4.6A THE RELATIONSHIP BETWEEN STRENGTH OF TURBULENCE AND BACKSCATTERED RADAR POWER AT HF AND VHF

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ABSTRACT

The formulae relating turbulence and other atmospheric parameters to backscattered power for radar observations are reviewed. The paper considers primarily the case of scatter from turbulent irregularities which have scales corresponding to the range of isotropic, inertial range turbulence, although some brief discussion of the applicability of this assumption is given. A new formula is introduced for the mesosphere which relates ionospheric electron densities to backscattered power.

INTRODUCTION

Discussions and the relationship between the intensity of turbulence and backscattered radar signal strengths have, in recent literature, been largely based upon the Kolmogoroff theory of inertial range isotropic turbulence (e.g., BATCHELOR, 1953; TATARSKI, 1961, 1971). This is not to say, however, that this is the only possible approach. For example, BOOKER and GORDON (1950) and STARAS (1952) adopted an alternative procedure for examination of turbulence (e.g., see review by GAGE and BALSLEY,1980). This second approach has not been as extensively applied as that due to Kolmogoroff, but, as pointed out by GAGE and BALSLEY (1980), it does allow extensions to conditions of anisotropic turbulence. Whether in fact the assumptions of inertial range, isotropic turbulence are valid for the atmosphere is to some extent an unresolved topic. For example the inertial range theory requires high Reynolds numbers (BATCHELOR, 1953, p 116), and Reynolds numbers in the atmosphere tend to be only moderate. Furthermore, observations of turbulence in the stratosphere often show very thin (≈50-200 m thick) well-defined layers of turbulence (e.g., CRANE, 1980). This is not predicted by the Kolmogoroff theory. Nevertheless, it is normally assumed that Kolmogoroff theory still applies within the layer, at scales smaller than the layer thickness. BOLGIANO (1968) has proposed a turbulence model in which thin well-mixed layers of turbulence form, and in this model radio-wave backscatter is not produced by the turbulence within the layer but by discontinuities in refractive index at its top and bottom. The scatter from such discontinuities should be very different in character to turbulent scatter. It should show an aspect sensitivity, with most scatter coming from the vertical, and should have slow fading times. Such "specular reflections" are well known to occur in the stratosphere at VHF (e.g., GAGE and GREEN, 1978; ROTTGER and LIU, 1978), but whether the mechanism proposed by Bolgiano explains these reflections has not been resolved. Other refinements to Kolmogoroff theory have been presented by some authors (e.g., HILL and CLIFFORD, 1978; WEINSTOCK, 1978a).

Despite these potential problems, however, the Kolmogoroff theory of inertial range turbulence appears to model the atmosphere reasonably well in the range of scales for which it is applicable. Therefore this model will be the main one discussed in this short essay.

A short introduction of the formulae of the inertial range theory will first be given, and then it will briefly be shown how these formulae extend to radio-wave scattering. Some discussion on the accuracy of these formulae will then follow.

It will be assumed initially that the radar looks at the atmosphere at an off-zenith angle, so that the role of specular reflectors can be ignored. The complexities introduced by specular scatter will not be discussed in detail; more complete discussions can be found in, for example, HARPER and GORDON (1980), and ROTTGER (1980a,b).

INERTIAL RANGE TURBULENCE

Atmospheric turbulence causes random fluctuations of various atmospheric parameters, such as density, velocity, refractive index, etc. The statistics of the turbulence is usually described using one of these parameters. However, the parameter chosen to describe the fluctuations must be a passive tracer. This means that its statistical properties must not depend on the position in the turbulence patch. For example, density is not a good passive tracer, as displacement of a parcel of air vertically alters its density. This matter was discussed more deeply by TATARSKI (1961), and will also be considered again shortly. Potential temperature is a good tracer. So are the velocity fluctuations.

For the present, let this passive tracer by a scalar, denoted by θ.

