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ERROR REDUCTION PROGRAM*
A PROGRESS REPORT

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The combustion chamber of the aircraft gas turbine has to be designed to satisfy the stringent dual engine requirements of reduced fuel consumption and increased durability. Combustor development based on conventional methods to meet these requirements is proving to be increasingly costly and time consuming. A mathematical model of the combustor would be very helpful in reducing cost and time of the design cycle as it would permit most of the work to be done on a computer. Such models are already being used in a limited manner in the industry. More widespread use of these models will be accelerated by the removal of some of the known deficiencies in the present codes. One of these deficiencies is the numerical error, or numerical diffusion, that can be introduced into the calculations under certain conditions. This numerical error is due to the finite differencing scheme, usually upwind or hybrid, used in these codes.

The objective of the Error Reduction Program is to evaluate available finite difference schemes for minimum numerical diffusion, and to incorporate the best available scheme into a three-dimensional combustor performance code.

As a first step toward this end, five schemes (refs. 1 through 5) were selected for initial evaluation. Some of these schemes were combined with bounding schemes to eliminate physically unrealistic undershoots and overshoots. Two basic criteria were used to judge these schemes: they should be conservative, and should produce solutions that exhibit no extraneous maxima or minima (boundedness property). The accuracy of the schemes was evaluated by performing the truncation error analysis, and running one- and two-dimensional test cases and comparing the calculated solutions against the exact solutions. Based on this evaluation, two schemes were selected: QUDS, Quadratic Upstream Differencing Scheme, and BSUDS2, Bounded Skew Upstream Differencing Scheme Two. The first scheme was proposed by Leonard (ref. 1), and the second scheme is the one proposed by Raithby (ref. 2), which is bounded by a new bounding scheme.

The selected two schemes were coded into a two-dimensional computer code, 2D-TEACH, and their accuracy and stability were evaluated by running several test cases (refs. 6 through 8). It was found that BSUDS2 was more stable than QUDS. It was also found that the accuracy of both schemes is dependent on the angle that the streamlines make with the mesh, QUDS being more accurate at smaller angles and BSUDS2 being more accurate at larger angles. However, both schemes, at all angles, were more accurate than the existing hybrid scheme. BSUDS2 was selected to be extended into three dimensions, primarily because it was more stable. This scheme is currently being incorporated into a three- dimensional code, 3D-TEACH.

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1. Leonard, B. P., "A Stable and Accurate Convective Modeling Procedure Based on Quadratic Upstream Interpolation", Computer Methods in Applied Mechanics and Engineering, Vol. 19, 1979, pp. 59-98.

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4. Rubin, S. G. and Graves, Jr., R. A., "Viscous Flow Solutions with a Cubic Spline Approximation", Computers and Fluids, Vol. 3, 1975, pp. 1-36.

5. Glass, J. and Rodi, W., "A Higher Order Numerical Scheme for Scalar Transport", Computer Methods in Applied Mechanics and Engineering, Vol. 31, 1982, pp. 337-358.

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7. Johnson, B. V. and Bennett, J. C., "Mass and Momentum Turbulent Transport Experiments with Confined Coaxial Jets", NASA CR-165574, November 1981.

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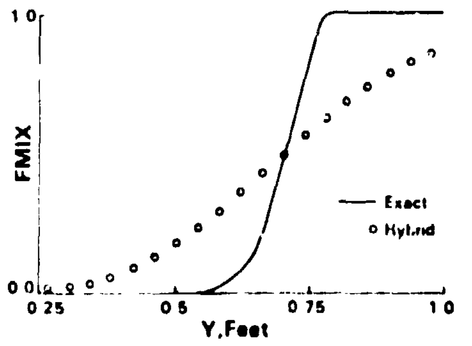
NEED FOR ERROR REDUCTION

