An Update on the Life Analysis of Spur Gears

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Gears used in aircraft and other applications may fail from scoring, tooth fracture due to bending fatigue, or surface pitting fatigue. Figure 1 illustrates a few of the failure modes that have occurred during experimental gear testing at the Lewis Research Center. The scoring type of failure is usually lubrication related and can be corrected by proper lubricant selection and/or changes in gear operating conditions (ref. 1). Tooth breakage is caused by tooth loads that produce bending stresses above the endurance limit of the material (ref. 2). It is usually accepted that the endurance limit, if it does exist, can be predicted from available stress-life (S-N) curves for the material being used (ref. 3). Current methods (ref. 4) of predicting gear surface-pitting failures are similar to those used for predicting the bending fatigue limit. According to the method of reference 4, the maximum surface contact stress (Hertz stress) should be limited to a value less than the surface endurance limit of the gear material. It is commonly believed that the gears would then have an infinite surface-pitting life. But, based on gearing studies (refs. 5 and 6) and on rolling-element bearing-life studies (ref. 7), there is no real evidence to support the concept of a surface-fatigue limit under normal operating conditions of bearings and gears. Rather, it appears that all gears, even if designed properly to avoid failure by scoring and tooth-bending fatigue, will eventually succumb to surface pitting in much the same way as rolling-element bearings.

In references 8 to 10 a method of predicting the surface-fatigue lives of low-contact-ratio spur gears was presented. The method was based on the commonly accepted Lundberg-Palmgren theory that has been used for many years to predict the lives of rolling-element bearings. Fatigue testing of spur gears of various materials, using different methods of manufacture and lubricants, has been in progress for the past 10 years at Lewis.

The objective of the information presented in this paper is to summarize and update the spur-gear life analysis and to relate the analysis to some of the experimental results from NASA tests.

Symbols

b	half width of Hertzian contact, m (in.)
C	gear center distance, m (in.); constant of proportionality, see eq. (14)
c	orthogonal shear stress exponent
D_a	rolling-element diameter, m (in.)
E	Young's modulus, Pa (psi)
e	Weibull exponent
f	face width of tooth in contact, m (in.)
G_{10}	10 percent life of a gear, millions of revolutions
h	depth to critical stress exponent
K	proportionality constant
L_{10}	10 percent life, millions of revolutions (Mr), or, equivalently, hr
L_i	gear rotations in millions
l	involute profile arc length, m (in.)
M	contact ratio
m_g	gear ratio, N_1/N_2

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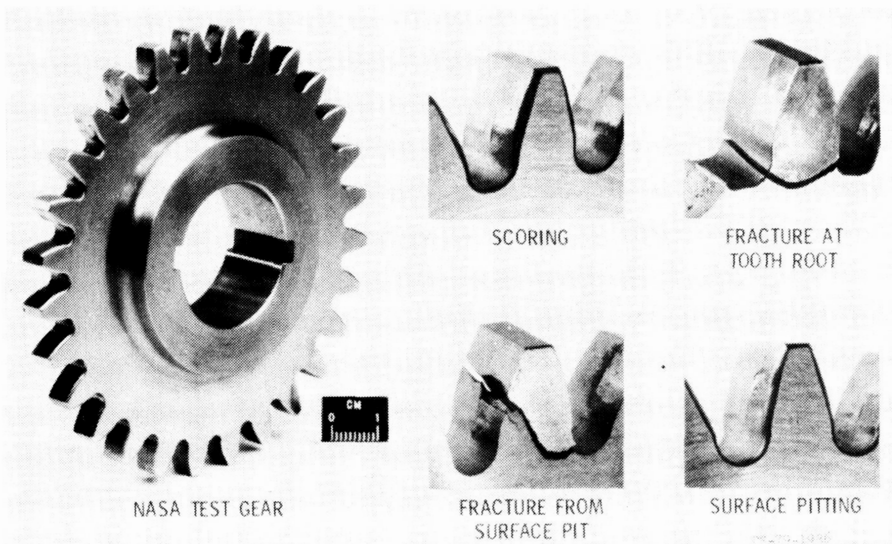

Figure 1. - Typical gear tooth failure modes. From tests run on NASA Lewis spur gear tester.

