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NON-GUASSIAN STATISTICAL MODELS OF SURFACE WAVE FIELDS FOR REMOTE SENSING APPLICATIONS

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Based on the complete Stokes wave model with the bias term and using a simple mapping approach and an iteration solution method, we established a formula for the joint probability density fuction of the surface slope-elevation of a nonlinear random wave field. The formula requires three parameters to define the whole density function: the rms surface elevation and slope values and the significant slope. This model represents the dynamics of the wave in a more direct way than the Gram-Charlier approximation. Based on this new statistical model and laboratory experiments, formula and numerical values of EM bias and dynamics bias are derived. The results inidicate that various biases should be considered seriously if accuracy of the altimeter measurement is required in centimeter range.

1. INTRODUCTION

Wind waves are always random. The randomness is the result of the generating forces as well as the consequence of the dynamic processes in wave evolution which induce different kinds of instabilities. Consequently, the only meaningful descriptions of the wind wave fields are the various statistical measures; among them the probability density functions of the surface elevation and slope are the most basic ones. With the recent development of active microwave remote sensing the techniques, the need for the statistical description of the ocean becomes more urgent, for the return signals of the radars are various convolution of radar signals and the ocean surface and these return signals are our only information sources. Successful extraction of the geogphysical parameter, therefore, depends critically on our knowledge of the ocean surface statistical properties. For the remote sensing applications, the most important statistical measure is the joint probability density function of the slope and elevation of the ocean surface.

This specific probability density function is necessary for evaluating the backscattering equation for a near nadir-looking radar as stated by Barrick (1972), Brown (1978), Valenzuela (1978), and Jackson (1979). Unfortunately, due to the nonlinear nature of the ocean waves, a realistic functional form has been elusive. Faced with this difficulty, past investigators have adopted a simple model that assumes all the statistical processes to be Gaussian and joint Gaussian. With this assumption, the statistical properties of the ocean surface become totally independent of the controlling dynamic processes. Consequently, the treatment of the ocean surface statistics is no different from any other random Gaussian process such as the noise of electronic circuitry. Indeed, most of the present statistical results used in describing the ocean surface can be traced to the classical set of papers by S. O. Rice (1944, 1945 and 1948), with certain extensions into two dimensions by Longuet-Higgins (1957, 1963, for example). Because the randomness of the ocean surface wave is the consequence of the generating forces as well as the controlling nonlinear dynamic processes which include various types of instabilities (see, for example, Phillips, 1977 and Yuen and Lake, 1982), the resulting surface geometry can not be adequately described by a linear superposition of statistically independent events. Thus the statistical process can not be Gaussian. Therefore, any results based on a Guassian assumption, though perhaps useful for some qualitative purposes, do not satisfy the need of our present and future engineering and remote sensing requirements.

The nonlienar effect of gravity waves on statistical properties of the wave field has been investigated first by Longuet-Higgins (1963), who treated the surface statistics as nearly Gaussian and represented it by a Gram-Charlier series in which the skewness of the surface elevation was determined through rigorous dynamic equations. This analysis was later extended by Jackson (1979) to the joint probability density distribution of slope and elevation. These results represented a major breakthough in the non-Gaussian statistical description of the ocean surface. Successful as they were, there are certain shortcomings: the approximation gives negative density values and it requires higher moments to implement. These shortcomings put a limitation the applications of the results.

Recently we established a mapping technique to model the non-Gaussian process of the ocean surface. This technique was first used by Tayfun (1980), and was later successfully modified and applied to the non-Gaussian processes described by the nonlinear surface wave elevation density function (Huang et al, 1983) and joint slope and elevation density function (Huang et al. 1984). In this paper we will summarize the technique used in constructing the non-Gaussian density functions and discuss the specific applications to the remote sensing techniques.

2. THE NEW STATISTICAL MODEL

The new statistical model for a nonlinear deep water wave field is based on the Stokes expansion, i.e.

$$\zeta = \frac{1}{2}a^2k + a\cos\chi + \frac{a^2k}{2}\cos 2\chi + \frac{3a^3k^2}{8}\cos 3\chi + \cdots . \qquad (2.1)$$

Let us start by considering the linear Gaussin case for which the sea surface can be represented by

$$\zeta 1 = a \cos \chi , \qquad (2.2)$$

with χ as the phase function

$$\chi = kx - nt + \phi ,$$

where k is the wave number, n is the frequency and ϕ is an arbirary phase shift uniformly distributed. It is well known that if a is Rayleight, the $\zeta 1$ is zero-mean Gaussian (see, for example, Papoulis, 1965). For the same amplitude and phase function,

$$\zeta 2 = a \sin \chi , \qquad (2.3)$$

is also Gaussian. Furthermore, $\zeta 1$ and $\zeta 2$ are statistically orthogonal to each other.

