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ELASTIC-PLASTIC ANALYSIS OF ANNULAR PLATE PROBLEMS USING NASTRAN

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SUMMARY

The plate elements of the NASTRAN code are used to analyze two annular plate problems loaded beyond the elastic limit. The first problem is an elastic-plastic annular plate loaded externally by two concentrated forces. The second problem is stressed radially by uniform internal pressure for which an exact analytical solution is available. A comparison of the two approaches together with an assessment of the NASTRAN code is given.

INTRODUCTION

The piecewise linear analysis option of the NASTRAN code can be used to analyze quite complicated elastic-plastic plane-stress problems (refs. 1 and 2). The reliability of this code has been well demonstrated in the linear range but not so in the nonlinear range of loadings. One major reason is because exact analytical solutions for elastic-plastic problems are usually not available for comparison with approximate NASTRAN solutions.

In this paper, the plate elements of the NASTRAN code are used to solve two annular plate problems loaded beyond the elastic limit. The first problem considered is an elastic-plastic annular plate loaded externally by two concentrated forces. There is no analytical solution for this two-dimensional plane-stress problem and the NASTRAN code is used to obtain numerical results. The second problem considered is an elastic-plastic annular plate radially stressed by uniform internal pressure. This problem is chosen because an exact analytical solution is available for comparison. For ideally plastic materials, the stress solution for this statically determinate problem was first obtained by Mises (ref. 3) and the corresponding two strain solutions were obtained by the present author on the basis of both J_2 deformation and flow theories (ref. 4). For elastic-plastic strain-hardening materials, an exact complete solution was recently reported in reference 5. Analytical expressions were derived on the basis of J_2 deformation theory, the Hill's yield criterion, and a modified Ramberg-Osgood law. The validity of the above solution has been established by satisfying the Budiansky's criterion.

In the following, the theory of elastic-plastic plate elements as used in NASTRAN is briefly reviewed. In its present form, NASTRAN cannot be used for problems involving ideally plastic materials. It is shown that this limitation can be easily removed by making minor changes. For an elastic-plastic strain-hardening material, the NASTRAN solution is reported here and compared with the exact solution. The results are presented graphically and an assessment of the NASTRAN code is made.

The theoretical basis of two-dimensional plastic deformation as used in NASTRAN is that developed by Swedlow (ref. 6). In the development, a unique relationship between the octahedral stress, τ_o , and the plastic octahedral strain, ϵ_o^P , is assumed to exist and the use of ideally plastic materials is excluded. The total strain components (ϵ_x , ϵ_y , ϵ_z , and γ_{xy}) are composed of the elastic, recoverable deformations and the plastic portions (ϵ_x^P , ϵ_y^P , ϵ_z^P , and γ_{xy}^P). The rates of plastic flow, ($\dot{\epsilon}_x^P$, etc.), are independent of a time scale and are simply used for convenience instead of incremental values. The definitions of the octahedral stress and the octahedral plastic strain rate for isotropic materials are:

$$\tau_o = \sqrt{(s_{11}^2 + 2s_{12}^2 + s_{22}^2 + s_{33}^2)/3} \quad (1)$$

$$\dot{\epsilon}_o^P = \sqrt{[(\dot{\epsilon}_{11}^P)^2 + 2(\dot{\epsilon}_{12}^P)^2 + (\dot{\epsilon}_{22}^P)^2 + (\dot{\epsilon}_{33}^P)^2]/3} \quad (2)$$

where

$$\begin{aligned} s_{11} &= \frac{1}{3} (2\sigma_x - \sigma_y) \quad , \quad \epsilon_{11}^P = \epsilon_x^P \\ s_{22} &= \frac{1}{3} (2\sigma_y - \sigma_x) \quad , \quad \epsilon_{22}^P = \epsilon_y^P \\ s_{33} &= -\frac{1}{3} (\sigma_x + \sigma_y) \quad , \quad \epsilon_{33}^P = \epsilon_z^P \\ s_{12} &= T_{xy} \quad , \quad \epsilon_{12}^P = -\gamma_{xy}^P \end{aligned} \quad (3)$$