N	number of teeth
p_b	base pitch, m/tooth (in./tooth)
Q	load normal to involute profile, N (lb)
Q_c	dynamic capacity for bearing, N (lb)
Q_m	dynamic capacity of gear mesh, N (lb)
q	maximum Hertz stress, Pa (psi)
r	pitch circle radius, m (in.)
r_a	addendum circle radius, m (in.)
r_b	base circle radius, m (in.)
S	probability of survival
T_{10}	10 percent life of a tooth, millions of cycles
U	number of contact cycles per bearing revolution
u	stress cycles per revolution
V	volume, m^3 (in.^{3})
x,y	Cartesian coordinates, m (in.)
Z	contact path length, m (in.)
z	depth of occurrence of critical shearing stress, m (in.)
β	roll angle increment, rad
δ	precontact roll angle, rad
η	millions of stress cycles
θ	base circle roll angle, rad
μ	Poisson's ratio
ρ	curvature radius, m (in.)
$\Sigma\rho$	curvature sum, m^{-1} (in.$^{-1}$)
τ	maximum subsurface orthogonal reversing shear stress, Pa (psi)
φ	pressure angle, rad

Subscripts:

H	high load
i	index representing 1 or 2

L low load
M mesh of pinion and gear
1 reference to high speed member, pinion
2 reference to low speed member, gear

Theory

Theory of Gear Tooth Life

The fatigue-life model proposed in 1947 by Lundberg (ref. 11) is the commonly accepted theory to determine the fatigue life of rolling-element bearings. The probability of survival as a function of stress cycles is expressed as

$$\log \frac{1}{S} \propto \frac{\tau^c \eta^e V}{z^h} \tag{1}$$

Hence, if the probability of survival is specified, the life η for the required reliability can be considered a function of the stressed volume V, the maximum critical stress τ and the depth to the critical shearing stress z. As a result, the proportionality can be written as

$$\eta \propto \frac{z^{h/e}}{\tau^{c/e} V^{1/e}} \tag{2}$$

The above formula reflects the idea that greater stress shortens life. The depth below the surface z at which the critical stress occurs is also a factor. A microcrack beginning at a point below the surface requires time to work its way to the surface. Therefore, we expect that life varies by an inverse power of depth to the critically stressed zone.

The stressed volume V is also an important factor. Pitting initiation occurs near any small stress-raising imperfection in the material. The larger the stressed volume, the greater the likelihood of fatigue failure. By the very nature of the fatigue-failure phenomenon, it is the repetitive application of stress that causes cumulative damage to the material. The greater the number of stress cycles, the greater the probability of failure. By experience it has been found that the failure distribution follows the Weibull model.

As an expression for the stressed volume let

$$V \propto f z l \tag{3}$$

where l is the involute length across the heavy load zone as defined in the appendix. The choice of this length is a simplification, in that a more complicated treatment would be to integrate across the entire involute length, which is composed of both heavy and light load zones. The simplification is justifiable, however, since operation under lighter loads greatly diminishes the probability of failure. Another reason for choosing the heavily loaded zone is that all observed failures on the NASA gear tests occurred in that zone, mostly where the Hertz load on the pinion is greatest (ref. 12). It is also assumed that the most severe Hertz stress generated in the contact of the teeth is acting over the entire load zone of single-tooth contact. The calculation of stresses is considered next. From Hertz theory (ref. 13) the maximum stress at the surface where line contact is assumed is given by

$$q = \frac{2Q}{\pi f b} \tag{4}$$

The semiwidth of the contact strip is

$$b = \sqrt{\frac{8Q(1-\mu^2)}{\pi f E \Sigma\rho}} \tag{5}$$

where $\Sigma\rho$ is the curvature sum, such that

$$\Sigma\rho = \frac{1}{\rho_1} + \frac{1}{\rho_2} \tag{6}$$

The expression for radius of curvature, ρ, is given in the appendix. According to reference 11, the cause of fatigue flaking of bearings is the maximum reversing orthogonal shear stress that occurs at a depth z below the surface and has an amplitude that varies between $\pm\tau$ where

$$z = \frac{b}{2} \tag{7}$$

and

$$\tau = \frac{q}{4} \tag{8}$$

Using the foregoing expressions for stress, stressed volume, and depth of stress allows equation (1) to be written as

$$\log \frac{1}{S} \propto Q^{(c-h+1)/2} f^{-(c-h-1)/2} \Sigma\rho^{(c+h-1)/2} l \eta^{e} \tag{9}$$

Assuming a probability of survival of 90 percent and making use of the foregoing expressions for z, τ, and V, we may express life as

$$\eta \propto Q^{-(c-h+1)/2e} f^{(c-h-1)/2e} \Sigma\rho^{-(c+h-1)/2e} l^{-1/e} \tag{10}$$

Evaluation of Exponents

It has been shown in reference 14 that life is inversely proportional to the 4.3 power of load and that the dispersion in life (Weibull exponent), e, equals 2.5 for AISI 9310 steel gears. From reference 11 the dependence of dynamic capacity on bearing size leads to the following proportionality:

$$Q_c \propto D_a^{(c+h-3)/(c-h+1)} \tag{11}$$

where Q_c is the dynamic capacity and is defined as the load that gives a bearing life of 1 million stress cycles at a 90-percent reliability level and where D_a represents ball size and as such is a measure of the size effect. In reference 11 the exponent of D_a is 1.07. Using the foregoing values, c and h are calculated to be 23.2525 and 2.7525, respectively. The life equation then becomes

$$\eta = KQ^{-4.3} f^{3.9} \Sigma\rho^{-5} l^{-0.4} \tag{12}$$

where K is a constant of proportionality and η is tooth life in millions of stress cycles. Based on the data presented in reference 14, it has been calculated that $K = 3.72 \times 10^{18}$ when the units used in the equation are pounds and inches.