Based on $\zeta 1$ and $\zeta 2$, we can define the normalized random variables

$$Z_1 = a\cos\chi/(\overline{a^2}/2)^{1/2} \quad ; \quad Z_2 = a\sin\chi/(\overline{a^2}/2)^{1/2} , \qquad (2.4)$$

with the overbar indicating mean quantity, and the joint probability of Z_1 and Z_2 is simply

$$p(Z_1, Z_2) = \frac{1}{2\pi} \exp\left\{-\frac{1}{2}(z_1^2 + z_2^2)\right\}. \tag{2.5}$$

If the surface wave profile is represented by the Stokes wave, a normalized zero-mean random variable can be defined as

$$\eta = \frac{\zeta - \bar{\zeta}}{(\overline{\zeta^2} - \bar{\zeta}^2)^{1/2}} = \frac{\zeta - \bar{\zeta}}{\sigma} , \tag{2.6}$$

where σ is the standard deviation of the surface elevation. From (2.1) we can also get the surface slope

$$\zeta_x = - ak \sin\chi - a^2k^2 \sin 2\chi - \frac{9}{8} a^3k^3 \sin 3\chi - \cdots . \tag{2.7}$$

Again, we can define another normalized zero-mean random variable from (2.7) as

$$\xi = \zeta_x / (\overline{\zeta_x^2})^{1/2}. \tag{2.8}$$

In terms of Z_1 and Z_2, η and ξ become

$$\eta = \frac{Z_1}{N} + \frac{Z_1^2}{N^2}\sigma k + \frac{3}{8}\frac{Z_1^3 - 3Z_1Z_2^2}{N^3}\sigma^2k^2 - \sigma k ; \quad \xi = -\frac{Z_2}{M} - \frac{2Z_1Z_2}{MN}\sigma k - \frac{9}{8}\frac{3Z_1^2Z_2 - Z_2^3}{MN^2}\sigma^2k^2 , \tag{2.9}$$

in which

$$N = (1 + 8\pi^2 \S^2)^{1/2} ,$$

$$M = (1 + 16\pi^2 \S^2)^{1/2} ,$$

and $\S$ is the significant slope of the wave field defined as

$$\S = (\overline{\zeta^2})^{1/2} / \lambda ,$$

with $\lambda = 2\pi/k$.

Since we know the joint density function of Z_1 and Z_2, it is easy to write the joint density function of any two functions of Z_1, and Z_2 through a mapping technique known as the fundamental theorem of probability (Papoulis, 1965), i.e.

$$p(z_1, z_2)\, dz_1 dz_2 = p\{Z_1(\eta, \xi), Z_2(\eta, \xi)\} J\left(\frac{Z_1, Z_2}{\eta, \xi}\right) d\eta\, d\xi , \tag{2.10}$$

in which J() is the Jacobian of the transformation.

Following this method, Huang et al. (1984) obtained the joint slope-elevation density function as:

$$p(\eta, \xi) = \frac{NM}{2\pi}\left\{1 - 4\eta\sigma k + \left[(23\eta^2/2 - 4) + 9M^2\xi^2/2N^2\right]\sigma^2k^2\right\} \cdot \exp\left\{-\frac{1}{2}\left[\begin{array}{l} N^2\eta^2 + M^2\xi^2 - [2\eta(\eta^2 - 1)N^2 + 4\eta\xi^2M^2]\sigma k \\ + \begin{bmatrix}(17\eta^4/4 - 6\eta^2 + 1 + 9M^2\eta^2\xi^2/4N^2)N^2 \\ (37\eta^2\xi^2/4 - 4\xi^2 + 9M^2\xi^4/4N^2)M^2\end{bmatrix}\sigma^2k^2 \end{array}\right]\right\} . \tag{2.11}$$

Statistically, the joint specular point and elevation distribution is the conditional distribution of

$$p(\zeta \mid \nabla\zeta = 0). \tag{2.12}$$

For a long crested wave field or a narrow band of energy spread case,

$$p(\zeta \mid \nabla\zeta = 0) \simeq p(\zeta \mid \zeta_x = 0). \tag{2.13}$$

Thus a good approximation of the specular point distribution can be obtained by simply setting $\xi = 0$ in (2.11). Hence, in terms of the normalized variable,

$$p(\eta \mid \xi = 0) = \frac{MN}{2\pi\tilde{a}}\left[1 - 4\eta\sigma k + \left(\frac{23}{2}\eta^2 - 4\right)\sigma^2 k^2\right] \cdot \exp\left\{-\frac{1}{2}\left[\begin{array}{l} N^2\eta^2 - 2\eta(\eta^2-1)N^2\sigma k \\ + \left(\frac{17}{4}\eta^4 - 6\eta^2 + 1\right)N^2\sigma^2 k^2 \end{array}\right]\right\} \tag{2.14}$$

where $\tilde{a}$ is the normalized factor to guarantee

$$\int_{-\infty}^{\infty} p(\eta \mid \xi = 0)\, d\eta = 1 .$$

From equation (2.11) and 2.14) we can see that the density functions are obviously skewed with respect to η. This skewness is caused by the harmonic distortion of the wave profile described by Stokes expansion. Comparison between the throretical and experimental results have been presented by Huang et al. (1983, 1984). The agreements are all good.