The s_{ij} is called the deviator of the stress tensor; σ_x , σ_y , and T_{xy} are the cartesian stresses. The isotropic material is assumed to obey the Mises yield criterion and the Prandtl-Reuss flow rule. The matrix relationship for the plastic flow is:

$$\begin{Bmatrix} \dot{\epsilon}_x^P \\ \dot{\epsilon}_y^P \\ \dot{\gamma}_{xy}^P \end{Bmatrix} = \frac{1}{6\tau_o^2 M_T(\tau_o)} \begin{bmatrix} s_{11}^2 & s_{11}s_{22} & 2s_{11}s_{12} \\ s_{11}s_{22} & s_{22}^2 & 2s_{22}s_{12} \\ 2s_{11}s_{22} & 2s_{11}s_{12} & 4s_{12}^2 \end{bmatrix} \begin{Bmatrix} \dot{\sigma}_x \\ \dot{\sigma}_y \\ \dot{T}_{xy} \end{Bmatrix} \quad (4)$$

where

$$2M_T(\tau_o) = \dot{\tau}_o / \dot{\epsilon}_o^P \quad (5)$$

The plastic modulus, $M_T(\tau_o)$, can be related to the slope E_T , of the effective stress-strain curve by:

$$\frac{1}{3M_T(\tau_o)} = \frac{1}{E_T} - \frac{1}{E} \quad (6)$$

The total strain increments, obtained by adding the plastic and linear elastic parts, are:

$$\{\Delta\epsilon\} = ([D^P] + [G]^{-1})\{\Delta\sigma\} = [G_T]^{-1}\{\Delta\sigma\} \quad (7)$$

0.05n) pounds for $n = 1, 2, \dots, 13$. The vertical displacement at the point of application (V_D) and the horizontal displacement (U_B) at point B (grid 11) are shown in figure 2 as functions of the applied force F . The major principal stresses in elements near the x and y -axes are shown in figure 3 for $F = 600$ and 1205.38 pounds, respectively. The results indicate that the maximum tensile stress occurs at point C.

ONE-DIMENSIONAL PROBLEM

Consider a one-dimensional annular plate stressed radially by uniform internal pressure. The plate geometry and material properties are the same as the two-dimensional one. Utilizing the condition of axial symmetry, we need only a sector of the annular plate for the finite element model. There are only 10 elements with 22 grids. All grid points are constrained in the tangential direction. The applied load is the internal pressure p . If $p = 1000$ psi, the equivalent nodal force at each of the two interior grids is $Q = 4.62329$ pounds in the radial direction. The true pressure corresponding to initial yielding for this problem is $p_0 = 23,571$ psi. The NAST AN results will depend on the users' choice of element sizes and load increments. In the elastic range, the NASTRAN results based on two finite-element models were compared with the exact solution. For a ten-element model, the maximum error is 0.36 percent in displacements and 0.95 percent in stresses. For a twenty-element model, the maximum errors are reduced to 0.09 percent and 0.24 percent, respectively. Since we are satisfied with one percent error, the ten-element model is chosen for incremental analysis beyond the elastic limit.

In the plastic range, the user has to choose the load increments properly in order to obtain good results at reasonable cost. The values of the load factors can be normalized so its unit value corresponds to the limit of elastic solution, i.e., $p_0 = 23,571$ psi. The load increments can be uniform or nonuniform. It seems that the size of load increments depends on the material properties and sizes of elements. In order to determine the influence of load factors on the displacements and stresses in the plastic range, four sets of load factors are chosen. The load increments for three of them are uniform with $\Delta p/p_0 = 0.20, 0.10, \text{ and } 0.05$, respectively. The influence of load factor, p/p_0 , on the inside radial displacement, u_1 , is shown in figure 4. The effect of load factors on the major principal stresses in elements is shown in figure 5. We also show in these two figures the corresponding analytical results. On the basis of these comparisons, we can make the following conclusions. In the earlier stages of plastic deformation, larger load increments can be used to give very good results. As plastic deformation becomes bigger, smaller load increments should be used in order to get a reasonably good answer. However, for very large plastic deformation, it seems that we cannot improve the NASTRAN results much better by choosing even smaller increments. This is because there are other built-in errors in the NASTRAN program, e.g., the linear displacement function is assumed.

