the view period rise time for S/C i and station j be r_{ij} and the view period set time for S/C i and station j be s_{ij}. The tracking time, T_{ij}, of S/C i at station j is

$$T_{ij} = (s_{ij} - w_{ij}) - (r_{ij} + z_{ij}) \tag{1}$$

and the total tracking time for S/C i over all stations is

$$TT_i = \sum_{j=1}^{3} T_{ij} \qquad i = 1, 2, \cdots, n . \tag{2}$$

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The objective function is then formulated as

$$\max \sum_{i=1}^{n} c_i \, TT_i \tag{3}$$

or

$$\max \left\{ \sum_{i=1}^{n} - c_i \left[\sum_{j=1}^{3} (w_{ij} + z_{ij}) \right] + c' \right\} \tag{4}$$

C. Constraints

The constraints for the 24-hour linear programming model included the following:

(1) Minimum tracking period.

(2) No overlap of tracking periods for a given S/C.

(3) No overlap of S/C tracking periods for a given station.

(4) Weighting of S/C tracking time inequality constraints.

As an example of a constraint formulation, the minimum tracking period constraint is presented. The minimum tracking period constraint states that the tracking periods for S/C i at station j have duration $\geqslant \delta_{ij}$. The constraint is

$$T_{ij} = (s_{ij} - w_{ij}) - (r_{ij} + z_{ij}) \geqslant \delta_{ij} \tag{5}$$

or

$$z_{ij} + w_{ij} \leqslant - r_{ij} + s_{ij} - \delta_{ij} \tag{6}$$

$$i = 1, 2, \cdots, n$$

$$j = 1, 2, 3.$$

The constraints for the 24-hour linear programming model are described in detail in Ref. 1.

IV. Mixed-Integer Linear Programming *N*-Day Scheduling Model

A. Continuous Variable Allocation Model

The variables for the linear programming model are the continuous linear programming rise variable z and the set variable w. The 24-hour model is actually an allocation model in that the order of S/C tracking is assumed; in this case all S/C are always tracked at each station in the order of rising over the Goldstone station. Also, the objective function and all of the constraints are linear in the variables, z and w, of the problem. The allocation problem here is the assignment of specific acquisition of signal (AOS) and loss of signal (LOS) times as determined by the optimal values of z and w in the linear programming solution.

The linear programming allocation problem may be written as

$$\begin{aligned} &\min c'x \\ &Ax \leqslant b \\ &x \geqslant 0 \end{aligned} \tag{7}$$

where c is an n-vector of weighting coefficients (c' is the transpose of c), x is an n-vector of the continuous linear programming variables z and w, A is a $m \times n$ matrix of coefficients of the constraint equations, and b is a m-vector of the right-hand side constants of the constraint equations. The problem may be easily converted from a minimization problem to a maximization problem by using the fact that *max* $g(x)$ = − *min* − $g(x)$. The x-vector is non-negative because negative value of either z or w would mean that tracking begins outside the view period, which is not realistic.

B. *N*-Day Sequencing Problem

The N-day scheduling problem, where $N > 1$, is actually an allocation problem for each arbitrary 24-hour period and a sequencing (or S/C ordering) problem over the N-days. This is so because in the general case, over any N-day scheduling period, the number of tracking passes for any S/C may be $< N$. Therefore, optimal decisions must be made concerning which days to track which S/C. It is no longer applicable to track all S/C simply in the order of Goldstone rise as for the 24-hour model.

