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AN ALTERNATIVE METHOD OF ANALYSIS FOR BASE ACCELERATED DYNAMIC RESPONSE IN NASTRAN

V. Elchuri, A. Michael Gallo, G. C. C. Smith
Bell Aerospace Textron, Buffalo, New York

SUMMARY

An alternative method of analysis to determine the dynamic response of structures subjected to base accelerations is presented. The method is exact as opposed to the approximate technique of using unusually large masses and loads to enforce desired base accelerations. This paper presents the relevant equations of motion, ALTERs for direct and modal frequency-, random- and transient-response rigid formats, and illustrative examples.

INTRODUCTION

Dynamic environments of industrial structures and structural components are frequently specified in terms of base accelerations. The dynamics analysis capabilities of NASTRAN, however, provide for the specification of loads rather than accelerations. An approximate technique is therefore, generally practiced wherein the structural degrees of freedom with known (base) accelerations are assigned very large masses (or mass moments of inertias, of the order of 10^6 to 10^{12} times the total structural mass) and subjected to correspondingly large loads to enforce the desired base accelerations (ref. 1). This method can, in some instances, lead to erroneous results as shown by the frequency response function in Figure 1. The function represents $\sigma_{xx,1}$ in element 2 when the cantilevered plate, shown in Figure 2, is subjected to a unit base acceleration $\ddot{z}\ (f) = 1$. A concentrated mass of 10^8 units was used in the z (translation) degree of freedom at grid point 1. The solution was obtained using the direct frequency and random-response rigid format DISP RF 8. A viscous damping matrix proportional to the structural stiffness matrix, BDD = 2.0 E-6 KDD, was used enabling a direct comparison of the results with those from the alternative method.

The method discussed in the following sections avoids the use of fictitious large masses thereby eliminating any associated conditioning problems. For a modal formulation of the problem, the modal basis currently available in NASTRAN rigid formats is used. These modes are the base-relaxed modes including rigid body modes, and constitute a plausible basis for base excited dynamic response calculations. The prevailing boundary conditions are satisfied by all modes. The user directly specifies base accelerations on existing NASTRAN bulk data cards.

NOMENCLATURE

Note: A consistent set of FLT units has been used throughout this paper.

	Damping matrix, $FL^{-1}T$
	Viscous damping, $FL^{-1}T$
f	Frequency, T^{-1}
g	Modal damping coefficient
i	$\sqrt{-1}$
K	Stiffness matrix FL^{-1}
k	Stiffness, FL^{-1}
M	Mass matrix, $FL^{-1}T^2$
m	Mass, $FL^{-1}T^2$
P	Load vector, F
p	Derivative operator $\frac{d}{dt}$, T^{-1}
t	Time, T
u	Displacement vector, L
$\ddot{y},\ddot{z}$	Translational accelerations, LT^{-2}
ω	Circular frequency = $2\pi f$, T^{-1}
ω_n^i	Circular frequency of ith natural mode, T^{-1}
ϕ	Modal matrix
$\sigma_{xx,1}$	Normal stress component in basic × direction, z- fiber, FL^{-2}

Subscripts:

a, d, h, ℓ, p, r NASTRAN displacement sets

$D \equiv d\text{-}r$ set

METHOD OF ANALYSIS

Direct Formulation

After the application of constraints and partitioning to both structural and direct input matrices, the equation of forced motion is

$$\left[[M_{dd}]p^2 + [B_{dd}]p + [K_{dd}]\right]\{u_d\} = \{P_d\ (t \text{ or } \omega)\}\ . \tag{1}$$

The displacement vector u_d is partitioned as

$$\{u_d\} = \left\{\begin{array}{c} u_D \\ \hline u_r \end{array}\right\}\ , \tag{2}$$

with u_r representing the base accelerated degrees of freedom. Equation (1) can now be rearranged into the following two equations:

$$\left[\left[\begin{array}{c|c} M_{DD} & 0 \\ \hline 0 & M_{rr} \end{array}\right] p^2 + \left[\begin{array}{c|c} B_{DD} & B_{Dr} \\ \hline 0 & 0 \end{array}\right] p + \left[\begin{array}{c|c} K_{DD} & K_{Dr} \\ \hline 0 & 0 \end{array}\right]\right]\left\{\begin{array}{c} u_D \\ \hline u_r \end{array}\right\}$$

$$= \left\{\begin{array}{c} P_D - [M_{Dr}]p^2\ \{u_r\} \\ \hline [M_{rr}]p^2\ \{u_r\} \end{array}\right\}\ , \text{ and} \tag{3}$$

$$\left[[M_{rD} \mid M_{rr}]p^2 + [B_{rD} \mid B_{rr}]p + [K_{rD} \mid K_{rr}]\right]\left\{\begin{array}{c} u_D \\ \hline u_r \end{array}\right\} = \{P_r\}\ . \tag{4}$$

Given the base accelerations p^2u_r, equation (3) is solved for u_d, pu_d and p^2u_d. Equation (4), in turn, can be solved for P_r— the loads required on the base degrees of freedom to cause the desired base accelerations.

Modal Formulation

The displacement vector u_d is written as

$$\{u_d\} = \left\{\begin{array}{c} u_D \\ \hline u_r \end{array}\right\} = \left[\begin{array}{c} \phi_{Dh} \\ \hline \phi_{rh} \end{array}\right]\{u_h\} = [\phi_{dh}]\{u_h\}\ , \tag{5}$$

where the modal matrix ϕ_{dh} has been appropriately expanded to include any extra points (ref. 2). Substituting equation (5) in equation (3), and premultiplying both sides by ϕ_{dh}^T, the resulting equation of motion is

$$\left[[M_{hh}]p^2 + [B_{hh}]p + [K_{hh}]\right]\{u_h\} = \{P_h\}\ , \tag{6}$$

where

$$\left.\begin{aligned} [M_{hh}] &= [\phi_{dh}]^T \left[\begin{array}{c|c} M_{DD} & 0 \\ \hline 0 & M_{rr} \end{array}\right][\phi_{dh}]\ , \\ [B_{hh}] &= [B^1_{hh}] + [\phi_{dh}]^T \left[\begin{array}{c|c} B_{DD} & B_{Dr} \\ \hline 0 & 0 \end{array}\right][\phi_{dh}]\ , \\ [K_{hh}] &= [\phi_{dh}]^T \left[\begin{array}{c|c} K_{DD} & K_{Dr} \\ \hline 0 & 0 \end{array}\right][\phi_{dh}]\ , \text{ and} \end{aligned}\right\} \tag{7}$$

(continued)

$$\{P_h\} = [\phi_{dh}]^T \left\{\begin{array}{c} P_D - [M_{Dr}]p^2\{u_r\} \\ \hline [M_{rr}]p^2\{u_r\} \end{array}\right\} \text{(continued)} \tag{7}$$

Consistent with the existing capabilities of NASTRAN rigid formats for modal frequency- and random-, and transient-response analyses (DISP RFs 11 and 12), the modal damping matrix B_{hh} consists of contributions due to,

1. the damping matrix B^1_{hh} proportional to the mass matrix as

$$[B^1_{hh}] = [\ \omega_n^h\ g(\omega_n^h)\ ]\ [\phi_{dh}]^T \left[\begin{array}{c|c} M_{DD} & M_{Dr} \\ \hline 0 & 0 \end{array}\right] [\phi_{dh}] \ , \tag{8}$$

with the elements of the diagonal matrix corresponding to all extra points set to zero, and

2. the direct input matrix.

Given the base accelerations p^2u_r, equation (6) is solved for u_h, pu_h and p^2u_h. Equation (5) is used to recover the displacement vector u_d and its rates pu_d and p^2u_d. Equation (4) can be used to recover the loads P_r. The damping terms B_{rD} and B_{rr} are partitions of the directly specified damping matrix B2PP.

IMPLEMENTATION IN NASTRAN

The method of analysis discussed in the previous section has been implemented in NASTRAN April 1982 release in the form of DMAP ALTERs. The ALTER packages for the displacement approach rigid formats 8, 9, 11 and 12 are given in the Appendix. Existing NASTRAN utility modules have been used to partition and merge various matrices for the rearranged equation of motion. The functional module FRRD2 for the frequency response rigid formats has been modified to solve coupled equations of motion. These modifications are also included in the Appendix.

In using these ALTER packages, the following points are to be considered.

1. The base accelerated degrees of freedom are specified on the SUPORT bulk data card.

2. The base accelerations are specified on RLOADi or TLOADi bulk data cards akin to specifying loads.

3. The base accelerated degrees of freedom must have non-zero mass (or mass moment of inertia). No fictitious large masses are required.