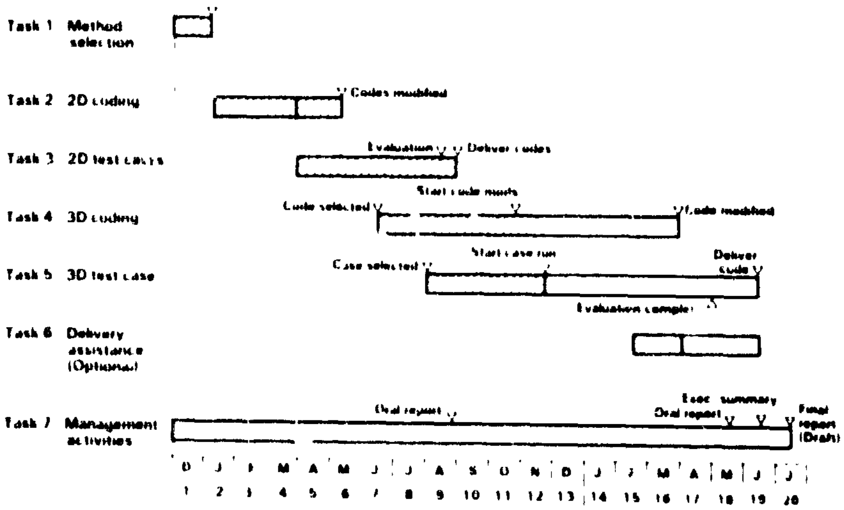


Convection and diffusion of a scalar step in constant property uniform inclined flow Pe_x Pe_y 60

OBJECTIVE

To identify and incorporate best available finite difference error reduction scheme into a 3D combustor performance code.

PROGRAM SCHEDULE



DISCRETIZATION SCHEMES EVALUATED

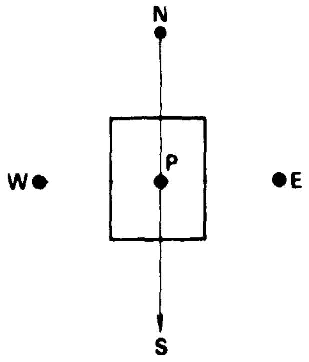
- **Agarwal Differencing Scheme (ADS)**
- **Quadratic Upwind Differencing Scheme (QUDS)**
- **Skew Upwind Differencing Scheme (SUDS)**
- **Spline/Hermetian Schemes**
  - **Cubic Spline Scheme (CSS)**
  - **Glass and Rodi Hermetian Scheme (GRHS)**
- **Flux Blending Schemes**
  - **Bounded-One**
  - **Bounded-Two**

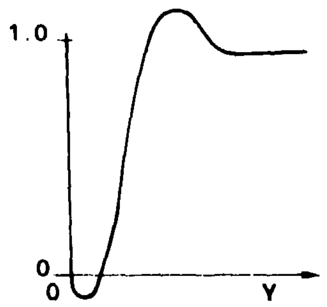
SCHEMES SELECTED

- **QUDS – Quadratic Upwind Differencing Scheme**
- **BSUDS2 – Bounded Skew Upwind Differencing Scheme Two**

WHAT IS BOUNDEDNESS

In the absence of sources the value at node P should be bounded by the values at surrounding nodes.





C-2

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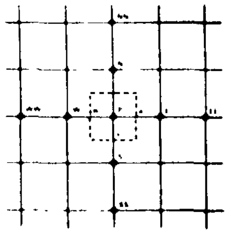
QUADRATIC UPWIND DIFFERENCING SCHEME (QUDS)

- Finite volume method
- Upwind biased quadratic interpolation for convection terms
- Central differencing for diffusion terms
- Employs extended nine point modecule

$$\phi_w = -\tfrac{1}{8}\phi_{WW} + \tfrac{3}{4}\phi_W + \tfrac{3}{8}\phi_P \; , u > 0$$

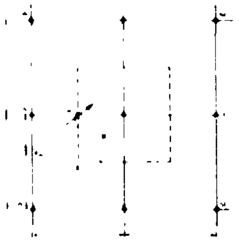
$$= -\tfrac{1}{8}\phi_E + \tfrac{3}{4}\phi_P + \tfrac{3}{8}\phi_W , u < 0$$



- Scheme is conservative
- Coefficients can become negative
- Solution can become unbounded