There are at least two ways to describe the statistical properties of the turbulence. One way is by means of the structure function, viz

$$D_\theta(\underline{r}) = \langle|\theta(\underline{x}) - \theta(\underline{x}+\underline{r})|^2\rangle. \tag{1}$$

Here, $\underline{x}$ represent a position vector, and $\underline{r}$ a spatial displacement. $\langle\rangle$ represents an average over space and time. It can be shown that for inertial range turbulence,

$$D_\theta(\underline{r}) = C_\theta{}^2 r^{2/3} \tag{2}$$

e.g. TATARSKI (1961), where $C_\theta{}^2$ depends on the intensity of turbulence. The turbulence fluctuations can also be expressed as the Fourier sum of wave numbers $\underline{k} = 2\pi/\Lambda$, Λ being the Fourier scale. Then TATARSKI (1961) showed that the spectrum of fluctuations is

$$\phi_\theta(\underline{k}) = 0.033\ C_\theta{}^2\ |\underline{k}|^{-11/3} \tag{3}$$

A normalization has been chosen such that $\iiint_{\infty}^{\infty} \phi_\theta(\underline{k})d\underline{k} = \langle\theta^2\rangle$.

It can be shown (TATARSKI, 1961) that $C_\theta{}^2$ is related to the outer scale of turbulence, L_o (i.e., the approximate transition scale between the inertial and buoyancy ranges of turbulence) by the relation

$$C_\theta{}^2 = a^2\alpha' L_o{}^{4/3} \left(\frac{d\overline{\theta}}{dz}\right)^2 \tag{4}$$

Here, a is a constant, ≈ 2.8 (e.g., VANZANDT et al., 1978), α' is a constant which is approximately 1., and $(d\overline{\theta}/dz)$ is the gradient of the mean quantity $\overline{\theta}$.

The formulae (3) and (4) form the basis of theories which relate backscattered radar power to turbulence. However, before discussing how this is done, some other spectral forms should be briefly discussed. It is important to note that the spectral form shown in (3) is not the only form which appears in the literature. It is the full three-dimensional spectrum. But at times the spectrum of wave numbers with magnitude $k = |\underline{k}|$ is given viz

$$E_\theta(k) = 4\pi k^2\phi_\theta(\underline{k}) = 0.132\pi\ C_\theta\ k^{-5/3}. \tag{5}$$

No distinction between the directions of the $\underline{k}$ vectors is made in this formula. Another very important spectrum is the spectrum of fluctuations which would be seen by a probe moving in a straight line through the turbulence. This is not the same as (3), since that only looks at 1 scale direction. But for a probe, all scales produce an effect along the path of the probe, but their "effective scales" change because they are not all orientated along the probe path. Then this produces a spectrum

$$S_\theta(k) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \phi_\theta(\underline{k})\,dk_y\,dk_z \qquad (6)$$

whence $S_\theta(k) \approx 0.25\ C_\theta{}^2 k^{-5/3}$ (7)

OTTERSTEN (1969) has emphasized the difference between (3) and (7), and pointed out that (3) is applicable for radar experiments, whilst (7) is applicable for in situ measurements. The two expressions should not be confused.

POTENTIAL REFRACTIVE INDEX GRADIENT

In considering backscatter of radio waves from the atmosphere, it is of course necessary to look at the fluctuations in refractive index caused by the turbulence. The refractive index n of air at centimetre and metre wavelengths is (TATARSKI, 1961).

$$n = 1 + 10^{-6} \times (79/T) \times (p + 4800\ e/T) \qquad (8)$$

where T is temperature (absolute), p is pressure (millibars) and e is the water vapour pressure. However, it is more convenient to express n as a function of the potential temperature H and the specific humidity, q [$= e/(1.62p)$]. Then

$$n = n(p,\ H,\ q) \qquad (9)$$

But n here is not a good passive tracer. This can be seen by the following consideration. Suppose an eddy moves from height z_1 to a height z_2. Suppose that at z_1 the eddy was in equilibrium with its environment, and that at this height there was pressure p_1, potential temperature H_1 and specific humidity q_1. Suppose that at height z_2, the atmospheric pressure is p_2, and the environmental H, q are H_2 and q_2. However, at z_2 the parcel itself has $H = H_1$ and $q = q_1$, since it is assumed to have moved adiabatically. Of course the pressure in the parcel is now p_2. Hence the difference in refractive index between the parcel and its environment at z_2 is

$$\Delta n = \underbrace{n(z_2,\ p_2,\ H_1,\ q_1)}_{\text{parcel}} - \underbrace{n(z_2,\ p_2,\ H_2,\ q_2)}_{\text{environment}}$$

$$\Delta n = \left(\frac{\partial n}{\partial H}\frac{\partial H}{\partial z} + \frac{\partial n}{\partial q}\frac{\partial q}{\partial z}\right)\Delta z \qquad (10)$$

where $\Delta z = z_2 - z_1$.

