ADAPTIVE CONTROL: MYTHS AND REALITIES

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ADAPTIVE CONTROL: MYTHS AND REALITIES

In recent years, the area of adaptive control has received a great deal of attention by both theoreticians and practitioners. Considerable theoretical progress was made in the design of globally asymptotically stable adaptive algorithms and in unifying different design philosophies under the same mathematical framework. In this vein, it was found that all currently existing globally stable adaptive algorithms have three basic properties in common [1]: (1) positive realness of the error equation, (2) square-integrability of the parameter adjustment law and, (3) need for "sufficient excitation" for asymptotic parameter convergence. Of the three, the first property is of primary importance since it satisfies a sufficient condition for stability of the overall system, which is a baseline design objective. The second property has been instrumental in the proof of asymptotic error convergence to zero, while the third addresses the issue of parameter convergence.

Positive-real error dynamics can be generated only if the relative degree (excess of poles over zeroes) of the process to be controlled is known exactly; this, in turn, implies "perfect modeling." This and other assumptions, such as absence of nonminimum phase plant zeros on which the mathematical arguments are based, do not necessarily reflect properties of real systems. As a result, it is natural to inquire what happens to the designs under less than ideal assumptions. In particular, this paper will be concerned only with the issues arising from violation of the exact modeling assumption which is extremely restrictive in practice and impacts the most important system property, stability.

THEME

A variety of adaptive control algorithms which reflect different philosophical approaches to control system design have been suggested in the literature.

Such algorithms as a rule tend to improve control system performance due to enhanced information obtained while the system is in operation. In addition, in the late 1970's, certain classes of adaptive algorithms representing the majority of those available (among which the self-tuning regulators, model reference adaptive controllers, and the so-called dead-beat algorithms) were proven theoretically to be globally asymptotically stable. In practice, however, such algorithms would almost surely result in unstable physical control systems, as recent research has indicated.

- MYTH

WE HAVE A VARIETY OF ADAPTIVE CONTROL ALGORITHMS THAT

(1) IMPROVE CONTROL SYSTEMS PERFORMANCE

(2) ARE PROVEN TO BE GLOBALLY STABLE

- REALITY

ABOVE ADAPTIVE ALGORITHMS ALMOST SURELY WOULD RESULT IN UNSTABLE PHYSICAL CONTROL SYSTEMS

BASIC PROBLEM

The basic problem behind the apparent inconsistency between theory and practice can be attributed to the fact that existing adaptive control algorithms have been focused almost exclusively upon performance improvement, without due consideration for system robustness as required in the presence of unmodeled dynamics and/or other unstructured modeling errors. In fact, fundamental system concepts, such as operating system bandwidths, have been ignored in the design of adaptive algorithms that tend to invariably adjust the parameters in response to output errors, regardless of their origin. As a result, although at first sight performance seems to be greatly improved, the system bandwidth grows without bound, and eventually the hard constraints imposed by the presence of high-frequency unmodeled dynamics are violated. The final result is violent instability of the controlled system that makes apparent at the same time the nonlinear nature of the overall feedback adaptive loop.

- MODELS HAVE LIMITATIONS; STUPIDITY DOES NOT
- EXISTING ADAPTIVE CONTROL ALGORITHMS HAVE FOCUSED UPON PERFORMANCE IMPROVEMENT
- STABILITY/ROBUSTNESS ISSUE WAS NEGLECTED
    - HIGH-FREQUENCY UNMODELED DYNAMICS IMPOSE HARD LIMIT UPON CONTROL SYSTEM BANDWIDTH
- ADAPTIVE CONTROL SYSTEM BANDWIDTH CAN GROW WITHOUT BOUND
    - PERFORMANCE LOOKS GREAT
    - SYSTEM EVENTUALLY BREAKS INTO VIOLENT INSTABILITY.

