COMPUTER PROGRAM FOR ANALYSIS OF SPLIT-STIRLING-CYCLE CRYOGENIC COOLERS*

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ABSTRACT

A computer program for predicting the detailed thermodynamic performance of split-Stirling-cycle refrigerators has been developed. The mathematical model includes the refrigerator cold head, free-displacer/regenerator, gas transfer line, and provision for modeling a mechanical or thermal compressor. To allow for dynamic processes (such as aerodynamic friction and heat transfer) temperature, pressure, and mass flow rate are varied by subdividing the refrigerator into an appropriate number of fluid and structural control volumes. Of special importance to modeling of cryogenic coolers is the inclusion of real gas properties, and allowance for variation of thermophysical properties such as thermal conductivities, specific heats and viscosities, with temperature and/or pressure. The resulting model, therefore, comprehensively simulates the split-cycle cooler both spatially and temporally by reflecting the effects of dynamic processes and real material properties.

The computer program was evaluated by modeling a small cryogenic cooler for which abundant test data was available. The accuracy of the computer program was verified by this comparison, and it provided an insight into the thermodynamic processes occurring in the cold regenerator. An important result of this effort is a clear idea of the further tests and research needed for those correlations which are not now well defined or whose application to small cryogenic coolers is questionable.

INTRODUCTION

In the late 1960's, it became obvious that a two-piece cryogenic refrigeration system offered a flexible approach to meeting the applications requirements, especially those typical of infrared sensor systems. In the split-cycle system, a primary power module is separated from the refrigeration module. Miniature, two-piece, split-Stirling-cycle refrigerators have been successfully developed and demonstrated.

The split-cycle refrigerator is a regenerative gas-cycle machine with its cyclic processes characterized by the Stirling cycle. The physical implementation of a split-cycle system requires a compression piston, an expander displacer, a thermal regenerator, and a gas transfer line to interconnect the expansion and compression spaces. Figure 1 is a schematic representation and typical pressure-volume curves of an actual split-cycle refrigerator. The

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amount of refrigeration produced is proportional to the enclosed area in the cold expansion volume diagram. The compression required of the drive motor is proportional to the area of the compression volume in the diagram for mechanical compressor. The refrigerator employs a valveless compressor with a motor/crankshaft mechanism by which pressure pulses are transmitted to an expander consisting of a displacer joined to a push rod. Since this rod is sealed, it experiences drive forces that are proportional to the differences between the magnitude of the pressure pulse and the mean pressure in the sealed volume. When the pressure pulse is high, the push rod tends to stroke the displacer into the mean pressure volume; when the pressure pulse is low, the converse occurs. Because the expansion displacer in these modules is pneumatically driven, no electric power is needed at the expander.

CYCLE ANALYSIS

The techniques currently in use* for analyzing cryogenic coolers are typically based on an ideal isothermal model, which assumes isothermal expansion and compression. This method evaluates ideal cycle performance for the actual cycle on the basis of a quasi-steady-state solution; it then reduces this ideal refrigeration by the sum of the parasitic heat losses. It is assumed that each of these losses acts independently of the others and that the temperature of the gas in any volume of the refrigerator is constant. The most notable disadvantage of this process of uncoupling the cold head regenerator losses from cooler performance is in predicting the conditions and performance of the cold head displacer regenerator. Since the efficiency of the regenerator is critical, the dynamic process cannot be adequately modeled by means of an isothermal model. A complete analysis is needed in order to treat the more general case of dynamic heat transfer in the regenerator matrix and of heat conduction in all the refrigerator components, including the regenerator matrix.

The objective of this effort was to conduct a comprehensive analysis of all thermodynamic and dynamic processes occurring within the cold head displacer assembly. The resulting mathematical model presented in this paper is a general, nonisothermal cold head displacer model, with the thermal, frictional, and pressure losses treated as an integral part of the dynamic and thermodynamic analysis. Special attention was given to incorporating empirical correlations and data whenever this step was needed to increase the accuracy of the analytical definition of the dynamic and thermodynamic processes.

NONISOTHERMAL MODEL

This section presents the final equations for the nonisothermal model. How these equations were developed is discussed in detail in the Cold Displacer

* As far as is known by the authors only the ideal isothermal model common to the iterative design process is currently in use. The more detailed analytical tool of a non-isothermal model, although common to Stirling engine analysis, has not been applied and evaluated for the special requirements of cryogenic coolers.

Study Final Report.* The following equations are based on a consideration of conservation of mass, momentum, and energy. The sign conventions used in these equations are presented in Figure 2.

For the variation of the temperature of the working fluid with time (with the compressor crank angle θ used as a reference) for the control volume n, we have

$$\frac{dT_n}{d\theta} = \frac{1}{C_v m}\left[e\,\Sigma G(T_w - T_n) + C_{P_{n,n-1}} T_{n,n-1} \frac{dm_{ni}}{d\theta} - C_{P_{n,n+1}} T_{n,n+1} \frac{dm_{(n+1)i}}{d\theta} - \frac{PdV}{d\theta} - C_v T \frac{dm}{d\theta}\right] . \qquad (1)$$

This equation neglects both potential and kinetic energy, and it assumes that the enthalpy and internal energy of the gas are provided by the ideal gas relations, i.e., by $C_p T$ and $C_v T$, respectively. These assumptions are very reasonable for the class of coolers to be modeled (relatively small and with cold expansion volume temperatures above 35K. In this equation, the term $\Sigma G\ (T_w - T_n)$ accounts for all of the heat transferred into the control volume, and the term $PdV/d\theta$ is the work performed on the control volume.