This is not simply the difference in refractive index at heights z_1 and z_2, which would be

$$\Delta n = \left[\frac{\partial n}{\partial p}\frac{\partial p}{\partial z} + \left(\frac{\partial n}{\partial H}\frac{\partial H}{\partial z} + \frac{\partial n}{\partial q}\frac{\partial q}{\partial z}\right)\right]\Delta z \qquad (11)$$

The formula discussed in the previous section can be applied for refractive index with $\theta = n$, but in equation (4), the term $(d\bar{\theta}/dz)$ should not simply be (dn/dz) as given by (11), but rather, from (10),

$$\frac{d\bar{n}}{dz} = \left(\frac{\partial n}{\partial H}\frac{\partial H}{\partial z} + \frac{\partial n}{\partial q}\frac{\partial q}{\partial z}\right) \tag{12}$$

This quantity is often denoted by M, and is called the generalized potential refractive index gradient.

For metre and centimetre scatter from the un-ionized atmosphere, (TATARSKI, 1961),

$$M = \frac{-79\times10^{-6}p}{T^2} \cdot \left(1 + \frac{15{,}500\,q}{T}\right)\left(\frac{dT}{dz} + \Gamma_a - \frac{7800}{\left(1+\frac{15{,}500}{T}\right)} \cdot \frac{dq}{dz}\right) \tag{13}$$

The term Γ_a is the adiabatic lapse rate.

In the stratosphere and mesosphere, $q = 0$. However, once heights of greater than 50-60 km are reached, scatter from turbulence is enhanced by the existence of free electrons (ionization), and in this case M needs modification.

For these circumstances

$$n = n(N,\ \nu_m) \tag{14}$$

Where N is the electron density, and ν_m is the collision frequency of electrons with neutral particles. (Pressure and temperature fluctuations also produce weak changes in n, as for the troposphere and stratosphere, but these effects are very weak compared to the effects of N and ν_m, and so can be ignored).

HOCKING (1980, 1981) has shown that the appropriate generalized refractive index gradient for the ionospheric D region is given approximately by

$$M_e = \frac{\partial n}{\partial N} \cdot \left[\frac{N}{T}\left(\frac{dT}{dz} + \Gamma_a\right) - \frac{dN}{dz} + \frac{N}{\rho}\frac{d\rho}{dz}\right] + \frac{\partial n}{\partial \nu_m}\frac{\partial \nu_m}{\partial z}, \tag{15}$$

where ρ is the neutral air density.
For the region 0-120 km, this equals

$$M_e = \frac{\partial n}{\partial N}\left[\frac{N}{T}\left(\frac{dT}{dz} + \Gamma_a\right) - \frac{dN}{dz} - (1.4\times10^{-4})N\right] + \frac{\partial n}{\partial \nu_m}\frac{\partial \nu_m}{\partial z} \tag{16}$$

At VHF in the D region n is related quite simply to N by the relation

$$n^2 = 1 - \pi^{-1} r_e N \lambda^2$$

where r_e is the classical electron radius, and λ is the radar wavelength, so $\partial n/\partial \nu_m = 0$, and

$$\frac{\partial n}{\partial N} = \frac{1}{2}\pi^{-1} r_e \lambda^2.$$

At HF and MF, the relation between n and N is more complex (e.g., BUDDEN, 1965). Thus M_e depends on the potential temperature gradient, the electron density gradient, and the neutral atmospheric density gradient.

In deriving (15), it was assumed that when a parcel of ionosphere is displaced, the ratio of electron density to neutral density remains constant, and that no change in the photochemical reaction rates occurs during such a displacement. HILL and BOWHILL (1979) have suggested that this might not be exactly true, but nevertheless equations (15) and (16) should provide a reasonable estimate of M_e.

