REGENERATION EFFICIENCY, SHUTTLE HEAT LOSS AND THERMAL CONDUCTIVITY IN EPOXY-COMPOSITE ANNULAR GAP REGENERATORS FROM 4K TO 80K [1]

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ABSTRACT

Several groups have built epoxy-glass Stirling-cycle cryocoolers which utilize the annular gap between the displacer and the stationary outer sleeve as a regenerator. The cooling power and ultimate cold temperature achieved with cryocoolers of this design are primarily limited by regeneration inefficiency at the low temperature end of the cryocooler. This is because of the rapid decrease in the specific heat and thermal conductivity of the epoxy composite at low temperatures.

A test apparatus designed to simulate a section of a cryocooler has been built. Measurements of regeneration efficiency, shuttle heat loss and thermal conductivity are reported for several regenerator test sections. The test composites were epoxy-glass, epoxy-glass with lead particles, epoxy-glass with activated charcoal and epoxy-graphite. Losses measured for these materials were approximately the same. Losses are in good agreement with those calculated theoretically for an epoxy-glass (G-10) composite. The implications of these results on cryocooler design are discussed.

INTRODUCTION

Stirling-cycle cryocoolers, in which the annular gap between a displacer and a stationary outer cylinder is used as a regenerator, have been demonstrated recently. Zimmerman and Sullivan have demonstrated a five-stage cyrocooler made from a commercial spun glass-epoxy (G-10) outer cylinder and a solid nylon displacer [1]. They report reaching 7K. We have demonstrated operation of a cryocooler at 9K [2]. Our cryocooler was a single-stage conical device with a hollow displacer. Both the displacer and the outer cylinder were made from an epoxy-glass cloth composite.

These research efforts are directed toward the development of a reliable, low-power, closed-cycle cryocooler for operating superconducting devices, especially SQUIDS. The temperatures achieved are already low enough to allow operation of some "high temperature" types of SQUIDS [3,4]. Nevertheless, it would be highly desirable to improve the cryocooler design and reach temperatures below the critical temperature of helium (5.2K). This would have two major advantages. One is that many other types of

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superconducting materials could be used. The second is that the superconducting apparatus could be immersed in a bath of liquid helium. A liquid bath would provide a large heat reservoir which would damp out rapid temperature fluctuations. If the liquid helium bath was connected to a sealed reservoir of gas at room temperature the bath temperature would be at the boiling point of liquid helium. Long term temperature drifts could be eliminated by monitoring the vapor pressure and using electronic feedback. The use of a liquid helium bath would therefore result in a large reduction in the noise, due to thermal fluctuations, observed in SQUIDS mounted directly on closed cycle cryocoolers [4].

This study of regeneration efficiency is directed toward the problem of designing a cryocooler capable of operation below 10K. The ultimate temperature of the cold end of a Stirling-cycle cryocooler is limited by various heat loss mechanisms. The ultimate temperature is reached when the refrigeration capacity of the volume of gas displaced at the cold end is equal to the heat transferred to the cold end by the loss mechanisms. The amount of heat lost by various mechanisms can be calculated theoretically and these calculations are consistent with experimental ultimate cryocooler temperatures observed [5,2]. Unfortunately, it is very difficult to study the loss mechanisms at low temperature in an experimental cryocooler. This is because the cryocooler performance depends on a combination of loss mechanisms over a wide range of temperatures, as well as on the refrigeration. We have built a special test facility to enable direct measurement of losses under conditions similar to those in an actual cryocooler.

HEAT LOSS MECHANISMS

Heat loss mechanisms present in a cryocooler include regeneration loss, shuttle heat loss, friction, conduction loss, radiation loss and the enthalpy deficit. Convection loss was also present in this test apparatus.