Gear Life

According to the relation for survival probability of a single gear tooth under constant service conditions,

$$\log \frac{1}{S} \propto \eta^{e} \tag{13}$$

For a 90-percent survival rate $S = 0.90$ and for the corresponding life $\eta = T_{10}$, a constant of proportionality C is defined by

$$\log \frac{1}{0.9} = CT_{10}^{e} \tag{14}$$

The life T_{10} is conceptually identical to the B_{10} life for rolling-element bearings. For any general survival rate the following equation holds:

$$\log \frac{1}{S} = \left(\frac{\eta}{T_{10}}\right)^{e} \log \frac{1}{0.9} \tag{15}$$

From basic probability theory the probability of survival for N teeth is the product of survival probabilities for the individual teeth. Then, the probability of survival for the gear "i" with N_i teeth is

$$S_i = S^{N_i} \tag{16}$$

and

$$\log \frac{1}{S_i} = N_i \left(\frac{uL_i}{T_{10}} \right)^e \log \frac{1}{0.9} \tag{17}$$

where L_i is gear rotations and u is stress cycles per gear revolution. Normally $u = 1$, but in the case of a bull gear used to collect power from a number of inputs, u would be set equal to the number of inputs, assuming that equal power was applied through each input. For a 90-percent survival for the gear $S_i = 0.9$, $L_i = G_{10}$, and

$$\log \frac{1}{0.9} = N_i \left(\frac{uG_{10}}{T_{10}} \right)^e \log \frac{1}{0.9} \tag{18}$$

Then the 10-percent life for the gear is related to the 10 percent life of a single tooth by the following equation:

$$\left(\frac{1}{G_{10}} \right)^e = N_i \left(\frac{u}{T_{10}} \right)^e \tag{19}$$

For the case of an idler gear, where power is transferred without loss from an input gear to an output gear through the idler gear, both sides of the tooth are loaded once per revolution. The stressed volume is doubled in this case, which is the same as doubling the number of teeth.

$$\left(\frac{1}{G_{10}} \right)^e = 2N_i \left(\frac{1}{T_{10}} \right)^e \tag{20}$$

For most circumstances the basic tooth life, as calculated by equation (12), can be applied to teeth on both member gears in a mesh. However, for cases where the tooth proportions of one gear member are different from those of the mating member, a separate calculation for each gear tooth must be made.

Life and Dynamic Capacity of the Mesh

From probability theory the probability of survival of the two gears in mesh is given by

$$S = S_1 \cdot S_2 \tag{21}$$

and, therefore,

$$\left(\frac{1}{L} \right)^e = \left(\frac{1}{L_1} \right)^e + \left(\frac{1}{L_2} \right)^e \tag{22}$$

where the subscript 1 denotes the higher speed gear (pinion) and 2 denotes the other gear. This equation is valid for a 90-percent probability of survival for the mesh combination of gear 1 and gear 2. In this equation it should be noticed that all lives L should be expressed in the same time base, for example, in hours or in rotations of gear 1.

The dynamic capacity Q_m for the mesh is defined as the normal tooth load that may be carried with a 90-percent probability of survival for one million revolutions of the highest speed shaft. Then for any other pitch line load, the corresponding mesh life is calculated from the following equation:

$$L_{10} = \left(\frac{Q_m}{Q}\right)^{4.3} \tag{23}$$

For example, imagine that gear 1 is on the higher speed gear shaft. Also assume that the teeth on the gear members are of the same proportions and are running in a simple mesh. In this case, based on the foregoing analysis, the dynamic capacity of the gear mesh is given by

$$Q_m = \left[\frac{K^{2.5} f^{9.75}}{\Sigma\rho^{12.5} l N_1 \left(1 + m_g^{1.5}\right)}\right]^{0.093} \tag{24}$$

Results and Discussion

Gear fatigue tests were conducted at the Lewis Research Center with gears made from AISI 9310 steel (ref. 14).

The test rig is shown in figure 2. The dimensions of the gears are given in table I.