If uniform load increments are used, a direct comparison of the analytical and NASTRAN results is not available. The solid curves shown in figures 4 and 5 were obtained indirectly since the analytical results of the displacements, stresses, and pressure were represented as functions of elastic-plastic boundary (ref. 5). We have obtained the analytical results when the elastic-plastic boundary is located at a radial distance of 0.05, 0.15, 0.25, 0.35, 0.45, 0.55, 0.65, 0.75, 0.85, and 0.95 inch from the inside surface. The corresponding values of the pressure factor (p/p_0) are 1.095, 1.266, 1.414, 1.540, 1.646, 1.734, 1.805, 1.860, 1.901, and 1.929, respectively. This set of values is used as load factors in the input deck of the

NASTRAN program. Some of the NASTRAN results together with the corresponding analytical results are shown in figures 6 and 7. A direct comparison of the two approaches in the plastic range can be seen. The effect of load factors on the distribution of radial displacements is shown in figure 6. The distributions of major principal stresses for three load factors are shown in figure 7. As can be seen in figures 6 and 7, a direct comparison of two approaches will support the following conclusion. Even if the results in the elastic range are in excellent agreement, the differences in the plastic range can be quite big for large values of load factors. This suggests more research efforts should be given to large plastic deformation.

CONCLUSIONS

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Two elastic-plastic annular plate problems have been analyzed by using NASTRAN plate elements. One problem is loaded externally by two concentrated forces and the other by uniform internal pressure. The NASTRAN results for the second problem have been compared with an exact analytical solution. It seems that the NASTRAN code in its present form is still a valuable tool because it can be used to solve quite complicated plane stress problems provided the plastic deformation involved is not too large. The limitation that the use of ideally plastic materials is excluded can be easily removed by making minor changes.

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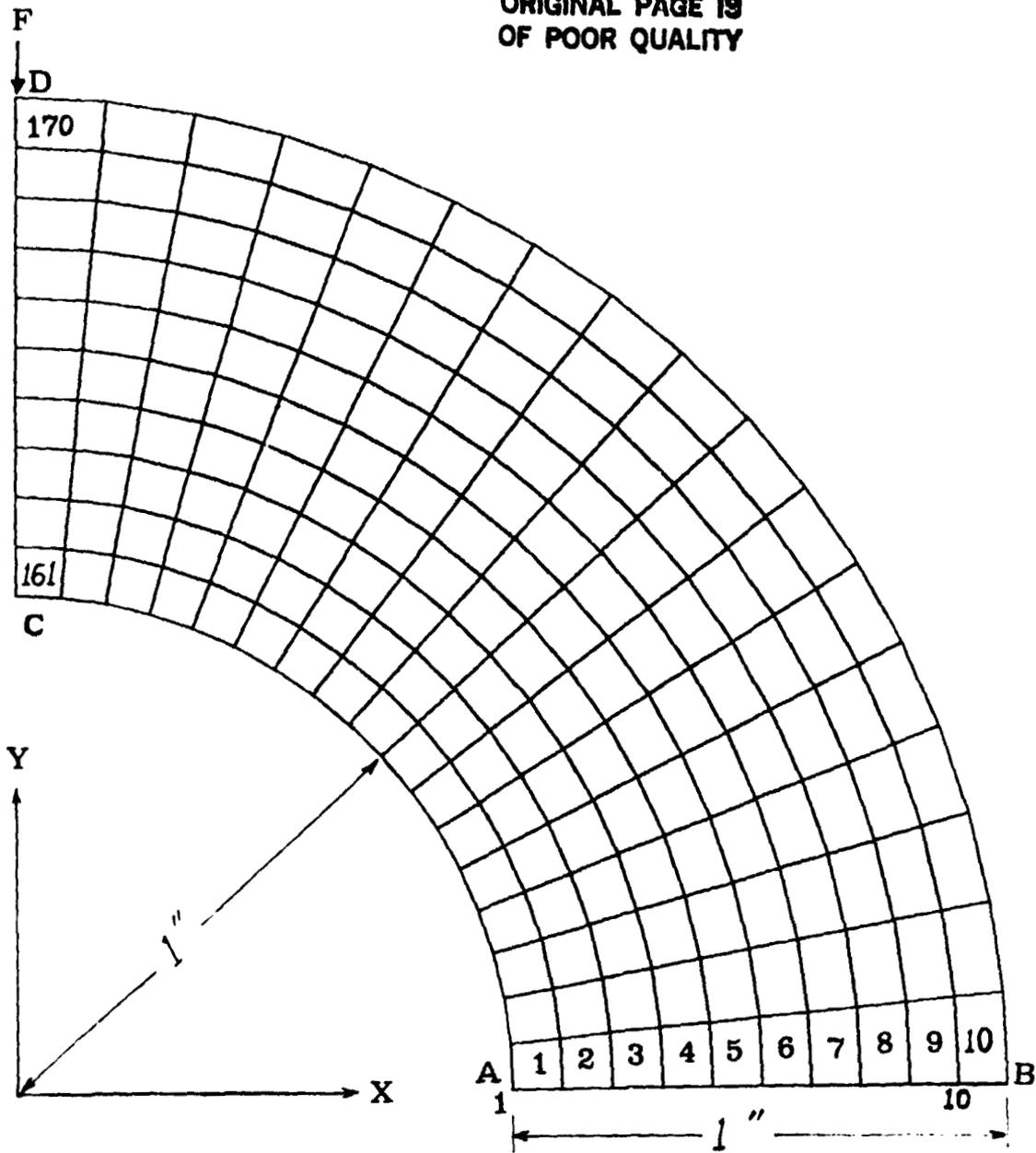