The variation in the pressure of the working fluid is

$$\frac{dP_n}{d\theta} = \frac{dm_i}{d\theta} \pm \left[\frac{1}{K}\left(P_n - P_{n+1}\right)\right]^{1/2} \frac{ZRTe}{V} - \frac{dV}{d\theta}\frac{P}{V} + \frac{dT}{d\theta}\frac{P}{T} + \frac{dZ}{d\theta}\frac{P}{Z} \quad . \qquad (2)$$

where the compressibility factor Z is used to define the equation of state, which is

$$m = \frac{PV}{ZRT} \qquad (3)$$

The pressure equation was derived by applying the Darcy flow resistance equation for mass flow between control volumes, whereby the mass flow rate is approximately proportional to a power function of the difference in the pressures of the control volumes. The flow resistances are denoted by the letter K.

Conservation of mass is expressed in terms of the mass entering a control volume. When the mass entering the first control volume is equal to zero (because the cooler is a closed system) the equation defining the mass flow into the n^{th} control volume is

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$$\frac{dm_{ni}}{d\theta} = - \sum_{j=1}^{j=n-1} \frac{dm_j}{d\theta} \tag{4}$$

The working control volumes are defined by assuming a sinusoidal variation in the compressor volume and then writing an equation for the dynamic motion of the cold displacer in order to determine the expander volumes. The motion of the cold displacer depends on the resultant gas pressure on the displacer, the seal drag, and the external vibration forces. Figure 3 presents the dynamic model for the free displacer together with the terms for the acceleration of the displacer in relation to the acceleration of the cylinder. The associated equation is

$$\frac{d^2u}{dt^2} = \frac{1}{m_P} \left[F_P - F_S \text{ Sign } (\dot{u}) - m_P \frac{d^2w}{dt^2} \right] \tag{5}$$

This equation is integrated to produce the velocity of the displacer in relation to that of the cylinder.

The final equation represents the variation in the temperature of the regenerator. When it is written in finite-difference form, this equation is a heat balance for node i, where G_{ij} is the heat flow path between nodes i and j. These paths can represent either conductive or convective heat transfer between the respective nodes. By expressing t as a function of compressor crank angle, this equation can be expressed as

$$\frac{dT_i}{d\theta} = \frac{e}{C_{p_i} \rho_i} \left(\sum_{j=1}^{n} G_{ij} (T_j - T_i) + Q_i \right) \tag{6}$$

The solution of this equation yields a temperature/time history for each regenerator node.

NODAL REPRESENTATION

The thermal model is of the nodal network type, which divides the refrigerator into discrete structure and working fluid nodes that are interconnected by heat transfer and mass transfer energy flow paths. A schematic cross section of a typical split-cycle cold head divided into a discrete nodal network is shown in Figure 4. Details of a typical regenerator control volume are given in Figure 5.

The gas system is divided into control volumes, and the machine structure is divided into elements of corresponding size. In this way all internal gas volumes, machine elements, and boundaries are modeled as variable- or constant-temperature regions that exchange energy with one another as defined by the fundamental equations of heat transfer. Modes of energy exchange between nodes include mass transport between the gas nodes, conduction across solid or gaseous material, and convection at gas/solid interfaces. Each volume of the nodal network is described by three sets of differential equations that are categorized as equations expressing the distribution of

1. Gas temperatures

2. Gas masses

3. Regenerator temperatures

The simultaneous digital solution of these three sets of equations by the nodal network representation is achieved by using the MIMIC program language (1). MIMIC is a high-level language used on a digital computer (in our case an Amdahl 580 and for Wright-Patterson a Control Data 6000) for simulation of continuous and sampled data systems. It is a simple way to express systems of ordinary differential equations since it allows the problem to be solved without requiring that the user formulate an algorithm. Working from this mathematical model, the user writes MIMIC statements describing the system; these statements can, in turn, be grouped into sets of statements that represent the various control volumes that make up the nodal network.

COMPUTER MODEL

To produce the computer program in final form, considerable attention was given to balancing the following three critical areas that affect the simulation accuracy and the size of the computer program needed.

1. The degree of complexity needed in the thermodynamic equations that define the conservation of energy and the conservation of mass of the working fluid.

2. The extent to which the nodal network should be divided, and what boundary conditions should be assumed.

3. The evaluation of the quantitative accuracy of the correlations used to define the various heat transfer mechanisms, gas-flow frictional pressure drops, and the frictional characteristics of displacer motion.

Each of these subjects was reviewed and was simplified as needed in order to devise a program that represents the physical system with an acceptable degree of accuracy and that is of manageable size (i.e., reasonable required core size for compiling and reasonable CPU time in which to conduct a complete simulation run).

THERMODYNAMIC EQUATIONS

Besides the simplifications made in the defining thermodynamic equations, already discussed, the thermophysical properties of the helium working fluid as well as the regenerator matrix properties and the extent to which these properties affect the cooler performance was also considered. Provision was made for the variations in thermophysical properties as the pressures and temperatures vary during the refrigeration cycle. These variations become increasingly critical as the control volume temperatures decrease. The computer program therefore includes a means of accessing important thermophysical properties data by using pressure and temperature information.

NODAL NETWORK

The number of nodes to be used in a cooler simulation depends on the lowest temperature that the cooler to be simulated is expected to reach. In the present form of the program six regions along the regenerator length were used. This number was adequate for simulating the base cooler used for program verification and kept the computer simulation time below 6 minutes CPU on the Amdahl. Isothermality was also assumed as a boundary condition for the compressor and the warm expansion volumes.

CORRELATION ACCURACY

Item (3) listed above is the pacing item in terms of program accuracy. The other two items can be varied to reduce simulation errors to the extent required by the investigation, but the accuracies of the correlations have not yet been thoroughly investigated. Their applicability to the conditions in a small cryogenic refrigerator have yet to be rigorously concluded.

