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APERTURE SYNTHESIS FOR MICROWAVE RADIOMETERS IN SPACE

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ABSTRACT

A technique is described for obtaining passive microwave measurements from space for remote sensing applications with high spatial resolution. The technique involves measuring the product of the signal from pairs of antennas at many different antenna spacings, thereby mapping the correlation function of antenna voltage. The intensity of radiation at the source can be obtained from the Fourier transform of this correlation function. Theory will be presented to show how the technique can be applied to large extended sources such as the earth when observed from space. As an example, details will be presented for a system with uniformly spaced measurements.

CONTENTS

ILLUSTRATIONS

I. INTRODUCTION

Microwave radiometers in space possess a significant potential for monitoring the earth's environment. Of current interest, for example, are microwave atmospheric sounders to complement infrared and visible sensors. Unlike infrared and visible radiation, microwave radiation can penetrate clouds and could provide meteorological data in the important regions beneath clouds and inside storms. However, microwave sensors in space require relatively large antenna systems and antenna size ultimately imposes limits on the spatial resolution practical with such sensors. This paper describes a technique to help overcome these limitations.

The technique involves coherently measuring the product of the signal from pairs of antennas with many different antenna spacings. Each antenna pair measures the correlation function of the antenna voltage, R(x), at an argument, x, determined by the distance between the antennas. For distant sources, it can be shown that the correlation function is proportional to the Fourier transform of the intensity of the source at a frequency which depends on the spacing, x. Thus, by making measurements at many different spacings one, in effect, determines the spectrum of the intensity of the source. The intensity can be obtained after all measurements are complete by inverting the transform. The resolution obtained in this manner is determined by how well the correlation function has been measured, not by the size of the antennas used. Thus, in principle, one can obtain very high resolution maps of the source by measuring at many different baselines using relatively small antennas.

This technique has been successfully employed by radio astronomers to obtain very high resolution maps of radio sources in what is called earth rotation synthesis (Swenson and Mathur, 1968; Brouw, 1975; Hewish, 1965). The Very Large Array in Socorro, New Mexico is an example. However, applying this technique to radiometers in space involves some problems which are different than those faced by the radio astronomer. For example, the objects to be mapped in radio astronomy are relatively small in angular extent and are quite distant whereas the earth is a relatively large and nearby source for a radiometer in space, even one in geostationary orbit. It will be shown below that the problems of mapping nearby extended sources can be resolved by controlling the field of view of the radiometer by proper choice of the antennas and filters used in the correlator. Several versions of this technique have also been proposed for remote sensing from space. These include a proposal (Schanda, 1976, 1979) in which the baseline is changed by making measurements at different frequencies, a concept (Mel'nik, 1972) involving a moving system with a single fixed baseline in which measurements must be made at several different time delays, and a modification which employs a scanning linear array which is rotated (C. Wiley, private communications). These systems can all be reduced to common principles which are the same as those employed in earth rotation synthesis.

The purpose of this paper is to develop the mathematical foundation for the concept of aperture synthesis as applied to remote sensing from space. Several alternative configurations for its implementation will be discussed, but the emphasis will be on the concepts and issues particular to microwave remote sensing from space rather than on the details of system design. In the sections to follow, the response of a single antenna to radiation from the source will be computed first. This will be done initially in the frequency domain (Fourier transform of Maxwell's equations with respect to time) by employing the vector Helmholtz equation, then the inverse transform will be taken to obtain the response in the time domain. In inverting the Fourier transform, it will be assumed that the bandwidth of the measurement is small compared to the nominal frequency of the system, an approximation which facilitates the mathematics and is a realistic approximation for most microwave sensors. Next, this solution for the response of a single antenna will be used to obtain an expression for the spatial correlation function (i.e., the average of the product of the signals from two antennas at different positions). In doing so, it will be assumed that the source and the maximum spacing between the antennas are both small compared to the distance between the source and antennas, and it will be assumed that the source electric fields are incoherent. With these approximations, it will be shown that the intensity of the source can be obtained by taking a Fourier transform of the spatial correlation function. Methods for measuring the correlation function and meeting the restrictions imposed by the approximations made in the theory will be discussed. Finally, a hypothetical system in which the correlation function is sampled uniformly on a Cartesian grid will be analyzed and expressions will be derived for the resolution and field of view obtained with this sampling strategy.

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II. ANTENNA RESPONSE IN THE FREQUENCY DOMAIN

Imagine two parallel planes, the "scene" plane which contains a source of radiation and the "image" plane where measurements are made. Also suppose an antenna is at position $\bar{r}$ in the image plane receiving radiation from the scene plane. In this section, it is desired to obtain an expression in the frequency domain for the output $V(\bar{r}, \nu)$ from the antenna in terms of the electric field, $\bar{E}_s(\bar{r}', \nu)$, in the scene plane. To begin, consider radiation from a small patch $\Delta x' \, \Delta y'$ at $\bar{r}'$ in the scene plane. The electric field $\Delta \bar{E}(\bar{r}/\bar{r}'; \nu)$ radiated from this small patch to the antenna can be obtained from the vector Helmholtz equation by assuming that the patch is a small aperture in an otherwise opaque screen on which the tangential components of the fields are zero. Following standard procedures (e.g., Tai, 1971), one obtains:

$$\Delta \bar{E}(\bar{r}/\bar{r}'; \nu) = -\Big\{\left[\hat{n} \times \bar{\nabla} \times \bar{E}_s(\bar{r}', \nu)\right] \cdot \bar{\bar{G}}(\bar{r}/\bar{r}') + \left[\hat{n} \times \bar{E}_s(\bar{r}', \nu)\right] \cdot \bar{\nabla} \times \bar{\bar{G}}(\bar{r}/\bar{r}')\Big\} \, \Delta x' \, \Delta y' \tag{1}$$

where $\hat{n} = \hat{z}$ is the unit vector normal to the scene plane and $\bar{\bar{G}}(\bar{r}/\bar{r}')$ is a dyadic Green's function satisfying the free space wave equation in the region $z > 0$. A convenient choice for $\bar{\bar{G}}(\bar{r}/\bar{r}')$ is:

$$\bar{\bar{G}}(\bar{r}/\bar{r}') = \left[g_0(\bar{r}/\bar{r}') - g_0(\bar{r}_I/\bar{r}')\right] \bar{\bar{I}} \tag{2}$$

where $\bar{\bar{I}}$ is the unit dyadic, $g_0(\bar{r}/\bar{r}') = e^{jk|\bar{r} - \bar{r}'|}/4\pi|\bar{r} - \bar{r}'|$ is the free space scalar Green's function and $\bar{r}_I = \bar{r} - 2z\hat{z}$ is the "image" of the antenna's position behind the scene plane. Now, the response of an antenna to radiation from the small patch can be expressed in terms of the voltage transfer function of the antenna, $\bar{A}(\bar{r}/\bar{r}', \nu)$, as follows:

$$\Delta V(\bar{r}/\bar{r}', \nu) = \Delta \bar{E}(\bar{r}/\bar{r}', \nu) \cdot \bar{A}(\bar{r}/\bar{r}', \nu) \tag{3}$$

The voltage transfer function is a vector quantity to account for the polarization of the antenna (Collin and Zucker, 1969) and can be obtained from the reciprocity theorem for antennas: One can show that $\bar{A}(\bar{r}/\bar{r}', \nu) \cdot \bar{A}^*(\bar{r}/\bar{r}', \nu) = A_e \, P_n(\bar{r}/\bar{r}')$ where A_e is the effective area of the receiving antenna and $P_n(\bar{r}/\bar{r}')$ is the normalized power pattern of the antenna (e.g., Kraus, 1966).