4. In rigid formats 8 and 11, external loads can be applied to all (p-r) set degrees of freedom.

5. In rigid formats 9 and 12, external loads can be applied to all (d-r) set degrees of freedom.

6. An OLOAD request for the base accelerated (r set) degrees of freedom in RFs 8 and 11 results in the loads on these degrees of freedom necessary to cause the

specified base accelerations. Such a request in RFs 9 and 12 will output the specified base accelerations. This is due to the non-availability of pu_d and p^2u_d in distinct data blocks.

7. If OLOADs are requested for the base accelerated degrees of freedom in RF8 and 11, a real diagonal matrix named FREQ must be input on DMI bulk data cards with entries sorted in an ascending order from the FREQ or FREQi bulk data card. The order of the FREQ matrix must be equal to the entries on the FREQ or FREQi bulk data card.

8. Mode acceleration method of data recovery is available both in RF11 and RF12.

9. The data recovery procedures in all the four rigid formats remain unchanged, with the exception of stress recovery. The stresses are computed using displacements (or modal displacements) relative to the base. This is due to the fact that in problems with specified base accelerations, the absolute displacements can become extremely large. The subsequent stress calculations, as a result, are based on small differences of large numbers, and can be in error as shown by the stress response at very low frequencies in Figure 1. It is to be noted that any limitations imposed by these ALTER packages are as a direct result of utilizing existing functional and utility modules, with the necessary exception of FRRD2. These limitations can easily be overcome by creating new (dummy) modules.

ILLUSTRATIVE EXAMPLES

Two problems (Figures 2 and 3) are considered to illustrate the alternative method of determining base accelerated dynamic response. The problem in Figure 3 is used to illustrate the accuracy of all the four ALTER packages. The problem in Figure 2 is used to compare response calculations with those shown in Figure 1.

Figure 3 shows a 2-degree of freedom system subjected to a known base acceleration at mass m_1. The problem is to determine the acceleration response of mass m_2. The following steps are followed in obtaining and cross-checking the solution by various ways:

1. RF8 with ALTERs is used to determine $\ddot{y}_2(f)$ and $P_1(f)$, given $\ddot{y}_1(f)$.

2. RF8 without ALTERs is used to determine $\ddot{y}_1(f)$ and $\ddot{y}_2(f)$, given $P_1(f)$.

3. Steps 1 and 2 are repeated with RF11.

4. RF9 with ALTERs is used to determine $\ddot{y}_2(t)$, given $\ddot{y}_1(t)$.

5. Step 4 is repeated with RF12.

Figures 4 and 5 show the results of steps 1 through 3 which compare well with the theoretical results given by

$$\ddot{y}_2(\omega) = \left[\frac{k + i\omega c}{k + i\omega c - \omega^2 m_2}\right] \ddot{y}_1(\omega) \quad ,\text{and} \quad P_1(\omega) = m_1\ddot{y}_1(\omega) + m_2\ddot{y}_2(\omega) \quad . \qquad (9)$$

Figure [illegible] of steps 4 and 5 with theory. The

$$\left.\begin{aligned}
&\ddot{y}_2(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \cos\left(\sqrt{1-\zeta^2}\,\omega_n t + \gamma_o'\right) , \\
\text{where}\quad &\zeta = c \,/\, (2\sqrt{km_2}) , \\
&\omega_n = \sqrt{k/m_2} \quad , \text{ and} \\
&\gamma_o' = \tan^{-1}\left(\frac{\zeta}{\sqrt{1-\zeta^2}}\right) , \\
\text{for}\quad &\ddot{y}_1(t) = 0 , \; t < 0 \\
&\qquad\;\; = 1 , \; t > 0 .
\end{aligned}\right\} \quad (10)$$

Figure 7 presents the frequency response function of Figure 1, using RF8 with base acceleration ALTERs. For comparison, a damping matrix proportional to the structural stiffness matrix was used in both solutions (BDD = 2.0 E-6 KDD). The response does not rely on the selection of any fictitious large masses. The error in the stress response at low frequencies is also eliminated.

CONCLUDING REMARKS

1. An alternative method of determining base accelerated dynamic response in NASTRAN has been presented and demonstrated, avoiding the use of fictitious large masses.

2. Although this paper discusses the problem and its solution in terms of absolute degrees of freedom, a number of variations can be simply achieved to suit particular problems. As an example, the introduction of the degrees of freedom relative to the base degrees of freedom, at least in the absence of extra points, can be easily accomplished by the transformation

$$\left.\begin{aligned}
&\{u_d\} \equiv \{u_a\} = \left\{\begin{array}{c} u_\ell \\ \hdashline u_r \end{array}\right\} = \left[\begin{array}{c:c} I & D \\ \hdashline 0 & I \end{array}\right] \left\{\begin{array}{c} u_{\ell,\text{rel.}} \\ \hdashline u_r \end{array}\right\} , \quad \text{(set e null)} \\
\text{with}\quad &[D] = -[K_{\ell\ell}]^{-1}\,[K_{\ell r}] .
\end{aligned}\right\} \quad (11)$$

This is useful in addressing shock spectrum response problems, and can lead to symmetric coefficient matrices and uncoupled (modal) equations of motion.

4. The method can form the basis for considering displacement and velocity base excitation problems in NASTRAN.

REFERENCES

1. MSC/NASTRAN Application Manual, Vol. I, January 1981.

2. NASTRAN Theoretical Manual, NASA SP-221(05), December 1978.

3. Myklestad, N. O., Fundamentals of Vibration Analysis, McGraw-Hill Book Company, Inc., 1956.

APPENDIX

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$$
$$ ALTERS TO SOL 8 (NASTRAN APRIL 1982) FOR BASE ACCELERATION.
$$
ALTER 87 $
RBMG1     USET,KAA, / KLL,KLR,KRR,,, $
RBMG2     KLL / LLL $
RBMG3     LLL,KLR,KRR / DM $
ALTER 121,122 $
FRLG      CASEXX,USETD,DLT,FRL,GMD,GOD,DIT,/PPF,PSF PDF,FOL,PHF/
          *DIRECT*/FREQY/*FREQ* $
EQUIV     PPF,PDF/NOSET $
VEC       USETD/VP/*P*/*COMP*/*R* $
VEC       USETD/VD/*D*/*COMP*/*R* $
PARTN     PPF,,VP/PPF1,PPF2,,/1 $
PARTN     PDF,,VD/PDF1,PDF2,,/1 $
PARTN     MDD,VD,/MDD11,MDD21,MDD12,MDD22 $
PARTN     BDD,VD,/BDD11,BDD21,BDD12,BDD22 $
PARTN     KDD,VD,/KDD11,KDD21,KDD12,KDD22 $
MERGE     MDD11,,,MDD22,VD,/ MDD1 /-1/0/1 $
MERGE     BDD11,,BDD12,,VD,/ BDD1 /-1/0/1 $
MERGE     KDD11,,KDD12,,VD,/ KDD1 /-1/0/1 $
MPYAD     MDD12,PPF2,PDF1/ PDFF1 /0/-1 $
MPYAD     MDD22,PPF2,     / PDFF2 /0 $
MERGE     PDFF1,PDFF2,,,,VD/ PDFF /1/0/2 $
FRRD2     KDD1,BDD1,MDD1,,PDFF,FOL/UDVF/0.0/0.0/-1.0 $
ADD       FREQ,/OMEGAI/(0.0,6.283185) $
ADD       FREQ,/OMEGA1/(6.283185,0.0) $
DIAGONAL  OMEGA1/OMEGA2/*SQUARE*/2.0 $
MERGE     MDD21,,MDD22,,VD, / MDD2 /1/0/2 $
MERGE     BDD21,,BDD22,,VD, / BDD2 /1/0/2 $
MERGE     KDD21,,KDD22,,VD, / KDD2 /1/0/2 $
SMPYAD    MDD2,UDVF,OMEGA2,,, / TEMP1 / 3/-1 $
SMPYAD    BDD2,UDVF,OMEGAI,,, / TEMP2 / 3 $
ADD       TEMP1,TEMP2 / TEMP3 $
MPYAD     KDD2,UDVF,TEMP3 / PPF2/ 0 $
MERGE     PPF1,PPF2,,,,VP / PPF / 1/0/2 $
EQUIV     PPF,PNEF/MPCF1 $
COND      LBL18A,MPCF1 $
VEC       USETD/VECNEM/*P*/*NE*/*M* $
PARTN     PPF,,VECNEM/PBNEF,PMF,, / 1 $
MPYAD     GMD,PMF,PBNEF/ PNEF / 1 $
LABEL     LBL18A $
EQUIV     PNEF,PFEF/SINGLE $
COND      LBL18B,SINGLE $
VEC       USETD/VECFES/*NE*/*FE*/*S* $
```