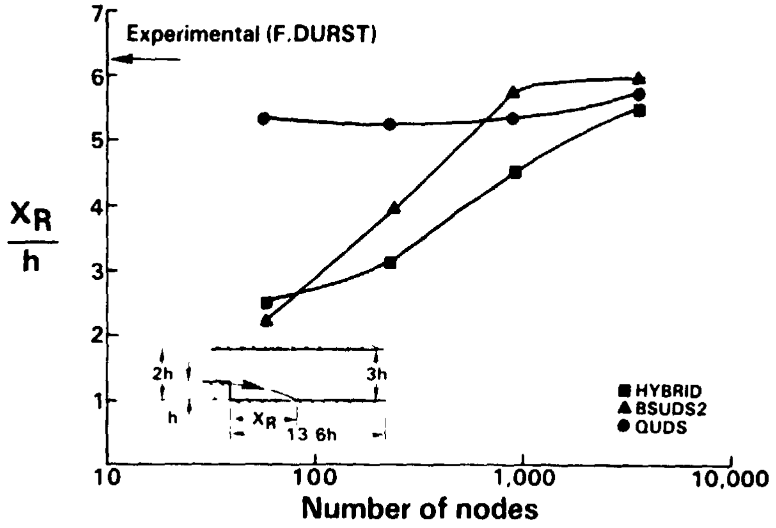
BOUNDED SKEW UPWIND DIFFERENCING SHCEME TWO (BSUDS2)

- Finite volume method
- Central differencing for diffusion terms at all Peclet numbers
- Central differencing for convection terms for absolute Peclet numbers less than two
- Blending of upwind and skewed upwind differencing for Peclet numbers greater than two
- Employs compact nine-point molecule
- Scheme is conservative
- Produces bounded solutions