The set of correlations used in the program are presented in Table 1. These equations were chosen from the literature in the expectation that when the results from the computer program were correlated with the test data any need for further refinements of certain correlations would become obvious. The equations for two relationships are not given in Table 1. These are correlations for seal drag and heat transfer coefficient for the cold expansion volume and will be discussed later.

BASELINE COOLER SIMULATION

In order to verify that the software accurately simulates the actual split-cycle operation, software-generated data and hardware test data had to be compared. In this comparison, a test cooler was chosen as the baseline cooler, and the computer program was used to generate simulation data for a representative number of steady-state refrigeration loads.

The baseline cooler was simulated by inputting the program information defining the mechanical configuration of the cooler and specific operating conditions. The following two pieces of data remained to be determined for this simulation because they are not inherent in the program.

- The magnitude of the regenerator seal drag

- The heat transfer film coefficient between the gas in the cold expansion volume and the expansion volume wall.

These items were given special attention because they cannot yet be accurately defined and they have a significant effect on the simulation results. No accurate correlation has been defined that will yield the dynamic friction force characteristics of the expander seal. It was therefore assumed that the seal drag did not vary with velocity or with pressure drop across the seal. The seal drag was therefore estimated by measuring the force required to manually move the displacer at constant velocity within an ambient expander cylinder. This measured value may deviate from actual operational drag force. The convective film coefficient at the cold expansion volume is another parameter that is not accurately defined because it has not been possible to analytically define the extremely complex and variable flow patterns in the cold expansion volume nor is there enough test data available to develop an empirical relation. A best estimate of 1000 W/M^2-K was chosen as a constant for this factor.

PRESSURE VOLUME DIAGRAM COMPARISON

The validation of the pressure model was particularly detailed because of the availability of data from the test cooler. This data is in the form of a pressure-volume diagram. The linear motion of the displacer was monitored by a detector at the ambient spring volume, and the pressure was monitored at the compressor. The resulting plot pressure versus stroke position (a multiplying factor is all that is needed to determine the swept volume) is displayed on an oscilloscope. During data acquisition, it was noted that as the refrigerator expansion volume cooled down (the ambient temperature remained constant), the displacer stroke became shorter. This phenomena is shown in Figure 6 for two cold volume temperature levels (case 2 is 30K warmer than the minimum temperature case). Note that the test data (dashed line) shows a 30% decrease in stroke at its minimum temperature level. The computer program closely predicts this decrease. It is not surprising to see some deviations between the test and simulation data since the simulated motion of the displacer is handicapped by the need to assume seal frictional characteristics. The incorporation of a more realistic seal drag correlation (other than an assumed constant value) should provide an even better simulation of pressure and motion profiles.

NET CAPACITY COMPARISON

The net capacity test data available includes mean cooler performance as well as a range of performance results. A comparison of the cooler simulation data cooler plotted in Figure 7 with the range of cooler test data (also plotted) shows that the data from the simulation is slightly conservative. Since (as previously discussed) there are other parametric values that affect cooler performance (e.g., seal drag and cold head heat transfer correlation) that were not adjusted at this time, it is expected that correlations that predict the values of these parameters more accurately will provide predicted performance data that is consistent with the test data. The appraisal of the applicability of the correlations used during the program will require additional cooler test data that can be obtained only from a completely instrumented test cooler.

Although the values of the thermodynamic design parameters require some additional fine tuning, the program can simulate cooler thermodynamic performance with great accuracy. The data obtained during these simulations for the cold regenerator and for the free displacer already provides a valuable insight into how these components affect cooler performance.

Additional simulation data profiles are presented in Figures 8 through 10 for one thermodynamic cycle (360° of rotation of the compressor drive). Figure 8 shows the variation in its pressure and volume of the cold end with time. The area within the cold expansion volume indicator diagram represents the gross cooling capacity available during a single thermodynamic cycle. After it is reduced by the thermal losses of the expander, this capacity represents the net refrigeration capacity. This net capacity is defined by the total heat flux across the cold expansion volume boundaries; see Table 2.

Figure 9 shows the variations in pressure with time during a complete cycle at the cold and ambient expansion volumes and at the compression volume. Aerodynamic frictional losses are the cause of the difference in the pressures of these volumes. Figure 10 shows how temperature, mass flow, and volume (displacement) at the cold expansion volume vary with time.

CONCLUSIONS AND RECOMMENDATIONS

The Cold Displacer Computer Program is an analytical tool that realistically simulates the performance and characteristics of cryogenic split-Stirling-cycle coolers. This program has already provided valuable qualitative information about cooler performance. The mathematical model employed appears to be entirely adequate for the tasks outlined in the abstract. It affords an opportunity for defining the intricate interrelationships of the thermodynamic and thermophysical properties and hence provides a better understanding of the performance of the split-cycle cooler. Quantitatively, this program simulates a test cooler very well. Further testing and research will make it possible to increase the accuracy of the program by providing data on the correlations that as yet are not well defined or whose application to small cryogenic coolers is questionable. The accuracy of the program as well as the extensive generalized modeling equations (especially those for the regenerator model) will give a realistic indication of how performance responds to design changes and to variations in the values of critical parameters.