```
PARTN     PNEF,,VECFES/PFEF,PSF,, / 1 $
LABEL     LBL18B $
EQUIV     PFEF,PDF/OMIT $
COND      LBL18C,OMIT $
VEC       USETD/VECDO/*FE*/*D*/*O* $
PARTN     PFEF,,VECDO/PBDF,POF,, / 1 $
MPYAD     GOD,POF,PBDF/ PDF/ 1 $
LABEL     LBL18C $
ALTER 123,123 $ USE FOL INSTEAD OF PPF
VDR       CASEXX,EQDYN,USETD,UDVF,FOL,XYCDB,/OUDVC1,/*FREQRESP*/
          *DIRECT*/S,N,NOSORT2/S,N,NOD/S,N,NOP/0 $
ALTER 139,139 $ USE FOL INSTEAD OF PPF
SDR2      CASEXX,CSTM,MPT,DIT,EQDYN,SILD,,,BGPDP,FOL,QPC,UPVC,EST,XYCDB,
          PPF/OPPC1,OQPC1,OUPVC1,,OEFC1,PUPVC1/*FREQRESP*/
          S,N,NOSORT2 $
VEC       USETD / VD2 / *D*/*A*/*E* $
VEC       USETD / VA2 / *A*/*L*/*R* $
PARTN     UDVF,,VD2 / UAVF,UEVF,, / 1 $
PARTN     UAVF,,VA2 / ULVF,URVF,, / 1 $
MPYAD     DM,URVF,ULVF / ULVFP / 0 / -1 $
UMERGE    USETD,ULVFP, / UAVFP / *A*/*L*/*R* $
UMERGE    USETD,UAVFP,UEVF / UDVFP / *D*/*A*/*E* $
EQUIV     UDVFP,UPVCP/NOA $
COND      LBL19A,NOA $
SDR1      USETD,,UDVFP,,,GOD,GMD,,,, / UPVCP,, / 1/*DYNAMICS* $
LABEL     LBL19A $
SDR2      CASEXX,CSTM,MPT,DIT,EQDYN,SILD,,,BGPDP,FOL,,UPVCP,EST,
          XYCDB,/,,,OESC1,, /*FREQRESP*/S,N,NOSORT2 $
ENDALTER
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$$ ALTERS TO SOL 9 (NASTRAN APRIL 1982) FOR BASE ACCELERATION.
$$
ALTER 87 $
RBMG1     USET,KAA, / KLL,KLR,KRR,,, $
RBMG2     KLL / LLL $
RBMG3     LLL,KLR,KRR / DM $
ALTER 125,125 $
VEC       USETD/VD/*D*/*COMP*/*R* $
PARTN     PD,,VD/PD1,PD2,,/1 $
PARTN     MDD,VD,/MDD11,MDD21,MDD12,MDD22 $
PARTN     BDD,VD,/BDD11,BDD21,BDD12,BDD22 $
PARTN     KDD,VD,/KDD11,KDD21,KDD12,KDD22 $
MERGE     MDD11,,,MDD22,VD,/ MDD1 /-1/0/1 $
MERGE     BDD11,,BDD12,,VD,/ BDD1 /-1/0/1 $
MERGE     KDD11,,KDD12,,VD,/ KDD1 /-1/0/1 $
MPYAD     MDD12,PD2,PD1/ PDD1 /0/-1 $
MPYAD     MDD22,PD2,    / PDD2 /0 $
MERGE     PDD1,PDD2,,,,VD/ PDD /1/0/2 $
TRD       CASEXX,TRL,NLFT,DIT,KDD1,BDD1,MDD1,PDD/
          UDVT,PNLD / *DIRECT*/NOUE/1/S,N,NCOL/C,Y,ISTART $
ALTER 139,139 $
SDR2      CASEXX,CSTM,MPT,DIT,EQDYN,SILD,,,BGPDP,TOL,QP,UPV,EST,
          XYCDB,PPT/OPP1,OQP1,OUPV1,,OEF1,PUGV/*TRANRESP* $
VEC       USETD / VD2 / *D*/*A*/*E* $
VEC       USETD / VA2 / *A*/*L*/*R* $
PARTN     UDVT,,VD2 / UAVT,UEVT,, / 1 $
PARTN     UAVT,,VA2 / ULVT,URVT,, / 1 $
MPYAD     DM,URVT,ULVT / ULVTP / 0 / -1 $
UMERGE    USETD,ULVTP, / UAVTP / *A*/*L*/*R* $
UMERGE    USETD,UAVTP,UEVT / UDVTP / *D*/*A*/*E* $
EQUIV     UDVTP,UPVTP/NOA $
COND      LBL19A,NOA $
SDR1      USETD,,UDVTP,,,GOD,GMD,,,, / UPVTP,, / 1/*DYNAMICS* $
LABEL     LBL19A $
SDR2      CASEXX,CSTM,MPT,DIT,EQDYN,SILD,,,BGPDP,TOL,,UPVTP,EST,
          XYCDB,/,,,OES1,, /*TRANRESP* $
ENDALTER
```

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$$
$$ ALTERS TO SOL 11 (NASTRAN APRIL 1982) FOR BASE ACCELERATION.
$$
ALTER 93 $
PARAM     //*ADD*/KDEK2/NOGENL/NOSIMP $
ALTER 95,96 $
PARAM     //*ADD*/NOBGG/-1/0 $
PARAM     //*ADD*/NOK4GG/-1/0 $
GKAD      USETD,GM,GO,KAA,,MAA,,K2PP,M2PP,B2PP/
          KDD,BDD,MDD,GMD,GOD,K2DD,M2DD,B2DD/*FREQRESP*/*DISP*/
          *DIRECT*/0.0/0.0/0.0/NOK2PP/NOM2PP/NOB2PP/MPCF1/SINGLE/
          OMIT/NOUE/NOK4GG/NOBGG/KDEK2/V,Y,MODACC=-1 $
EQUIV     B2DD,BDD/NOBGG/M2DD,MDD/NOSIMP/K2DD,KDD/KDEK2 $
FRLG      CASEXX,USETD,DLT,FRL,GMD,GOD,DIT,/PPF,PSF,PDF,FOL,PHF/
          *DIRECT*/FREQY/*FREQ* $
EQUIV     PPF,PDF/NOSET $
VEC       USETD/VP/*P*/*COMP*/*R* $
VEC       USETD/VD/*D*/*COMP*/*R* $
PARTN     PPF,,VP/PPF1,PPF2,,/1 $
PARTN     PDF,,VD/PDF1,PDF2,,/1 $
PARTN     MDD,VD,/MDD11,MDD21,MDD12,MDD22 $
PARTN     BDD,VD,/BDD11,BDD21,BDD12,BDD22 $
PARTN     KDD,VD,/KDD11,KDD21,KDD12,KDD22 $
MERGE     MDD11,,,MDD22,VD,/ MDD1 /-1/0/1 $
MERGE     BDD11,,BDD12,,VD,/ BDD1 /-1/0/1 $
MERGE     KDD11,,KDD12,,VD,/ KDD1 /-1/0/1 $
MPYAD     MDD12,PPF2,PDF1/ PDFF1 /0/-1 $
MPYAD     MDD22,PPF2,     / PDFF2 /0 $
MERGE     PDFF1,PDFF2,,,,VD/ PDFF /1/0/2 $
GKAM      USETD,PHIA,MI,LAMA,DIT,,,,CASEXX/
          MXHH,BXHH,KXHH,PHIDH/NOUE/C,Y,LMODES=0/
          C,Y,LFREQ=0.0/C,Y,HFREQ=-1.0/-1/-1/-1/
          S,N,NONCUP/S,N,FMODE $
MPYAD     PHIDH,PDFF,/PHF/1/1/0/0 $
SMPYAD    PHIDH,MDD1,PHIDH,,,/MHH/3/1/1/0/1 $
SMPYAD    PHIDH,KDD1,PHIDH,,,/KHH/3/1/1/0/1 $
PURGE     B2HH/NOB2PP $
COND      LBL13A,NOB2PP $
SMPYAD    PHIDH,BDD1,PHIDH,,,/B2HH/3/1/1/0/1 $
LABEL     LBL13A $
EQUIV     MAA,M1DD/NOUE $
COND      LBL13B,NOUE $
VEC       USETD/VD1/*D*/*A*/*E* $
MERGE     MAA,,,,VD1,/ M1DD /-1/0/1 $
LABEL     LBL13B $
PARTN     M1DD,VD,/M1DD11,,M1DD12, $
MERGE     M1DD11,,M1DD12,,VD,/ M1DD2 /-1/0/1 $
SMPYAD    PHIDH,M1DD2,PHIDH,,,/M1HH2/3/1/1/0/1 $
DIAGONAL  MXHH/MINV/*SQUARE*/-1.0 $
```

```
SMPYAD   MINV,BXHH,M1HH2,,, / TEMP1 / 3 $
ADD      TEMP1,B2HH / BHH $
PARAM    //*ADD*/NONCUP/1/0 $
ALTER    99,100 $
FRRD2    KHH,BHH,MHH,,PHF,FOL/UHVF/0.0/0.0/-1.0 $
DDR1     UHVF,PHIDH/UDVF $
ADD      FREQ,/OMEGAI/(0.0,6.283185) $
ADD      FREQ,/OMEGA1/(6.283185,0.0) $
DIAGONAL OMEGA1/OMEGA2/*SQUARE*/2.0 $
MERGE    MDD21,,MDD22,,VD, / MDD2 /1/0/2 $
MERGE    BDD21,,BDD22,,VD, / BDD2 /1/0/2 $
MERGE    KDD21,,KDD22,,VD, / KDD2 /1/0/2 $
SMPYAD   MDD2,UDVF,OMEGA2,,, / TEMP2 / 3/-1 $
SMPYAD   BDD2,UDVF,OMEGAI,,, / TEMP3 / 3 $
ADD      TEMP2,TEMP3 / TEMP4 $
MPYAD    KDD2,UDVF,TEMP4 / PPF2 / 0 $
MERGE    PPF1,PPF2,,,,VP / PPF / 1/0/2 $
EQUIV    PPF,PNEF/MPCF1 $
COND     LBL15A,MPCF1 $
VEC      USETD/VECNEM/*P*/*NE*/*M* $
PARTN    PPF,,VECNEM/PBNEF,PMF,, / 1 $
MPYAD    GMD,PMF,PBNEF/ PNEF / 1 $
LABEL    LBL15A $
EQUIV    PNEF,PFEF/SINGLE $
COND     LBL15B,SINGLE $
VEC      USETD/VECFES/*NE*/*FE*/*S* $
PARTN    PNEF,,VECFES/PFEF,PSF,, / 1 $
LABEL    LBL15B $
EQUIV    PFEF,PDF/OMIT $
COND     LBL15C,OMIT $
VEC      USETD/VECDO/*FE*/*D*/*O* $
PARTN    PFEF,,VECDO/PBDF,POF,, / 1 $
MPYAD    GOD,POF,PBDF/ PDF/ 1 $
LABEL    LBL15C $
ALTER 101,101 $ USE FOL INSTEAD OF PPF
VDR      CASEXX,EQDYN,USETD,UHVF,FOL,XYCDB,/OUHVC1,/*FREQRESP*/
         *MODAL*/S,N,NOSORT2/S,N,NOH/S,N,NOP/FMODE $
ALTER 116,116 $ USE FOL INSTEAD OF PPF
DDR2     USETD,UDV1F,PDF,K2DD,B2DD,MDD,FOL,LLL,DM/
         UDV2F,UEVF,PAF/*FREQRESP*/NOUE/REACT/FRQSET $
ALTER 122,122 $ USE FOL INSTEAD OF PPF
SDR2     CASEXX,CSTM,MPT,DIT,EQDYN,SILD,,,BGPDP,FOL,QPC,UPVC,EST,
         XYCDB,PPF/OPPC1,OQPC1,OUPVC1,OESC1,OEFC1,PUGV/*FREQ*/
         S,N,NOSORT2 $
ALTER 128,128 $
SDR2     CASEXX,CSTM,MPT,DIT,EQDYN,SILD,,,,LAMA,QPH,PHIPH,EST,XYCDB,/
         ,IQP1,IPHIP1,,IEF1,/*MMREIG*/S,N,NOSORT2 $
ALTER 129,129 $ USE FOL INSTEAD OF PPF
SDR2     CASEXX,,,,EQDYN,SILD,,,,FOL,,,,XYCDB,PPF/
         OPPCA,,,,,/*FREQ* $
VEC      USETD / VD2 / *D*/*A*/*E* $
```

```
VEC       USETD / VA2 / *A*/*L*/*R* $
PARTN     PHIDH,,VD2 / PHAVF,PHEVF,, / 1 $
PARTN     PHAVF,,VA2 / PHLVF,PHRVF,, / 1 $
MPYAD     DM,PHRVF,PHLVF / PHLVFP / 0 / -1 $
UMERGE    USETD,PHLVFP, / PHAVFP / *A*/*L*/*R* $
UMERGE    USETD,PHAVFP,PHEVF / PHDVFP / *D*/*A*/*E* $
SDR1      USETD,,PHDVFP,,,GOD,GMD,,,, / PHIPHF,, / 1/*DYNAMICS* $
SDR2      CASEXX,CSTM,MPT,DIT,EQDYN,SILD,,,,LAMA,,PHIPHF,EST,
          XYCDB,/,,,IES1,, /*MMREIG*/S,N,NOSORT2 $
ALTER 134,134 $ USE FOL INSTEAD OF PPF
DDRMM     CASEXX,UHVF,FOL,IPHIP2,IQP2,IES2,1EF2,XYCDB,EST,MPT,DIT/
          ZUPVC2,ZQPC2,ZESC2,ZEFC2, $
ALTER 138,138 $ USE FOL INSTEAD OF PPF
DDRMM     CASEXX,UHVF,FOL,IPHIP1,IQP1,IES1,IEF1,,EST,MPT,DIT/
          ZUPVC1,ZQPC1,ZESC1,ZEFC1, $
ENDALTER
```

```
$$
$$ ALTERS TO SOL 12 (NASTRAN APRIL 1982) FOR BASE ACCELERATION.
$$
ALTER 88 $
PARAM    //*ADD*/KDEKA/NOUE/NOK2PP $
PARAM    //*ADD*/KDEK2/NOGENL/NOSIMP $
EQUIV    KAA,KDD/KDEKA $
ALTER 90,91 $
PARAM    //*ADD*/NOBGG/-1/0 $
PARAM    //*ADD*/NOK4GG/-1/0 $
GKAD     USETD,GM,GO,KAA,,MAA,,K2PP,M2PP,B2PP/
         KDD,BDD,MDD,GMD,GOD,K2DD,M2DD,B2DD/*TRANRESP*/*DISP*/
         *DIRECT*/C,Y,G=0.0/C,Y,W3=0.0/0.0/NOK2PP/NOM2PP/NOB2PP/
         MPCF1/SINGLE/OMIT/NOUE/NOK4GG/NOBGG/KDEK2/V,Y,MODACC=-1 $
EQUIV    B2DD,BDD/NOGPDT/M2DD,MDD/NOSIMP/K2DD,KDD/KDEK2 $
GKAM     USETD,PHIA,MI,LAMA,DIT,,,,CASECC/
         MXHH,BXHH,KXHH,PHIDH/NOUE/C,Y,LMODES=0/
         C,Y,LFREQ=0.0/C,Y,HFREQ=-1.0/-1/-1/-1/
         S,N,NONCUP/S,N,FMODE $
VEC      USETD/VD/*D*/*COMP*/*R* $
EQUIV    MAA,M1DD/NOUE $
COND     LBL12,NOUE $
VEC      USETD/VD1/*D*/*A*/*E* $
MERGE    MAA,,,,VD1,/ M1DD /-1/0/1 $
LABEL    LBL12 $
PARTN    M1DD,VD,/M1DD11,,M1DD12, $
MERGE    M1DD11,,M1DD12,,VD,/ M1DD2 /-1/0/1 $
SMPYAD   PHIDH,M1DD2,PHIDH,,,/M1HH2/3/1/1/0/1 $
DIAGONAL MXHH/MINV/*SQUARE*/-1.0 $
SMPYAD   MINV,BXHH,M1HH2,,,/B1HH/3 $
ALTER 99,99 $
TRLG     CASEXX,USETD,DLT,SLT,BGPDT,SIL,CSTM,TRL,DIT,GMD,GOD,,EST,
         MGG/PPT,PST,PDT,PD,,TOL / S,N,NOSET/NCOL $
ALTER 101,101 $
PARTN    PD,,VD/PD1,PD2,,/1 $
PARTN    MDD,VD,/MDD11,MDD21,MDD12,MDD22 $
PARTN    BDD,VD,/BDD11,BDD21,BDD12,BDD22 $
PARTN    KDD,VD,/KDD11,KDD21,KDD12,KDD22 $
MERGE    MDD11,,,MDD22,VD,/ MDD1 /-1/0/1 $
MERGE    BDD11,,BDD12,,VD,/ BDD1 /-1/0/1 $
MERGE    KDD11,,KDD12,,VD,/ KDD1 /-1/0/1 $
MPYAD    MDD12,PD2,PD1/ PDD1 /0/-1 $
MPYAD    MDD22,PD2,    / PDD2 /0 $
MERGE    PDD1,PDD2,,,,VD/ PDD /1/0/2 $
MPYAD    PHIDH,PDD,/PH/1/1/0/0 $
SMPYAD   PHIDH,MDD1,PHIDH,,,/MHH/3/1/1/0/1 $
SMPYAD   PHIDH,KDD1,PHIDH,,,/KHH/3/1/1/0/1 $
EQUIV    B1HH,BHH/NOB2PP $
COND     LBL13A,NOB2PP $
SMPYAD   PHIDH,BDD1,PHIDH,,,B1HH/BHH/3/1/1/0/1 $
```

```
LABEL     LBL13A $
PARAM     //*ADD*/NONCUP/1/0 $
TRD       CASEXX,TRL,NLFT,DIT,KHH,BHH,MHH,PH/UHVT,PNLH/
          *MODAL*/NOUE/NONCUP/S,N,NCOL/C,Y,ISTART $
ALTER 128,128 $
SDR2      CASEXX,CSTM,MPT,DIT,EQDYN,SILD,,,,LAMA,QPH,PHIPH,EST,
          XYCDB,/,IQP1,IPHIP1,,IEF1,/*MMREIG* $
ALTER 129 $
VEC       USETD / VD2 / *D*/*A*/*E* $
VEC       USETD / VA2 / *A*/*L*/*R* $
PARTN     PHIDH,,VD2 / PHAVT,PHEVT,, / 1 $
PARTN     PHAVT,,VA2 / PHLVT,PHRVT,, / 1 $
MPYAD     DM,PHRVT,PHLVT / PHLVTP / 0 / -1 $
UMERGE    USETD,PHLVTP, / PHAVTP / *A*/*L*/*R* $
UMERGE    USETD,PHAVTP,PHEVT / PHDVTP / *D*/*A*/*E* $
SDR1      USETD,,PHDVTP,,,GOD,GMD,,,, / PHIPHP,, / 1/*DYNAMICS* $
SDR2      CASEXX,CSTM,MPT,DIT,EQDYN,SILD,,,,LAMA,,PHIPHP,EST,
          XYCDB,/,,,IES1,, /*MMREIG*/S,N,NOSORT2 $
ENDALTER
```

```
./ CHANGE NAME=FRD2B
./ NUMBER SEQ1=70,NEW1=71,INCR=1,INSERT=YES
C
C     COMMON FRD2BC IS INITIALIZED BY ROUTINE FRRD2.
C
      COMMON /FRD2BC/ IH
./ NUMBER SEQ1=410,NEW1=411,INCR=1,INSERT=YES
C
C     IF IH=0, USE COMPLEX DOUBLE PRECISION ARITHMETIC.
C
      IF (IH .EQ. 0) ITY = 4
./ NUMBER SEQ1=500,NEW1=510,INCR=1,INSERT=YES
      CALL MAKMCB(MC,OUT,IROW,IFO,ITY)
./ DELETE SEQ1=510,SEQ2=510
./ CHANGE NAME=FRD2C
./ NUMBER SEQ1=120,NEW1=121,INCR=1,INSERT=YES
C
C     COMMON FRD2BC IS INITIALIZED BY ROUTINE FRRD2.
C
      COMMON /FRD2BC/ IH
./ NUMBER SEQ1=240,NEW1=241,INCR=1,INSERT=YES
C
C     IF IH=0, USE COMPLEX DOUBLE PRECISION ARITHMETIC.
C
      IF (IH .EQ. 0) IOUT = 4
./ NUMBER SEQ1=370,NEW1=371,INCR=1,INSERT=YES
C
C     IF IH=0, THEN DO NOT USE INCORE CAPABILITIES BECAUSE FOR IH=0
C              COMPLEX DOUBLE PRECISION ARITHMETIC WILL BE REQUESTED
C              AND SUBROUTINE INCORE CALLED BY FRD2C IS WRITTEN ONLY
C              FOR COMPLEX SINGLE PRECISION MATRICES.
C
      IF (IH .EQ. 0) GO TO 102
./ NUMBER SEQ1=1070,NEW1=1071,INCR=1,INSERT=YES
C
C     IF IH=0, USE COMPLEX DOUBLE PRECISION ARITHMETIC.
C
      IF (IH .EQ. 0) IOUT = 4
./ CHANGE NAME=FRRD2
./ NUMBER SEQ1=270,NEW1=271,INCR=1,INSERT=YES
C
C     COMMON FRD2BC WILL BE USED BY ROUTINES FRD2B AND FRD2C.
C
      COMMON /FRD2BC/ IH
./ NUMBER SEQ1=440,NEW1=441,INCR=1,INSERT=YES
C
C     IF QHHL IS PURGED AND MACH NUMBER IS NEGATIVE THEN
C     SOLVE THE COUPLED EQUATION = (-W**2.M + IW.B + K)U = P
C     USING COMPLEX DOUBLE PRECISION ARITHMETIC.
C     VARIABLE IH WILL BE USED TO CONTROL SOLUTION LOGIC IN
```