Regeneration loss is the net heat carried by the cycling of helium gas at constant pressure between the warm end and the cold end. When the regenerator is operating efficiently the majority of the heat carried by the helium gas is absorbed and released by the plastic walls of the displacer and cylinder as the helium flows in the annular gap between them. Theoretical formulae for regeneration loss are given in the appendix. Note that, for a fixed pressure and a fixed stroke, the regeneration loss is a strong function of temperature. Both regeneration loss terms are proportional to the square of the mass of helium gas transferred in one cycle. One regeneration loss term is inversely proportional to the conductivity of helium gas and inversely proportional to the gap between the displacer and the cylinder. The other regeneration loss term is inversely proportional to the square root of the product of the heat capacity and thermal conductivity of the plastic. The total regeneration loss therefore has a temperature dependence between T^{-3} and T^{-4}. Regeneration loss is the dominant loss term for temperatures below 20K.

Shuttle heat loss is the net heat carried by the specific heat of the displacer and transferred to the cylinder by the movement of the displacer. It varies approximately as T and is the dominant loss term above 20K. Over a wide range of pressures the shuttle heat loss is independent of the pressure of the working gas, since the thermal conductivity of the gas does not depend on pressure. Losses measured at low pressures in the test apparatus were therefore primarily shuttle heat loss.

Friction was a significant source of heat in our test facility. It was approximately independent of pressure and temperature and was usually in the neighbourhood of 1 milliWatt. Friction is also very significant in a cryocooler. A common failure mode of cryocoolers is heating due to an increase in friction caused by solidification of impurities in the gap at the low temperature end.

Conduction loss and radiation loss are insignificant at the cold end (<20K) of a properly designed cryocooler. Conduction loss was also measured with this test apparatus.

The enthalpy deficit occurs in a non-ideal gas because the total enthalpy flow from the high temperature to the low temperature is greater than the enthalpy flow in the reverse direction due to pressure variations [5]. This is due to changes in the specific heat with pressure. It can be calculated from the physical properties of helium [6]. As our measurements were done at constant pressure they do not include the enthalpy deficit.

Convection of helium gas is a heat loss mechanism in our test apparatus which is not normally present in a cryocooler (unless the cryocooler is operated upside down). Unfortunately, our test apparatus was "upside down", with the cold end above the warm end. At high pressures convection losses were significant in our tests. The heat loss measurements, reported in Figs. 3 and 4, have been corrected for convection losses, as explained in the section on experimental results.

EXPERIMENTAL APPARATUS

A diagram of the heat loss test apparatus is shown as Fig. 1. The apparatus simulates a section of an actual cryocooler. A displacer (C) is moved up and down with a sinusoidal cycle by means of a stainless steel rod going to room temperature. When the displacer is moved, helium gas in the working space (5) is forced to slide past the displacer, in the gap between the displacer and the outer cylinder (B). The upper working volume is maintained at a fixed temperature by means of an electronic temperature controller. The controller is connected to a heater and a carbon glass thermometer in the copper plate (A). A known amount of heat is applied to the lower working volume by means of a heater in the copper end cap of the cylinder (E). In operation, the lower end of the cylinder (E), is hotter than the upper end (D). We define the "effective thermal conduction" or "heat loss" of the test section as the heat input at (E) divided by the temperature difference between (E) and (A). This "heat loss" is a function of the pressure of helium in the working space (5), the frequency and stroke of the displacer, the mean gap between the displacer and the outer cylinder and the mean temperature of the test section.

The test section is surrounded by a pumped evacuated space (3) and a copper radiation shield at the temperature of the upper (cold) end of the test section. Tests may be made over a wide temperature range by using either liquid nitrogen or liquid helium as a low temperature bath (1). Depending on the temperature desired, different pressures of exchange gas (2) were used to facilitate the electronic temperature regulation.

This design has the special feature that the test section is attached to the flange (A) with bolts and an indium O-ring to facilitate changing test sections. The displacer is fitted with copper end caps (F) and (G) which are always within the copper sections, (D) and (E) respectively, of the cylinder. This feature ensures that the temperature of the ends of the displacer remain fixed at the same values as the ends of the cylinder.