Three groups of vacuum arc remelted (VAR) AISI 9310 case carburized and hardened steel gears were fatigue tested under loads of 463×10^3, 578×10^3, and 694×10^3 N/m (2645, 3305, and 3966 lb/in.), which produced maximum Hertz stresses of 1.531, 1.710, and 1.875 GPa (222 000, 248 000, and 272 000 psi). The lubricant was a superrefined naphthenic mineral oil with a 5 percent extreme-pressure additive package. Failure of the gears was due to surface fatigue pitting. Test results were statistically evaluated using the methods of reference 15. The results of these tests are plotted on Weibull coordinates in figure 3. Weibull coordinates are the log-log of the reciprocal of the probability of survival graduated as the statistical percent of specimens failed (ordinate) against the log of stress cycles to failure or system life (abscissa). In each test group there were 19 failures out of 19 tests. All failures occurred in the zone of single tooth loading at or just below the pitch diameter.

The 90-percent confidence interval limits were determined for each group of test data (fig. 3). The interpretation of these limits is that the true life determined from an extremely large sample of gears running at each stress condition will fall between these confidence limits 90 percent of the time. Were these confidence limits to overlap, the life differences as determined from the test would not be considered statistically significant. The consistent trend of decreasing life with increasing stress indicates good statistical significance in the data that resulted from these three test series.

TABLE I. - SPUR GEAR DATA

[Gear tolerance per ASMA class 12.]

Tooth width in contact, cm (in.)	0.25 (0.10)
Tooth width, cm (in.)	0.64 (0.25)
Number of teeth	28
Diametral pitch	8
Circular pitch, cm (in.)	0.9975 (0.3927)
Whole depth, cm (in.)	0.762 (0.300)
Addendum, cm (in.)	0.318 (0.125)
Chordal tooth thickness reference, cm (in.)	0.485 (0.191)
Pressure angle, deg	20
Pitch diameter, cm (in.)	8.890 (3.500)
Outside diameter, cm (in.)	9.525 (3.750)
Root fillet, cm (in.)	0.102 to 0.152 (0.04 to 0.06)
Measurement over pins, cm (in.)	9.603 to 9.630 (3.7807 to 3.7915)
Pin diameter, cm (in.)	0.549 (0.216)
Backlash reference, cm (in.)	0.0254 (0.010)
Tip relief, cm (in.)	0.001 to 0.0015 (0.0004 to 0.0006)