ORIGINAL PAGE IS
OF POOR QUALITY

```
C     ROUTINES FRRD2, FRD2B AND FRD2C.
C
      IF (MCB(1).LE.0 .AND. M.LT.0.0) NONCUP = 1
C
./ NUMBER SEQ1=720,NEW1=721,INCR=1,INSERT=YES
C
C     IF IH=0, THEN DO NOT USE INCORE CAPABILITIES BECAUSE FOR IH=0
C              COMPLEX DOUBLE PRECISION ARITHMETIC WILL BE REQUESTED
C              AND SUBROUTINE INCORE CALLED BY FRD2C IS WRITTEN ONLY
C              FOR COMPLEX SINGLE PRECISION MATRICES.
C
      IF (IH .EQ. 0) GO TO 20
./ NUMBER SEQ1=1210,NEW1=1211,INCR=1,INSERT=YES
C
C     IF IH = 0, THEN CREATE NULL TRAILERS FOR SCR2(QHR) AND SCR3(QHI).
C                THESE DATA BLOCKS ARE NORMALLY GENERATED BY FRD2A IF
C                IH IS NOT EQUAL TO ZERO EACH TIME THRU THE LOOP ON
C                NFREQ IN THIS ROUTINE. SINCE FRD2A IS NOT EXECUTED IF
C                IH EQUALS ZERO THEN AFTER THE FIRST PASS THE TRAILERS
C                FOR SCR2 AND SCR3 WOULD BE INCORRECT SINCE SCR2 AND
C                SCR3 ARE ALSO USED BY FRD2C AS SCRATCH DATA SETS.
C
      IF (IH .NE. 0) GO TO 38
      CALL MAKMCB(MCB,SCR2,0,0,0)
      CALL WRTTRL(MCB)
      MCB(1) = SCR3
      CALL WRTTRL(MCB)
   38 CONTINUE
C
./ NUMBER SEQ1=1480,NEW1=1481,INCR=1,INSERT=YES
C
C     CREATE A PSEUDO FRL DATA BLOCK ON SCR1 FROM DATA BLOCK FOL FOR
C     INPUT TO ROUTINE FRD2F. (NO TRAILER IS NECESSARY).
C     THE FREQUENCIES FROM FOL HAVE BEEN READ INTO IZ(1) DURING THE
C     SET-UP AT THE BEGINING OF THIS ROUTINE. THESE FREQUENCIES MUST
C     BE CONVERTED TO RADIAN FREQUENCIES FOR FRL (W = 2PI*F).
C
      CALL GOPEN(SCR1,IZ(IBUF1),1)
      DO 210 I = 1,NFREQ
      Z(I) = Z(I) * TWOPI
  210 CONTINUE
      CALL WRITE(SCR1,Z,NFREQ,1)
      CALL CLOSE(SCR1,1)
      CALL FRD2F(MHH, BHH, KHH, SCR1, 1, NLOAD, NFREQ, PHF, UHVF)
./ DELETE SEQ1=1490,SEQ2=1490
```