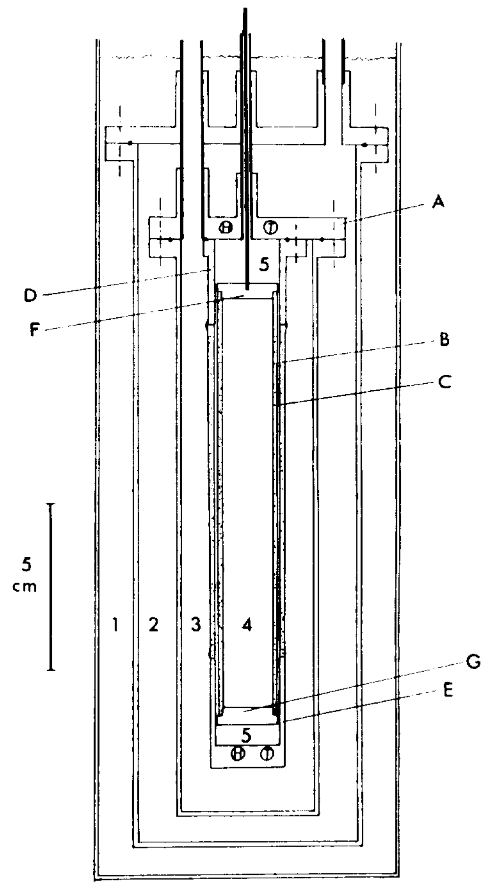

Fig. 1: Regenerator test apparatus.

A Upper copper mounting flange. It is held at a fixed temperature by heater (H) and thermometer (T).
B Conical glass-epoxy cylinder under test. (The taper of the cone is too slight to be noticable in the figure.)
C Conical glass-epoxy displacer under test.
D Copper cylinder flange mounted with an indium seal.
E Copper cylinder end with heater (H) to generate a temperature gradient and a thermometer (T).
F,G Copper displacer end plugs.

The volumes are filled as follows: 1 - Liquid helium, 2 - Exchange gas (helium), 3 - Hard vacuum, 4 - Vacuum (air frozen out) and 5 - Helium working gas.

The impedance of the regenerator to the flow of gas is low. Therefore, the difference in gas pressure between one end of the regenerator and the other is always negligible. The working gas (5) is connected to a large room temperature volume of helium gas. The pressure of the working gas is, therefore, to a good approximation, constant.

This design imposed some limitations. The gas line for the working volume goes through the liquid helium bath. This limited the pressure of the working gas to less than one atmosphere.

MATERIALS TESTED

The test displacer and cylinder sections were made by laying up the epoxy-composite material on a tapered mandril. The wall thicknesses of both the displacers and the cylinders were 1.3 mm. The inside diameter of the cylinder varied from 2.03 cm to 1.78 cm along its 7.6 cm length. Four materials were tested: epoxy-glass, epoxy-glass with 10wt% activated charcoal, epoxy-glass with 60wt% powdered lead and epoxy-graphite cloth. In all cases the displacer was sealed full of air, hence would be evacuated at temperatures below the freezing point of air. The epoxy-graphite displacer was filled with air but was stuffed with glass wool and layers of aluminized mylar to eliminate any convection of gas inside the displacer.