ADAPTIVE ALGORITHMS CONSIDERED

The algorithms considered in the present study can be classified into one of the following three categories: (1) model reference adaptive control algorithms, otherwise known as direct adaptive control [2-7]; (2) self-tuning regulators, otherwise known as minimum variance controllers [8-10]; (3) dead-beat algorithms designed for discrete-time systems, otherwise referred to as projection or least-squares algorithms [11].

They all differ in the parameterization of the controller and, hence, the form of the resulting error equations, in the way they synthesize the control input and in the specific realization of the parameter adjustment laws. The common features of the above seemingly fundamentally different algorithms include the assumption of minimum phase plant zeros, the basic signal correlation of the learning mechanism, and the exact knowledge of the plant relative degree. The latter has proven crucial in obtaining global asymptotic stability proofs for all the above-mentioned algorithms.

- COMMON THEME: GLOBAL STABILITY PROOFS AVAILABLE

- MODEL REFERENCE ADAPTIVE CONTROL

  MONOPOLI *ET AL*, NARENDRA *ET AL*, MORSE *ET AL*, LANDAU *ET AL*

- SELF-TUNING CONTROLLERS

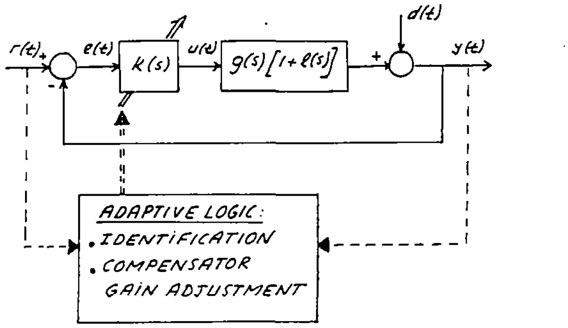
  ASTROM *ET AL*, EGARDT, LANDAU AND SILVIERA

- DEAD-BEAT ADAPTIVE CONTROL

  GOODWIN, RAMADGE, AND CAINES

STRUCTURE OF ADAPTIVE CONTROL

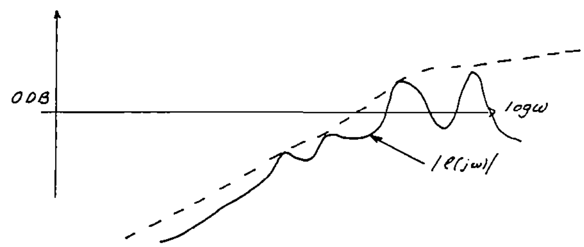
The general structure of an adaptive control system is shown in the first figure below. The term g(s) represents the nominal (low-frequency) plant transfer function, whose parameters are considered unknown. The term K(s) represents the compensator whose parameters are adjusted on-line on the basis of information generated by the adaptive logic block; its basic component is the "learning mechanism" that adjusts the compensator gains either directly or by identifying them first on the basis of the error e(t) and the signals r(t), u(t), and y(t), along with their associated auxiliary state variables. The term $\ell(s)$ represents a multiplicative high-frequency modeling error whose frequency profile is shown in the second figure below. The term $\ell(s)$ has been implicitly assumed to be identically zero at all frequencies in all the algorithms that have been proven to be globally asymptotically stable.



g(s): LOW-FREQUENCY MODEL OF PLANT

ℓ(s): HIGH-FREQUENCY MODELING ERROR

K(s): COMPENSATOR WITH ADJUSTABLE PARAMETERS



INSTABILITY MECHANISMS

When the ideal assumption of exact modeling is violated, at high frequencies, i.e., $\ell(s) \neq 0$, two mechanisms of instability were identified in the algorithms studied [12]. The first is the so-called "phase instability" that arises as a result of high-frequency inputs to the plant. For sufficiently high frequencies, the unmodeled dynamics contribute enough lag so that the total phase shift of the overall loop reaches 180°, at which point the feedback becomes positive and instability occurs. The second mechanism is referred to as "gain instability" and is due to persistent unmeasurable output disturbances and/or nonzero steady-state errors. In this case, the adaptive control system feedback gains keep drifting to increasingly larger values with a resulting increase in bandwidth; as a result, the high-frequency dynamics get excited, and the closed-loop system becomes unstable.