3. REMOTE SENSING APPLICATIONS

A particularly interesting aspect of the specular point density functions is the location of the mean. More specifically, the difference between the mean of the specular point density and the mean of the surface elevation density functions. This difference is the critical electromagnetic bias (Jackson, 1979; Walsh et al., 1983), for the mean sea surface measured by any nadir looking electromagnetic device is not the true mean out the mean of the reflecting facets or the specular points. Based on our observational data, the mean is determined and plotted as a function of the significant slope in Figure 3.1. The data show an almost linear increasing of the bias. The mean value of the theoretical model can be calculated by the ratio of the first moment to the zeroth moment of the density function. The value of the bias is the difference between the corresponding values of the mean in specular point density and elevation density functions. The numerical value calculated based on (2.14) is also given in figure 3.1. Although they do not exactly conincide, the trend is unmistakeable. Better model with more realistic wave profile in two-dimension may be needed.

Another aspect of the nonlinear effects in the wave profile is the non-zero mean - the deviation of the local mean surface under waves from the undisturbed calm surface. The existence of this non-zero mean in a nonlinear wave train has been shown by Stokes (1880), Rayleigh (1917), De (1955) and, more recently, Schwartz (1974) and Longuet-Higgins (1975). In the past, the term $a^2k/2$ in equation (2.1) has been treated as a negligible constant in all wave studies. It is discarded by simply shifting the coordinate system up by the exact amount of $a^2k/2$. This is permissible as long as our interest in the oscillatory part of the motion. But the coordinate shifting is no longer permissible for our present interest because the amount shifted is precisely the dynamic sea state bias which could have far reaching consequence in the accurate determination of the mean sea surface using an altimeter.

The existence of this non-zero mean in a nonlinear wave train can be explained by mass balance in the wave motion. It has been shown by Stokes that a non-zero mean mass transport always accompanies the wave motion. For deep water waves, this