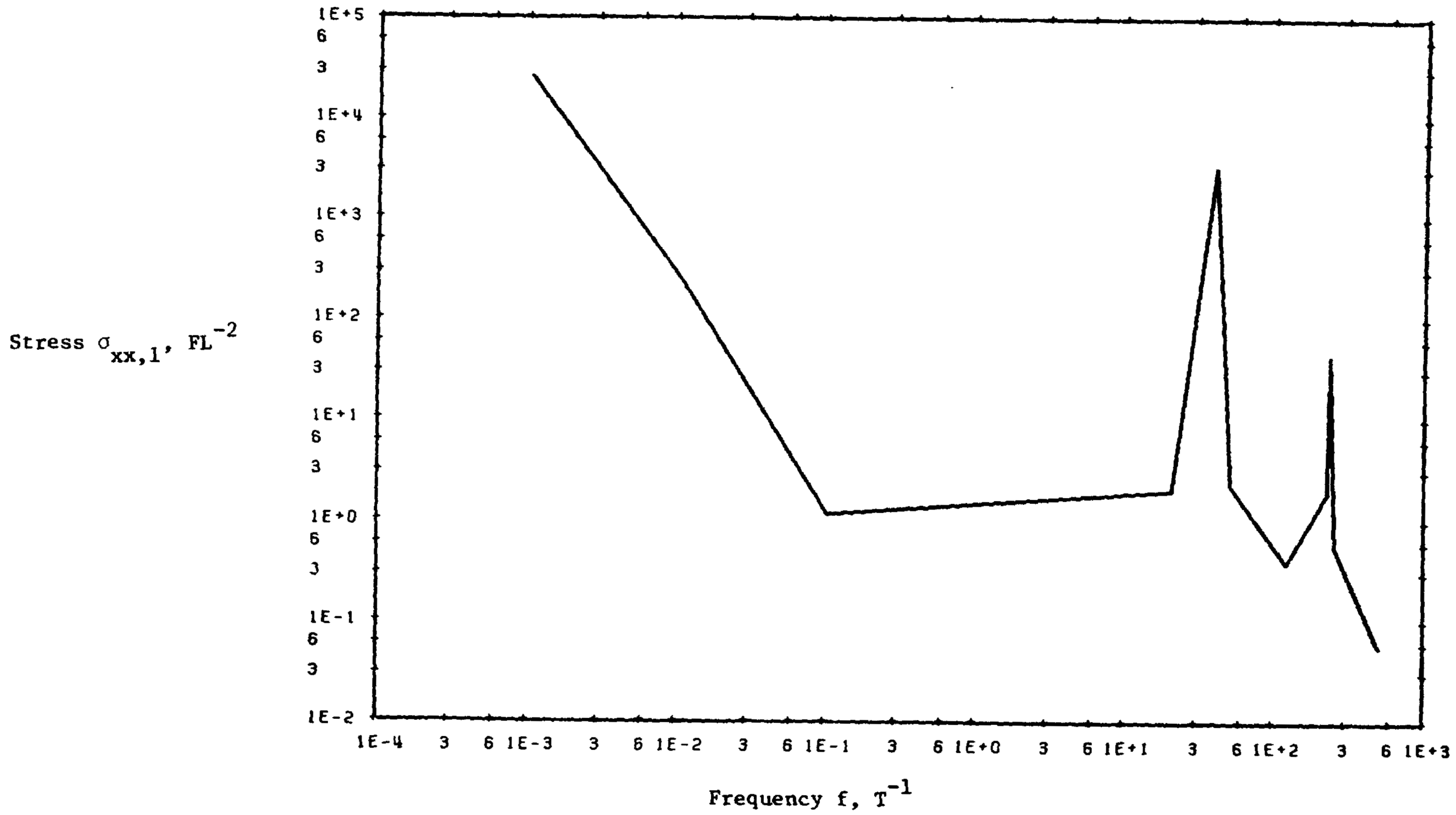
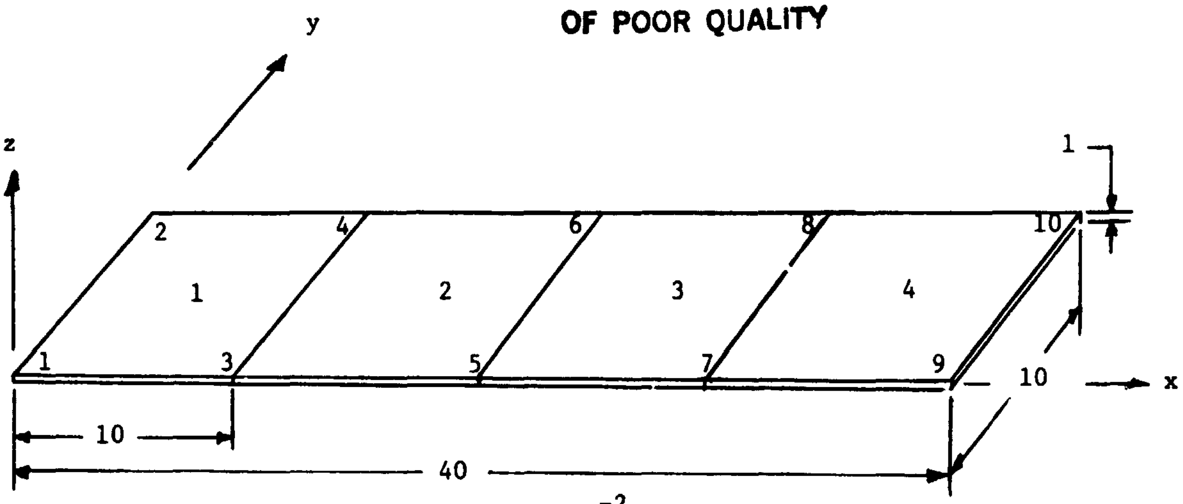