- INSTABILITY DUE TO HIGH-FREQUENCY INPUTS
  - HIGH-FREQUENCY DYNAMICS YIELD $\pm 180°$ PHASE SHIFT

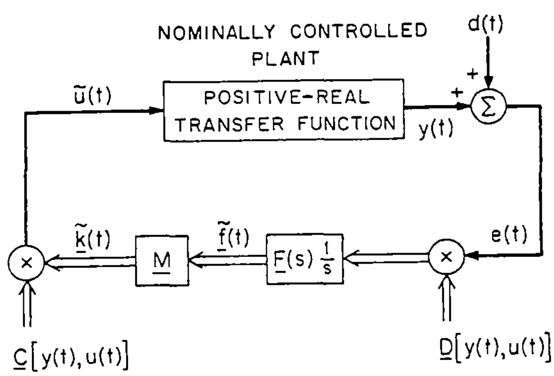
- INSTABILITY DUE TO PERSISTENT UNMEASURABLE OUTPUT DISTURBANCES
  - ADAPTIVE CONTROL SYSTEM FEEDBACK GAINS DRIFT AND GET LARGE
  - CONTROL SYSTEM BANDWIDTH INCREASES
  - HIGH-FREQUENCY DYNAMICS GET EXCITED
  - CLOSED-LOOP SYSTEM BECOMES UNSTABLE

ESSENCE OF PROOFS

The previously discussed error mechanisms can be better visualized in the figure below which is a generic representation of the error system complete with adjustment mechanism in the feedback path. Here C[y(t), u(t)] and D[y(t), u(t)] are linear operators that generate auxiliary state variables through stable filters, F(s) can represent a shaping filter, and M represents a matrix of constants.

The essence of global stability proofs is captured in the figure shown, with a positive-real transfer function in the forward path and a (passive) adaptation mechanism in the feedback [1]. All globally stable adaptive algorithms construct a positive-real function based upon the nominal plant model, which governs the error dynamics [1]. However, the positive-real condition is always violated in real applications due to unmodeled dynamics.

- REF: VALAVANI, PROC. JACC, 1980 (REF. 1)
- ALL GLOBAL STABILITY PROOFS AND ASSOCIATED ALGORITHMS CONSTRUCT A POSITIVE-REAL FUNCTION BASED UPON NOMINAL PLANT MODEL



- PITFALL:

  POSITIVE-REAL CONDITION ALWAYS VIOLATED IN REAL APPLICATIONS

  - CAUSE: UNMODELED HIGH-FREQUENCY DYNAMICS

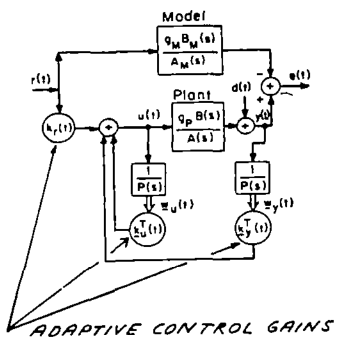
SIMPLEST MODEL REFERENCE ADAPTIVE CONTROL STRUCTURE

For the sake of example, the simplest model reference adaptive control structure is depicted below [3]. The model and plant transfer functions are given respectively by

$$\frac{g_M B_M(s)}{A_M(s)} \quad \text{and} \quad g_p \frac{B(s)}{A(s)}$$

Stable filters $\frac{1}{P(s)}$ generate from u(t) and g(t) auxiliary state variable vectors $\underline{w}_u(t)$ and $\underline{w}_y(t)$, which multiply feedback gain vectors $\underline{k}_u(t)$ and $\underline{k}_y(t)$, as shown. These gains, along with $k_r(t)$, are adjusted according to the equation in the square. The overall scheme looks indeed very simple for real-time implementation!