To obtain the response of the antenna to all sources, it is now necessary to sum over all of the scene plane. In the limit of infinitesimally small patches the sum approaches an integral and one obtains:

$$V(\bar{r}, \nu) = \iint_{\text{scene}} \Delta\bar{E}(\bar{r}/\bar{r}', \nu) \cdot \bar{A}(\bar{r}/\bar{r}', \nu)\, d\bar{r}'$$

$$= \iint_{\text{scene}} \Big\{ \Big[\hat{n} \times \bar{\nabla} \times \bar{E}_s(\bar{r}', \nu) \Big] \cdot \bar{\bar{G}}(\bar{r}/\bar{r}') + \tag{4}$$

$$+ \Big[\hat{n} \times \bar{E}_s(\bar{r}', \nu) \Big] \cdot \bar{\nabla} \times \bar{\bar{G}}(\bar{r}/\bar{r}') \Big\} \cdot \bar{A}(\bar{r}/\bar{r}', \nu)\, d\bar{r}'$$

Now using Equation 2 for $\bar{\bar{G}}(\bar{r}/\bar{r}')$ and recognizing that the following relationships apply when $\bar{r}'$ is on the scene plane ($z' = 0$):

$$\bar{\bar{G}}(\bar{r}/\bar{r}') = 0 \tag{5a}$$

$$\bar{\nabla} \times \bar{\bar{G}}(\bar{r}/\bar{r}') = \left(\hat{y}\frac{\partial G}{\partial z}\right)\hat{x} + \left(-\hat{x}\frac{\partial G}{\partial z}\right)\hat{y} \tag{5b}$$

$$\frac{\partial}{\partial z} G = -2\left(jk - \frac{1}{R}\right)\frac{z}{R}\, g_0(\bar{r}/\bar{r}') \tag{5c}$$

$$R = |\bar{r} - \bar{r}'| \tag{5d}$$

$$\hat{n} = \hat{z} \tag{5e}$$

Equation 4 becomes:

$$V(\bar{r}, \nu) = 2\iint_{\text{scene}} \bar{E}_t(\bar{r}', \nu) \cdot \bar{A}(\bar{r}/\bar{r}', \nu) \left[\left(jk - \frac{1}{R}\right)\frac{z}{R}\, g_0(\bar{r}/\bar{r}')\right] d\bar{r}' \tag{6}$$

where $\bar{E}_t(\bar{r}', \nu)$ are the electric field components on the scene plane which are tangent to the plane. Equation 6 is an expression for the voltage response to radiation from the scene plane, $V(\bar{r}, \nu)$, of an antenna with power pattern $P_n(\bar{r}/\bar{r}') = (1/A_e)\, \bar{A}(\bar{r}/\bar{r}', \nu) \cdot \bar{A}^*(\bar{r}/\bar{r}', \nu)$. The solution as given in Equation 6 is in the frequency domain. The time domain response is desired for the analysis to be performed here and will be derived in the following section.

III. THE TIME DOMAIN ANTENNA RESPONSE

To obtain the time dependent form of the antenna response $V(\bar{r},t)$ it is necessary to take the Fourier transform of Equation 6. In doing so, it is convenient to introduce the complex analytic voltage $V_c(\bar{r}, t)$ defined as follows (e.g., Born and Wolf, 1959):

$$V_c(\bar{r}, t) = 2\int_0^{\infty} V(\bar{r}, \nu)\, e^{-j2\pi\nu t} d\nu \qquad (7)$$

The real part of $V_c(\bar{r}, t)$ is the actual (i.e., physically measurable) voltage out of the antenna. To make the integration in Equation 7 tractable, the following three approximations will be made.

First, it will be assumed that the distance between the image and scene planes is much greater than the maximum extent of either. The geometry is illustrated in Figure 1. The origin of the coordinate system is in the scene plane at $x' = y' = z' = 0$ and the position of the antenna is measured from an arbitrary point (X_o, Y_o, Z_o) in the image plane by the local coordinates (η_x, η_y, η_z). Antenna positions are being described with respect to (X_o, Y_o, Z_o) to accommodate antennas which view the scene obliquely with their main beam pointing along R_o. Z_o is the distance between the image plane and scene plane and the assumption being made here is that $(x')^2 + (y')^2$ and $\eta_x^2 + \eta_y^2 + \eta_z^2$ are small compared to Z_o^2. This allows the distance R between the antenna and an arbitrary point on the scene plane to be approximated by:

$$\begin{aligned} R(\bar{r}/\bar{r}') &= \left\{(X_o + \eta_x - x')^2 + (Y_o + \eta_y - y')^2 + (Z_o + \eta_z)^2\right\}^{1/2} \\ &\cong R_o + \Psi(\bar{\eta}, \bar{r}') \qquad (8) \\ \Psi(\bar{\eta}, \bar{r}') &= (\eta_x - x')\cos\alpha + (\eta_y - y')\cos\beta + \eta_z\cos\gamma + \frac{1}{2R_o}\left[(\eta_x - x')^2 + (\eta_y - y')^2 + \eta_z^2\right] \end{aligned}$$

where $\cos\alpha = X_o/R_o$, $\cos\beta = Y_o/R_o$ and $\cos\gamma = Z_o/R_o$ are the direction cosines of the vector from the origin in the scene plane to the reference point (X_o, Y_o, Z_o) in the image plane. With this approximation and assuming that $kR_o >> 1$ Equation 6 can be written:

$$V(\bar{r}, \nu) \approx 2jk \cos\gamma \frac{e^{jkR_0}}{4\pi R_0} \iint_{\text{scene}} \bar{E}(\bar{r}', \nu) \cdot \bar{A}(\bar{r}/\bar{r}', \nu)\, e^{jk\Psi(\bar{r}, \bar{r}')} d\bar{r}' \tag{9}$$

Second, it will be assumed that the signal from the antenna passes through a filter, $h(\nu)$, representing the effective bandwidth of the system. This might be an actual filter placed in the system or might represent the effective bandwidth of the detector, integrator or other piece of hardware in the system. In either case, it will be assumed that the frequency limitations of the system can be described by a filter at the antenna terminals. In addition, it will be assumed that $h(\nu)$ is a narrow bandpass filter centered about frequency ν_0. That is

$$h(\nu) = \begin{cases} \tilde{H}(\nu - \nu_0) & |\nu - \nu_0| < \Delta \\ 0 & |\nu - \nu_0| > \Delta \end{cases} \tag{10}$$

where $\Delta / \nu_0 << 1$. This approximation means that the receiving system is tuned to receive energy only in a narrow band about the center frequency ν_0. With this assumption the output of the antenna-filter system is $V(\bar{r}, \nu)\ \tilde{H}(\nu - \nu_0)$.

Finally, it will be assumed that in the frequency interval passed by the filter, the time and space dependence of the electric fields on the source are separable and that the characteristics of the antenna don't change appreciably: That is, $\bar{E}_t(\bar{r}', \nu) = \bar{E}_t(\bar{r}')\,\tilde{B}(\nu - \nu_0)$ and $k\,A(\bar{r}/\bar{r}'; \nu) \approx k_0\,A(\bar{r}/\bar{r}'; \nu_0)$ where $k_0 = 2\pi \nu_0/c$ is the wave number at the center frequency of the filter.

Now, with the preceding approximations the integration in Equation 7 can be done analytically. Substituting Equation 9 into Equation 7 and making the change variables $\xi = \nu - \nu_0$ one obtains:

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$$V_c(\bar{r},t) = 4je^{-j2\pi\nu_o\hat{t}} \int_{-\nu_o}^{\infty} \iint_{scene} V(\bar{r}/\bar{r}', \nu_o)\, \widetilde{B}(\xi)\, \widetilde{H}(\xi) \exp\left\{-j2\pi\xi\left[\hat{t} - \frac{1}{c}\Psi(\bar{\eta}, \bar{r}')\right]\right\} d\bar{r}'d\xi \tag{11}$$

where

$$V(\bar{r}/\bar{r}', \nu_o) = \frac{k_o \cos\alpha}{4\pi R_o} \bar{E}_t(\bar{r}') \cdot \bar{A}(\bar{r}/\bar{r}', \nu_o)\, e^{jk_o\Psi(\bar{\eta}, \bar{r}')} \tag{12}$$

and $\hat{t} = t - R_o/c$ is time "retarded" by the propagation delay between the origin in the scene plane and the reference point (X_o, Y_o, Z_o) in the image plane. Since $\widetilde{H}(\xi)$ is zero for $|\xi| > \Delta$ and since $\Delta << \nu_o$ the lower limit in Equation 11 can be formally extended to infinity. In this case the integral over ξ becomes a Fourier transform and one obtains:

$$V_c(\bar{r},t) = 2je^{-j2\pi\nu_o\hat{t}} \iint_{scene} V(\bar{r}/\bar{r}', \nu_o)\, B(\hat{t} - \Psi/c) * H(\hat{t} - \Psi/c)\, d\bar{r}' \tag{13}$$

where the asterisk (*) denotes a convolution and the tilda (~) over B and H has been removed to indicate a Fourier transformed quantity (i.e., the Fourier transform of $B(t)$ is $\widetilde{B}(\nu)$).