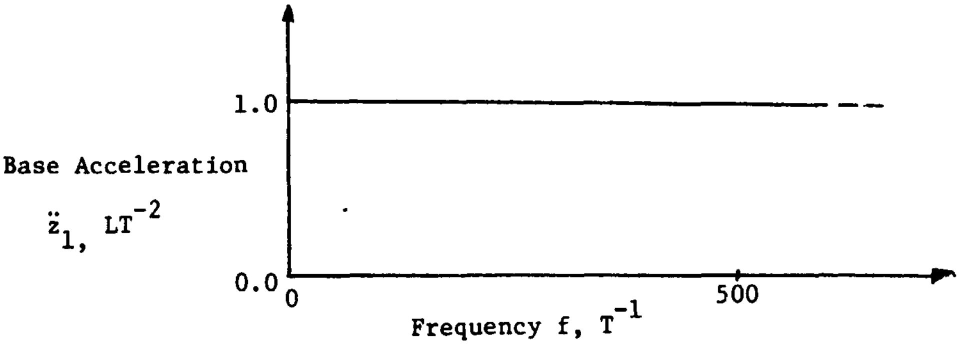
Figure 1. Stress Response $\sigma_{xx,1}$ in Element 2 of Example 1 (Figure 2), RF8 With Large Mass.