- STUDIED BY NARENDRA AND VALAVANI



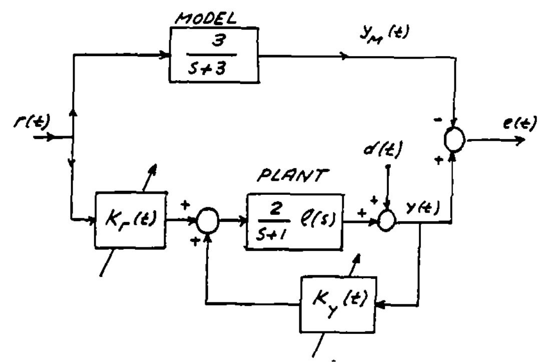
- ADAPTIVE CONTROL GAIN ADJUSTMENT

$$\frac{d}{dt}\,\underline{k}(t) = \underline{\Gamma}\,\underline{w}(t)e(t)$$

$$\underline{\Gamma} = \underline{\Gamma}' \geq 0$$

NUMERICAL EXAMPLE

The simple structure outlined on the previous page was employed for the adaptive control of a nominally first-order plant with a pair of complex poles. The reference model, the nominal plant complete with unmodeled dynamics, and the adjustment laws were as described below. The digital simulation results for a number of different reference input and disturbance combinations corroborate the foregoing discussion.



$\dot{k}_r(t) = \gamma_1 r(t)e(t) \quad \dot{k}_y(t) = \gamma_2 y(t)e(t)$

• UNMODELED DYNAMICS

- $\ell(s) = \ell_1(s) = \dfrac{229}{s^2+30s+229}$

  POLES AT: $s=-15\pm j$

- $\ell(s) = \ell_2(s) = \dfrac{100}{s^2+8s+100}$

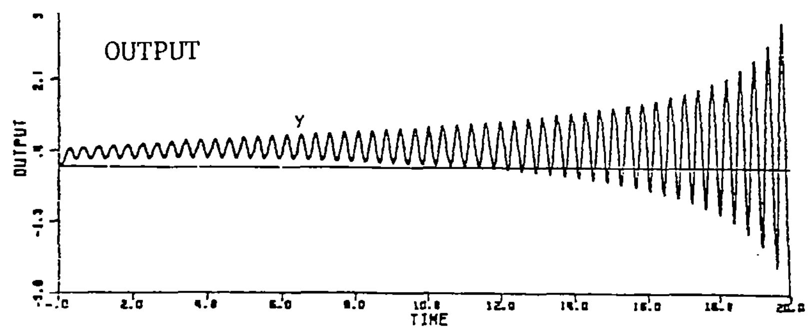
  POLES AT: $s=-4\pm j8.33$

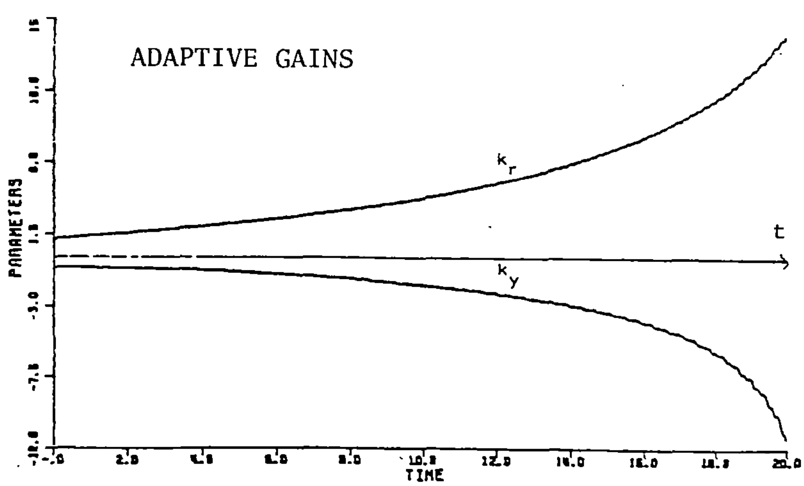
INSTABILITY DUE TO HIGH-FREQUENCY INPUT

The figures below show the plant output and adaptive gain evolution, respectively, when the reference input was chosen to have a d.c. and a sinusoidal component as shown below, in the absence of any output disturbance. The input frequency was precisely the frequency at which the "nominally controlled plant," with unmodeled dynamics $\ell_1(s)$, has 180° phase shift. The output displays an exponential-type oscillatory growth, while the parameters keep drifting and finally diverge.