SCALES OF THE INERTIAL RANGE

Before proceeding to show how these turbulence formulae relate to radar backscatter, it is important to illustrate over what scales they can be applied.

At very small scales, the kinetic energy density contained by the eddies is diminished due to viscous effects, and much of the turbulent energy is dissipated as heat. This small scale range is often called the "viscous range". At very large scales, buoyancy effects become important, and turbulent eddies taken on a "pan-cake"-like appearance, with horizontal scales much larger than their vertical dimensions.

An important scale for determining the boundary of the inertial-range to viscous range transition is the Kolmogoroff microscale, defined by

$$\eta = (\nu^3/\varepsilon)^{1/4}. \tag{17}$$

Here, ν is the kinematic viscosity, and ε is the turbulent energy dissipation rate. This is a scale well within the viscous range. The scale

$$\ell_o = 7.4\eta \tag{18}$$

is known as the "inner scale" (e.g., HILL and CLIFFORD, 1978) and defines the approximate transition scales between the inertial and viscous ranges. (The constant 7.4 is only relevant for air.)

The scale for determining the transition region between the inertial and buoyancy ranges is (WEINSTOCK, 1978b)

$$L_B = (2\pi/0.62)\ \varepsilon^{1/2}\ \omega_B^{-3/2}, \tag{19}$$

where ω_B is the Brunt-Vaisala period of the atmosphere at the height of the turbulence. This L_B should not be confused with L_o in (4): they are different parameters, as will be seen later.

The inertial range of turbulence strictly only applies for scales somewhat less than L_B and larger than ℓ_o.

Approximate values of L_B and ℓ_o are shown in Figure 1. For a radar wavelength λ, backscatter occurs for scales of $\lambda/2$. Thus if a 50 MHz radar is used (λ = 6 m), then scatter should be possible from isotropic inertial range turbulence up to altitudes of about 65-70 km. Figures similar to Figure 1 have appeared elsewhere in the literature (e.g., GAGE and BALSLEY, 1980), and show similar values for η and ℓ_o.

RADIO-WAVE BACKSCATTER

Having illustrated some appropriate formulae for relating refractive index fluctuations to turbulence parameters, it is now necessary to determine how these formulae relate to radio-wave backscatter.

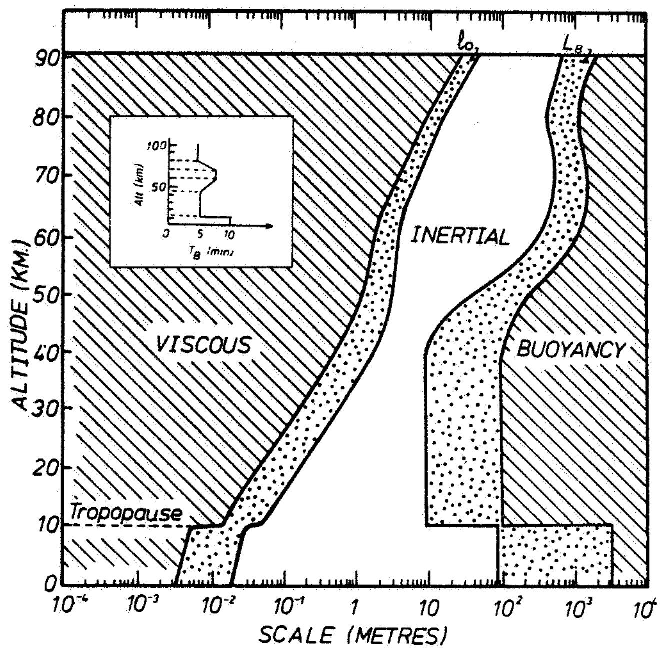

Figure 1. Typical inner and outer scales (ℓ_o and L_B respectively) for inertial-range turbulence in the atmosphere. The formulae used are given in the text. The profile of the Brunt-Vaisala period is also shown. It was assumed that the mean value of ε was 10^{-1} W kg^{-1} at 90 km, decreasing exponentially to 10^{-2} W kg^{-1} at 80 km. Between 80 and 60 km, this mean was taken at 10^{-2} W kg^{-1}. In the region 60-90 km, the bounds of the dotted areas correspond to turbulent energy dissipation rates of 1/3 rd and 3 times these mean values. Below 40 km, and downto the tropopause, the upper and lower limits of ε were taken as 10^{-3} and 10^{-5} W kg^{-1}. L_B and ℓ_o were assumed to vary smoothly between 40 and 60 km. (The region between 30 and 80 km is perhaps the most uncertain part of the graph.) Below the tropopause, ε was assumed to be limited between 10^{-4} and 10^{-1} W kg^{-1}. Larger ε values correspond to smaller ℓ_o and larger L_B values (i.e. the inertial range widen at both ends, when ε increases).