NOMENCLATURE

A_w -Cross-sectional area of wall

BDC -Bottom-dead-center

C -Cylinder circumference

C_p -Specific heat at constant pressure

C_v -Specific heat at constant volume

D_c -Diameter, cylinder

D_p -Diameter, sphere

D_t -Diameter, regenerator bed

F_f -Force, mechanical friction

F_p -Force, resultant

F_s -Force, seal drag

F_v -Force, vibration and/or shock

G -Conductance

K -Flow friction coefficient

k_{cy} -Thermal conductivity, cylinder

k_{eff} -Thermal conductivity, effective

k_g -Thermal conductivity, gas

k_s -Thermal conductivity, matrix

L_{cy} -Length, cylinder

m -Mass of gas

m_p -Mass of displacer

N -Cyclic speed

P -Pressure

Q -Heat Flow

R -Gas constant

Re_h -Reynold's number based on hydraulic diameter

Re_s -Reynold's number based on sphere diameter

S -Radial clearance

T -Temperature

TDC -Top-dead-center

U -Velocity

V -Volume

W -Work

Y -Stroke

Z -Compressibility factor

Greek Letters

ε -Porosity

ρ -Density

θ -Crank angle

General Subscripts

n -node

a -ambient

c -cold

cv -control volume

e -exit

i -into

w -wall

Table 1. Correlations used within the cold displacer computer program

Item	Equation	Source
Regenerator Effective Conductivity for Sphere Matrix (bed axial heat conduction)	$k_{eff} = k_g * 1.72 \left(\frac{k_s}{k_g}\right)^{0.26} + \frac{0.071\ (Re_s)^{0.69}}{\left(\frac{D_p}{D_t}\right)^{0.5} (1 - \varepsilon)^{0.19}}$	Reference 2
Regenerator Effective Conductivity for Screen Matrix	$k_{eff} = k_g\ 1.10 \left(\frac{k_s}{k_g}\right)^{0.25}$	Curve fit of Figure 11-7 Reference 2
Colburn Heat Transfer Modulus for Sphere Matrix	$j = 0.23\ Re_s^{-0.3}$	Curve fit of data in Reference 3
Colburn Heat Transfer Modulus for Screen Matrix	$j = 0.6 + Re_h^{-0.4}$	Curve fit of data in Reference 3
Darcy Friction Factor for Sphere Matrix	$f = 4 \left(0.43 + \frac{57.75}{Re}\right)^{0.97}$	Curve fit of data in Reference 3
Darcy Friction Factor for Screen Matrix	$f = 4 \left(0.4 + \frac{45}{Re}\right)$	Curve fit of data in Reference 3
Friction Loss of Perforated Header Plate	$\Delta P = \frac{A_D V_D Y}{A_f C} \frac{PD \left(1 - A_f A_D^2\right)}{2g}$	Reference 4 (see reference for nomenclature)
Pumping Heat Transfer Loss	$Q = \left[\frac{2\pi D_c L_{cy} S^3 (T_a - T_c)}{3.2kg}\right] * \left(\frac{(P_{max} - P_{min})\ C_p N}{R\ \frac{T_a + T_c}{2}}\right)^2$	Reference 5

Table 1 (Cont.)

Item	Equation	Source
Seal Frictional Heating Loss	$Q = 2\ Y\ N\ F_f$	Reference 5
Thermal Conduction Along Cylinders, Displacers and Other Structural Walls	$Q = \frac{k_{cy}\ A_w}{L_{cy}} (T_a - T_c)$ where $k_{cy} = \frac{1}{(T_a - T_c)} \int_{T_c}^{T_a} k_{cy}\ dT$	Reference 5
Shuttle Heat Transfer Loss	$Q = 0.186 * Y^2 \frac{K_g}{S} \frac{(T_a - T_c)}{L_{cy}}$	Reference 5

TABLE 2. RELATIVE CYCLIC ENERGY BALANCE FOR COLD EXPANSION VOLUME

WORK OF VOLUME CHANGE	(GROSS CAPACITY)	100%
THERMAL LOAD	(NET CAPACITY)	-36.6%
THERMAL LOSSES		
REGENERATOR INEFFICIENCY	35.0%	
CYLINDER CONDUCTION	8.7%	
DISPLACER CONDUCTION	0.6%	
SHUTTLE	8.9%	
PUMPING	10.2%	
SUM		-63.4%
CHANGE IN INTERNAL ENERGY FOR ONE COMPLETE CYCLE (STEADY-STATE ACHIEVED)		0.0%

*NUMBERS LISTED ARE FOR A SAMPLE SIMULATION OF THE TEST COOLER UNDER THE FOLLOWING CONDITIONS:
AMBIENT TEMPERATURE 22°C
MAXIMUM RATED HEAT LOAD
MOTOR SPEED 20Hz