- DATA: $\ell(s) = \ell_1(s)$

  $r(t)=0.3 + 1.85 \sin 16.1t;\ d(t)=0$



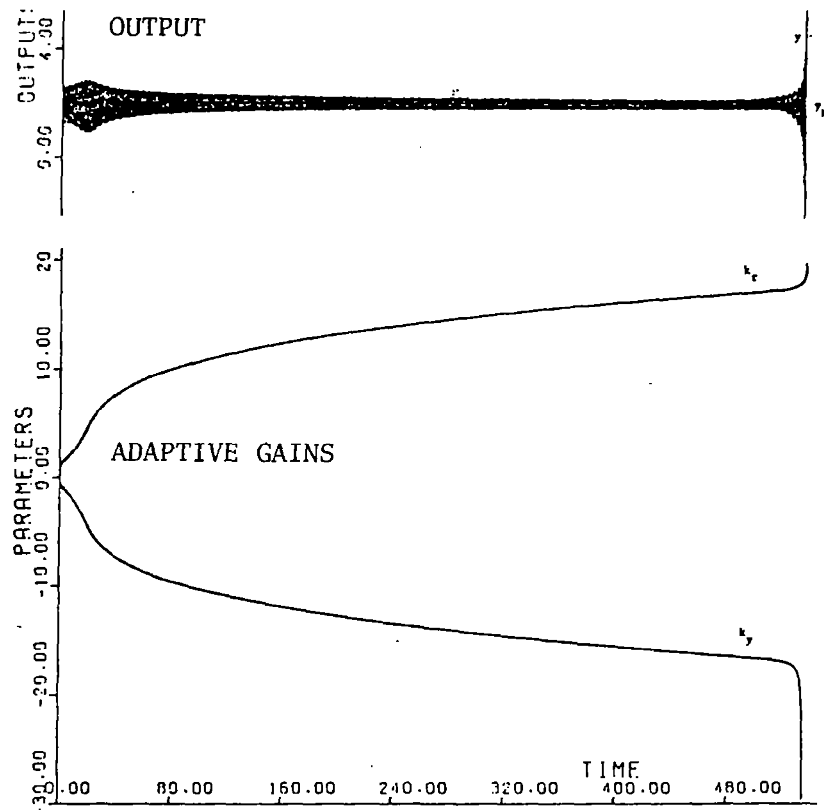


INSTABILITY DUE TO SINUSOIDAL DISTURBANCE

With the same reference model, plant, and unmodeled dynamics, the reference input was now chosen to be a simple d.c. input of amplitude 2 (units); an output disturbance was added to the experiment. The frequency of the disturbance was lower than that which would cause a 180° phase lag. The figure below shows the plant output and parameter evolution, respectively. For a very extended period of time, it looks as if the plant output has converged to within a satisfactory deviation from the desired output; however, the parameters keep drifting. Finally, both plant and parameters break into abrupt instability. This is the gain instability mechanism that is indicative of the nonlinear nature of the overall adaptive loop.