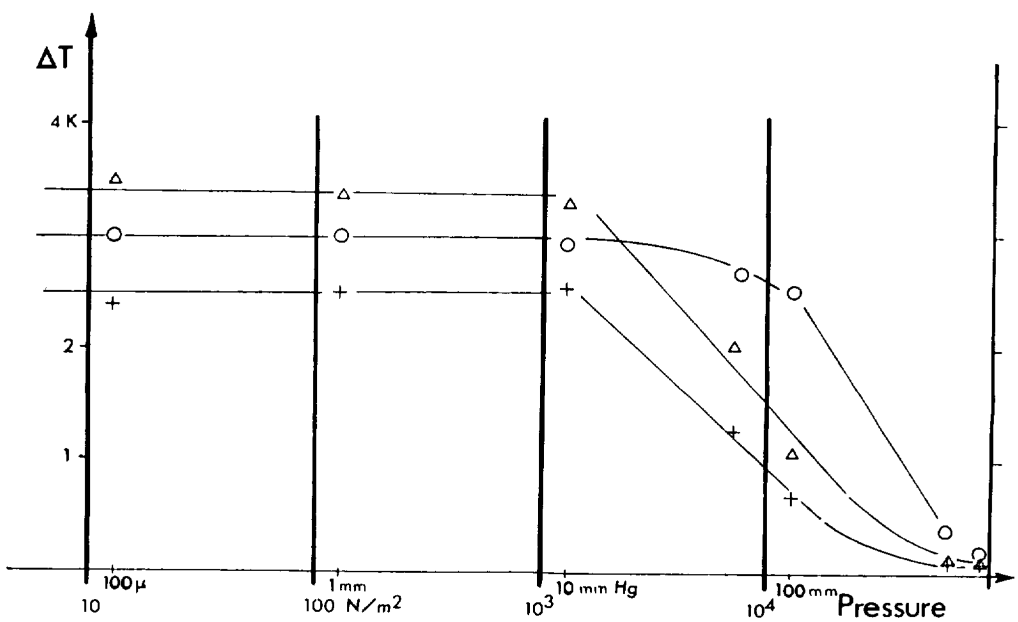

Fig. 2.

Temperature difference between the upper and lower ends of the cylinder, as a function of helium gas pressure, with the upper end held at a temperature of 6.0K.

o Displacer stationary, heater on (0.65 mW), ΔT_1.

+ Displacer moving 0.625Hz 1.3cm stroke, heater off, ΔT_2.

▲ Displacer moving 0.625Hz 1.3cm stroke, heater on (0.65mW), ΔT_3.

EXPERIMENTAL RESULTS

Typical experimental measurements are shown in Fig. 2 for an epoxy-glass displacer and an epoxy-glass cylinder. The temperature of the upper end of the cylinder was 6K for all of the data points shown. At each pressure three data points were taken. First, 0.65mW was applied to the lower heater, with the displacer held stationary. The temperature difference ΔT_1 between the two ends of the cylinder is plotted in the figure. This is related to the sum of thermal conductivity in the glass-epoxy and to the convection of helium gas in the working space. Measurements at low pressures have negligible convection, and allow calculation of the thermal conductivity of the glass-epoxy.

A second set of data points was taken with the heater turned off, but with the displacer moving up and down at 0.63 Hz, with a stroke of 1.3 cm. As a result of the slight taper of the displacer and cylinder the annular gap varied from 0.2 to 0.4 mm throughout the stroke. The temperature difference ΔT_2 between the upper and lower ends of the cylinder was again measured. This measurement is utilized to determine the amount of frictional heating which resulted from the movement of the displacer.

A third set of data points were taken with the heater turned on (0.65mW) and with the displacer moving giving the temperature differences ΔT_3.

These measurements allow us to estimate the sizes of the various heat loss mechanisms. First, the effective thermal conductivity of the cylinder and displacer (plus convection losses significant at higher pressures) is $\dot{Q}_c/\Delta T_1 = 0.65\text{mW}/\Delta T_1$. The values of thermal conductivity measured for the four materials are in close agreement with the published values for epoxy-glass G-10 [1]. The amount of frictional heating may be determined from the ratio of the temperature rise caused by friction, to that caused by electrical heating. The frictional heat is therefore: $H_f = 0.65\text{mW}\ \Delta T_2/(\Delta T_3 - \Delta T_2)$. The effective conductivity between the upper and lower ends of the cylinder due to regenerator inefficiency and shuttle heat transfer is:

$$\frac{\dot{Q}_{rs}}{\Delta T_3} = \frac{0.65\text{mW} + H_f - \dot{Q}_c}{\Delta T_3} = \frac{0.65\text{mW}}{(\Delta T_3 - \Delta T_2)} - \frac{0.65\text{mW}}{\Delta T_1} \tag{1}$$

The term on the right corrects for conduction and convection losses. It is also possible to determine the shuttle and regeneration losses separately from this data. The regeneration loss is proportional to the pressure (helium mass flow) squared, whereas the shuttle heat loss is almost independent of pressure for pressures above 10 pascals (0.1 torr). Fig. 3 shows the measured pressure dependence of the sum of regeneration and shuttle heat losses, for a fixed temperature.