Equation 13 is the complex analytic form for the response of the antenna to radiation incident from the scene plane. It reduces to standard results in special cases. For example, consider radiation from a single point source at $\bar{r}' = \bar{r}_o'$. In this case $V_c(\bar{r}, t) = 2\,\widetilde{V}(\bar{r}/\bar{r}_o', \nu_o) B(\tau_o) * H(\tau_o) \cdot \exp(-j2\pi\nu_o\tau_o)$ where $\tau_o = t - R_o/c - \Psi(\eta, \bar{r}_o')/c$ is time delayed by the propagation time from the source at $\bar{r}_o'$ to the antenna and $\widetilde{V}(\bar{r}/\bar{r}_o', \nu_o) = 2j\left[k_o \cos\gamma / 4\pi R_o\right] \hat{p} \cdot \bar{A}(\bar{r}/\bar{r}_o', \nu_o)$ is, except for a phase factor, the frequency domain response of the antenna to a point source at $\bar{r}_o'$ with polarization $\hat{p}$. When the filter is very narrow $\left[\text{i.e., } \widetilde{H}(\xi) = \delta(\xi)\right]$ then $V_c(\bar{r}, t) \cong 2\,\widetilde{V}(\bar{r}/\bar{r}_o', \nu_o)\widetilde{B}(\nu_o) \exp(-j2\pi\nu_o\tau_o)$ which is a sinusoid at the center frequency of the filter (ν_o) with an amplitude equal to the response of the antenna at frequency ν_o times the spectrum of the source at ν_o. In the other extreme, when the filter is very broad [i.e., $H(\tau) = \delta(\tau)$], then $V_c(\bar{r}, t) \cong 2\,\tilde{V}(\bar{r}/\bar{r}'_o, \nu_o)\,B(\tau_o)\exp(-j2\pi\nu_o\tau_o)$ which again is a sinusoid at the center frequency of the filter but in this case modulated by the time dependence of the radiation from the source.

IV. SPATIAL CORRELATION FUNCTION

Now consider two antennas at positions $\bar{r}_1$ and $\bar{r}_2$ in the image plane and suppose that the signals $V_c(\bar{r},t)$ from each antenna are multiplied and averaged to form the spatial correlation function $< V_c(\bar{r}_1,t)V_c^*(\bar{r}_2\,,t)>$. In forming the averages it will be assumed that the spatial and temporal variations of the source are independent random processes and that each random process by itself is uncorrelated. That is, it will be assumed that:

$$< \widetilde{B}(\xi)\widetilde{B}^*(\xi')> = \widetilde{T}(\xi)\,\delta(\xi - \xi') \tag{14}$$

and that

$$< \bar{E}_t(\bar{r})\bar{E}_t^*(\bar{r}')> = \bar{\bar{S}}(\bar{r})\,\delta(\bar{r} - \bar{r}') \tag{15a}$$

$\bar{\bar{S}}(\bar{r})$ in the preceding expression is a dyadic correlation function defined by

$$\bar{\bar{S}}(\bar{r}) = \begin{bmatrix} S_{xx}(\bar{r})\,\hat{x}\,\hat{x} & S_{yx}(\bar{r})\,\hat{y}\,\hat{x} \\ \\ S_{xy}(\bar{r})\,\hat{x}\,\hat{y} & S_{yy}(\bar{r})\,\hat{y}\,\hat{y} \end{bmatrix} \tag{15b}$$

where $S_{ij}(\bar{r})$ is the cross correlation function for the i and j components of the electric field on the source.

Now using Equations 14 and 15 and Equation 11 for $V_c(\bar{r},t)$ and doing the integrations over the delta functions, one obtains:

$$< V_c(\bar{r}_1,t)\,V_c^*(\bar{r}_2,t)> = \left[\frac{k_o\cos\gamma}{\pi R_o}\right]^2 \iint_{\text{scene}} d\bar{r}' \int_{-\infty}^{\infty} d\,\xi \left[\bar{A}(\bar{r}_1/\bar{r}', \nu_o)\cdot\bar{\bar{S}}(\bar{r}')\cdot\bar{A}^*(\bar{r}_2/\bar{r}',\nu_o)\right]$$

$$\widetilde{T}(\xi)\widetilde{H}^2(\xi)\exp\left\{ j\frac{2\pi}{c}(\nu_o - \xi)\left[\psi(\bar{\eta}_1, \bar{r}') - \psi(\bar{\eta}_2, \bar{r}')\right]\right\} \tag{16}$$

where $\widetilde{H}^2(\xi) = \widetilde{H}(\xi)\widetilde{H}^*(\xi)$. Equation 16 can be simplified by letting $\frac{1}{c}\left[\Psi(\bar{\eta}_1, \bar{r}') - \Psi(\bar{\eta}_2, \bar{r}')\right] = \Delta\tau$ and noting that

$$\Delta\tau = \frac{\nu_x}{\nu_o}\left[X_o - x'\right] + \frac{\nu_y}{\nu_o}\left[Y_o - y'\right] + \frac{\nu_z}{\nu_o}Z_o + \Omega(\bar{\eta}_1, \bar{\eta}_2) \tag{17}$$

where

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$$\nu_x = \frac{\eta_{x_1} - \eta_{x_2}}{c R_o} \nu_o \tag{18a}$$

$$\nu_y = \frac{\eta_{y_1} - \eta_{y_2}}{c R_o} \nu_o \tag{18b}$$

$$\nu_z = \frac{\eta_{z_1} - \eta_{z_2}}{c R_o} \nu_o \tag{18c}$$

$$\Omega(\eta_1, \eta_2) = \frac{\left(\eta_{x_1}^2 - \eta_{x_2}^2\right) + \left(\eta_{y_1}^2 - \eta_{y_2}^2\right) + \left(\eta_{z_1}^2 - \eta_{z_2}^2\right)}{2 c R_o} \tag{18d}$$

$\Delta\tau$ is the difference in time of arrival at the two antennas of radiation from a point on the scene.

Doing the integration over ξ (a Fourier transform), Equation 16 becomes

$$\langle V_c(\bar{r}_1,t) V_c^*(\bar{r}_2,t) \rangle = \left[\frac{k_o \cos\gamma}{\pi R_o}\right]^2 e^{jk_o \Omega} \iint_{\text{scene}} \left[\bar{A}(\bar{r}_1/\bar{r}', \nu_o) \cdot \bar{\bar{S}}(\bar{r}') \cdot \bar{A}^*(\bar{r}_2/\bar{r}', \nu_o)\right] \cdot$$

$$T(\Delta\tau) * H^2(\Delta\tau) \exp\left\{j2\pi\left[\nu_x(X_o - x') + \nu_y(Y_o - y') + \nu_z Z_o\right]\right\} d\bar{r}' \tag{19}$$

where $T(\Delta\tau)$ is the temporal correlation function of sources on the scene plane and $H^2(\Delta\tau) = H(\Delta\tau) * H^*(\Delta\tau)$ is the convolution of the temporal response of the filter with itself.