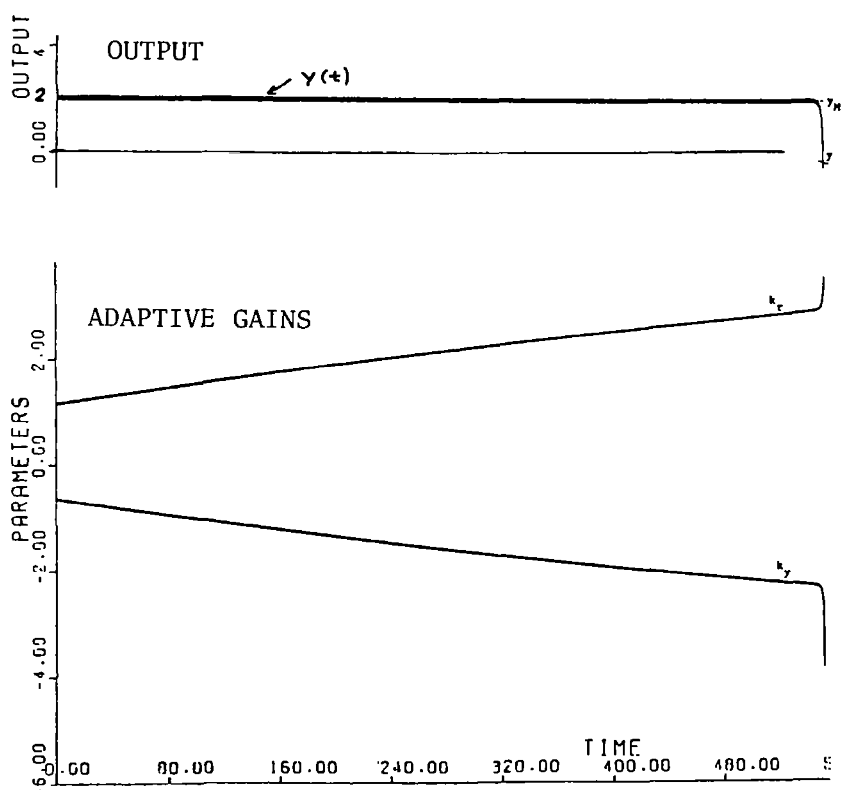
• DATA: $\ell(s) = \ell_1(s)$

$r(t)=2.0 \quad d(t)=0.5 \sin 8t$



In the present experiment, the only difference from the preceding example is that the magnitude of the disturbance is smaller (by a factor of 5). Again, the same trends are observed, although now the plant output deviates much less from the reference model output and the parameter evolution is different in shape and magnitude. However, the final instability comes about in an almost identical manner.

- DATA: $\ell(s) = \ell_1(s)$
- $r(t)=2.0 \quad d(t)=0.1 \sin 8t$

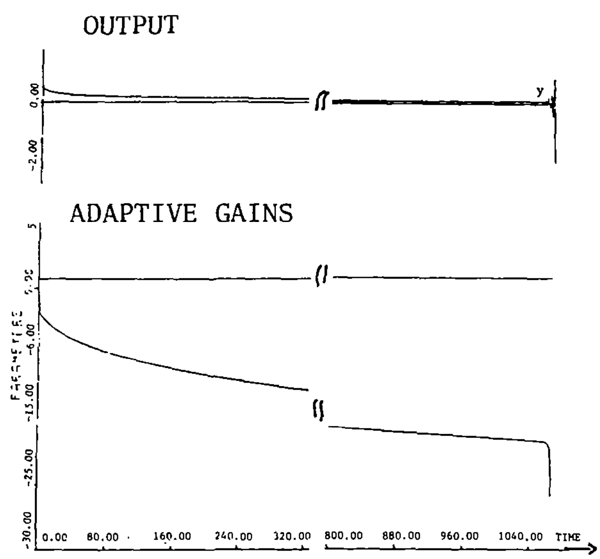


INSTABILITY DUE TO CONSTANT DISTURBANCE

This is a regulator-type experiment for the same setup as before. There is a zero reference input and a d.c. disturbance of amplitude 3 (units). Notice that the output behaves almost ideally for an infinitely long period of time compared to the system time constants. However, after about 1000 seconds some arrhythmia-type behavior is observed, after which point violent instability occurs in both plant output and parameter values. We remark that the parameters have continued to drift while the output was displaying a satisfactory behavior. The fact that the gain instability has taken such a long time to develop may have been a reason why the phenomenon was not discussed in the literature earlier. However, it was indeed observed in some applications to chemical processes [13].