BOOKER (1956) has shown that the power backscattered from refractive index fluctuations with mean square value $N(\underline{k})d\underline{k}$ at scales $\Lambda = 2\pi/k$, per unit solid angle, per unit incident power density, and per unit volume (i.e., the cross section of backscatter) is

$$\sigma = [4\,\pi^2/\Lambda^2]N(\underline{k}). \tag{20}$$

(this expression is true at VHF, but at lower frequencies may not be) $P(\underline{k})$ is similar to $\phi_n(k)$ in equation (3) (θ=n), but BOOKER (1955) used the normalization

$$(2\pi)^{-3} \iiint_{-\infty}^{\infty} N(\underline{k})dk_1\, dk_2\, dk_3 = |\overline{\Delta n}|^2$$

Hence $N(\underline{k}) = (2\pi)^3\, \phi_n(\underline{k})$. (21)

For radar backscatter at wavelength λ, $k = 4\pi\, \lambda^{-1}$, and so using (21) and (29), and using (3) for $\phi_n(\underline{k})$,

$$\sigma = .00654\pi^{4/3}\, C_n^{\,2}\, \lambda^{-1/3} \tag{22}$$

Sometimes an alternative definition of backscatter cross section is used. This is the total power which would be scattered if power were scattered isotropically with an intensity equal to that of the backscattered radiation, per per unit incident power density, per unit volume of scatterer. It is often denoted by η, and although this can at times be confused with the Kolmogoroff microscale, this convention will be maintained here. Then

$$\eta = 4\pi\ \sigma, \tag{23}$$

and hence

$$\eta = 0.38 C_n^{\ 2}\ \lambda^{-1/3}, \tag{24}$$

as also derived by OTTERSTEN (1969). It should be noted that, at least for the ionosphere, the wavelength dependence for η is more complex than $\lambda^{-1/3}$, since $C_n^{\ 2}$ is itself a function of wavelength. This can be seen by considering a given patch of turbulence in the ionosphere. For this patch, there is a constant electron density structure constant, which we may denote by $C_N^{\ 2}$. Then $C_N^{\ 2}$ $= (\partial n/\partial N)^2\, C_N^{\ 2}$ (e.g., HOCKING and VINCENT, 1982), and $(\partial n/\partial N)$ is strongly wavelength dependent, as has already been seen.

Now it is necessary to show how σ (or η) relates to the power received in a backscatter experiment. Consider scatter from a height h. Then the peak power per unit area incident at h is

$$P_h = (P_T G_T e_T)/(4\pi h^2), \tag{25}$$

where P_T is the transmitter peak power, G_T is the transmitter array directivity and ε_T is the transmitter efficiency. To obtain the power backscattered per unit steradian, we simply multiply this by σV, where V (the "radar volume") is the volume defined by the locus of the half-power points of the radar, and the pulse length. The receiving array subtends a solid angle A_R/h^2 to this scattering region at height h, so the peak power received by the receiver is (ignoring absorption)

$$P_R = \frac{P_T\ G_T\ V\sigma\ e_T\ e_R\ A_R}{4\pi\ h^4} \tag{26}$$

where e_R is the receiver efficiency and A_R is the effective area of the receiving array.