Notice that if the antennas are identical and close together compared to R_o, conditions likely to be encountered in remote sensing from space, then Equation 19 can be simplified. In particular, assuming that both antennas are identical with polarization in the $\hat{p}$ - direction and so close together that their antenna patterns on the scene are essentially the same (i.e., $\bar{A}(\bar{r}_1/\bar{r}', \nu_o) = \bar{A}(\bar{r}_2/\bar{r}', \nu_o) = A(\bar{r}_o/\bar{r}', \nu_o)\,\hat{p}$ where $\bar{r}_o$ is the position vector of the reference point (X_o, Y_o, Z_o) in the image plane) one has:

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$$
\begin{aligned}
\bar{A}(\bar{r}_1/\bar{r}', \nu_0) \cdot \bar{\bar{S}}(\bar{r}') \cdot \bar{A}^*(\bar{r}_2/\bar{r}', \nu_0) &= \\
&\cong A(\bar{r}_0/\bar{r}', \nu_0)\, A^*(\bar{r}_0/\bar{r}', \nu_0) \left[\hat{p} \cdot \bar{\bar{S}}(\bar{r}) \cdot \hat{p}\right] \qquad (20) \\
&= A_e\, P_n(\bar{r}_0/\bar{r}') \left[\hat{p} \cdot \bar{\bar{S}}(\bar{r}') \cdot \hat{p}\right]
\end{aligned}
$$

where $P_n(\bar{r}/\bar{r}')$ is the normalized power pattern for the antenna and A_e is its effective receiving area (Kraus, 1966). Also notice that at microwave frequencies 100 MHz or so is typical of the bandwidth of radiometers but thermal sources such as the earth have a very much broader bandwidth. Hence, the assumption, $T(\Delta\tau) * H^2(\Delta\tau) = H^2(\Delta\tau)$ ought to be reasonable for microwave radiometers viewing the earth. Assuming the antennas are close together and letting $T(\Delta\tau) * H^2(\Delta\tau) = H^2(\Delta\tau)$ Equation 19 can be written in the final form:

$$
\begin{aligned}
\langle V_c(\bar{r}_1,t)\, V_c^*(\bar{r}_2,t) \rangle &= 4 \frac{\cos^2\gamma}{\Omega_a R_0^2}\, e^{jk_0\Omega} \iint_{\text{scene}} H^2(\Delta\tau)\, P_n(\bar{r}_0/\bar{r}') \left[\hat{p} \cdot \bar{\bar{S}}(\bar{r}') \cdot \hat{p}\right] \\
&\cdot \exp\left\{ j2\pi \left[(X_0 - x')\nu_x + (Y_0 - y')\nu_y + \nu_z Z_0 \right] \right\} d\bar{r}' \qquad (21)
\end{aligned}
$$

where $\Omega_a = \lambda^2/A_e$ is called the antenna solid angle (Kraus, 1966).

V. SOLUTION FOR INTENSITY ON THE SCENE

It is possible with restrictions which are reasonable for microwave sensor systems to invert Equation 21 to obtain an expression for $\hat{p} \cdot \bar{\bar{S}}(\bar{r}') \cdot \hat{p}$. To do so, first introduce the function $P(\bar{r}') = P_n(\bar{r}_o/\bar{r}')\, A(\bar{r}')$ where $A(\bar{r}') = 1$ for $|\bar{r}'| < b$ and zero otherwise. $P(\bar{r}')$ is an idealized antenna pattern which neglects sidelobes which are sufficiently far from the main beam. Secondly, assume that $H^2(\Delta\tau) = H^2(0)$ for $r' < b$. This assumption means that the filter response is essentially constant over the entire field of view of the antenna. It imposes an upper limit on the bandwidth of the measurement. This upper limit depends on the maximum spacing between the antennas and varies from about 3 GHz ÷ (antenna separation in meters) when $R_o = Z_o$ to about .3 GHz ÷ (antenna separation in meters) when $R_o >> Z_o$. With the first of these two approximations the limits of integration in Equation 21 may formally be extended to infinity and by imposing the second approximation the resultant integral takes the form of a Fourier transform on ν_x and ν_y. With these two approximations Equation 21 is a Fourier transform which can be inverted to obtain an expression for $\hat{p} \cdot \bar{\bar{S}}(r') \cdot \hat{p}$. One obtains

$$\hat{p} \cdot \bar{\bar{S}}(\bar{r}') \cdot \hat{p} = \frac{C_o}{P(\bar{r}')} \iint_{-\infty}^{\infty} e^{-jk_o\Omega}\, e^{-j2\pi\nu_z Z_o} < V_c(\bar{r}_1,t) V_c^*(\bar{r}_2,t) > \exp\left\{-j2\pi\left[(X_o - x')\nu_x + (Y_o - y')\nu_y\right]\right\} d\nu_x\, d\nu_y \qquad (22)$$

where $C_o = \Omega_a R_o^2 / \left[4 \cos^2\gamma\, H^2(0)\right]$.

Each of the terms in the integrand in Equation 22 (i.e. ν_x, ν_y, ν_z and Ω) can be determined from the positions $\bar{r}_1$ and $\bar{r}_2$ of the antennas. Consequently, if $< V_c(\bar{r}_1,t)\, V_c^*(\bar{r}_2,t) >$ was known as a function of position, the integrand in Equation 22 would be determined and the integration could be performed (e.g. numerically). In fact, $< V_c(\bar{r}_1,t)\, V_c^*(\bar{r}_2,t) >$ can be measured using a coherent detector such as shown in Figure 2. Referring to Figure 2, $V_{1,2}(t) \cos(\omega t + \phi_{1,2}) = \mathrm{Re}\, V_c(\bar{r}_{1,2}, t)$ are the real (i.e. physically measurable) voltages at the antenna terminals and the desired correlation function is $< V_c(\bar{r}_1, t)\, V_c^*(\bar{r}_2, t) > = < V_1(t)\, V_2(t)\, [\cos(\phi_1 - \phi_2) +$

$j\sin(\phi_1 - \phi_2)\,]\rangle$. To obtain the correlation function the real voltages from each antenna are initially coherently mixed with the local oscillator signal to shift them to a carrier ω_{IF}. Then the signals are multiplied together and low pass filtered to obtain $V_1(t)\,V_2(t)\cos(\phi_1 - \phi_2)$ which is the real part of $V_c(\bar{r}_1, t)\,V_c^*(\bar{r}_2, t)$ and also multiplied after one signal has been shifted in phase by $\pi/2$ radians and then filtered to obtain the imaginary part of the correlation function. Assuming that $V_c(\bar{r},t)$ is a stationary, ergodic random process, the ensemble average (denoted by the pointed brackets $\langle\,\rangle$) can be obtained as a time average. Thus, given a measurement in the image plane, the ν and Ω are determined by the position of the antennas and $\langle V_c(\bar{r}_1, t)\,V_c^*(\bar{r}_2, t)\rangle$ is determined by averaging the signal from a coherent detector such as shown in Figure 2. Given these quantities the integral on the right side in Equation 22 can be evaluated numerically to determine the intensity of radiation on the scene plane.

Figure 3 illustrates how this procedure might be implemented to do remote sensing from space. Imagine two identical antennas, one spiraling about the other, and suppose that the signal from each is received at a common point and processed as indicated in Figure 2. At discrete points indicated by the dashes in Figure 3, the average $\langle V_c(\bar{r}_1,t)\,V_c^*(\bar{r}_2,t)\rangle$ is recorded. Since each point corresponds to a different baseline, and therefore to different frequencies ν_x, ν_y, this sytem is in effect mapping the correlation function $\langle V_c(\bar{r}_1, t)\,V_c^*(\bar{r}_2, t)\rangle$ as a function of ν_x and ν_y. If the coordinates η_x, η_y, η_z are known at each point, then all the terms in Equation 22 are determined. After all the measurements have been made the integral can be evaluated numerically to determine the intensity of radiation on the scene plane. The accuracy with which this can be accomplished will depend on the range of ν_x, ν_y over which measurements have been made (the diameter of the spiral in Figure 3) and the spacing between measurements, but it does not depend on the size of the antenna used for this measurement. The antenna must be chosen large enough so that the paraxial ray approximation (Equation 8) is valid, but otherwise the antenna size does not affect the resolution of the measurement. A large antenna reduces the field of view (area imaged) but does not determine the resolution.