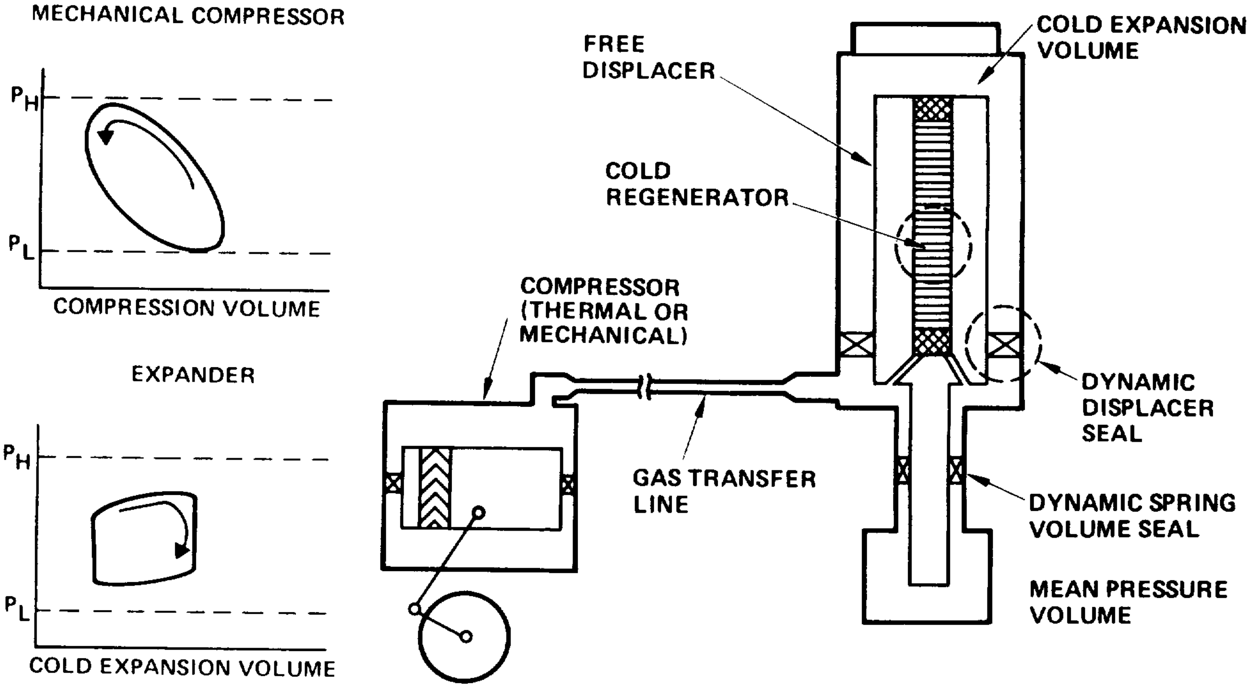

FIGURE 1. SCHEMATIC REPRESENTATION AND PRESSURE/VOLUME RELATIONSHIP OF SPLIT-CYCLE REFRIGERATOR

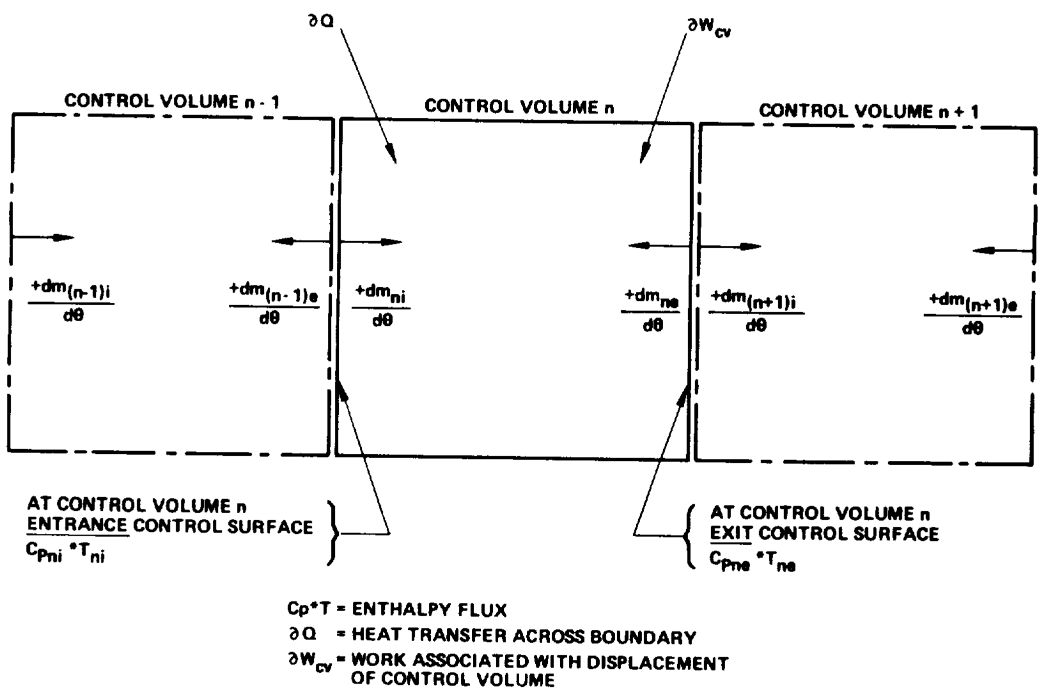

FIGURE 2. SIGN CONVENTION FOR THE ENERGY BALANCE OF A CONTROL VOLUME AND MASS FLOW CONVENTION FOR SEQUENTIAL CONTROL VOLUMES