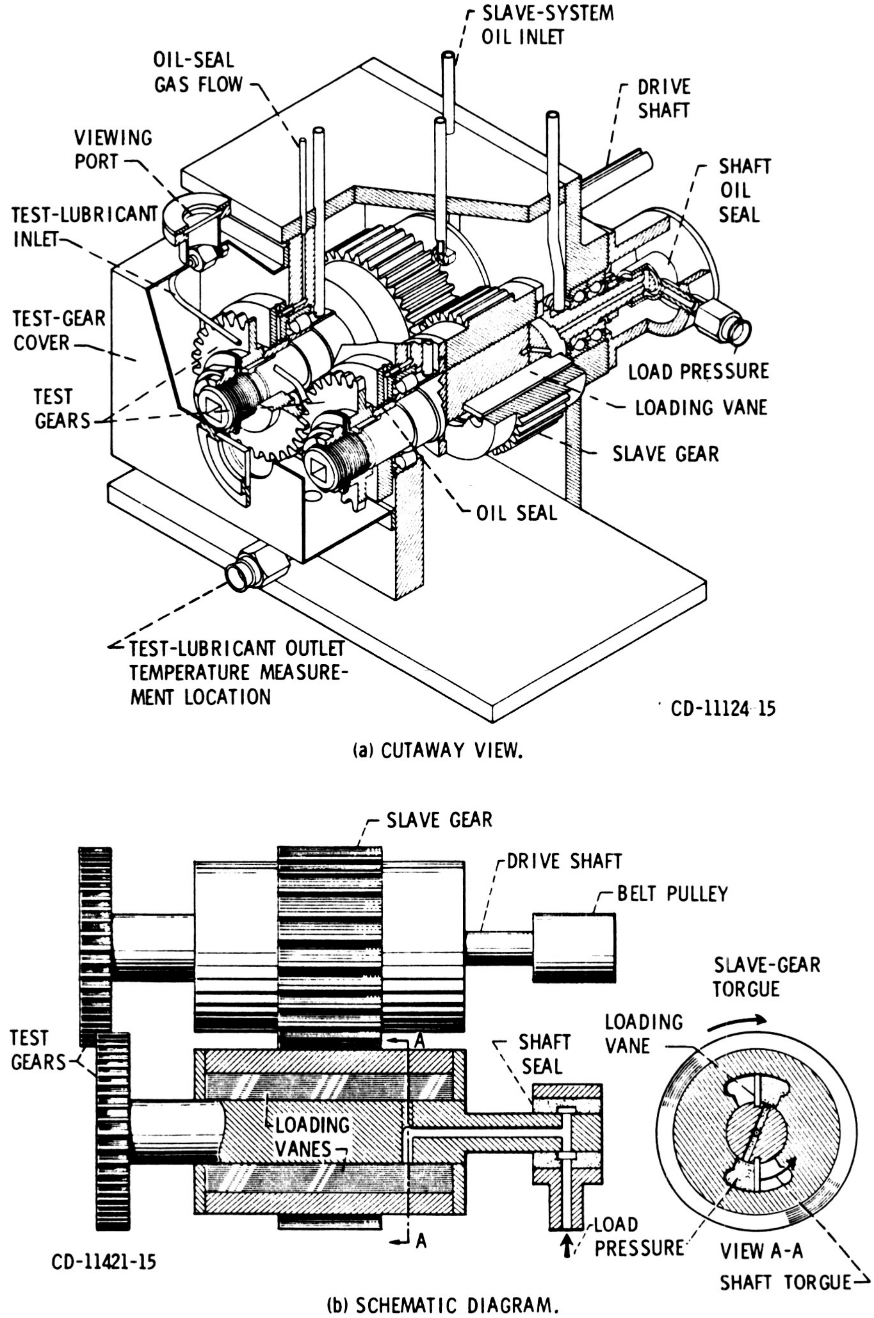

Figure 2. - NASA Lewis Research Center's gear fatigue test apparatus.

The American Gear Manufacturers Association (AGMA) has published two standards for tooth surface fatigue, AGMA 210.02 and AGMA 411.02 (refs. 4 amd 16). AGMA 210.02 provides for an endurance limit for surface fatigue below which it is implied that no failure should occur. In practice, there is a finite surface fatigue life at all loads. AGMA 411.02 recognizes this finite-life condition. Therefore, it does not contain an endurance limit in the load-life curve but does show a continuous decrease in life with increasing load. Both AGMA standards are illustrated in figure 4. The AGMA load-life curves shown are for a 99-percent probability of survival or the L_1 life. The experimental L_1, L_{10}, and L_{50} lives from the data of figure 3 are plotted for comparison.

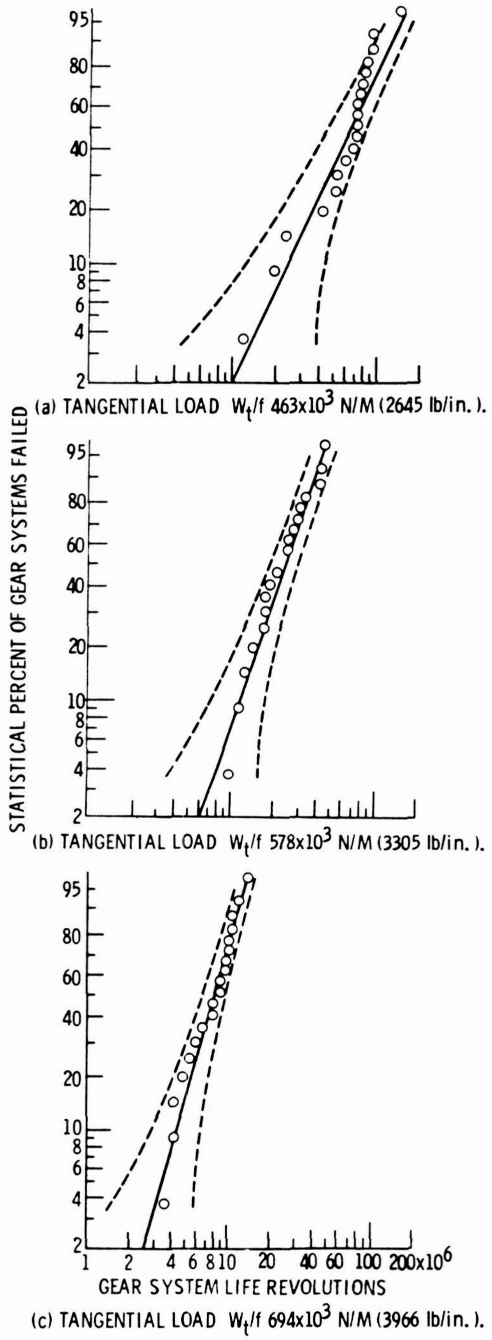

Figure 3. - Comparison of life-prediction theory with experimental results for VAR AISI 9310 steel spur gears. Speed, 10 000 rpm.

It is evident that the load-life relation used by AGMA is different from the experimental results reported herein. The difference between the AGMA life prediction and the experimental lives could be the result of differences in stressed volume. The AGMA standard does not consider the effects of stressed volume, which may be considerably different from that of the test gears used herein. The larger the volume of material stressed, the greater the probability of failure or the lower the life of a particular gearset. Therefore, changing the size or contact radius of a gearset, even though the same contact stress is maintained, would have an effect on gear life.