The sequencing or, in this case, the S/C ordering problem is, in general, a nonlinear problem and linear programming is no longer applicable. This is because the ordering possibilities must be modeled using discrete integer variables; i.e., a S/C is either tracked within a particular view period or it is not. Because it is a yes/no decision, the problem may be set up so that the variables take on only 0 or 1 values. These variables are commonly called indicator variables and, for the N-day scheduling problem, indicate whether or not a particular S/C is tracked within each specified view period. So, for a tracking situation with i S/C, j stations, and k days the number of indicator variables required is ijk. As a typical example, for a scheduling problem with 6 S/C, 3 stations and 7 days, 126 integer indicator variables are required. Each set of values of the indicator variables represents a particular S/C order. The number of possible S/C orderings is 2^{ijk}. For the example given, the number of possible S/C orderings is 2^{126} or 8.5×10^{37}. For each order specified by the indicator variables, a S/C allocation problem must be solved in order to determine its objective function value so that the optimal schedule may be found. Obviously, the N-day optimal scheduling problem is a large-scale optimization problem with potentially severe computational considerations.

C. Formulation of Switching Constraints

The purpose of the switching constraints is to set up the lower and upper bounds on the tracking pass lengths for each possible view period. For a given view period (recall there is one for each S/C, station, and day), let LB denote the lower bound and UB denote the upper bound, and y be the 0,1 indicator variable. The constraints are called switching constraints because the set (LB, UB) goes to (0,0) as the indicator variable steps from 1 to 0. In the latter case the tracking pass is switched out and the S/C is not tracked within this view period. For this particular view period, let T be the tracking pass length where, from the previous section,

$$T = (s - w - r - z) \tag{8}$$

The set of switching constraints for this view period is

$$\begin{aligned} &T \geqslant 0 \\ &T \leqslant UB \\ &T \leqslant LB + (y - 1)M \\ &T \leqslant yM \\ &y = 0 \text{ or } 1 \end{aligned} \tag{9}$$

where M is an arbitrarily large number. Note that zero always satisfies the upper bound constraint but not the lower bound constraint. Therefore, the lower bound must either be some value $LB > 0$ or included as zero if no lower bound is specified. Also, if no minimum or maximum tracking pass length is specified, $LB = 0$ and UB = length of the tracking pass $(s - r)$. In this case there are redundant equations in the constraint set.

D. Formulation of Other Constraint Types

The form of the switching constraints is such that it includes both the continuous variables z and w and the discrete 0.1 variable y. It is possible to write all of the constraint types mentioned in terms of these variables. This includes tracking on consecutive days and stations, number of tracking passes required during N days, station and S/C overlap requirements, antenna switching options, and minimum elevation angle requirements. Also, since the equations or inequalities representing these constraints are additively separable, the formulation of the problem may be divided into two subproblems, one containing only the continuous variables z and w, and the other containing only the discrete 0,1 variables.

E. Formulation of the N-Day Scheduling Model

A math programming problem which contains both continuous and discrete variables is called a mixed-integer program. Since the optimal scheduling problem also has a linear objective function and only linear constraints in the continuous variables, it is a mixed-integer linear programming problem. Mixed-integer linear programming (MILP) problems are a specific category of optimization problems in the field of operations research, and there are several solution techniques available for these problems in the literature. These problems are often of a large-scale nature and are typically computationally difficult.

The form of the constraint set suggests that by rearranging all equations and inequalities, so that the continuous variable and discrete 0.1 variables are separate, the problem may be solved by dealing with the continuous and discrete subproblems separately. The objective function also may be separated into a continuous variable and discrete variable portion. There are several solution procedures which take advantage of a separable structure. These procedures frequently couple two individual solution procedures and iterate between them. In this case, the continuous subproblem may be solved by linear programming and the discrete 0,1 variable subproblem solved by a specialized 0,1 integer programming algorithm.

Let x be an n-vector of z and w continuous variables and y be a p-vector of discrete 0,1 variables; c is an n-vector of weighting coefficients for the continuous variables and c is a p-vector of weighting constraints for the 0,1 discrete variables. Rearranging continuous and discrete variables, the problem form is

$$\begin{gathered} \min c'x + \bar{c}'y \\ \text{subject to } Ax + \bar{A}y = b \\ x \geqslant 0, y \in Y \end{gathered} \tag{10}$$

where A and $\bar{A}$ are $m \times n$ and $m \times p$ coefficient matrices, respectively, for the constraint set and b is a m-vector of right-hand side constants of the constraint set. Y is the subset of the set of integer variables which contains only the values 0 and 1. Recall that the left-hand subproblem is a continuous-variable linear problem and the right-hand subproblem is a nonlinear problem.