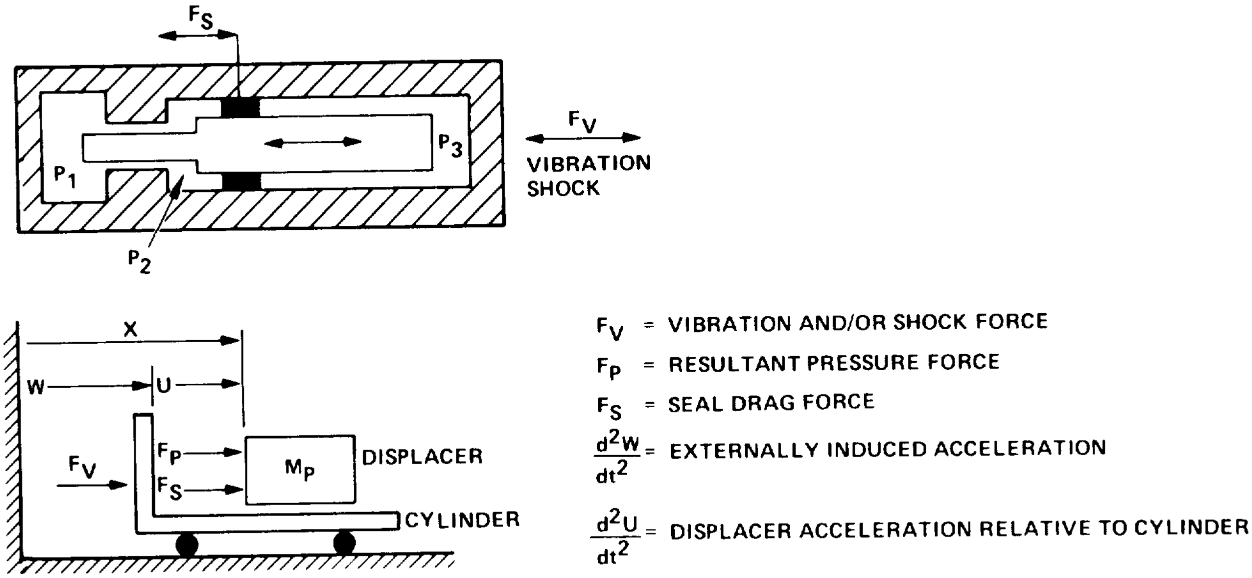

FIGURE 3. DYNAMIC MODEL OF THE FREE DISPLACER

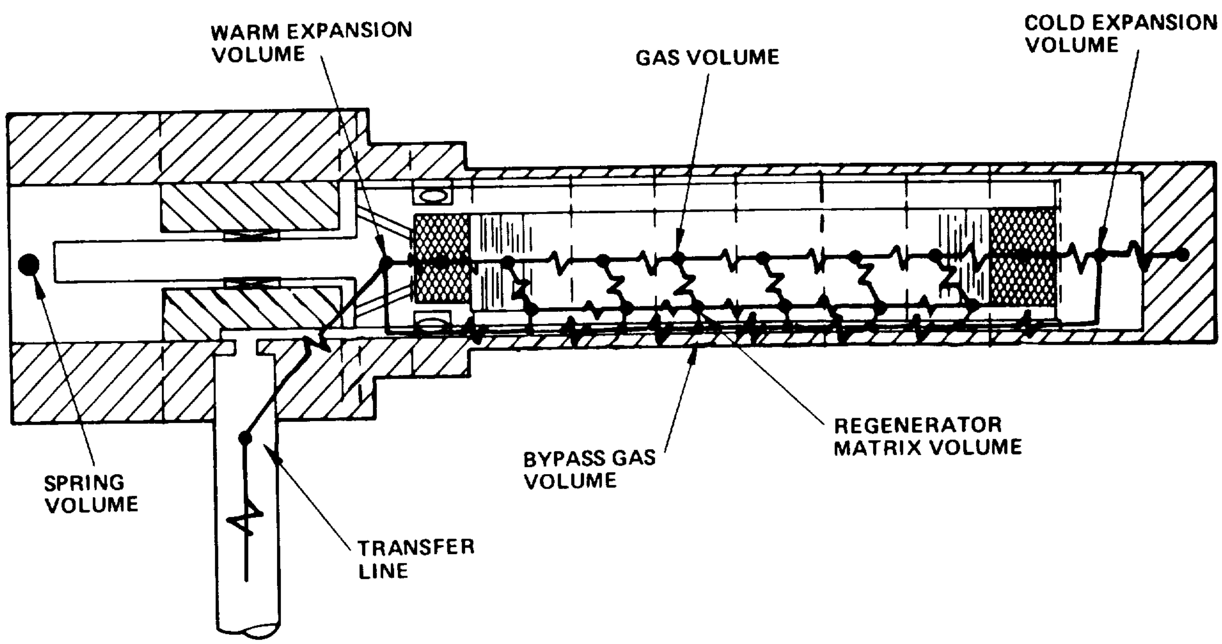

FIGURE 4. COLD HEAD NODAL NETWORK

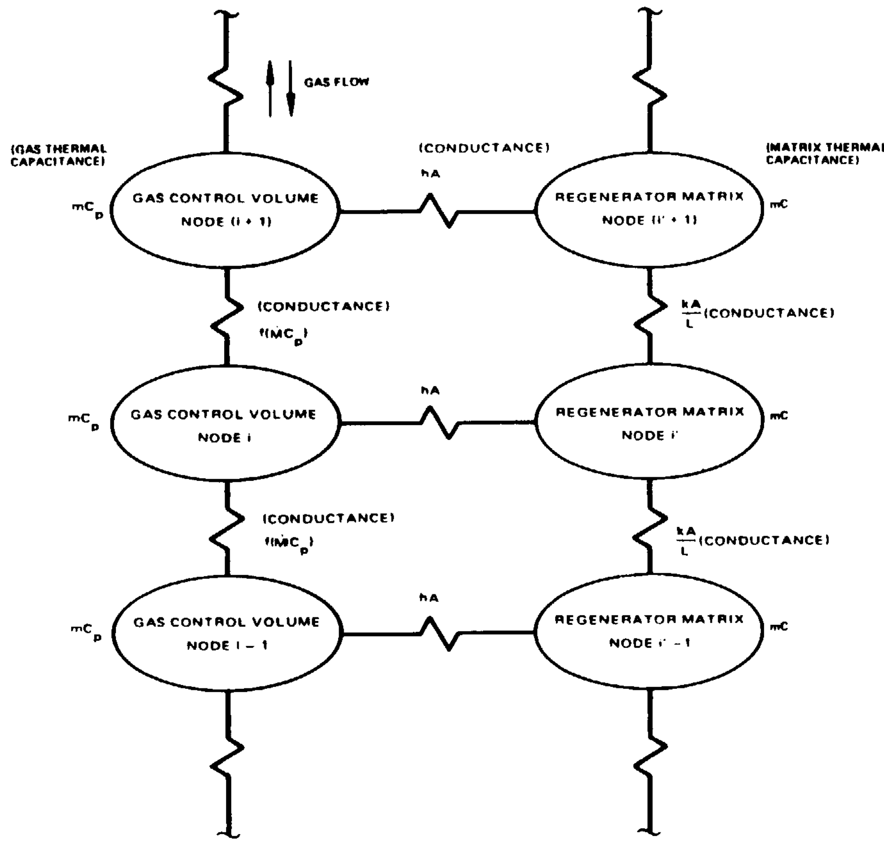

FIGURE 5. DETAILED NODAL NETWORK OF A TYPICAL REGENERATOR CONTROL VOLUME

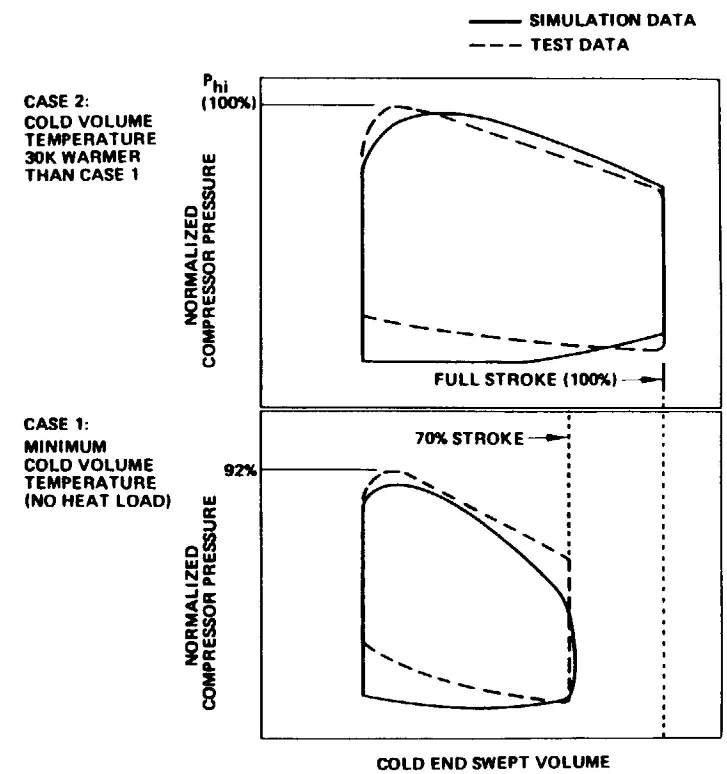