Thus by (22) and (26),

$$C_n^{\ 2} = \frac{16\pi^2\ P_R\ h^4\ \lambda^{1/3}}{0.38V\ P_T\ G_T\ e_T\ e_R\ A_R} \tag{27}$$

In the case of a circularly symmetric array, $V \simeq \pi(h\theta_{1/2})^2 \cdot L$, where L is the pulse-length and $\theta_{1/2}$ is the half-power-half-width of the polar diagram. If the same array is used for transmission and reception, $e_T = e_R$, and $A_R = G_T\lambda^2/4\pi$, so

$$C_n^{\ 2} = \frac{1662.25\ P_R\ h^2\ \lambda^{-5/3}}{P_T\ G_T^{\ 2}\ e_T^{\ 2}\ \theta_{1/2}^{\ 2}\ L} \tag{28}$$

For a radar, it can be shown that $G_T\ \theta_h^{\ 2} \approx \pi^2/4$, where θ_h is the half-power-half-width. For the case in which the same radar is used for transmission and reception, the effective half-power width reduces by $\sqrt{2}$, so $\theta_{1/2}=\theta_h/\sqrt{2}$, and $G_T\ \theta_{1/2}^2 = \pi^2/8$. Then

$$C_n^{\ 2} = \frac{128\ \lambda^{1/3}\ h^2\ P_R}{0.38\pi\ e^{\ 2}\ A\ \ L\ P} \qquad (29)$$

Either (30) or (31) can be used to estimate $C_n^{\ 2}$ from absolute measurements of received power. This has been done, for example, by NASTROM et al. (1982) and GOOD et al. (1982).

APPLICATION OF THE FORMULAE IN THE REAL ATMOSPHERE

Clearly (28) or (29) can be used to estimate $C_n^{\ 2}$ with a radar, but unfortunately this does not give the structure constant C_n^2 for the turbulence itself. It would, if the turbulence filled the radar volume, but in reality turbulence appears to occur in thin horizontal layers, with depths of 10s to 100s of meters (e.g., VANZANDT et al., 1978; CRANE, 1980; WEINSTOCK, 1981, and references therein). Thus the scattering within the radar volume is usually from a few thin turbulent layers, and the effective volume V should not be $\pi h^2\theta_{1/2}^{\ 2}L$ as proposed earlier, but $(\pi h^2\theta_{1/2}^{\ 2}L)\cdot F$, where F represents the fraction of volume within the radar volume which is filled with turbulence. VANZANDT et al. (1978) obtained a formula enabling F to be determined from a knowledge of the mean wind shear (taken with a resolution of about a kilometer or so), the standard deviation of the fine-scale shear, and a "critical wind shear" S_c. Then, if $\overline{C_n^2}$(turb) is the refractive index structure constant for the turbulence, and $\overline{C_n^2}$(radar) is the value measured by the radar,

$$\overline{C_n^{\ 2}}(\text{turb}) = \overline{C_n^{\ 2}}(\text{radar})/F. \qquad (30)$$

VANZANDT et al.(1978) used meteorological data to estimate M (equation 13) and then applied (4) and (30) to estimate $\overline{C_n^2}$(radar). They found that with a value of L_o equal to 10 m, good agreement occurred between the model and radar observations, particularly in the stratosphere. Agreement was not so good in the lower troposphere, because the model did not account for humidity fluctuations.

VANZANDT et al. (1981) improved the theory of VANZANDT et al. (1978) by considering these humidity fluctuations, by considering small-scale fluctuation in potential temperature, by letting the layer thicknesses be non-constant, and also by using a more realistic distribution for the wind shears.

VANZANDT et al. (1978) compared $\overline{C_n^{\ 2}}$(radar) estimates from their model to experimental radar values, assuming L_o= 10 m. GAGE et al. (1980) applied this principle in reverse, using radar estimates of C_n^2 to effectively estimate L_o (through equation 14). They then drew on an equation relating L_o and the turbulent energy dissipation rate ε, to estimate ε. This relation was (TATARSKI, 1961)

$$\varepsilon = b\ S^{3/2}\ L_o^{\ 2} \qquad (31)$$

where b was taken as a constant equal to unity, and $S = (d\bar{u}/dz)^2$ is the shear in the mean wind. By replacing S with ω^2B/R_i, where R_i is the Richardson number and ω_B is the Brunt-Vaisala frequency, and assuming that turbulence exists if $R_i = R_i$(crit), (a critical value), they obtained, using $L_o$from (4),

$$\varepsilon_{turb} = [C_n^{\ 2}(\text{turb}) \cdot (a^2\alpha' R_i(\text{crit})\ \omega_{\bar{B}}^{\ 2}\ M^2)^{-1}]^{3/2}. \qquad (32)$$