The scheme presented in Figure 3 is conceptually what must be done to obtain the intensity on the scene plane using aperture synthesis. The essential requirement is to make measurements at different inteferometer baselines and the important flexibility is that these measurements do not have to be made at the same time. Thus, one can move a single antenna pair sequentially from one baseline to the next as in Figure 3, or one could have placed antennas at all the dashes and obtained measurements at all baselines at once. There are many possibilities between these extremes. For example, antennas uniformly spaced along two orthogonal axes, perhaps in the form of a +, an arrangement called a Mills cross, can measure all the baselines in a rectangular grid of the same dimensions as the cross. Additional baselines would be possible if the Mills cross was slowly rotating. It is also possible to keep the antenna positions fixed and change the baseline by changing the frequency of the measurement (Equation 18). Aperture synthesis using this approach has been suggested for remote sensing from space by Schanda (1976), but it requires measurements over a large frequency range to obtain good resolution. Other variations of the basic concept have also been proposed (Mel'nik, 1972; Wiley, private communication).

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VI. AN EXAMPLE

To illustrate the procedure consider a system in which measurements are uniformly spaced on a Cartesian grid. That is, assume that the antennas in the image plane can be at positions ($\eta_x = n_x d_x$; $\eta_y = n_y d_y$; $\eta_z = 0$) where n_x and n_y are integers. Then assuming that $|n_{x,y}| \leq N_{x,y}$ and letting $X_o = Y_o = 0$ for convenience one can write Equation 22 in the following form:

$$\hat{I}_{pp}(x',y') = \frac{\Delta x \Delta y}{P(\bar{r})} \sum_{n_x=-N_x}^{N_x} \sum_{n_y=-N_y}^{N_y} \tilde{V}(n_x \Delta_x, n_y \Delta_y)\, e^{j2\pi\left[n_x \Delta_x x' + n_y \Delta_y y'\right]} \tag{23}$$

where $\tilde{V}(\nu_x, \nu_y) = C_o\, e^{-jk_o \Omega} < V_c(\bar{r}_1,t)\, V_c^*(\bar{r}_2,t) >$ and $\Delta_{x,y} = \frac{d_{x,y}}{c R_o} \nu_o$. Comparing Equations 22 and 23, one sees that $\hat{I}_{pp}(x',y')$ is the intensity on the scene plane with polarizations $\hat{p}$ obtained with this particular sampling scheme and $\tilde{V}(\nu_x, \nu_y)$ is the Fourier transform of the actual intensity on the surface multiplied by the normalized antenna pattern. That is, $F^{-1}\left[\tilde{V}(\nu_x, \nu_y)\right] = P(\bar{r}')\left[\hat{p} \cdot \bar{\bar{S}}(\bar{r}') \cdot \hat{p}\right] = P(\bar{r}')\, I_{pp}(x', y')$ where $I_{pp}(x',y')$ is the true intensity on the scene plane with polarization in the $\hat{p}$ - direction and F^{-1} denotes an inverse Fourier transform. The significance of Equation 23 can be made more apparent by introducing delta functions $\delta(\nu_{x,y} - n_{x,y} \Delta_{x,y})$. Doing so one obtains:

$$\hat{I}_{pp}(x',y') = \frac{\Delta x \Delta y}{P(\bar{r})} \sum_{n_{x,y}} \iint d\nu_x\, d\nu_y \tilde{V}(\nu_x, \nu_y) e^{-j2\pi\left[\nu_x x' + \nu_y y'\right]}\, \delta(\nu_x - n_x \Delta_x)\, \delta(\nu_y - n_y \Delta_y)$$

$$= P^{-1}(\bar{r}') \iint d\nu_x d\nu_y \tilde{V}(\nu_x,\nu_y)\left[\Delta_x \Delta_y \sum_{n_{x,y}} \delta(\nu_x - n_x\Delta_x)\, \delta(\nu_y - n_y\Delta_y)\right] e^{-j2\pi\left[\nu_x x' + \nu_y y'\right]}$$

$$= P^{-1}(\bar{r}')\, F^{-1}\left[\tilde{V}(\nu_x, \nu_y)\right] * D(x',y') \tag{24}$$

$$= I_{pp}(x', y') * D(x', y')$$

The asterisk (*) denotes a convolution and $D(x', y')$ is the Fourier transform of the sum $\Delta_x \Delta_y \sum_{n_{x,y}} \delta(\nu_x - n_x \Delta_x)\, \delta(\nu_y - n_y \Delta_y)$ which can be written in the following form:

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$$D(x',y') = h(x')\,g(y')$$
$$= \left\{\Delta_x \frac{\sin[\pi x'\Delta_x(2N_x+1)]}{\sin[\pi x'\Delta_x]}\right\}\left\{\Delta_y \frac{\sin[\pi y'\Delta_y(2N_y+1)]}{\sin[\pi y'\Delta_y]}\right\} \qquad (25)$$

Notice that Equation 24 has the form associated with the response of an antenna with effective (power) pattern $D(x',y')$ (e.g., Kraus, 1966; Collin and Zucker, 1969). In fact, $h(x')$ and $g(y')$ are just the array factors encountered in antenna engineering for a uniformly spaced, linear array (Jordan and Balmain, 1968). Thus, in this example the measurement described in Equation 22 yields an image which has the same resolution obtained with a planar array of $(2N_{x,y} + 1)$ elements on a side spaced $d_{x,y}$ meters apart. The important point to notice, however, is that this resolution is determined by the pattern of measurements taken in the image plane, and not by the antenna used in the measurements (i.e. not by $P_n(\bar{r}/\bar{r}')$). This means that measurements with high resolution can be made even with small antennas by making measurements over a sufficiently large area in the image plane.

Finally, notice that in the limit $\Delta_{x,y}\, N_{x,y} \to \infty$, the functions $h(x')$ and $g(y')$ become delta functions at $x' = y' = 0$. In this case $\hat{I}_{pp}(x', y')$ is equal to the actual intensity on the surface, $I_{pp}(x', y')$. In this case the measurement is perfect. In the more general case $h(x')$ and $g(y')$ have peaks of finite width $(N_{x,y}\Delta_{x,y})^{-1}$ and $\hat{I}_{pp}(x', y')$ is a blurred image of the true intensity, $I_{pp}(x', y')$. The width of the functions $h(x')$ and $g(y')$ is a direct measure of the blurring and can be used as a figure of merit for the resolution of the system. Thus, using the width to the first zero to define the resolution of the synthesized antenna aperture, one has:

$$\rho_{x,y} = \left[N_{x,y}\,\Delta_{x,y}\right]^{-1} = \frac{c\,R_o}{N_{x,y}\,d_{x,y}\,\nu_o} \qquad (26)$$

Although the antennas used in the measurement don't determine the resolution, they do determine the field of view (FOV) of the synthesized antenna aperture. Assuming measurements made with antennas with dimensions $D_{x,y}$ in the x- and y-directions, respectively, the portion of the scene plane from which energy is received is approximately:

$$FOV_{x,y} = \frac{\lambda}{D_{x,y}} R_o = \frac{c R_o}{\nu_o D_{x,y}} \quad (27)$$

where R_o is the distance from the antenna to the scene plane. However, several restrictions have been imposed on this field of view in the course of developing the theory. First, the field of view must be much smaller than the distance between the image plane and scene plane to satisfy the sagittal approximation (Equation 8) made in the derivation of the antenna response (Equation 9). Using Equation 27 this restriction may be stated as:

$$\frac{c}{D_{x,y}\, \nu_o} << 1 \quad (28)$$

(The sagittal approximation also limits the size of the array possible in the image plane, but considering that R_o is large for sensors in space, this is not likely to be a significant restriction.) Second, the array functions $h(x')$ and $g(y')$ have grating lobes which affect the field of view. These grating lobes occur every $1/\Delta_{x,y}$ meters; hence, for unambiguous measurements the field of view must be restricted to be less than this distance:

$$FOV_{x,y} \leq \frac{1}{\Delta_{x,y}} = N_{x,y}\, \rho_{x,y} \quad (29)$$

Comparing Equations 27 and 29, one finds that $d_{x,y} < D_{x,y}$ to avoid grating lobes.