FIGURE 6. COMPARISON OF SIMULATION AND TEST DATA FOR VARIATION IN COLD VOLUME TEMPERATURE

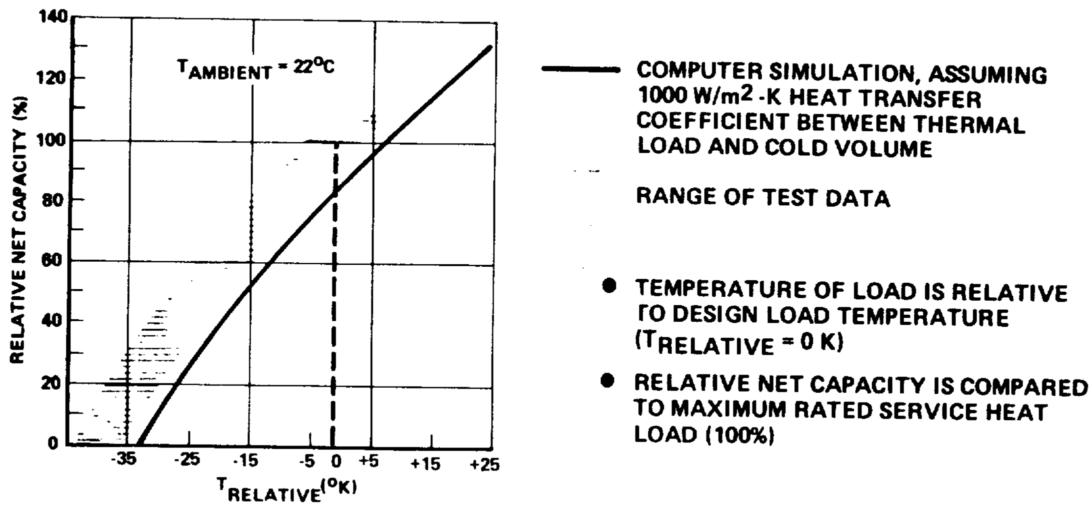

FIGURE 7. CORRELATION OF COMPUTER SIMULATIONS TO AVAILABLE DATA OF A SPLIT-CYCLE TEST COOLER

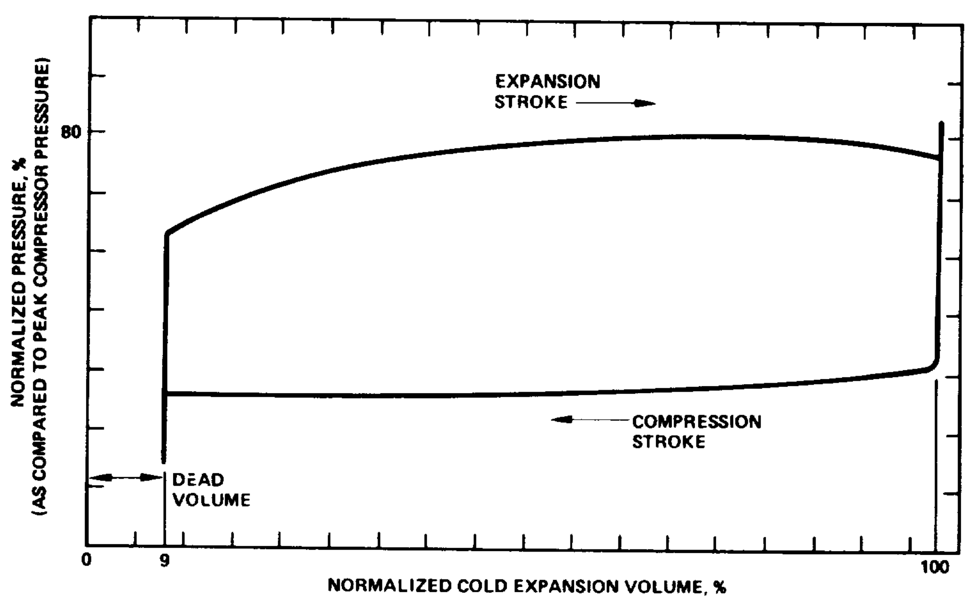

FIGURE 8. PRESSURE VERSUS VOLUME PLOT FOR THE COLD EXPANSION VOLUME