V. Results of Prototype Study

A prototype computer model was developed using the mixed-integer linear programming model presented. A standard mixed-integer subroutine was used to solve typical example DSN scheduling problems. The computer program was designed to be run in an interactive graphics mode. This enables a user to sit at an interactive graphics terminal and conveniently and efficiently make scheduling decisions. The effects of potential changes to the current schedule are quickly presented on the video display.

The prototype study positively demonstrated the feasibility of developing a mathematical model and interactive computer tool to aid in making complex DSN scheduling decisions. The following advantages of such an automated tool were identified:

(1) Scheduling decisions could be made very efficiently using graphics terminals in an interactive mode:

(2) Better schedules would result in terms of network utilization or other selected criteria.

(3) Less effort and time would be required by the schedulers.

(4) Conflicts could be resolved in a systematic way if desired.

(5) Clear-cut criteria for conflict resolution choices could be provided.

(6) Manual scheduling will always be possible and automated decisions may be overridden.

As a result of this study it was also determined that a more powerful solution procedure needed to be developed to solve complex scheduling problems more efficiently. It is out of this need that the proposed development of a solution procedure using Benders' Method and a heuristic starting algorithm evolved. The theoretical basis of Benders' Method and its application to the DSN optimal spacecraft scheduling problem are presented in an appendix.

VI. Further Development

Plans for further development of the optimal spacecraft scheduling model include the following steps:

(1) Develop a mixed-integer linear programming computer model which uses Benders' Method for solution of the DSN scheduling problem.

(2) Develop an efficient heuristic starting algorithm to use in the Benders' Model.

(3) Present test data for the model using actual DSN scheduling examples.

(4) Provide favorable comparisons, both in efficiency and results, between the research model and the current DSN scheduling procedure.

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1. Webb, W. A., "Scheduling of Tracking Times for Interplanetary Spacecraft on the Deep Space Network," Joint National Meeting of the Institute of Management Sciences and Operations Research Society of America, New York, May 1978.

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Table 1. Definition of problem variables and parameters

r_{ij}	View period rise time for S/C i and station j
s_{ij}	View period set time for S/C i and station j
z_{ij}	Linear program rise variable for S/C i and station j
w_{ij}	Linear program set variable for S/C i and station j
T_{ij}	Tracking time for S/C i at station j
TT_i	Total tracking time for S/C i over all stations
δ_{ij}	Minimum tracking period duration for S/C i and station j
c_i	Objective function tracking time weighting for S/C i

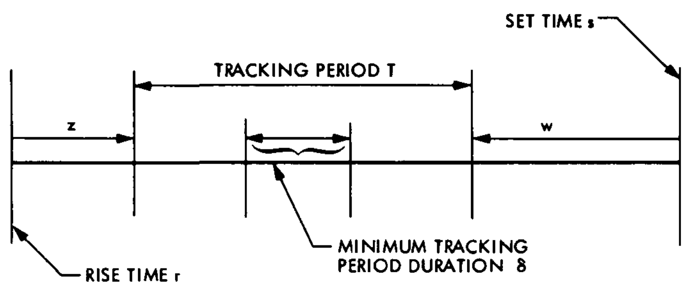

Fig. 1. Tracking period and linear programming variables

Appendix

Solution of the N-Day Scheduling Problem by Benders' Method

The form of the N-day scheduling problem as formulated by a mixed-integer linear programming model lends itself very well to solution by Benders' partitioning algorithm for mixed-variable optimization problems. Benders' Method may be used on mixed-integer programming problems which are separable into the linear and nonlinear (integer) portions. The separable subproblems may then be solved individually to determine their individual optimal solutions and summed to determine the optimum of the original problem. This partitioning procedure takes up less computer storage and allows the usage of specialized techniques for each portion of the problem. For example, linear programming may be used for the linear part of the problem. The method also allows the use of a good initial schedule which may be determined by a heuristic algorithm and, thus, save considerable time in arriving at optimal solution.