Combining these results, one obtains the following parameters for the image formed with this measurement scheme. Assuming measurements from an altitude, R_o, at a frequency, ν_o, with antennas of size, $D_{x,y}$ on a side, and assuming $2N_{x,y} + 1$ measurements uniformly spaced $d_{x,y}$ meters apart on each side, one obtains:

Resolution

$$\rho_{x,y} = \frac{cR_o}{(N_{x,y}\, d_{x,y})\, \nu_o} \quad (30a)$$

Field of View

$$FOV_{x,y} = \frac{cR_o}{D_{x,y}\, \nu_o} \quad (30b)$$

For no Grating Lobes

$$d_{x,y} < D_{x,y} \tag{30c}$$

To satisfy the Sagittal Approximation

$$\frac{c}{\nu_o D_{x,y}} << 1 \tag{30d}$$

VII. CONCLUSIONS

Notice that in the preceding example, Equations 30a-c are just the parameters associated with the antenna beam of a linear array with uniform element spacing $d_{x,y}$ and $N_{x,y}$ elements on a side. That is, the image synthesized with the uniform sampling scheme described in the example will be the same as one formed by scanning with a linear array which occupies the same region in the image plane as was used to obtain the synthesized image (Equation 22). However, the unique advantage of the synthesis procedure is that all of the antennas do not need to be present for each measurement. In principle, the image could be formed with just two antennas which are moved to appropriate positions in the image plane.

Also notice the special role played by system bandwidth in the synthesis scheme. The field of view obtained with aperture synthesis and with a linear array with an equivalent resolution are determined by the elemental antennas used in the measurements; however, in the case of aperture synthesis the field of view is also affected by the bandwidth of the measuring system. For example, consider an ideal filter with response $H^2(\Delta\tau) = H^2(0)$ for $\Delta\tau < \tau_c$ and 0 otherwise. Radiation from a point symmetrically located below a pair of antennas (i.e. equally distant from each) arrives at each antenna at the same time (i.e. $\Delta\tau = 0$); however, as one moves away from this position the time delay increases. When the delay exceeds τ_c radiation from this position no longer is passed by the filter. Consequently, the filter has the effect of restricting the field of view. To a first approximation ($X_o = Y_o = 0$ and R_o much larger than the field of view) this distance is $cR_o/d_{x,y}\,\Delta$ where Δ is the system bandwidth ($\Delta \cong \tau_c^{-1}$) and $d_{x,y}$ is the distance between antennas. In the analysis presented above it was assumed that $H^2(\Delta\tau) = H^2(0)$ for points within the field of view determined by the antennas. That is, the field of view set by the filter was chosen to be much greater than that set by the antennas themselves. However, it is also possible to use the filter itself to control the field of view. The difficulty is that $\Delta\tau$, and therefore the field of view, depend on the spacing between the antennas. Consequently, to keep the field of view constant, the filter must be changed for each measurement. This may not be so impractical as it seems if, for example, the data from each antenna was recorded digitally with a fixed bandwidth and then processed later to obtain the correlation function. The filtering in this case could be done numerically as part of the processing. Such a procedure would have very great flexibility in choosing the field of view, but it requires very large band width during the initial recording process.

Another point to consider is the sensitivity of the synthesized aperture (i.e., the accuracy with which the source can be determined with a given averaging time per measurement). To give an idea of the sensitivity achieved with aperture synthesis a comparison with the sensitivity of a linear array has been made. This is presented in the Appendix. It is shown in the Appendix that for point sources the real aperture is more sensitive because its narrow beam restricts the background against which the source must be contrasted, whereas in the case of aperture synthesis the antennas view the entire field of view on each measurement. However, for extended sources such as the earth, which fill the field of view, the real aperture and synthetic apertures receive the same power density on each measurement and therefore are equally sensitive. Hence, for remote sensing of the earth from space, aperture synthesis can provide sensitivity comparable to a real aperture.

Finally, consider the effects of antenna position errors on the image obtained with the synthesis scheme. An error $\Delta\eta_i$ in antenna separation along the i-th coordinate is equivalent to an error $\Delta\nu_i$ in spatial frequency (Equation 18). In order for this error to be have a small effect on the image one requires (Equation 22) that $\Delta\nu_i X_i = \left(\frac{\Delta\eta_i}{\lambda_o}\right)\frac{X_i}{R_o} << 1$ where λ_o is the wavelength at which the measurement is made and $X_i \in \left\{(X_o + x'), (Y_o + y'), Z_o\right\}$. However, to satisfy the sagittal approximation (Equation 7) it has been assumed that $x'/R_o << 1$ and $y'/R_o << 1$. Hence, the condition that position errors have small effect on the image can be written $\left(\frac{\Delta\eta_i}{\lambda_o}\right)\frac{X_i}{R_o} << 1$ where $X_i \in \left\{X_o, Y_o, Z_o\right\}$. Notice that $X_o/R_o = \cos\alpha$ and $Y_o/R_o = \cos\beta$ are measures of the angles that the beams of the antennas used in the synthesis procedure make with nadir. Hence, the synthetic aperture can be made insensitive to errors in the position parallel to the image plane ($\Delta\eta_x$ and $\Delta\eta_y$) by restricting the antennas to point near nadir. On the other hand, except in extreme cases $Z_0 \cong R_0$. Consequently, errors in the position perpendicular to the image plane are particularly critical and need to be much less than a wavelength. Similar comments pertain to the linear array. However, notice that the requirement in the synthesis procedure is only for knowledge of position, and that the position itself is not particularly important. If the position is known, even if it is many wavelengths from the desired position, then appropriate corrections can be made in processing the data by adjusting the phase terms in Equation 22.

VIII. SUMMARY

A theoretical basis has been presented for synthesizing antenna apertures for microwave radiometers in space. The technique consists of measuring the product of the signal from pairs of antennas at many different antenna spacings, thereby mapping the correlation function of the antenna voltage. The intensity of radiation at the source can be obtained from this correlation function by means of a Fourier transformation. A variety of procedures are possible to measure the correlation function, including a pair of antennas with variable baseline, an array of antennas which simultaneously make measurements at many baselines, measurements with fixed spacing but at several frequencies, and combinations of these. For mapping the earth from space all of these schemes must limit the field of view in order to guarantee a Fourier transform relationship between the intensity on the scene and the correlation function. The resolution obtained with these schemes is similar to that of a real linear array of the same dimensions in the image plane. The major differences being that the synthesized image is obtained by signal processing after all measurements are made and that the field of view of the synthesized aperture must be restricted to meet the paraxial ray approximation. The field of view can be limited by the actual antennas employed in the measurement or by controlling the effective bandwidth of the measurement. The latter requires large bandwidth but is potentially the most flexible system.

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APPENDIX

Although the synthetic aperture has the same resolution as a linear array of comparable size, the images are formed in a much different manner with each system. The linear array maps by forming individual beams which receive energy from each resolution cell in the map (i.e., one beam per pixel). In contrast, the synthetic aperture makes all its measurements with a small antenna which receives energy from the entire field of view during each measurement. This raises the question of the relative sensitivity of the two approaches: that is, given an available integration time per measurement, are there differences in the temperature resolution of the images formed with aperture synthesis as compared to that obtained with a real array. This question is addressed in the paragraphs to follow where it is shown that for point sources the real antenna is more sensitive but that for extended sources which fill the field of view, aperture synthesis and the real array are equally sensitive.