They took $R_{i(crit)}$ = 1/4. Then ε_{turb} is the mean turbulent energy dissipation rate. GAGE et al. (1980) also calculated a quantity which they denoted by $\bar{\varepsilon}$, which was the mean turbulent energy dissipation rate averaged over the radar volume. They took

$$\bar{\varepsilon} = F^{-1}\varepsilon_{turb} \tag{33}$$

From radar measurements GAGE et al. (1980) estimated $\overline{C_n^2}$(radar). Then they made some reasonable assumptions concerning F, and so were able to estimate ε_{turb} and $\bar{\varepsilon}$ from their radar data. M was calculated from meteorological measurements of T and p, and it was assumed that the humidity terms in M were unimportant. It should also be noted that F is dependent on ω_B, although this may not be obvious in the simplified discussion given above. This dependence of F on ω_B can cause some problems in estimating F, but GAGE et al. (1980) were careful to reduce this error as much as was reasonable.

The technique outlined above is, at least in principle, the primary means by which ε is obtained for the atmosphere using VHF radars. Variations on the details of these formulae have been presented (e.g., CRANE, 1980; WEINSTOCK, 1981), but the principle remains similar - namely, to determine the fraction of the radar volume actually filled by turbulence, and then to correct C_n^2 values measured by the radar to give ε_{turb} and $\bar{\varepsilon}$.

In their model calculations, VANZANDT et al. (1978) chose $L_o = 10$ m. It should be noted that L_o is not equivalent to L_B in (19). If $S = \omega_B^2/R$ is substituted in (31), as was proposed, then

$$L_o = 0.35\ \varepsilon^{1/2}\ \omega_B^{-3/2} \tag{34}$$

and comparison with (19) shows that

$$L_o = .035\ L_B \tag{35}$$

The difference arises because of the different definition used to define these "outer scales". L_B is probably a better measure of the transition scale between the inertial and buoyancy subranges, but L_o is quite appropriate wherever the formulae of TATARSKI (1961) are applied. This of course means that (31) and (4) are only applicable for L_o as defined by TATARSKI (1961). The choice of $L_o = 10$ m used by VANZANDT et al. (1978) corresponds to a choice of L_B of about 290 m. WEINSTOCK (1981) developed his theory relating ε and C_n^2(radar) using L_B as an estimate of the sum of the thicknesses of the turbulent layers in the radar volume, and achieved numerical results similar (to within a factor of 2) to those of GAGE et al. (1980).

It is also possible to apply (32) for the mesosphere, using M_e (equation 16) in place of M. However, there are some problems in estimating F for this case. For example, CZECHOWSKY et al. (1979), using a 150 m resolution radar, have shown that at mesospheric altitudes of ≈80 km, the scattering layers can be quite thick (up to ≈1 km) and so F may approach unity. Further, at VHF the appropriate scattering scales may be within the viscous range, so (3) and hence (29) may not be applicable. At HF and MF radar wavelengths (e.g., $\lambda = 150$ m), however, scatter should be from the inertial range and these formulae should be appropriate.

DISCUSSION

Interestingly, RASTOGI and BOWHILL (1976) presented some formulae relating turbulence parameters to backscattered power, and concluded that for the mesosphere the backscattered power was independent of ε. They based their conclusions on dimensional arguments. However, these arguments were nowhere near as rigorous as those presented in this paper, and it is felt that (32) more appropriately represents the relation between ε and C_n^2. The appropriate generalized refractive index gradient can be obtained from (13) (troposphere and stratosphere) or (16) (mesosphere), provided that scatter can be assumed to be from within the inertial range of turbulence. It will be noted that (32) is indeed dimensionally correct, since C_n^2 has units of $m^{-2/3}$.

Direct, independent measurements of ε and C_n^2(radar) have not been extensively made, so it is difficult to comment on the validity of these theories. Certainly, however, the estimates of ε presented by GAGE et al. (1980) are of the correct order of magnitude.