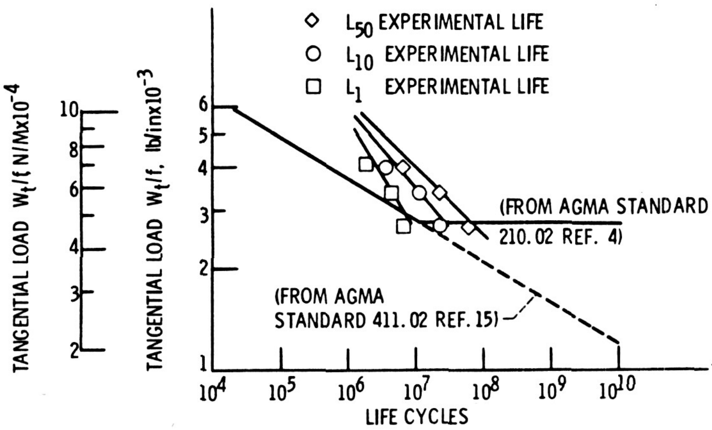

Figure 4. - Comparison of experimental life for VAR AISI 9310 spur gears with AGMA life prediction. Speed, 10 000 rpm; lubricant superrefined naphthenic mineral oil with additive package.

It should be mentioned here that the Weibull slope e was assumed to be independent of the stress level in the original work by Lundburg and Palmgren. There is some evidence to suggest that the Weibull slope is a function of the applied stress. This was noticed by Schilke (ref. 6); and the tests conducted at Lewis (ref. 14) at various stress levels gave Weibull slopes from 1.9 to 2.9. The Weibull slope $e = 2.5$ that is suggested for use in this report is an average value that will serve in most applications where the stress is not unusually high or low. When life calculations are made using the method outlined in this paper, the predicted life can be considered a reasonably good engineering approximation to what may be expected in a practical gear application. However, the theoretical prediction does not consider material and processing factors such as material type, melting practice, or heat treatment, nor does it consider environmental factors such as lubrication and temperature. All these factors are known to be extremely important in their effect on rolling-element bearing life (ref. 17). There is no reason why the effects used to determine bearing life should be significantly different from those used to determine gear life. Figure 5 is a collection of life data based on gear tests conducted at the Lewis Research Center. The lives are compared with a base-line life of unity for AISI 9310 gears. The mode of failure was pitting. More test data obtained with gear specimens under various test conditions and different materials and lubricants continue to be required to establish and/or affirm the material and processing factors and the exponents c, h, and e for gears, as they are used in various applications. However, the general methods presented herein explain and support the use of the statistical methods to predict spur-gear fatigue life. Table II gives a sample calculation for a simple mesh of two gears.

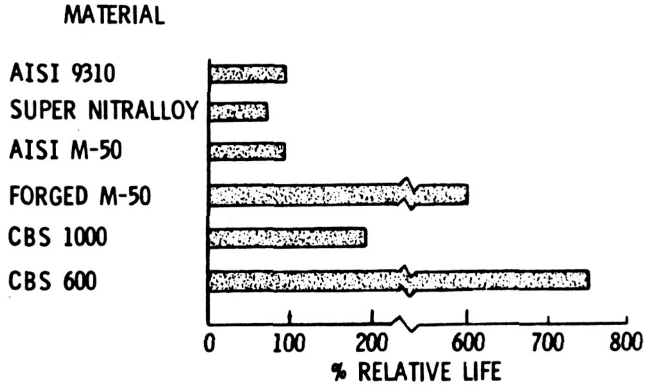

Figure 5. - Summary of gear fatigue lives based on NASA spur gear tests with various materials.

TABLE II. - SAMPLE CALCULATION

Symbol	Definition	Equation	Result
Q	Normal load	Given	1610 N (363 lb)
φ	Pressure angle		20°
N_1	Number of teeth on gear 1		28
N_2	Number of teeth on gear 2		28
f	Face width		2.79 mm (0.11 in.)
r_1	Pitch radius		4.45 cm (1.75 in.)
r_2	Pitch radius		4.45 cm (1.75 in.)
r_{a1}	Addendum radius		4.76 cm (1.88 in.)
r_{a2}	Addendum radius		4.76 cm (1.88 in.)
r_{b1}	Base radius		4.17 cm (1.64 in.)
r_{b2}	Base radius		4.17 cm (1.64 in.)
Z	Length of contact path	$\sqrt{r_{a1}^2 - r_{b1}^2} + \sqrt{r_{a2}^2 - r_{b2}^2} - (r_1 + r_2)\sin\varphi$	1.63 cm (0.641 in.)
p_b	Base pitch	$2\pi r_{b1}/N_1$	9.35 mm (0.368 in.)
β_{L1}	Roll angle increment	$(Z - p_b)/r_{b1}$	0.166 rad
β_{H1}	Roll angle increment	$(2p_b - Z)/r_{b1}$	0.058 rad
δ_1	Pre contact roll angle	$\dfrac{(r_1 + r_2)\sin\varphi - \left(r_{a2}^2 - r_{b2}^2\right)^{1/2}}{r_{b1}}$	0.169 rad
ℓ_1	Length of involute during single tooth contact	$r_{b1}\beta_{H1}\left(\delta_1 + \beta_{L1} + \frac{1}{2}\beta_{H1}\right)$	0.879 mm (0.035 in.)
ρ_1	Radius of curvature	$r_{b1}(\delta_1 + \beta_{L1})$	1.39 cm (0.549 in.)
ρ_2	Radius of curvature	$(r_1 + r_2)\sin\varphi - \rho_1$	1.65 cm (0.648 in.)
$\Sigma\rho$	Curvature sum	$(1/\rho_1) + (1/\rho_2)$	8.55 cm^{-1} (3.36 $in.^{-1}$)
T_{10}	Tooth life	$3.72\times10^{18}Q^{-4.3}f^{3.9}\Sigma\rho^{-5}\ell^{-0.4}$	59.6 Mr*
G_{10}	Gear life	$\left[N_1(1/T_{10})^{2.5}\right]^{-1/2.5}$	15.7 Mr*
L_{10}	Mesh life	$\left[2(1/G_{10})^{2.5}\right]^{-1/2.5}$	11.9 Mr*