The temperature dependence of these two losses is shown plotted in Fig. 4 for a fixed pressure (0.65×10^5 Nt/m^2). Experimental data points are shown for the four types of materials tested. The theoretical shuttle heat loss and regeneration loss, calculated with the equations given in the appendix, are also shown plotted in the figure. The theory is in good agreement with the experimental data points. The four materials tested gave approximately the same losses in spite of their compositional differences.

This is because the thermal properties of the four materials were all dominated by the properties of their major constituent, epoxy. It should be noted that for a mean gap of 0.3 mm the thermal conductivity of helium is low enough that it imposes a significant limitation on regeneration, even if the specific heat of the plastic was higher.

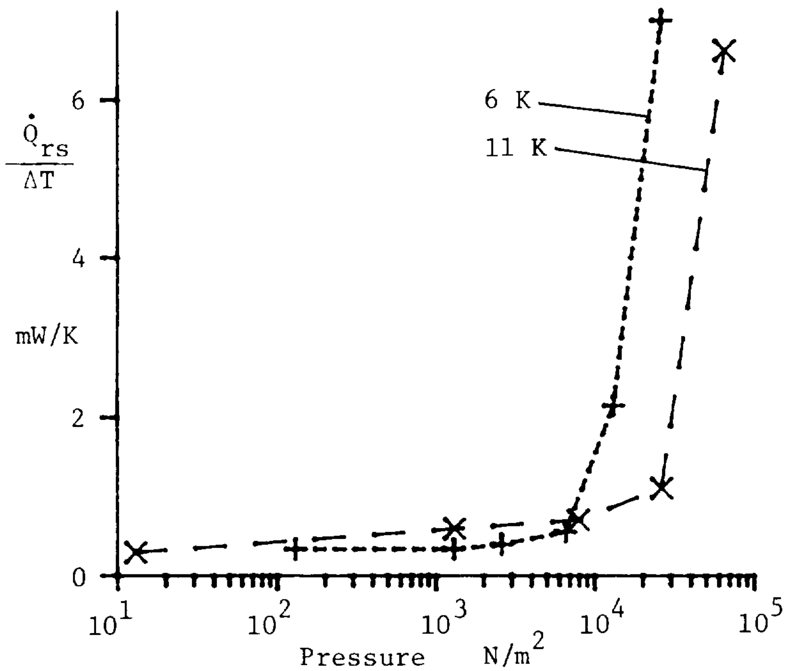

Fig. 3.

Regeneration and shuttle heat loss as a function of helium gas pressure, for two temperatures. Cylinder was epoxy-glass and the displacer was epoxy-glass plus 60% by wt. lead.

CONCLUSIONS

The good agreement obtained between experimental measurements of losses and the theoretical calculations encourages us to examine the implications of the theoretical loss calculations on cryocooler design. Examination of Fig. 4 shows that for temperatures above 20K the shuttle loss is dominant, whereas below 20K regeneration losses are dominant. The equations in the appendix show that an increase in the radial gap between the displacer and the cylinder causes an increase in the regeneration loss and a decrease in the shuttle heat loss. This suggests that the total loss due to regeneration and shuttle heat transfer can be reduced by increasing the radial gap in the warmer parts of the cryocooler and decreasing it at the cold end. This can be effected in a conical cryocooler by decreasing the pitch of the cone at the cold end, since the pitch of the cone is related to the mean gap if the displacer and outer cylinder touch everywhere at the bottom of a stroke.

Unfortunately, the poor regeneration at low temperatures will not be greatly improved by reducing the radial gap as it is limited by the low heat capacity of the epoxy. The addition of lead to the epoxy should improve this somewhat, but the most we were able to add was 60wt%, which did not make a huge difference in the specific heat.