I. Theoretical Basis of Benders' Method

Benders' Method may be applied to an optimization problem of the form

$$\left.\begin{aligned} &\min c'x + f(y) \\ &\text{subject to } Ax + F(y) = b \\ &x \geqslant 0, y \in Y \end{aligned}\right\} \text{problem} \qquad \text{(A-1)}$$

where $min\ g(x) = -\ max - g(x)$.

For this discussion, the matrix A is $m \times n$, x and c are n-vectors (c' is the transpose of c), y is a p-vector, f is a scalar-valued function of y, F is an m-vector whose components are functions of y, b is an m-vector, and Y an arbitrary subset of E^q. The functions f and F need not be linear; in fact, for the scheduling problem Y is the set of variables in E^q with integral values components. Problem P will be referred to as the primal problem. For the scheduling problem, x is the set of continuous variables which determine the allocation of tracking passes and Y is the vector of integer variables which describe the tracking frequency and order.

Since P is linear in x for fixed values of y, it is natural to attempt to solve it by fixing y, solving a linear program in x, obtaining a "better" y, etc. Of course, only values of y for which there exist x satisfying the resulting linear constraints may be considered. That is, y must lie in the set

$$\bar{Y} = \{y \in Y \mid \exists\ x \geqslant 0 \ni Ax = b - F(y)\} \qquad \text{(A-2)}$$

$$= \{y \in Y \mid \gamma^j (b - F(y) \leqslant 0, j = 1, \cdots, K\} \qquad \text{(A-3)}$$

where γ^j are all (K of these) of the simplex multipliers of Phase I of the simplex method. The simplex method is a two-phase procedure for finding an optimal solution to linear programming problems. Phase I finds an initial basic feasible solution if one exists, or gives the information that one does not (in which case the constraints are inconsistent and the problem has no solution). Phase II uses this solution as a starting point and either finds a minimizing solution or yields the information that the minimum is unbounded (i.e., $-\infty$). So $\bar{Y}$ is the set of all $y \in Y$ such that you get feasible constraints for problem P. A feasible right-hand side does not exist unless that right-hand side makes an obtuse angle with every one of the Phase I simplex multipliers.

Define the function $\sigma(b,y)$ as

$$\begin{aligned} \sigma(b - F(y)) &= \min c'x \\ \text{subject to } Ax &= b - F(y) \qquad \text{(A-4)} \\ x &\geqslant 0. \end{aligned}$$

Now if $y \in \bar{Y}$ then $\sigma(b - F(y) < \infty$, and if $y \in Y \sim \bar{Y}$ (i.e., if y is not feasible), then $\sigma = +\infty$. (A-5)

Recall from linear programming duality theory that if y is feasible then

$$\min c'x = \max \pi\ [b - F(y)] \qquad \text{(A-6)}$$

where π is the set of dual variables corresponding to the primal problem $min\ c'x$. *Let* $\{\pi', \cdots, \pi^J\}$ be *all* possible *optimal* simplex multiplier vectors from Eq. (A-6). Then for $y \in Y$,

$$\sigma\ [b - F(y)] = \max \pi^i\ [b - F(y)] \qquad \text{(A-7)}$$

$$i = 1, \cdots, J.$$

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Rewrite problem P as

$$\min_{y \in \bar{Y}} f(y) + \min c'x \quad \text{subject to } Ax = b - F(y), \quad x \geqslant 0. \tag{A-8}$$