Recently, HOCKING (1983a,b) has presented an alternative method for measurement of turbulent energy dissipation rates with radars. This utilizes not the signal strength backscattered, but the spectral widths of the received signal. The principle of the method has been known for many years (e.g., ATLAS, 1964; FRISCH and CLIFFORD, 1974; FRISCH and STRAUCH, 1976) but the major advance presented by HOCKING (1983a,b) was the accurate removal of both (i) spectral broadening due to the motion of the mean wind across the finite beamwidth and (ii) spectral "broadening" (or "narrowing" in some cases) due to vertical wind shears in the horizontal wind. These two factors have previously been considered separately (e.g., ATLAS, 1964), but never coherently. HOCKING (1983a) also illustrated that there was a necessity to distinguish between vertical and horizontal fluctuating motions, and showed how this could be done. This technique was illustrated using an HF radar to measure energy dissipation rates in the mesosphere.

The technique can readily be applied at VHF, and the author is currently doing this with the "SOUSY" radar (ROTTGER et al., 1978) in West Germany. The estimates of ε appear to be of the correct order of magnitude, and will be reported separately in a later paper. Application of this method can allow independent comparisons of ε and C_n^2(radar), and therefore can check the equation (32). This new method of obtaining involves less assumptions than (32), and may prove to be a more reliable method in the future. Previously, some authors made comparisons of signal fading time and received power (e.g., FUKAO et al., 1980a,b; RASTOGI and BOWHILL, 1976b), but it is difficult to decide how much the fading time (or equivalently the spectral width) is contaminated be beam- and wind-shear broadening. Therefore these measurements cannot really be regarded as comparison of ε and C_n^2.

For approximate estimates of the effects of beam-broadening, the following formula may be useful. If $\theta_{1/2}$ is the half-power-half-width of the effective radar beam, and V is the mean velocity of the scatterers tangential to the beam (usually this amounts to the horizontal velocity), then the half-power spectral half-width due to beam broadening is

$$f_{1/2} \approx (1.0) 2/\lambda \ \theta_{1/2} V \tag{36}$$

This is very nearly exact, provided beam widths of less than 3° - 4° are used. A similar formula was presented by ATLAS (1964), and was originally derived by HITSCHFELD and DENNIS (1956). [Atlas gives an equation $\sigma_v = 0.3\ \theta V$. However, this equation is for the case in which the same radar is used for transmission and reception, and θ is the half-power-full-width for the transmitter (or receiving) polar diagram only. Thus θ in this equation is equal to $2\sqrt{2}$ times $\theta_{1/2}$ in (36), since $\theta_{1/2}$ there is the half-width for the effective polar diagram (transmitter and receiver polar diagram included).] For proper removal of beam-broadening and shear-broadening, however, the complete treatment presented by HOCKING (1983a) is recommended.

As discussed in the introduction, there may be problems with the assumption of inertial range turbulence, and it is useful to list some of these. Specular reflection has already been mentioned, and the cause of this has not been fully explained. It is sometimes assumed to be a process separate from turbulence, but this may not be. For example, the model proposed by BOLGIANO (1968), which was discussed earlier, may be important. In this case, tilting the radar beam from the vertical may cause the layer to disappear, since one of the assumptions of Bolgiano's model was that turbulence mixes the layer so well that no parameters such as density vary with height within the layer. Thus the generalized refractive index gradient within the layer is close to zero, and very little radio-wave scatter from the turbulence itself can occur. If such layers do exist, and are not seen by tilted VHF radars, this could lead to biases in estimates of $\bar{\varepsilon}$ for the atmosphere. For vertically beamed radars, the relation between the specular scatter and the degree of turbulence may not be simple. Investigations of this matter await more experiments. The possibility that turbulence could be anisotropic even at scales of meters has also been briefly mentioned. Multifrequency experiments may help resolve some of these issues.

CONCLUSIONS

If it is assumed that radio-wave scatter is from inertial range turbulence, then the back-scattered power and the energy dissipation rate can be simply related through equations (28) (or 29), (30), (32), (16), (19) and (33). The derivation of these equations assumed inertial range isotropic turbulence, and the scales within which this is probably true are indicated in Figure 1. Considerable experimental work remains to be done to determine when these relations are valid, and when they break down.

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