• DATA: $\ell(s) = \ell_1(s)$

$r(t)=0 \quad d(t)=3$



RULES OF THUMB

In general, although the time at which instability occurs may be large, the fact remains that it does occur. Factors that can prolong its onset are increased frequency separation between nominal plant model and unmodeled high-frequency dynamics and decreasing disturbance amplitudes. At present, there is no systematic way to prevent such a phenomenon from occurring. The reason is because the adaptive loop is highly nonlinear and, therefore, rigorous mathematical analysis is not easy. However, certain cures have been proposed, such as the use of dither signals in the reference inputs to stabilize the drifting process, exponential forgetting factors, and dead-zones to prevent the adaptation mechanisms from causing the parameters to drift [14-19]. None of the suggested methods, however, seems to hold general validity at the present time.

- TIME AT WHICH INSTABILITY SHOWS UP CAN BE LARGE
  - BUT SYSTEM IS UNSTABLE
- INSTABILITY TIME INCREASES
  - AS FREQUENCY SEPARATION OF LOW-FREQUENCY MODELED DYNAMICS AND HIGH-FREQUENCY UNMODELED DYNAMICS INCREASES
  - AS DISTURBANCE MAGNITUDE DECREASES
- CERTAIN CURES MAY WORK
  - DITHER SIGNALS
  - EXPONENTIAL FORGETTING
  - DEAD ZONE

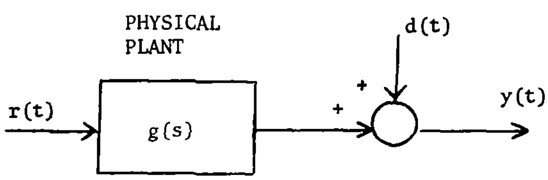
  NOT CLEAR OF GENERAL VALIDITY
- ADAPTIVE LOOP HIGHLY NONLINEAR
  - ANALYSIS NOT EASY

FUTURE RESEARCH DIRECTIONS

Clearly, a lot more research is needed to understand the complex interplay between time-domain and frequency-domain quantities in an on-line adaptation mechanism. Key concepts such as finite-time identification need to be understood and developed further; input constraints and disturbance constraints have to be formulated and taken into consideration as part of the overall design along with a more precise mathematical representation of modeling uncertainty, perhaps in terms of a modeling error $\ell_M^T(\omega)$ as suggested below. To summarize, maximum allowable system bandwidth should be reflected in any design problem either explicitly or implicitly, and the mathematics of any adaptive algorithm should try not to violate the hard constraints imposed by it.

- EXAMINE FUNDAMENTAL ISSUES
  - INTEGRATED TIME-DOMAIN AND FREQUENCY-DOMAIN APPROACH

- FINITE-TIME IDENTIFICATION



$0 \leq t \leq T$

- INPUT CONSTRAINTS: $r(t) \in R$
- DISTURBANCE CONSTRAINTS: $d(t) \in D$

NOMINAL MODEL: $\hat{g}_T(s)$

MODEL ERROR BOUND:

$$\ell_M^T(\omega) > |g(j\omega) - \hat{g}_T(j\omega)|$$

NEED $\ell_M^T(\omega)$ TO LIMIT BANDWIDTH

MYTHS AND REALITIES

In conclusion, adaptive control algorithms as they now stand are deceptively simple and promising; they are proven theoretically stable and they improve performance by overcoming the conservativeness that nonadaptive designs typically have to contend with. Unfortunately, however, the advantages that these algorithms enjoy are based on mathematical assumptions that are always violated in practice. Moreover, the "ideal" properties that they seem to possess are very nonrobust to even subtle violations of the underlying assumptions. Consequently, they may result in unstable systems when applied in real engineering problems.

- MYTH:

  LET US RUSH TO IMPLEMENT ADAPTIVE CONTROLLERS: GOOD PERFORMANCE, PROVEN STABILITY

- REALITY:

  THE MATHEMATICAL ASSUMPTIONS THAT LEAD TO GLOBAL STABILITY PROOFS ALWAYS VIOLATED IN REAL LIFE.

  WATCH OUT: INSTABILITY MAY EVENTUALLY SET IN.