Young's Modulus = 1E8 FL^{-2}

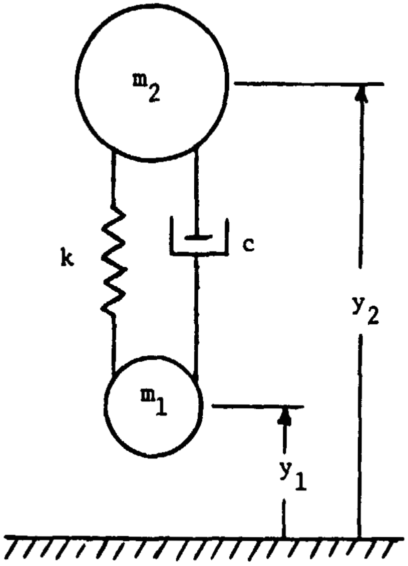
Poisson's Ratio = 0.4

Material Density = 8E-4 $FL^{-4}T^{2}$



Base Acceleration Input for Figures 1 and 7

Figure 2. Example 1

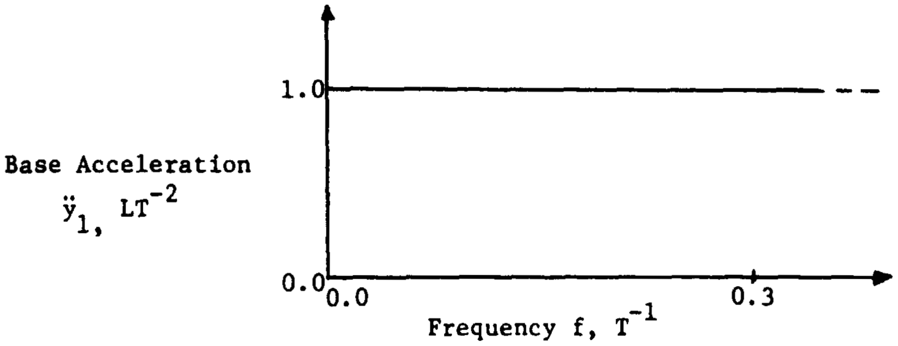


$m_1 = 2 \quad FL^{-1}T^2$

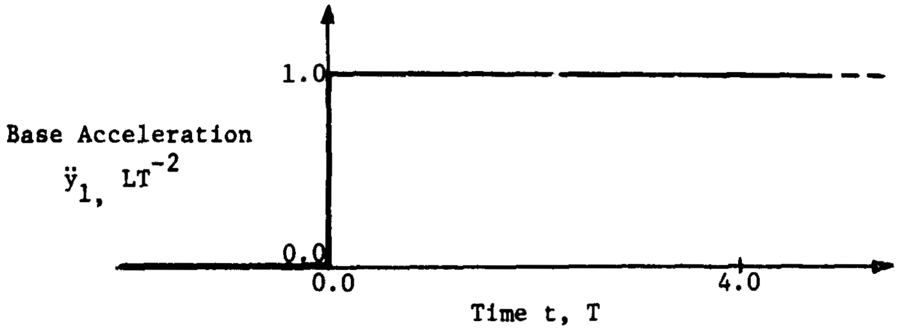
$m_2 = 8 \quad FL^{-1}T^2$

$k = 4 \quad FL^{-1}$

$c = 0.6 \quad FL^{-1}T$



Base Acceleration Input for Figures 4 and 5



Base Acceleration Input for Figure 6

Figure 3. Example 2

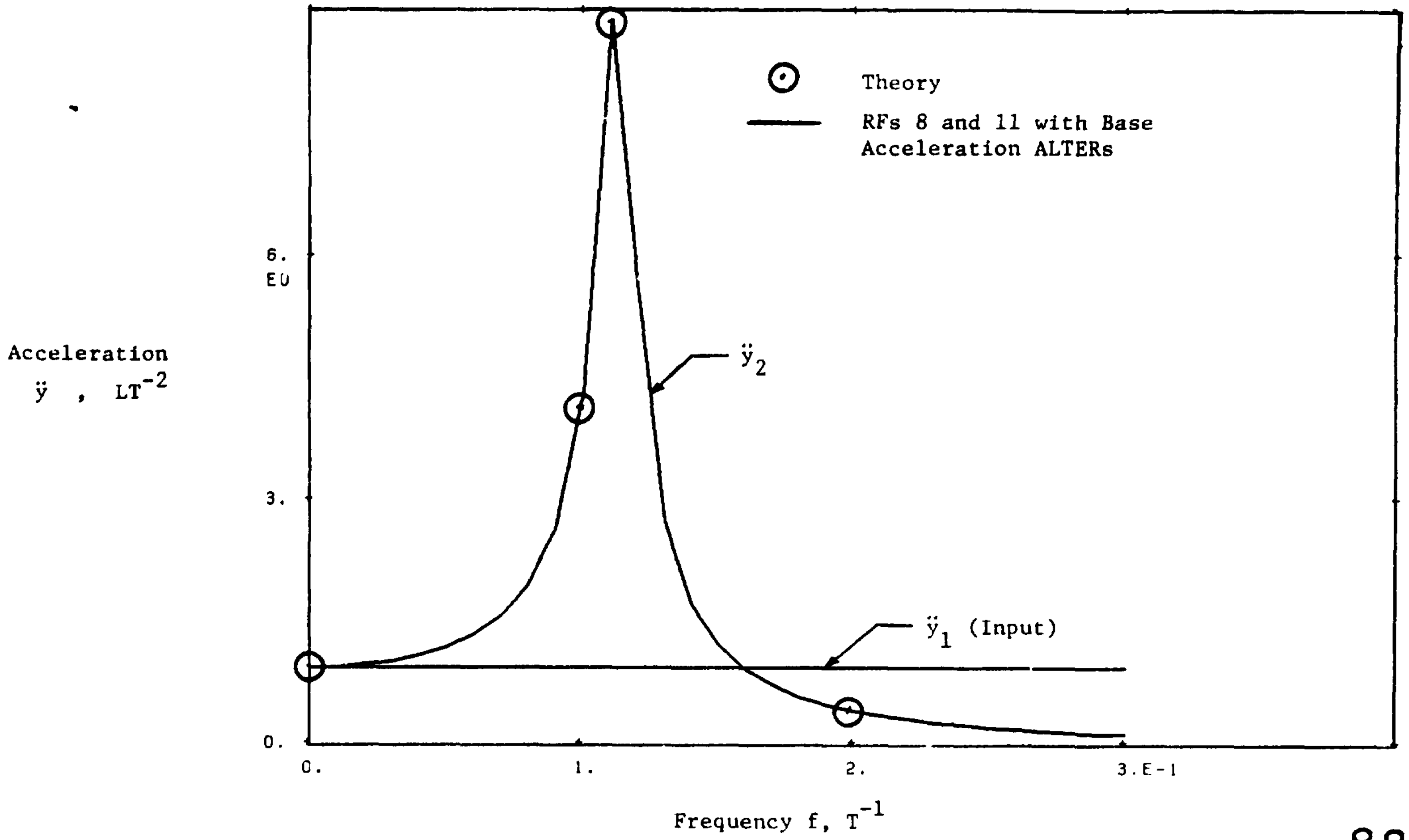


Figure 4. Acceleration Response of Mass m_2 of Example 2 (Figure 3), Frequency Response