Recall from classical optimization theory that

$$\min_{x,y} f(x, y) = \min_{y} \min_{x} f(x, y). \tag{A-9}$$

The form of problem (A-8) now becomes

$$\min \{f(y) + \sigma [b - F(y)]\} \tag{A-10}$$

subject to

$$\gamma^j [b - F(y)] \leqslant 0 \quad j = 1, \cdots, K$$

$$y \in Y.$$

Substituting further from duality theory

$$\min \{f(y) + \max_{i = 1, \cdots, J} \pi^i [b - F(Y)]\} \tag{A-11}$$

subject to

$$\gamma^j [b - F(y)] \leqslant 0 \quad j = 1, \cdots, K$$

$$y \in Y.$$

Recalling that

$$\min_{x \in X} f(x) = \min \eta \quad \eta - f(x) \geqslant 0, \quad x \geqslant 0, \tag{A-12}$$

problem (A-11) may be written as

$$\min \eta$$

subject to

$$\eta \geqslant f(y) + \max \pi^i [b - F(y)] \quad i = 1, \cdots, J$$

$$0 \geqslant \gamma^j [b - F(y)] \quad j = 1, \cdots, K$$

$$y \in Y \tag{A-13}$$

or

$$\min \eta$$

subject to

$$\left.\begin{array}{lll} \eta \geqslant f(y) + \pi^i [b - F(y)] & & \\ 0 \geqslant \gamma^j [b - F(y)] & i = 1, \cdots, J & \\ y \in Y & j = 1, \cdots, K & \end{array}\right\} \text{Problem } M. \tag{A-14}$$

Problem M will be referred to as the master problem and is equivalent to problem P. Problem M includes all of the simplex multipliers of Phases I and II of the simplex method applied to Problem P. In general, it has many constraints. Furthermore, the constraints are not known explicitly because the simplex multipliers π^i and γ^j are not known.

II. Application of Benders' Method to the N-Day Scheduling Problem

The original problem, Problem P, is restated from the previous subsection as

$$\left.\begin{array}{r} \min c'x + F(y) \\ \text{subject to } Ax + F(y) = b \\ x \geqslant 0, y \in Y \end{array}\right\} \text{Problem } P \tag{A-15}$$

This problem may be rewritten in the form of the DSN scheduling problem by introducing slack variables to convert equality constraints to inequality constraints and by using the fact that *max* $g(x) = -$ *min* $- g(x)$. The x-vector represents the continuous w and z variables which determine the tracking pass allocations for a given tracking order, and the y variables are the 0,1 switching variables which determine the order and frequency of tracking. Decomposition allows the continuous variable subproblem to be solved by linear programming and the 0,1 integer problem to be solved by specialized 0,1 integer programming algorithms. Solving each of the subproblems separately results in a savings of computer storage required by breaking a large problem into two smaller ones. Time savings in arriving at a solution results from taking advantage of linear programming for the continuous subproblem and a specialized algorithm for the 0,1 integer subproblem. Also, considerable computing time may be saved by making a good choice of the starting schedule to begin the Benders' procedure.

To solve the scheduling problem using Benders' Method, let $y^o \epsilon \bar{Y}$ be the initial scheduling order (e.g., the scheduling order resulting from the schedule determined by a heuristic algorithm). Solve the primal problem, Problem P, as a function of y^o where

$$\sigma(b - F(y^o)) \triangleq \min c'x \qquad \text{(A-16)}$$

subject to

$$Ax = b - F(y^o)$$

$$x \geqslant 0.$$

This subproblem represents the schedule allocation problem given the sequence specified by order y^o and may be solved by linear programming. Let x^o be an optimal solution, and let π^o be the vector of optimal simplex multipliers.