*Mr = million revolutions.

Summary of Results

An analytical method for predicting surface fatigue life of gears was presented. General statistical methods were outlined, showing the application of the general methods to a simple gear mesh. Experimentally determined values for constants in the life equation were given. Comparison of the life theory with NASA test results and AGMA standards was made. Gear geometry pertinent to life calculations was reviewed. The following results were obtained:

1. The life analysis resulted in the following fundamental equation for gear tooth life:

$$\eta = KQ^{-4.3}f^{3.9}\Sigma\rho^{-5}l^{-0.4} \tag{12}$$

where $K = 3.72 \times 10^8$ for calculations in pounds and inches.

2. For a simple mesh of one gear with another the dynamic capacity is

$$Q_m = \left[\frac{K^{2.5}f^{9.75}}{\Sigma\rho^{12.5}lN_1(1+m_g^{1.5})}\right]^{0.093} \tag{24}$$

Appendix—Gear Geometry

In figure 6 an involute tooth is shown. The involute curve may be thought of as the path traced by point A on the tangent line as it is rolled upon the base circle with radius r_b. The radius of curvature of the involute at point A is given by ρ.

$$\rho = r_b\theta \tag{A1}$$

The x and y coordinates of point A are given by the following equations:

$$\left.\begin{aligned} x &= r_b(\sin\theta - \theta\cos\theta) \\ y &= r_b(\cos\theta + \theta\sin\theta - 1) \end{aligned}\right\} \tag{A2}$$

The differential element of involute arc length dl is

$$dl = (dx^2 + dy^2)^{1/2} = r_b\theta\, d\theta \tag{A3}$$

The length of the path of contact Z is the distance between where the two addendum circles cross the line of action (as shown in fig. 7):

$$A = \sqrt{r_{a1}^2 - r_{b1}^2} + \sqrt{r_{a2}^2 - r_{b2}^2} - C\sin\varphi \tag{A4}$$

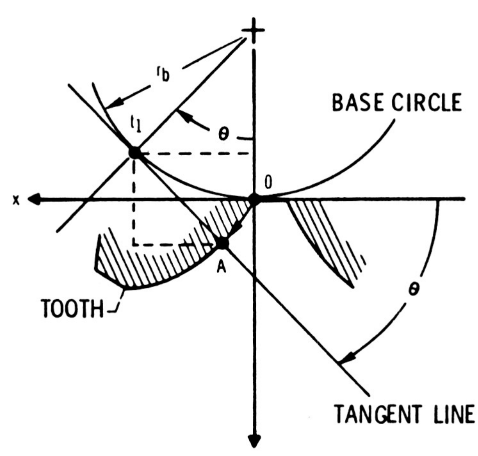

Figure 6. - Involute profile geometry.

where C is the distance between gear centers. The base pitch p_b is defined as the distance from a point on one tooth to the corresponding point on the next tooth measured along the base circle (see fig. 7). It may also be thought of as the distance from one tooth to the next measured along the line of action.

$$p_b = \frac{2\pi r_{b1}}{N_1} = \frac{2\pi r_{b2}}{N_2} \tag{A5}$$

The contact ratio M is defined as the average number of gear teeth in contact at one time. It is calculated by

$$M = \frac{Z}{p_b} \tag{A6}$$

When two pairs of teeth are in contact, the load is assumed to be equally shared. For low-contact-ratio gears ($1 < M < 2$) there are three load zones. The load zones for gear 1 are shown in figure 8. Two pairs of teeth are in contact during roll-angle increments β_{L1}, and only one pair during roll-angle increment β_{H1}. For gear 1 the roll-angle increment for which two tooth pairs are in contact is calculated by

$$\beta_{L1} = \frac{Z - p_b}{r_{b1}} \tag{A7}$$

And the roll angle movement for the heavily loaded zone is given by

$$\beta_{H1} = \frac{2p_b - Z}{r_{b1}} \tag{A8}$$

Finally δ_1, the precontact roll angle is defined as the roll angle from the base of the involute to the start of the zone of action as shown in figures 8 and 9. The angle δ_1 is given by the following equation:

$$\delta_1 = \frac{C \sin \varphi - (r_{a2}^2 - r_{b2}^2)^{1/2}}{r_{b1}} \tag{A9}$$

