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GENERALIZING THE TRAPRG AND TRAPAX FINITE ELEMENTS

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SUMMARY

The NASTRAN TRAPRG and TRAPAX finite elements are very restrictive as to shape and grid point numbering. The elements must be trapezoidal with two sides parallel to the radial axis. In addition, the ordering of the grid points on the element connection card must follow strict rules. The paper describes the generalization of these elements so that these restrictions no longer apply.

INTRODUCTION

Since NASTRAN's inception in the early 1970's, the axisymmetric trapezoidal ring element TRAPRG has been an accurate, efficient element used in solid axisymmetric problems. More recently, the TRAPAX element was introduced to handle non-axisymmetric loading for such structures. While these two elements usually perform very well, the restrictions imposed upon the user in the specification of the elements can be considered, at best, difficult, at worst, unreasonable, in light of today's automatic data generators. The restrictions require that the elements be trapezoidal, rather then generally quadrilateral, with the top and bottom edges parallel to the radial axis. Also, the specification of the grid points on the connection card must be given counterclockwise starting with the grid point with smaller radial coordinate of the two grid points with the smaller axial coordinate. This paper presents the reasons for these restrictions, how the restrictions have been removed, the imposition of a new, but less stringent restriction, and an example problem.

THEORY

The stiffness matrix for a finite element is usually represented as

$$[K] = \int_V [B]^T [D] [B] dv \qquad (1)$$

where [B] is the matrix of strain-displacement relations and [D] is the materials matrix describing the constitutive relations. The NASTRAN Theoretical Manual (ref. 1) defines [B] for TRAPRG and TRAPAX elements. The manual shows that, in order to evaluate the integral of Equation (1), the integrals

$$\int_A r^p z^q dr dz, \ p = -1, 0, 1, 2, 3 \text{ and } q = 0, 1, 2 \qquad (2)$$

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must be evaluated over the cross-sectional area of the element. The NASTRAN Programmer's Manual (ref. 2) describes how the integrals (2) are computed analytically. It is because of the nature of the expressions involved in the exact representation of these integrals that the original restrictions on the elements were imposed. The restrictions have been removed by replacing the exact integration with Gauss quadrature, as follows.

The three-point quadrature is given as

$$\int_A r^p z^q \, dr\, dz = \sum_{l=1}^{3} \sum_{m=1}^{3} r^p z^q(\xi_l, \eta_m)\, H_l H_m \, |J(\xi_l, \eta_m)| \qquad (3)$$

where

$$r = \sum_{i=1}^{4} N_i r_i$$

$$z = \sum_{i=1}^{4} N_i z_i$$

N_i = $1/4(1 + \xi\xi_i)(1 + \eta\eta_i)$, the linear isoparametric shape function over the square $\xi = -1$ to $+1$ and $\eta = -1$ to $+1$

r_i, z_i = r,z coordinates of the grid points at the four corners of the element

ξ_l, η_m = isoparametric coordinates at which $r^p z^q$ is evaluated in a three-point Gauss quadrature

H_l, H_m = quadrature weights corresponding to ξ_l, η_m

$|J(\xi_l, \eta_m)|$ = determinant of the Jacobian, $\begin{vmatrix} \frac{\partial r}{\partial \xi} & \frac{\partial z}{\partial \xi} \\ \frac{\partial r}{\partial \eta} & \frac{\partial z}{\partial \eta} \end{vmatrix}$, evaluated at (ξ_l, η_m)

With this formulation, the restrictions on the trapezoidal shape and grid point numbering can be removed.

There is, however, one situation which can cause numerical problems. Note that, in integrals (2), if $p = -1$ and $r \to 0$, then the integral $\to \infty$. Two cases can be examined. The first is the core element, a TRAPRG element with exactly two grid points with zero radius. (NASTRAN does not allow the TRAPAX element to be a core element since it does not allow a RINGAX ring to have a zero radius.) At these two points, the radial displacement must be zero. This condition can be assured as

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follows. If u and w are the radial and axial displacements, respectively, at some point within an element, then the assumed displacement functions are (ref. 1)

$$u = B_1 + B_2 r + B_3 z + B_4 rz \tag{4}$$

$$w = B_5 + B_6 r + B_7 z + B_8 rz \tag{5}$$

where the B_i are unknown coefficients. Then,

$$\begin{Bmatrix} u_1 \\ w_1 \\ \vdots \\ u_4 \\ w_4 \end{Bmatrix} = \begin{bmatrix} \\ H_{Bu} \\ \\ \end{bmatrix}^{-1} \begin{Bmatrix} B_1 \\ \vdots \\ B_8 \end{Bmatrix}$$

where u_i, w_i are the displacements of the ith grid point. If $r_i = 0$, then u_i must be 0, which means that the (2_i-1)th column of H_{Bu} can be zeroed. In addition, from Equation (4), $0 = u = B_1 + B_3 z$ for all z. Therefore, $0 = B_1 = B_3$, which can be assured by zeroing the first and third columns of H_{Bu}. However, the only terms of H_{Bu} which contain a form of (2) with $p = -1$ are in the first and third columns. Therefore, for a TRAPRG core element with $r_i = 0$, no numerical problems exist.