Regeneration losses at low temperatures will be very large for all regenerator materials. They can be reduced somewhat by decreasing the thermal gradient by extending the tip of the displacer. The result of this modification would be a shape somewhat like that of an exponential horn rather than the simple cone used in our earlier cryocooler.

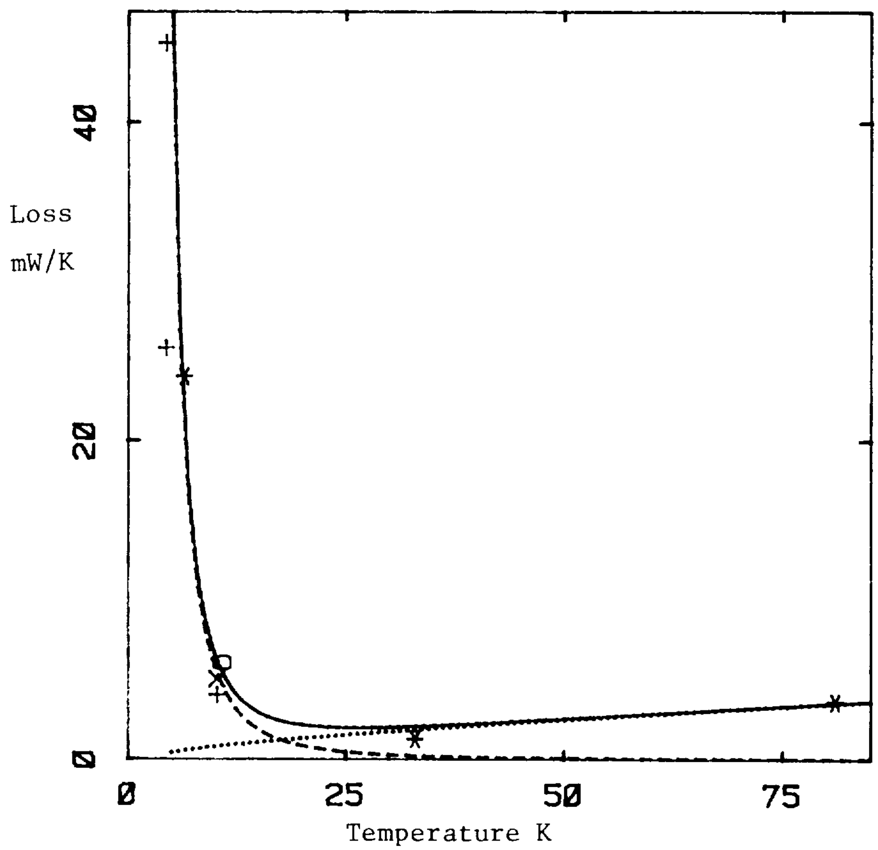

Fig. 4.

Regeneration and shuttle heat loss as a function of temperature for a pressure of 60000 Pascals (0.6 atmosphere).
———Theoretical regeneration plus shuttle heat loss.
·········Theoretical shuttle heat loss.
-----Theoretical regeneration loss.

Experimental data.
- * Epoxy glass.
- X Epoxy glass plus 60% by weight lead.
- + Epoxy glass plus 10% by weight charcoal.
- o Epoxy with graphite fibers.

APPENDIX: THEORETICAL HEAT LOSS EQUATIONS

The theoretical regeneration and shuttle heat losses plotted in Fig. 4 are derived from published equations for regeneration [2], shuttle heat loss [7], the physical properties of epoxy-glass G-10 [1], and the physical properties of helium [6]. The equations used are reproduced below for convenience.