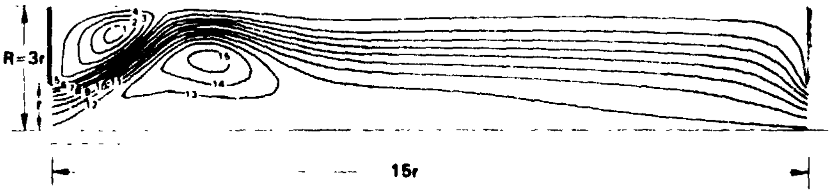


FIRST LAMINAR FLOW TEST CASE

QUDS more accurate over all

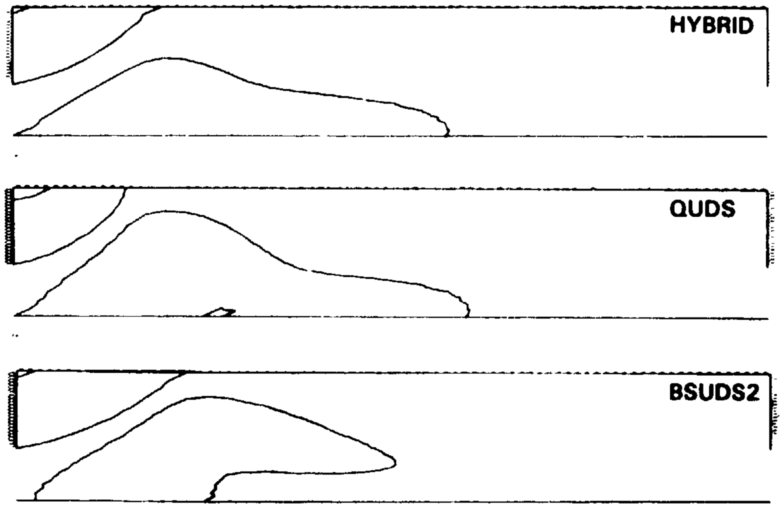


SECOND LAMINAR FLOW TEST CASE

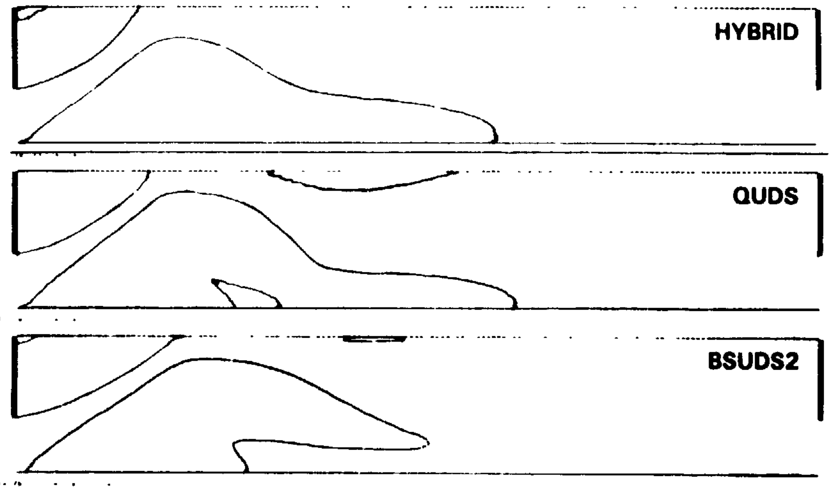


Flow in a sudden expansion with swirl; Re = 450, swirl no. = 0.66, at 45-degree vane angle, contraction ratio = 9.0

STAGNATION STREAMLINES FOR SECOND LAMINAR FLOW TEST CASE ON 40 x 20 MESH

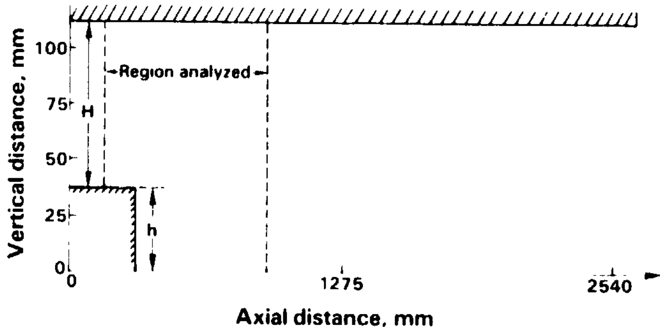


STAGNATION STREAMLINES FOR SECOND LAMINAR FLOW TEST CASE ON 78 x 40 MESH



FIRST TURBULENT TEST CASE

Flow over backward facing step



Inlet boundary conditions

Mean velocity	60.11 ft/sec	(18.32 m/s)
Temperature	59°F	(41.2°C)
Pressure	14.7 psia	(101.35 kN/m^2)
Boundary layer thickness	0.33 inch	(8.4 mm)
Turbulence intensity	0.003	

CALCULATED REATTACHMENT POINTS (MEASURED 7 STEP HEIGHTS)

Case	HYBRID	BSUDS2	QUDS
Dumb (15x17)	2.4	2.25	2.28
Coarse (26x29)	5.2	5.4	5.5
Fine (50x56)	5.7	5.9	Unstable
Fine adjusted (74x53)	5.8	5.8	Unstable

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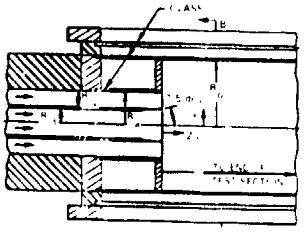
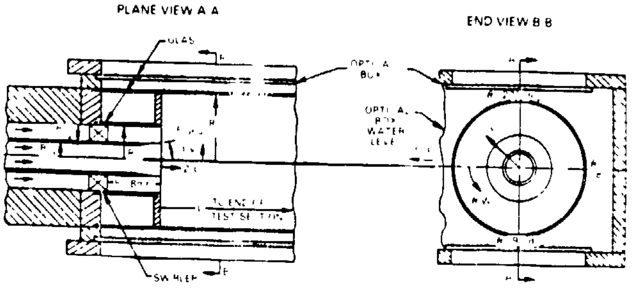
SECOND TURBULENT TEST CASE



30 DEG MEAN ANGLE SWIRL IN ANNULAR INLET DUCT

DIMENSION	R_1	R_2	R_4	R	
LENGTH mm	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
LENGTH	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]