Figure 1. Finite Element Model of an Annular Plate.

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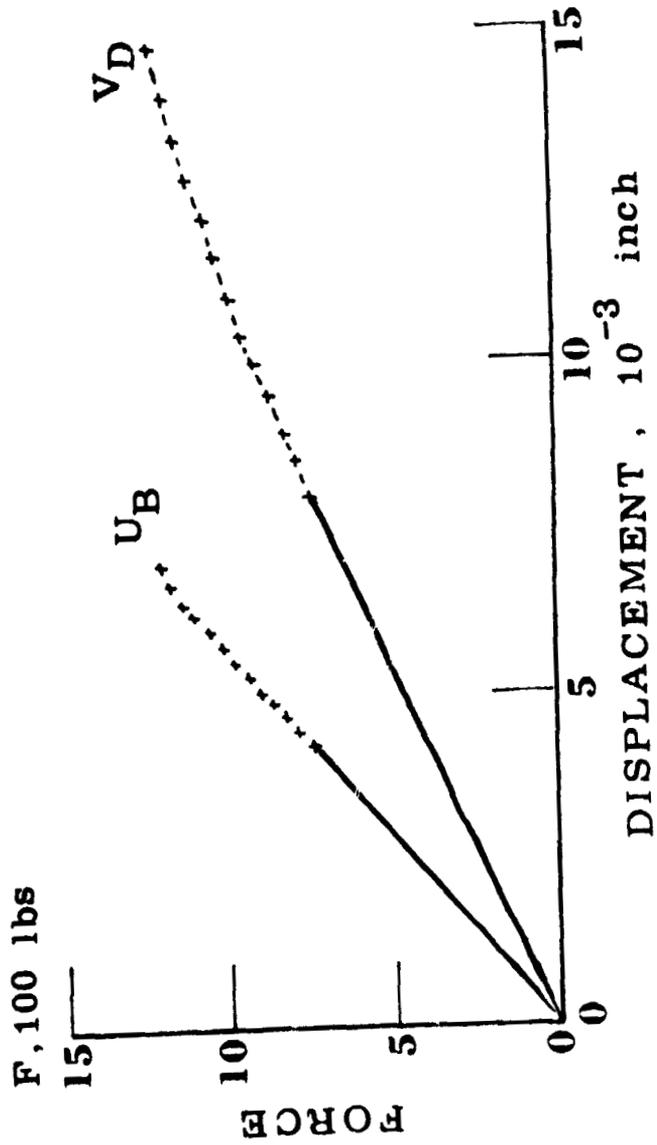


Figure 2. Vertical and Horizontal Displacements (V_D, U_B) as Functions of the Applied Force (F) - Problem 1.