REFERENCES

1. Valavani, L., "Stability and Convergence of Adaptive Algorithms: A Survey and Some New Results," Proceedings of the Joint Automatic Control Conference, Vol. 1, pp. WA2-B, 1980.

2. Monopoli, R. V., "Model Reference Adaptive Control with Augmented Error Signal," IEEE Transactions on Automatic Control, Vol. AC-19, pp. 474-484, October 1974.

3. Narendra, K. S. and Valavani, L. S., "Stable Adaptive Controller Design- Direct Control," IEEE Transactions on Automatic Control, Vol. AC-23, pp. 570-583, August 1978.

4. Feuer, A. and Morse, A. S., "Adaptive Control of Single-Input Single-Output Linear Systems," IEEE Transactions on Automatic Control, Vol. AC-23, pp. 557-570, August 1978.

5. Landau, I. D., "An Extension of a Stability Theorem Applicable to Adaptive Control," IEEE Transactions on Automatic Control, Vol. AC-25, pp. 814-817, August 1980.

6. Narendra K. S., Valavani, L. S. and Lin, Y. H., "Stable Adaptive Controller Design, Part II: Proof of Stability," IEEE Transactions on Automatic Control, Vol. AC-25, pp. 440-448, June 1980.

7. Landau, I. D. and Silveira, H. M., "A Stability Theorem with Applications to Adaptive Control," IEEE Transactions on Automatic Control, Vol. AC-24, pp. 305-312. April 1979.

8. Aström, K. J., Borisson, U., Ljung, L. and B. Wittenmark, "Theory and Applications of Self-Tuning Regulators," Automatica, Vol. 13, No. 5, pp. 457-476, September 1977.

9. Egardt, B., "Unification of Some Continuous-Time Adaptive Control Schemes," IEEE Transactions of Automatic Control, Vol. AC-24, pp. 588-592, August 1979.

10. Kreisselmeier, G., "On Adaptive State Regulation," IEEE Transactions on Automatic Control, Vol. AC-27, pp. 3-16, February 1982.

11. Goodwin, G. G., Ramadge, P. J. and Caines, P. E., "Discrete-Time Multivariable Adaptive Control," IEEE Transactions on Automatic Control, Vol AC-25, pp. 449-456, June 1980.

12. Rohrs, C. E., Adaptive Control in the Presence of Unmodeled Dynamics, Doctoral Dissertation, Dept. of Electrical Engineering and Computer Science, Massachusetts Institute of Technology, August 1982.

13. Dumont, G. and Belanger, P. R., "Successful Industrial Application of Advanced Control Theory to a Chemical Process," IEEE Control Systems Magazine, Vol. 1, No. 1, pp. 12-16, March 1980.

14. Krause, J., Adaptive Control in the Presence of High Frequency Modeling Errors, LIDS-TH-1314, M. S. Thesis, Dept. of Electrical Engineering and Computer Science, Massachusetts Institute of Technology, May 1983.

15. Ioannou, P. and Kokotovic, P., "Singular Perturbations and Robust Redesign of Adaptive Control," Paper presented at the IFAC Workshop on Singular Perturbations and Robustness of Control Systems, Lake Ohrid, Yugoslavia, July 1982.

16. Orlicki, D., Valavani, L. and Athans, M., "Comments on Bounded Error Adaptive Control," LIDS-P-1320, Massachusetts Institute of Technology, August 1983.

17. Peterson, B. and Narendra, K., "Bounded Error Adaptive Control," IEEE Transactions on Automatic Control, Vol. AC-27, No. 6, pp. 1161-1168, December 1982.

18. Anderson, B. D. O., "Exponential Stability of Linear Equations Arising in Adaptive Identification," IEEE Transactions on Automatic Control, Vol. AC-22, No. 1, pp. 83-88, February 1977.

19. Anderson, B. D. O. and Johnstone, R. M., "Adaptive Systems and Time-Varying Plants," International Journal of Control, Vol. 37, No. 2, pp. 367-377, February 1983.