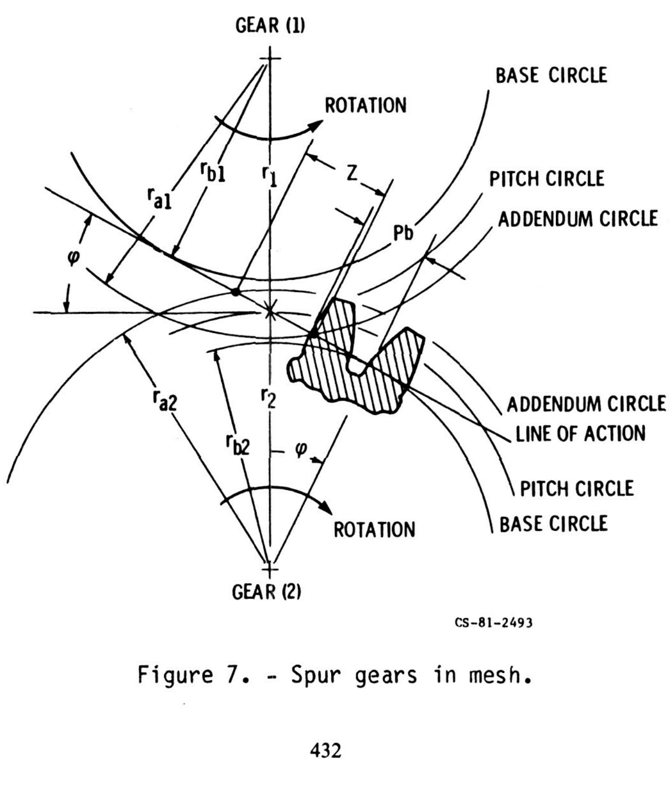

Figure 7. - Spur gears in mesh.

After integrating equation (A3) between the limits that bracket the heavy load zone, the result is the length of involute in the heavy load zone.

$$l_1 = r_{b1}\beta_{H1}\left(\delta_1 + \beta_{L1} + \frac{1}{2}\beta_{H1}\right) \tag{A10}$$

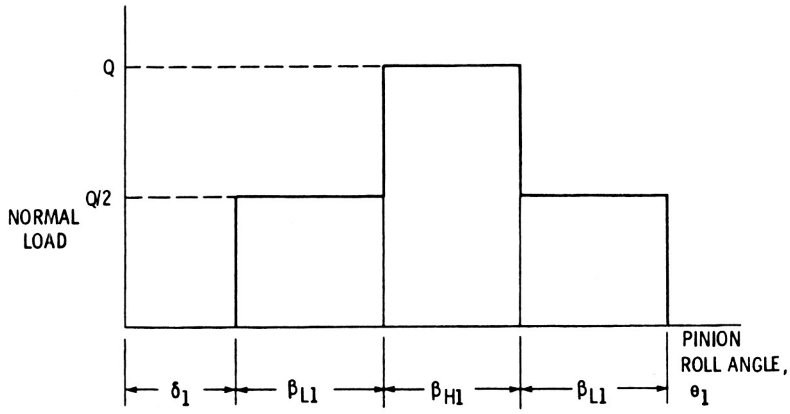

Figure 8. - Load sharing diagram. Load on tooth for low-contact-ratio gear depends on roll angle.

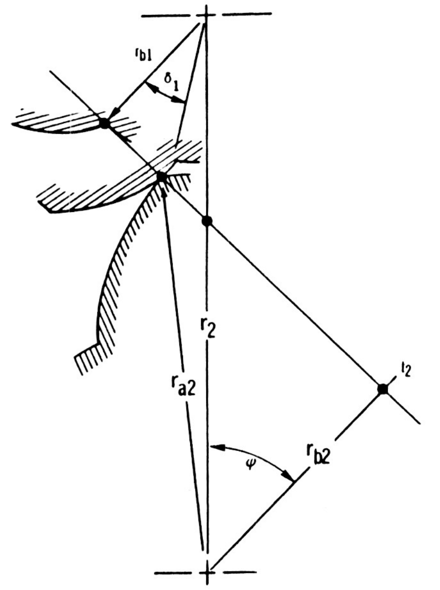

Figure 9. - Geometry for calculation of pre-contact roll angle, δ_1.

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