For thermal sources the power available from an antenna with normalized power pattern, $P_n(\theta, \phi)$ can be expressed in terms of the equivalent black body temperature of the sources in the following form (e.g., Kraus, 1966; this can also be derived from Equation 9 in the text):

$$T_a = \frac{\eta}{\Omega_a} \iint_{4\pi} T(\theta, \phi)\, P_n(\theta, \phi) \sin\theta \, d\theta \, d\phi \qquad \text{(A1)}$$

where Ω_a is called the antenna beam solid angle and equals (directivity/4 π $)^{-1}$, η is a constant called the antenna efficiency to account for losses in the antenna system, and T_a is the noise temperature of a resistor which has the same available power as the antenna. Assuming for simplicity that the temperature is constant over the source, $T(\theta, \phi) \cong T_o$, the above simplifies to:

$$T_a \cong \eta \left(\frac{\Omega_s}{\Omega_a} \right) T_o \qquad \text{(A2)}$$

where

$$\Omega_s \triangleq \iint_{\text{source}} P_n(\theta, \phi) \sin\theta \; d\theta \, d\phi \tag{A3}$$

The integration time τ required to measure T_a with an RMS error (i.e., variance) ΔT_a is determined by the system temperature T_{sys} and bandwidth Δ in the following form:

$$\delta T_a = \frac{\epsilon \; T_{sys}}{\sqrt{\Delta \tau}} \tag{A4}$$

where ϵ is a constant which depends on the type of receiver ($\epsilon \cong .7$ for a correlating receiver and $\epsilon \cong 2$ for a Dicke receiver). Combining Equations A2 and A4, one obtains the following expression for τ:

$$\tau = \left(\frac{\epsilon \; T_{sys}}{\eta \; \delta T_o}\right)^2 \left(\frac{\Omega_a}{\Omega_s}\right)^2 \frac{1}{\Delta} \tag{A5}$$

It is desired to compare the time required to form an image using a large real antenna and a synthetic aperture with the same resolution. To form the synthetic aperture assume that measurements are made uniformly on a cartesian grid with $2N_{x,y} + 1$ measurements on a side and in order to obtain the maximum unambiguous field of view assume that $d_{x,y} = D_{x,y}$ where $D_{x,y}$ are the dimensions of the antenna used in the synthesis. Let Ω_a be the solid angle of this antenna and let Ω_A be the solid angle of the equivalent real aperture. The real aperture is $2N_{x,y} \, d_{x,y}$ meters on a side and neglecting possible differences in side lobe levels $\Omega_A = \Omega_a / 4N_xN_y$. Using these definitions, the time required per measurement with the real antenna is:

$$\tau_A = \left(\frac{\epsilon_A T_{sys}}{\eta_A \; \delta T_o}\right) \cdot \left(\frac{\Omega_A}{\Omega_{SA}}\right)^2 \frac{1}{\Delta} \tag{A6}$$

and the time required for each measurement in forming the synthetic aperture is:

$$\tau_a = \left(\frac{\epsilon_a T_{sys}}{\eta_o \; \delta T_o}\right) \left(\frac{\Omega_a}{\Omega_{sa}}\right)^2 \frac{1}{\Delta} \tag{A7}$$

where Ω_{SA} and Ω_{sa} are the beam solid angles subtended by the source in the case of the large antenna and the antenna used in aperture synthesis, respectively.

The total time required for each system to obtain an image is obtained by multiplying by the required number of measurements. Assuming that these are the same in each case (actually, fewer measurements are needed in synthesis schemes which remove redundant measurements), and assuming that each system has the same bandwidth and approximately the same system temperature and efficiency, one has:

$$\frac{\tau_A}{\tau_a} = \left(\frac{\Omega_{sa}}{\Omega_{SA}}\right) \frac{1}{N^2} \tag{A8}$$

where it has been assumed that $2N_x = 2N_y = N$ for simplicity, and the relationship $\Omega_A = \Omega_a/N^2$ has been used.

The ratio in Equation A8 may have several different values depending on the size of the source. For example, when the source is smaller than the main beam of either antenna $\Omega_{SA} = \Omega_{sa}$; when the source fills the beam of the real antenna but is smaller than the beam of the antenna used in synthesis $\Omega_{SA} = \Omega_A$; and when the source is larger than the beam of either antenna $\Omega_{SA} = \Omega_A$ and $\Omega_{sa} = \Omega_a$. Thus, one obtains the following possibilities:

$$\tau_A / \tau_a = \begin{cases} 1/N^2 & \text{source smaller than either antenna beam} \\ (1/N^2)(\Omega_{sa}/\Omega_A) & \text{source fills beam of real antenna but less than FOV} \\ 1 & \text{source fills beam of both antennas} \end{cases}$$

Clearly, in the case of small (e.g., point) sources mapping by scanning with a single large antenna is a more efficient use of available energy than aperture synthesis and requires less integration time per measurement to obtain desired accuracy. However, when the source completely fills the beam of the antenna used in synthesizing an aperture, then aperture synthesis and scanning with a large antenna yield comparable sensitivity. This is the situation likely to be encountered in remote sensing of the earth's surface.

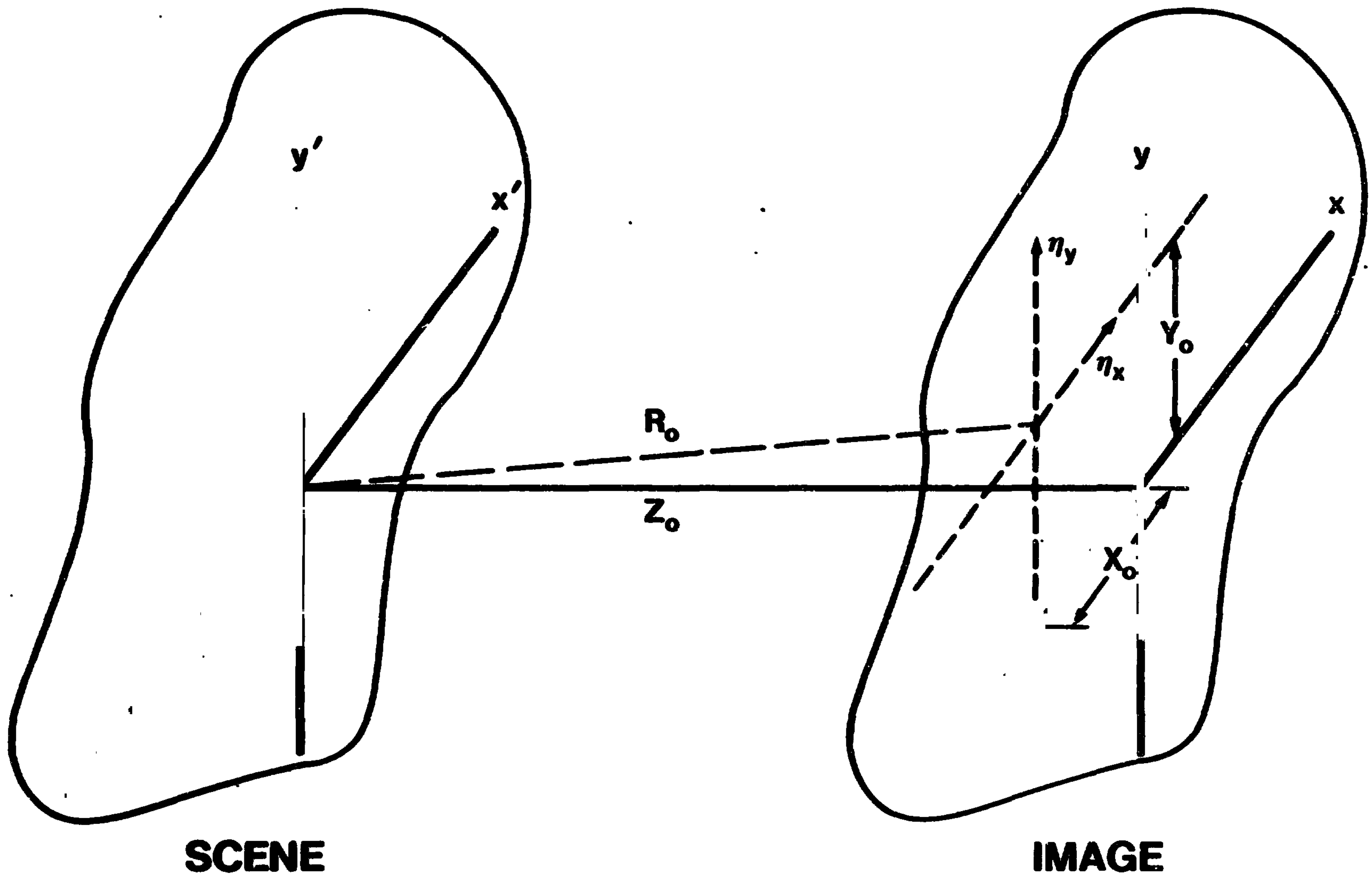

Figure 1. The geometry and coordinate system used in the calculations.