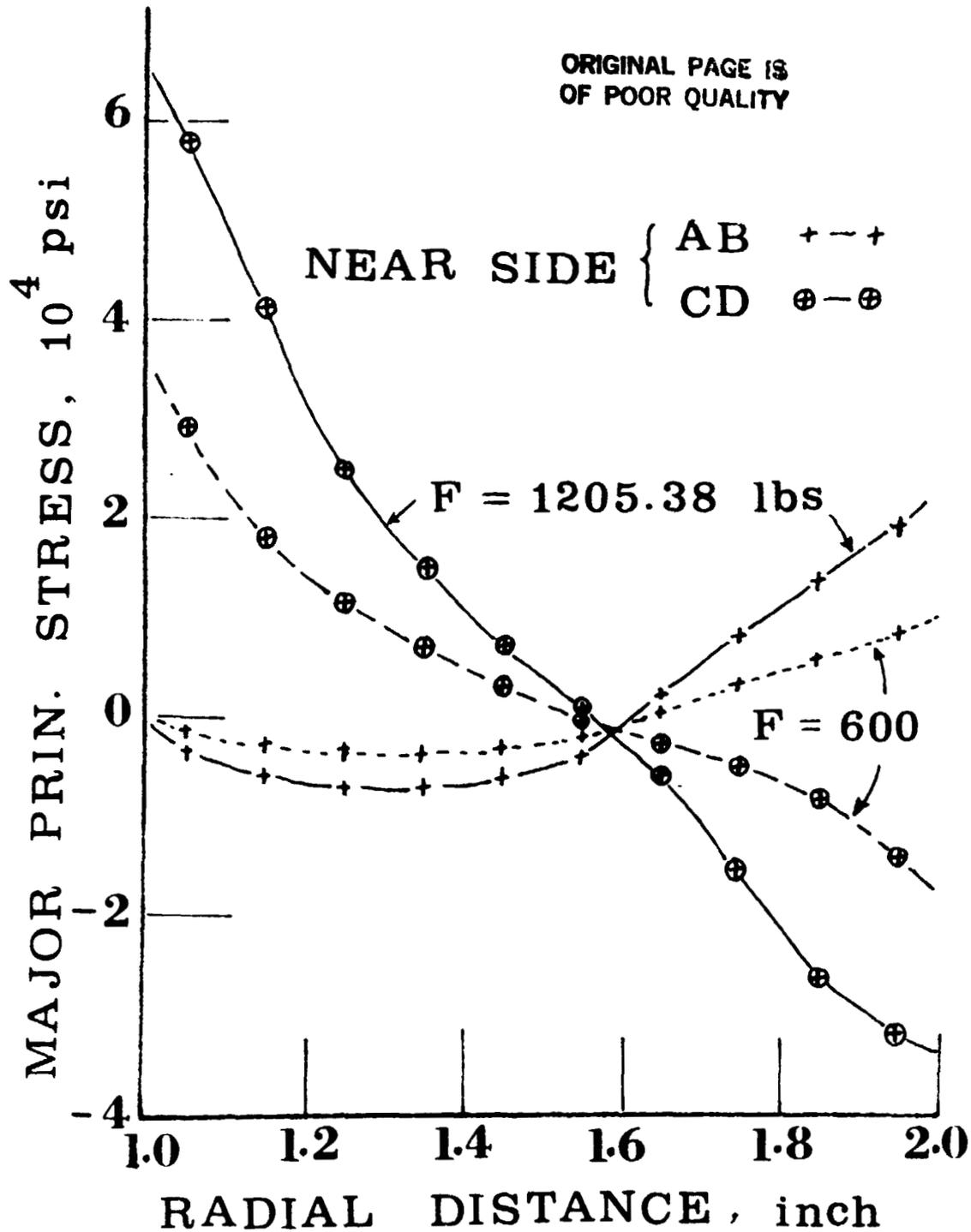


Figure 3. Distribution of Major Principal Stresses - Problem 1.

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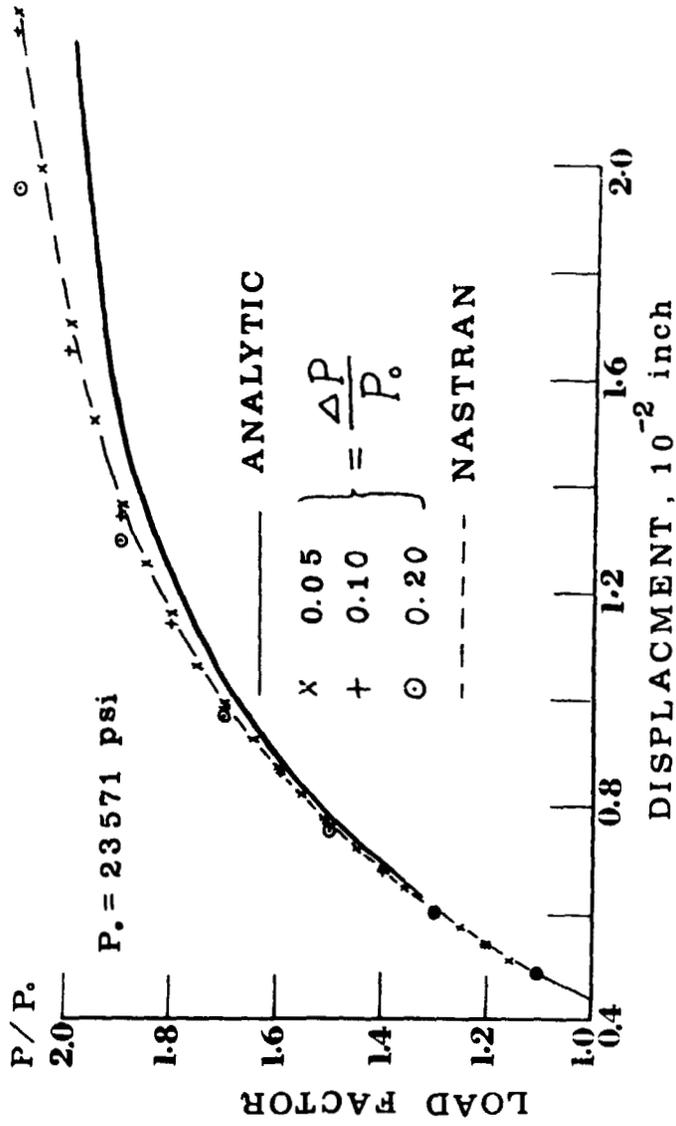