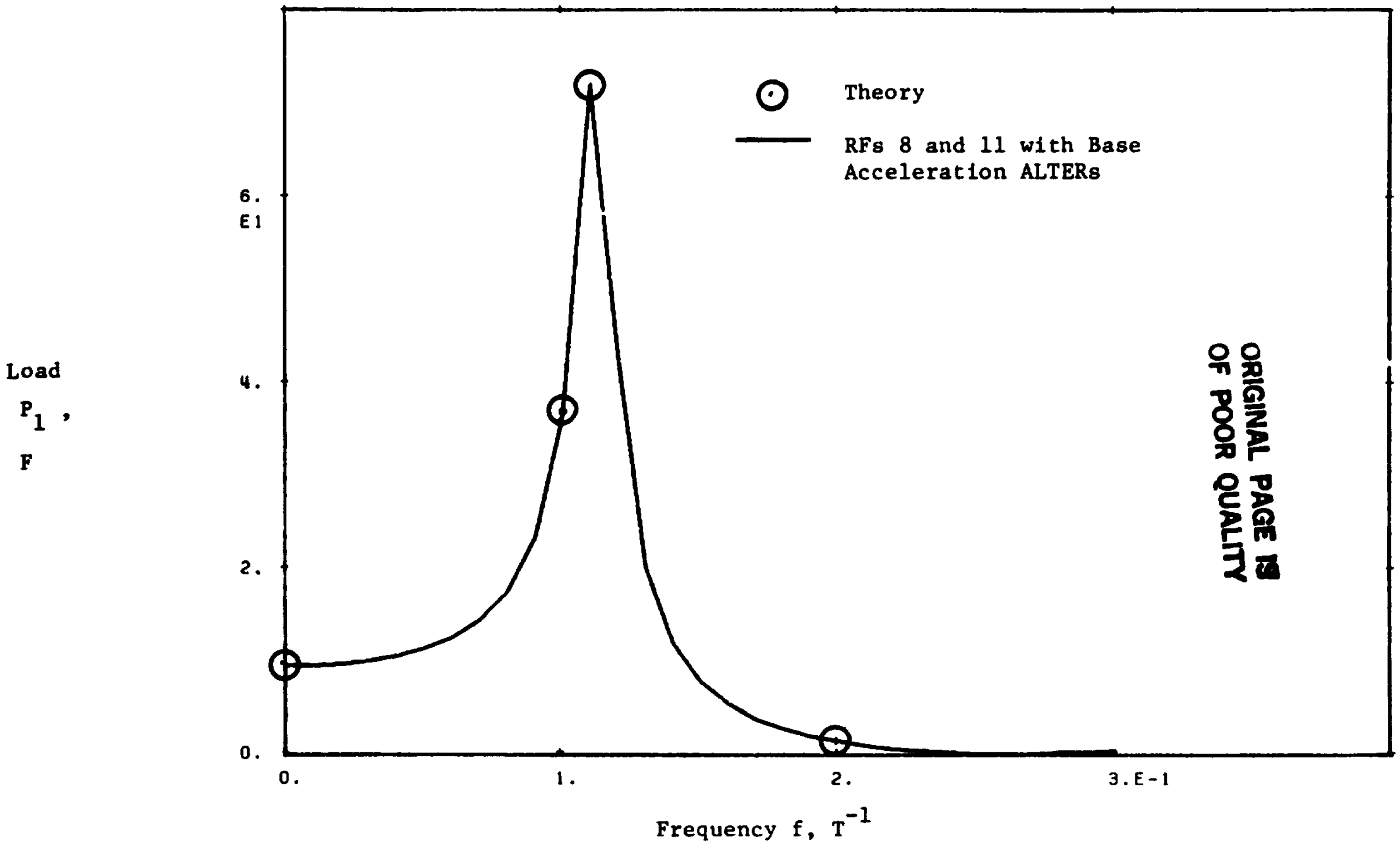


Figure 5. Load Required on Mass m_1 of Example 2 (Figure 3), Frequency Response

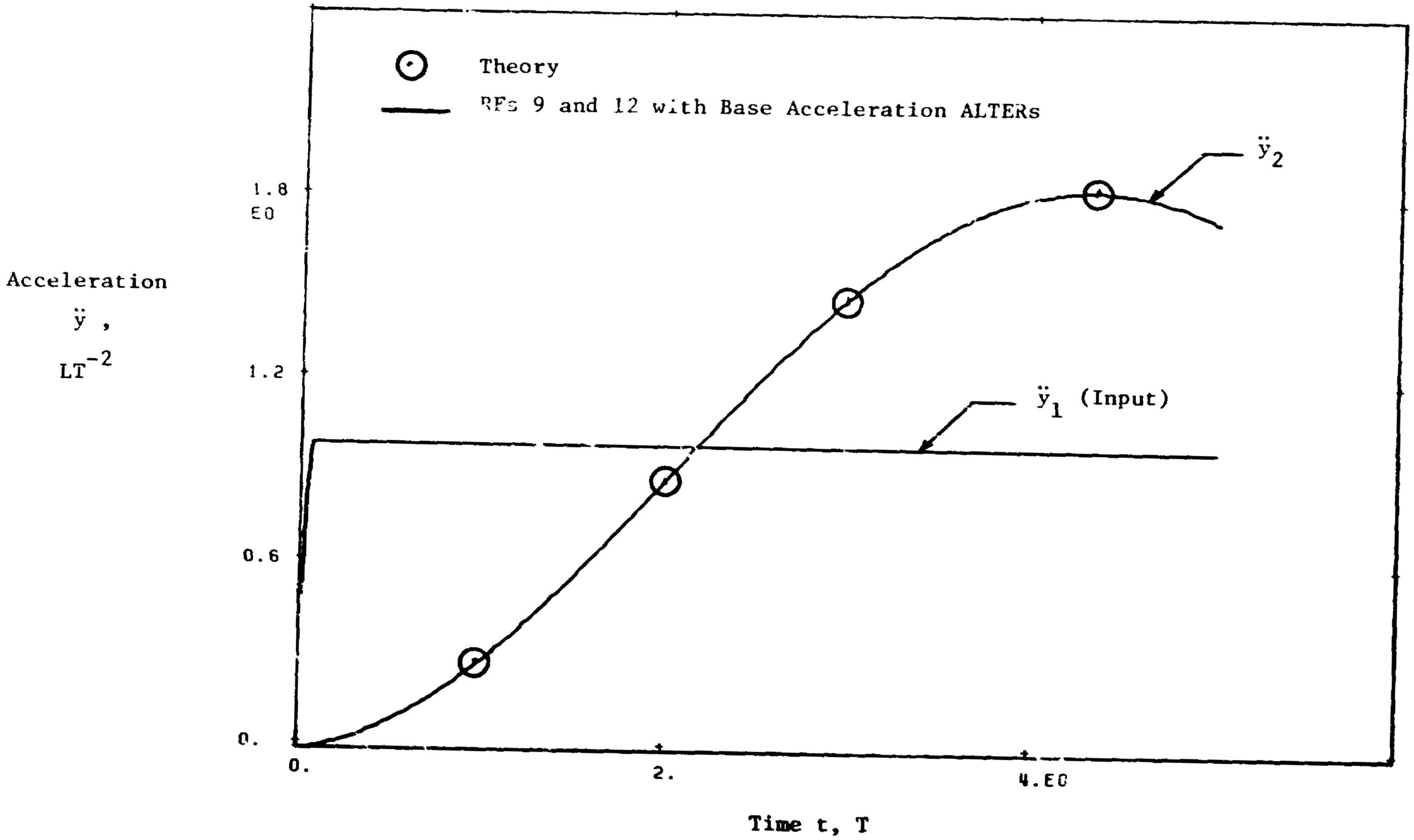


Figure 6. Acceleration Response of Mass m_2 of Example 2 (Figure 3), Transient Response

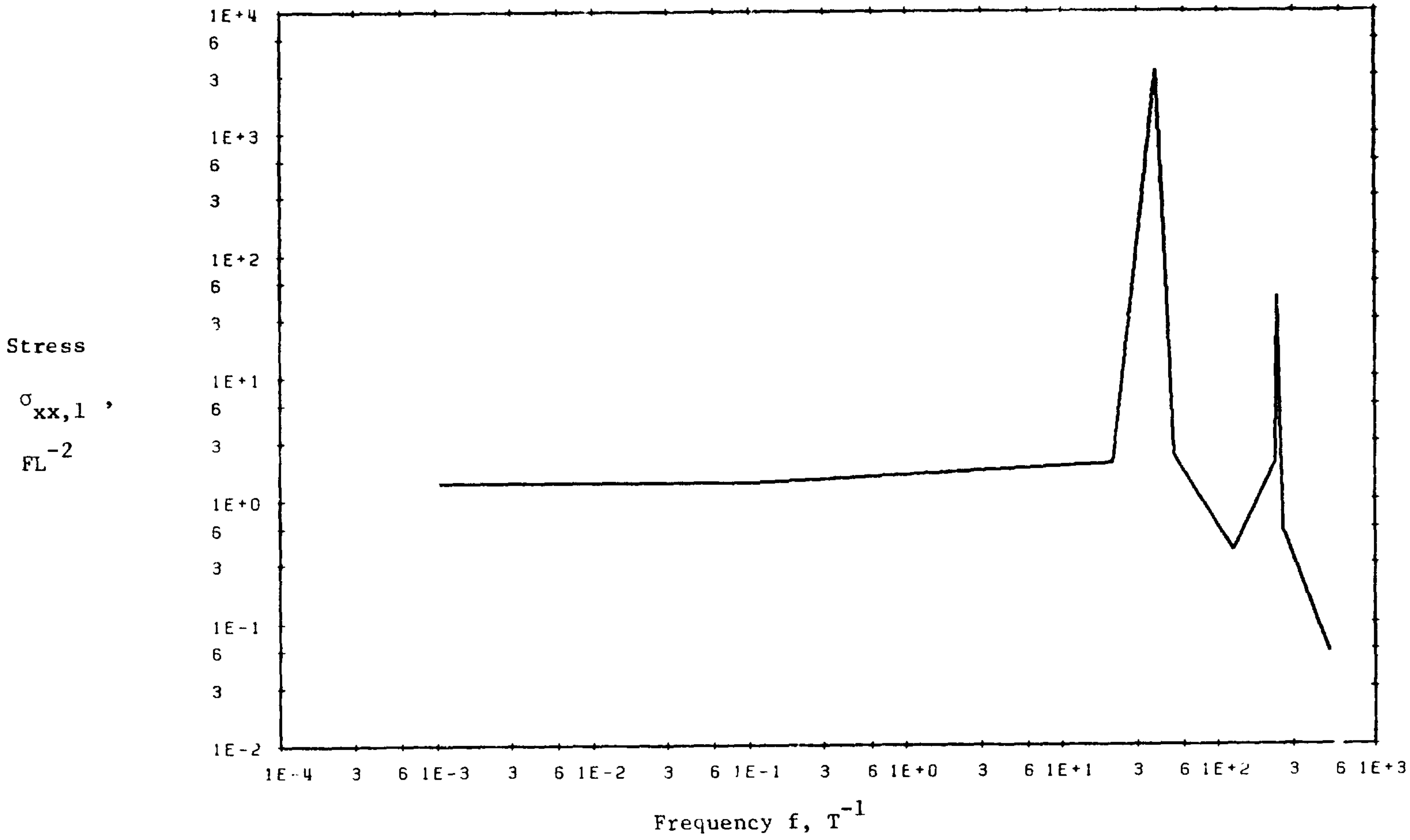


Figure 7. Stress Response $\sigma_{xx,1}$ in Element 2 of Example 1 (Figure 2), RF8 with Base Acceleration ALTERs and no Large Mass.