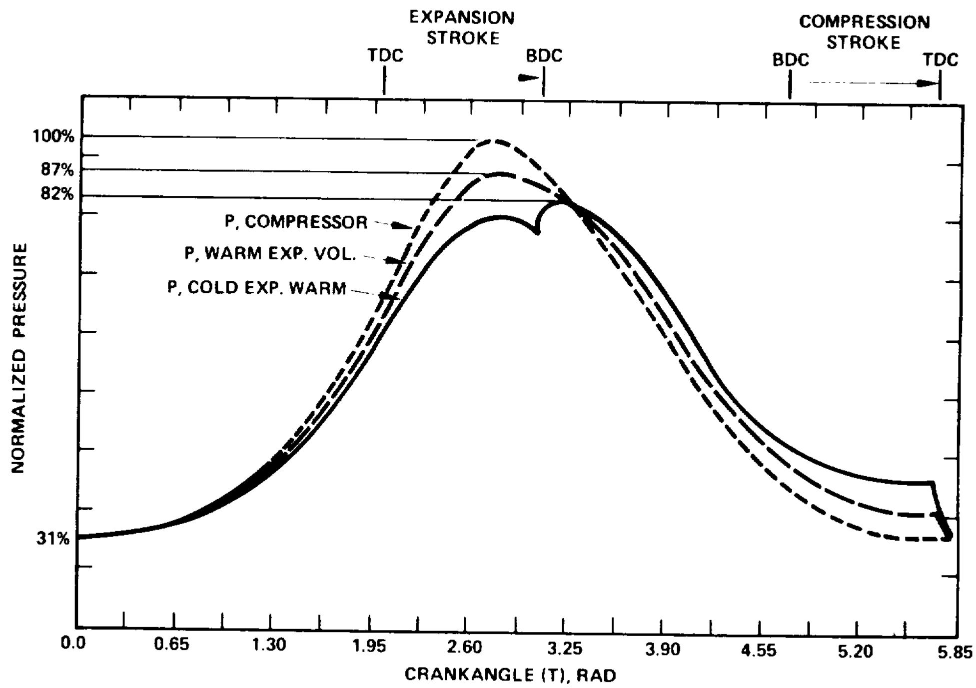

FIGURE 9. PRESSURE VERSUS TIME PLOTS FOR THE COMPRESSOR, AMBIENT EXPANSION VOLUME AND COLD EXPANSION VOLUME

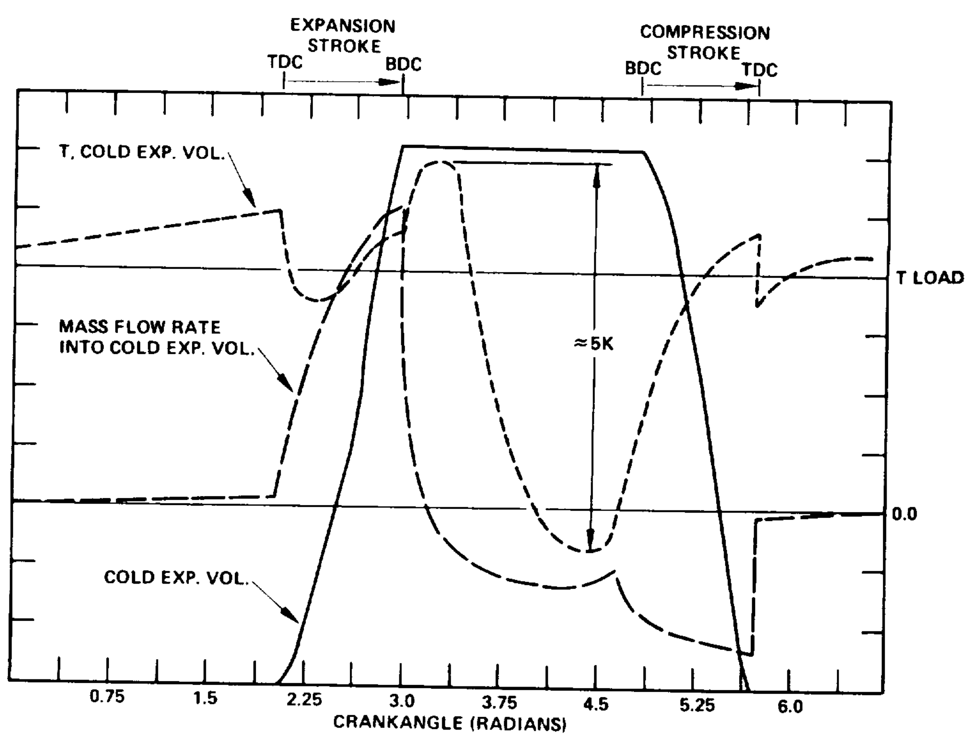

FIGURE 10. ENTRANCE MASS FLOW RATE, TEMPERATURE AND EXPANDER COLD WORKING VOLUME VERSUS TIME

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