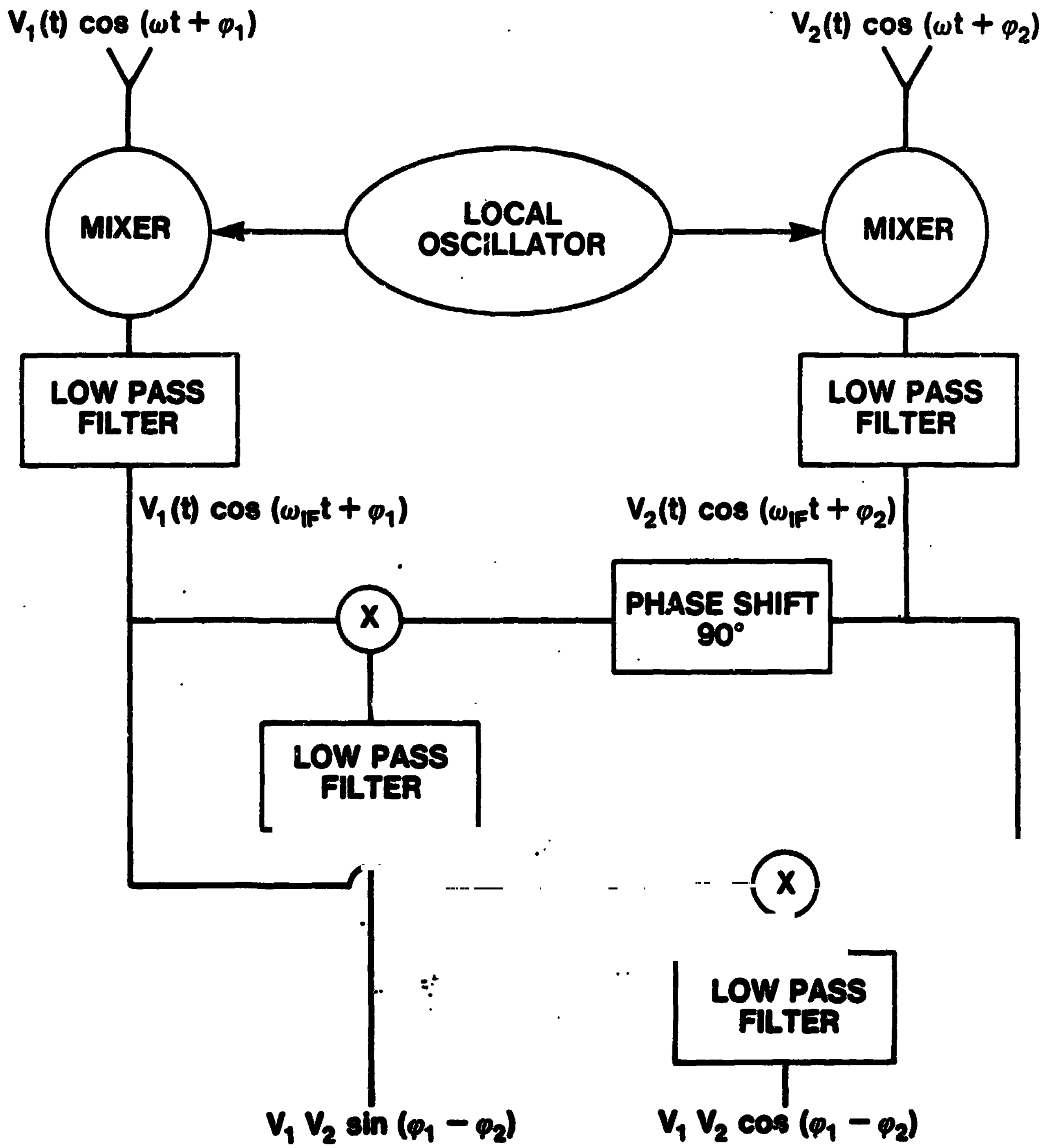

Figure 2. Receiver for measuring the correlation function.

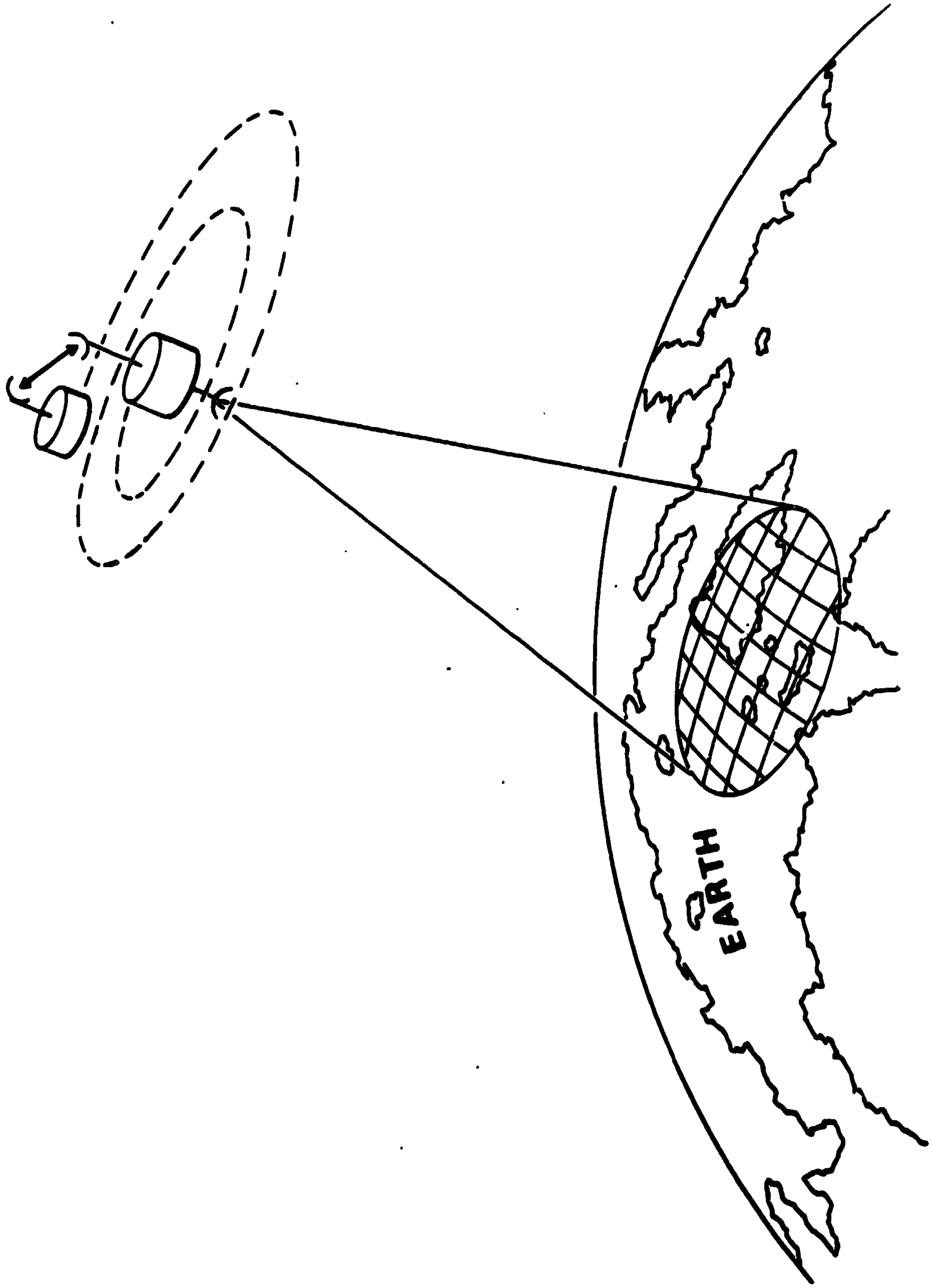

Figure 3. Schematic illustrating the concept of aperture synthesis from space.