Figure 4. Influence of Load Factor on the Inside Displacements - Problem 2.

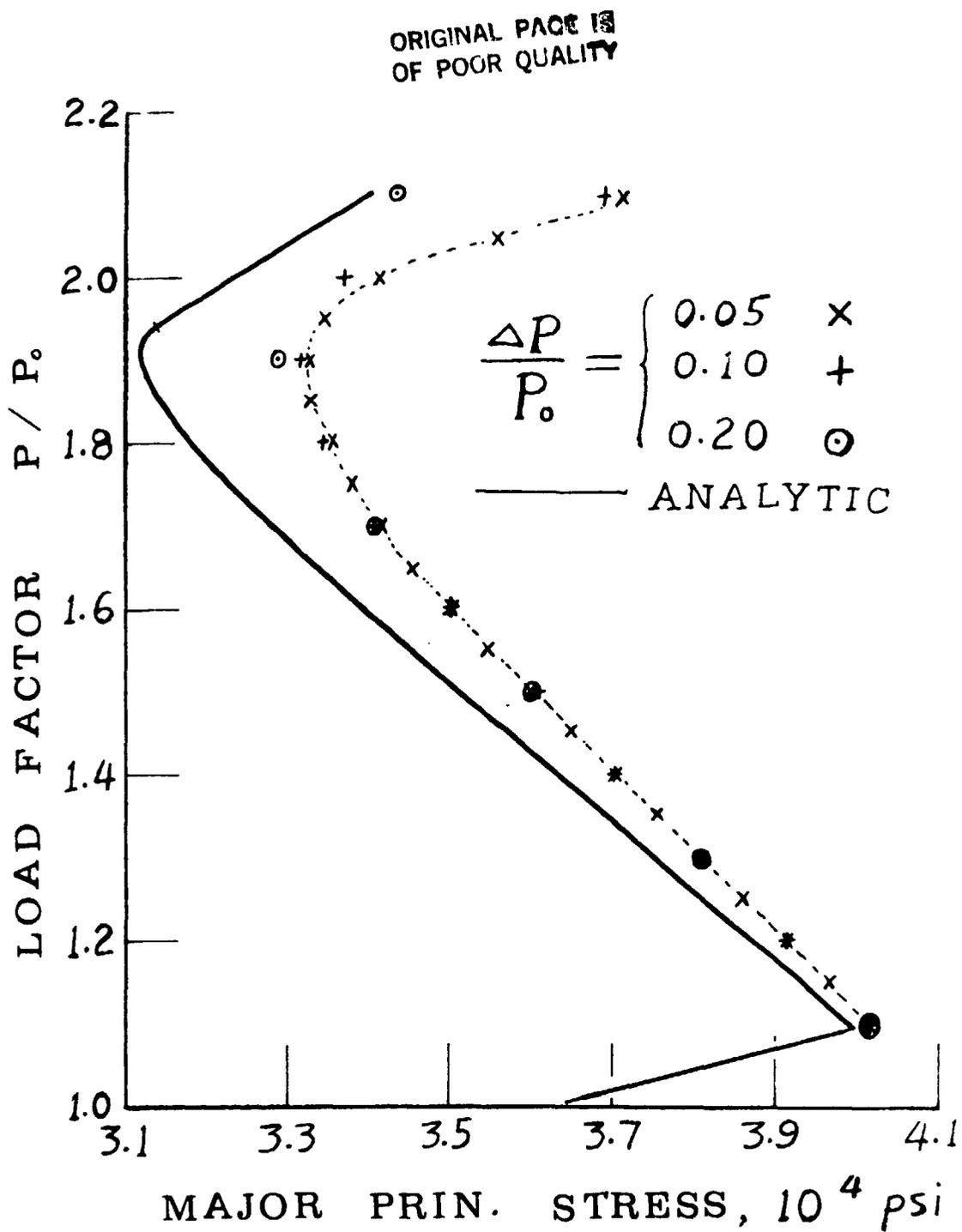


Figure 5. Influence of Load Factor on the Major Principal Stresses