Now consider a second case - one where some r_i is small compared with some other r_i in the element. Comparisons between the analytical integration and the Gauss quadrature show that if

$$\frac{r_{max}}{r_{min}} > 10 \tag{7}$$

then the Gauss quadrature results begin to significantly degrade ($r_{max} = \max r_i$, $i = 1, \ldots, 4$). Therefore, inequality (7) is not allowed. This restriction should impose a hardship mainly when a TRAPAX core element is desired. This hardship however should require only an extra element or two to transition to larger radii.

SAMPLE PROBLEM

The sample problem for this work was a normal modes analysis of a thick-walled cylinder of inner radius 5.0 inches, outer radius 6.25 inches, and length 3.5 inches. The finite element model for the unmodified version of NASTRAN is shown in Figure 1a and, for the version which removes the restriction, in Figure 1b. Both models used the TRAPAX element. The results are shown in Table 1. The comparison of the results shows a degradation at the highest mode computed. However, this can be expected because of the severity of the non-uniform mesh.

REFERENCES

1. The NASTRAN Theoretical Manual (Level 17.5), NASA SP-221 (05), Dec. 1978.
2. The NASTRAN Programmer's Manual (Level 17.5), NASA SP-223 (05), Dec. 1978.

TABLE 1. COMPARATIVE RESULTS

MODE	NATURAL FREQUENCIES (CPS)	
	RESTRICTIVE ELEMENTS	GENERAL ELEMENTS
1	1090.003	1090.000
2	1660.250	1660.864
3	1760.034	1760.101
4	1988.622	1989.121
5	2071.862	2071.920
6	5407.311	5209.751

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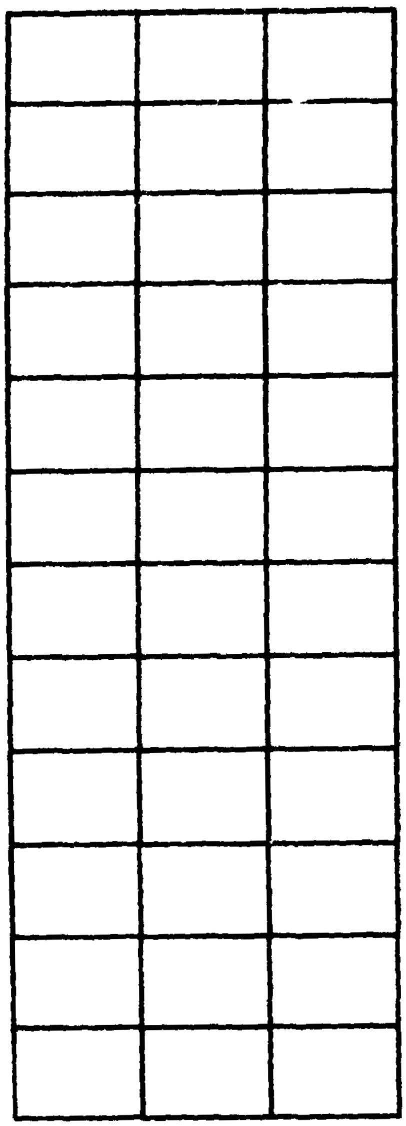

Figure 1a

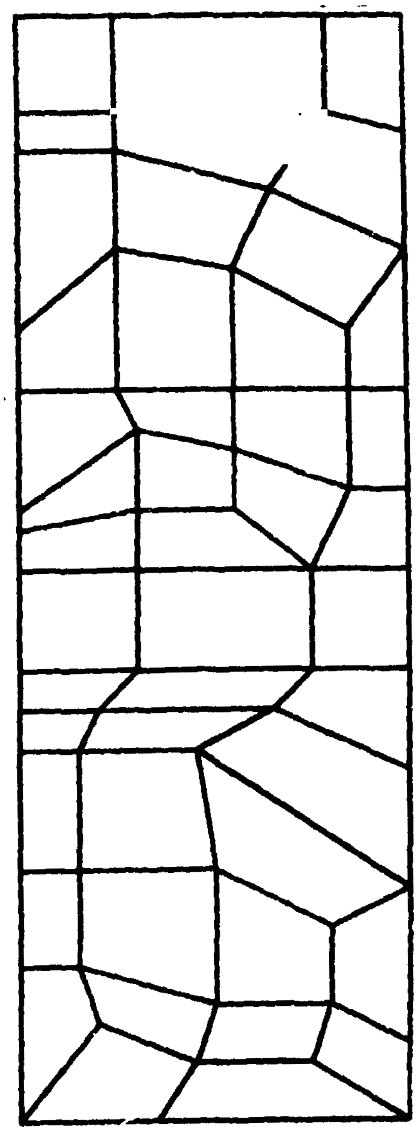

Figure 1b

Figure 1. Finite Element Models