Let $UB \triangleq f(y) + \sigma(b - F(Y))$ be defined as the upper bound on the objective function for the scheduling order specified by y. For the initial schedule y^o,

$$UB^o = f(y^o) + \sigma\,(b - F(y^o))\,. \qquad \text{(A-17)}$$

Now solve the first relaxed master problem,

$$\min \eta$$

$$\text{subject to } \eta \geqslant f(y) + \pi^o\,[b - F(y)]\,. \qquad \text{(A-18)}$$

$$y \,\epsilon\, Y$$

Recall that a relaxed master problem has fewer constraints than the master problem M. For the N-day scheduling problem this is a 0,1 integer program problem which may be solved by specialized techniques. Let (η', y') be the solution. Note that η' is a lower bound for P since the objective function is being minimized for more constraints at each iteration and must have an equal or higher value. Define the lower bound on problem P as

$$LB \triangleq n$$

where $\eta' \leqslant UB^o$. Now if $LB = UB$ at any iteration, an optimal solution has been found and the algorithm stops. If at any iteration $LB = \eta < UB$, solve the linear program,

$$\sigma\,[b - F(y')] = \min c'x$$

$$\text{subject to } Ax = b - F(y') \qquad \text{(A-19)}$$

$$x \geqslant 0$$

and determine a new x' which represents a new schedule allocation. The algorithm is continued in this manner until $LB = UB$ (or the bounds are sufficiently close to meet some tolerance criteria).

Suppose at some iteration the set of constraints

$$Ax = b - F(y') \qquad \text{(A-20)}$$

$$x \geqslant 0$$

is infeasible. In this case, Phase I of the simplex method will generate a γ' so that

$$\gamma'\,[b - F(y')] > 0\,. \qquad \text{(A-21)}$$

A new relaxed master problem is formulated as

$$\min \eta$$

$$\text{subject to } \eta \geqslant f(y) + \pi'\,[b - F(y)] \qquad \text{(A-22)}$$

$$0 \geqslant \gamma'\,[b - F(y)]$$

$$y \,\epsilon\, Y.$$

Let (η^2, y^2) be the optimal solution to this 0,1 integer programming problem. Note that η^2 must be at least as large as η^1 because another constraint has been added.

On the other hand, suppose $y' \epsilon \overline{Y}$ (i.e., is feasible); solve for

$$\sigma(b - F(y')) = \min c'x$$

$$\text{subject to } Ax = b - F(y') \qquad \text{(A-23)}$$

$$x \geqslant 0$$

and let x' be an optimal solution. Let π' be an optimal simplex multiplier vector. Set $UB' = min\ \{UB^o, f(y') + \sigma\ [b - F(y')]\}$ so that if the new value is smaller than the previous upper bound then it becomes the new upper bound. Note that while lower-bound values are monotonically increasing at each iteration, the upper bounds are not generally monotonically decreasing.

If at some iteration $LB' = \eta' = UB'$ and an optimal schedule has been found, then the y' vector of 0,1 integers specifies the order of spacecraft tracking throughout the N days and the x' vector specifies the acquisition and loss of signal times for each tracking pass. If $\eta' < UB'$, a new constraint is added to the relaxed master problem, a new y' is determined, and a new iteration is performed.

III. A Heuristic Starting Schedule for the Benders' Model

Recall that the first step in implementing Benders' Method for the N-day scheduling problem is to solve the subproblem

$$\sigma(b - F(y^o)) = \min c'x$$

$$\text{subject to } Ax = b - F(y^o) \qquad \text{(A-24)}$$

$$x > 0$$

where y^o is a vector which represents the order of S/C tracking during the scheduling period. This subproblem may generally be solved fairly easily by linear programming, even for large problems. The resulting solution, x, may then be used to determine a new y', and the iterative process continues.

The vector y^o is easily determined from the schedule which is determined using a heuristic algorithm. Note that the actual allocation of acquisition and loss of signal times is not of interest here and this fact may play a role in determining the heuristic algorithm. Also, note that the upper bound, UB, which is a function of the y vector,

$$UB = f(y) + \sigma\ [b - F(y)]\ , \qquad \text{(A-25)}$$

is not monotonic since the functions $f(y)$ and $F(y)$ are nonlinear. However, a well-designed heuristic algorithm which results in good suboptimal initial scheduling orders, specified by the y^o vector, may result in a saving of perhaps 80% or more of the computational time normally required for determining optimality starting with an arbitrarily chosen y^o.