in Element 1 - Problem 2.

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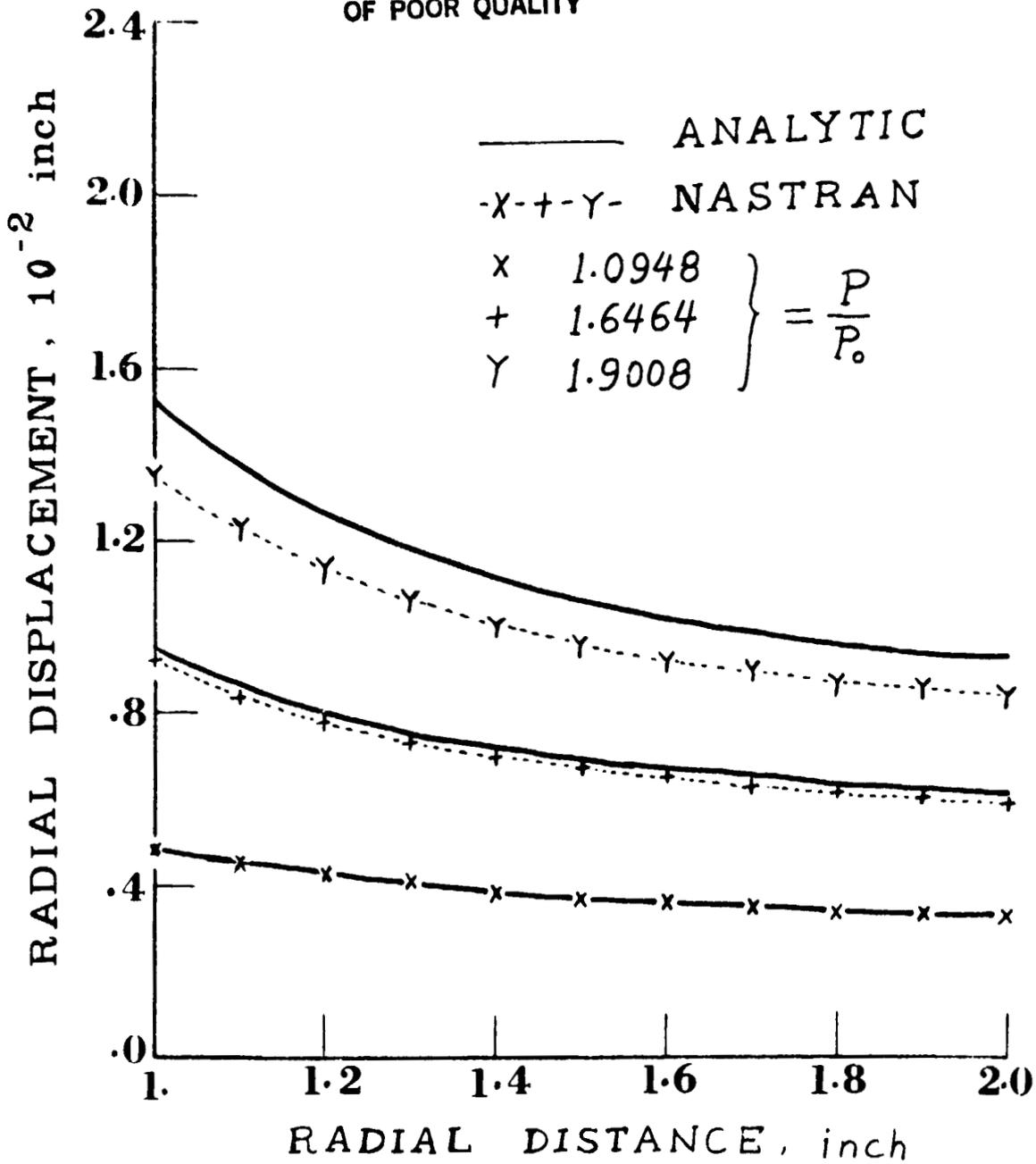


Figure 6. Distribution of Radial Displacements - Problem 2.