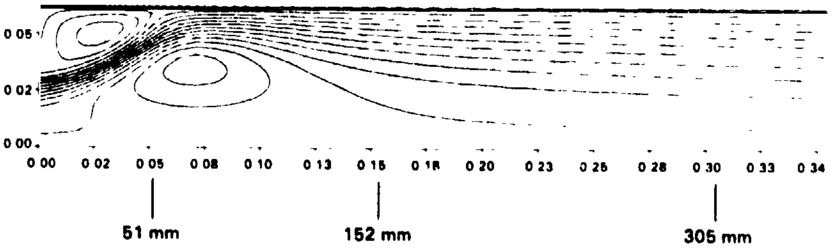
Co axial jets with and without swirl

SECOND TURBULENT TEST CASE WITH SWIRL

Inlet boundary conditions;

- **Measurements at x = 5mm were used**
- **This plane was considered as inlet plane**

SECOND TURBULENT TEST CASE WITH SWIRL

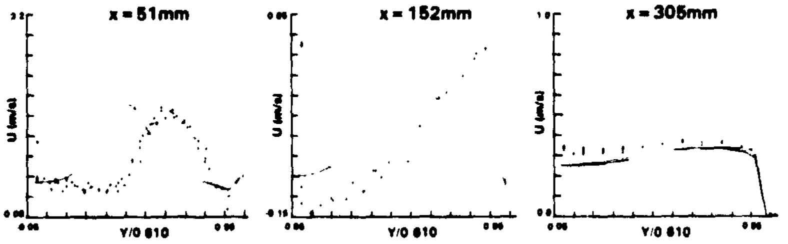


Stations at which comparisons have been made

SECOND TURBULENT TEST CASE WITH SWIRL COMPARISON OF AXIAL VELOCITY PROFILES

Coarse grid (40x35)

HYBRID ————
QUDS — — —
BSUDS2 - - - - - - - -



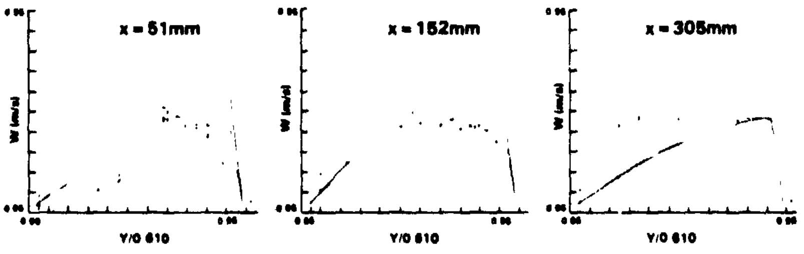
QUDS seems more accurate

No difference in accuracy for fine grid (47x43)

SECOND TURBULENT TEST CASE WITH SWIRL COMPARISON OF TANGENTIAL VELOCITY PROFILES

Coarse grid (40x35)

HYBRID ————
QUDS — — —
BSUDS2 - - - - - - - -



QUDS seems more accurate

No difference in accuracy for fine grid (47x43)

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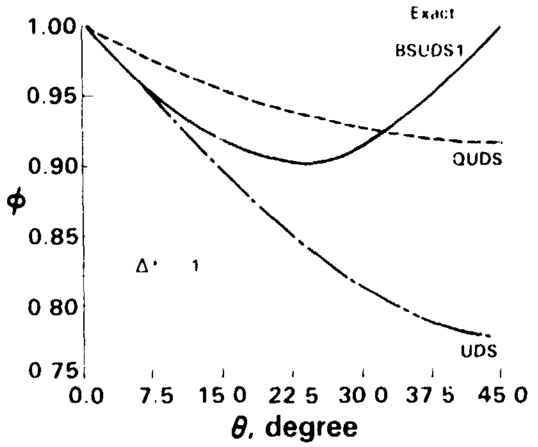
SINGLE CELL CALCULATIONS OF CONVECTION OF NORMAL DISTRIBUTION

$$\phi(x) = \exp(-\tfrac{1}{2}(\eta\Delta^*)^2)$$

$$(\eta = x/\Delta, \Delta^* = \Delta/\sigma)$$



Influence of the flow angle on the error for various difference schemes

OBSERVATIONS

- **QUDS unstable — needs a better solver**
- **No significant differences between HYBRID, QUDS and BSUDS2 for intelligent fine grids (1000-1500 nodes)**
- **QUDS more accurate than BSUDS2 most of the time**
- **BSUDS2 more accurate for second laminar test case — very strong streamline curvature**
- **Flows with swirl more sensitive to difference schemes**