The regeneration loss $\dot{Q}_{regeneration}$ is the sum of a term due to the finite effective heat capacity of the plastic ($\dot{Q}_{rP}$) and a term due to the limited thermal conductivity of the helium gas ($\dot{Q}_{rHe}$). The Reynolds number is about half that for the onset of turbulent flow even at the lowest temperatures, consequently the conductivity of the gas is calculated for laminar flow conditions.

$$\frac{\dot{Q}_{regeneration}}{\Delta T} = \frac{\dot{Q}_{rP}}{\Delta T} + \frac{\dot{Q}_{rHe}}{\Delta T} \tag{2}$$

$$\frac{\dot{Q}_{rP}}{\Delta T} = \frac{\omega}{16\pi DL} (C_{pHe}M_o)^2 \left(\frac{\omega}{2\rho_P C_P K_P}\right)^{\frac{1}{2}} \tag{3}$$

$$\frac{\dot{Q}_{rHe}}{\Delta T} = \frac{17}{1120} \frac{\omega^2 g}{\pi DLK_{He}} (C_{pHe}M_o)^2 \tag{4}$$

M_o is the mass of helium moved in one cycle. The mass flow is assumed to be sinusoidal, therefore the peak mass flow rates are +/- $\omega M_o/2$.

For our test conditions the $\dot{Q}_{rP}$ term was about twice the $\dot{Q}_{rHe}$ term between 5K and 10K.

The shuttle heat loss $\dot{Q}_{shuttle}$ is due to the effective conduction of the plastic $\dot{Q}_{sP}$ in series with the effective conduction of the helium gas in the gap $\dot{Q}_{sHe}$.

$$\frac{\dot{Q}_{shuttle}}{\Delta T} = \left(\frac{\Delta T}{\dot{Q}_{sP}} + \frac{\Delta T}{\dot{Q}_{sHe}}\right)^{-1} \tag{5}$$

$$\frac{\dot{Q}_{sP}}{\Delta T} = \frac{D\ S^2}{4L} (\omega K_P C_P)^{\frac{1}{2}} \tag{6}$$

$$\frac{\dot{Q}_{sHe}}{\Delta T} = \frac{K_{He}\ D\ S^2}{2gL} \tag{7}$$

For our test conditions the $\dot{Q}_{sHe}$ term dominated the $\dot{Q}_{sP}$ term by more than a factor of two. This indicates the importance of a large gap in reducing shuttle loss at high temperatures.

The thermal conductivity K_p in mW/(cm-K) and the specific heat per unit volume $\rho_p C_p$ in mJ/(K-cm^3) for epoxy-glass G-10 for temperatures from 5K to 300K may be expressed as [1] :

$$K_p(T) = 0.917 + 0.0319\ T - 3.67 \times 10^{-5}\ T^2, \text{ and} \tag{8}$$

$$\rho_p C_p(T) = -14.9 + 3.75\ T + 0.0335\ T^2 - 9.21 \times 10^{-5}\ T^3. \tag{9}$$

SYMBOLS:
D - diameter of the displacer (1.9 cm)
ω - 2π times the frequency of cycling (0.63Hz x 2π)
K_{He} - Thermal conductivity of helium gas.
C_{pHe} - Heat capacity per unit mass of helium gas.
K_p - Thermal conductivity of the plastic (G-10).
ρ_p - Density of the plastic.
C_p - Heat capacity per unit mass of plastic.
ΔT - Temperature difference between the ends of the test section.
g - Mean radial gap between the displacer and cylinder (0.3 mm)
L - Length of test section (7.6 cm).
S - Stroke (1.3 cm)

DISCUSSION

Ronald E. Sager - Was your radiation shield isothermal or did it match the temperature gradient of the experimental apparatus?
Calvin Winter - The radiation shield was isothermal. It was held at the temperature of the coldest (top) end of the regenerator under test. The effects of radiation were negligible for tests at 6K and 11K.
Donald B. Sullivan - Does variation of the gap affect the operation of tapered displacer cryocoolers?
Calvin Winter - Yes. The tip region of the displacer will have a smaller slope and therefore a smaller variation in the gap when the displacer is moved, than the base region of the displacer. A smaller average gap at the tip than at the base will tend to reduce losses. Unfortunately, it is not possible to make the gap at the tip small enough because the resultant friction will render the cryocooler inoperative.

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