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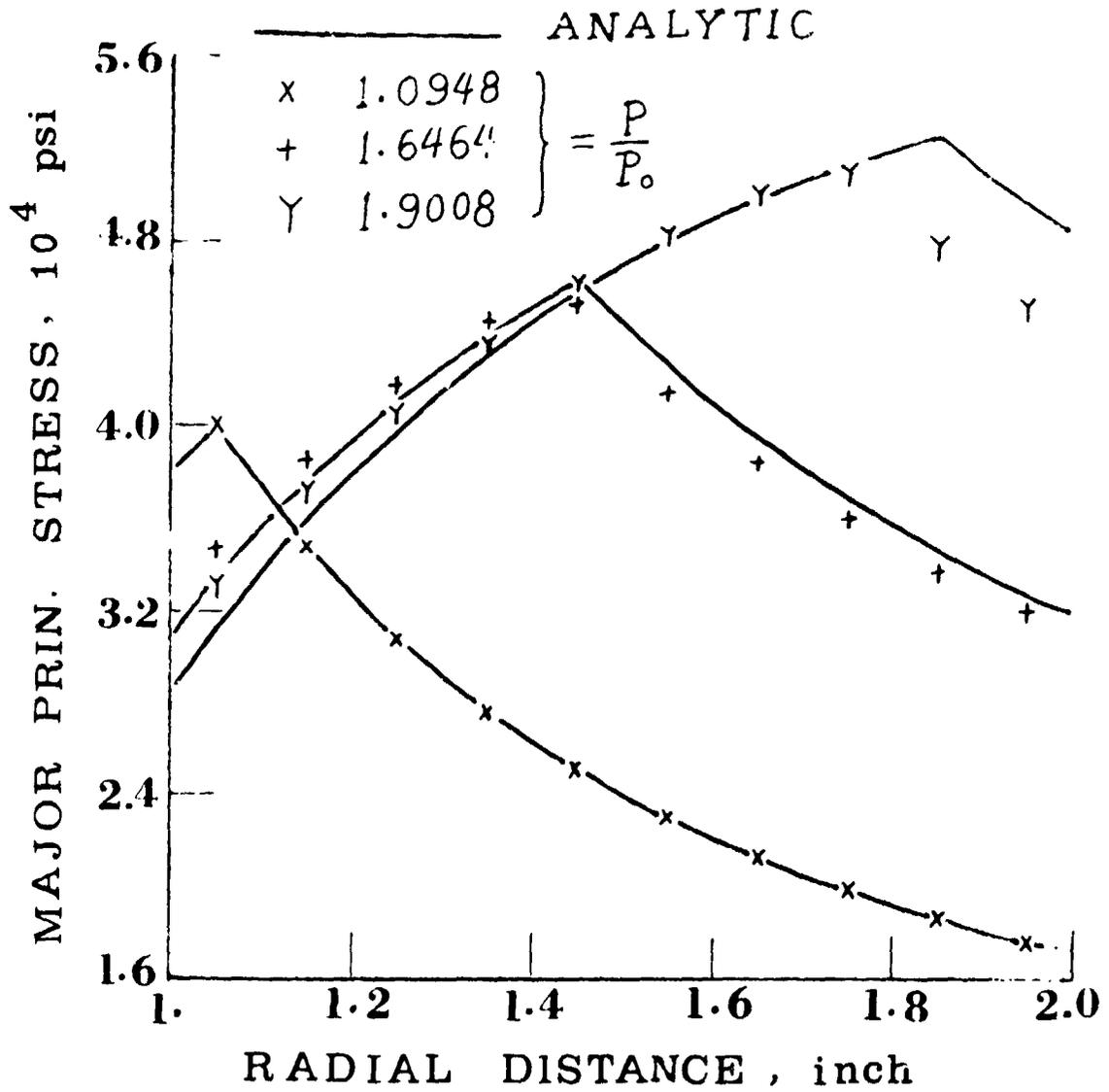


Figure 7. Distribution of Major Principal Stresses - Problem 2.