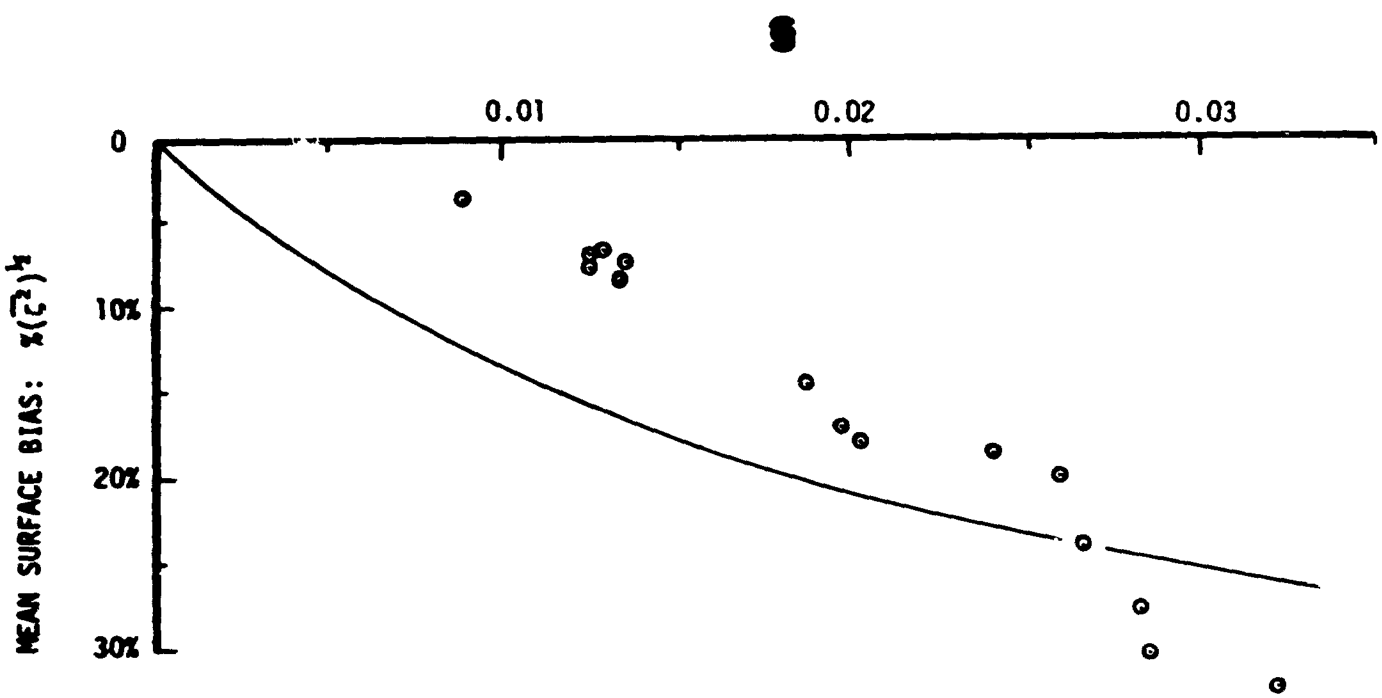

FIGURE 3.1 EM Bias as function of significant slope.
— numerical value according to the new statistical model
⊙ Laboratory simulated data.

velocity is

$$u_\ell = a^2 n\, k\, e^{2kz} \tag{3.1}$$

where Z is the vertical position of a particle. The rate of total mass transport through any vertical section is given by

$$Q = \int_{-\infty}^{0} a^2 n\, k\, e^{2kz}\, dz = a^2 n/2 \tag{3.2}$$

The surface elevation change caused by this mass transport can be calculated as

$$\Delta h_v = QT/\lambda \,, \tag{3.3}$$

in which T and λ are the wave period and wave length respectively. Using the definition of wave number and frequency, one can easily show the equivalence between (3.3) and the constant term in (2.1).

For a random wave field, the Stokes drift is given by Huang (1971) as

$$u_\ell = \iint_{\underset{\sim}{k}\, n} 2n\underset{\sim}{k}\, X(\underset{\sim}{k}, n)\, e^{2|\underset{\sim}{k}|z}\, d\underset{\sim}{k}\, dn \tag{3.4}$$

Therefore, the rate of the total mass transport is

$$Q = \int_{-\infty}^{0} u_\ell\, dz \tag{3.5}$$

and the bias is

$$\begin{aligned}\Delta h_w &= \int_{-\infty}^{0} \iint_{\underset{\sim}{k}\, n} \frac{|\underset{\sim}{k}|}{n}\, 2nk\, X(\underset{\sim}{k}, n)\, e^{2|\underset{\sim}{k}|z}\, d\underset{\sim}{k}\, dn\, dz \\ &= \iint_{\underset{\sim}{k}\, n} \underset{\sim}{k}\, X(\underset{\sim}{k}, n)\, d\underset{\sim}{k}\, dn \,,\end{aligned} \tag{3.6}$$

in which X(k,n) is the directional wave number frequency spectrum. In order to evaluate (3.6), a specific spectrum form has to be adopted. Since the quantity sought here in a scaler, Δh, we can reduce (3.6) by using the dispersion relationship to eliminate k and reduce X(k,n) to a simple frequency spectrum. Then adopting the simplified Wallops spectrum model (Huang et al., 1981), we obtain

$$\Delta h_w = 2\pi\, (\overline{\zeta^2})^{1/2}\, \S\, (m-1)/(m-3) \,, \tag{3.7}$$

where m is a simple function of the significant slope, §, defined by

$$m = \left| \log(\sqrt{2}\, \pi \S)^2 / \log 2 \right| ,$$

and

$$\S = (\overline{\zeta^2})^{1/2} / \lambda_0 \,,$$

with λ_0 as the wave length of the wave having a frequency at the peak of the spectrum, n_0.

If the significant wave height, $H_{1/3}$, is used as in most practical applications, (3.7) can be written as

$$\Delta h_w = \frac{\pi}{2}\, \S\, H_{1/3}\, (m-1)/(m-3) \tag{3.8}$$

The typical value of § in the open ocean is around 0.01, which gives the value of m around 10; the dynamic sea state bias, Δh, would be around 1.5% of the significant wave height. During a storm, the § value could reach 0.02 as an upper bound, and then Δh would be as high as 3% of $H_{1/3}$. For the sea state dominated by swell, however, the dynamic bias can almost be neglected because § would be of the order of 0.001 or less.

As an estimate of the magnitude of this bias, we use the empirical formula given by Neumann and Pierson (1965), then

$$H_{1/3} = 2.12\times10^{-2}\ W^2 \tag{3.9}$$

for a fully developed sea. Under such conditions the phase velocity of the energy containing waves equals the wind speed. Thus the sea state bias can be written as

$$\Delta h_W = 2.75\times10^{-4}\ W^2\,(m-1)/(m-3)\,. \tag{3.10}$$

This value is plotted in Figure 3.2 For all wind speed, the high sea state this bias is not negligible.

4. CONCLUSION

In this study, we summarized the non-Gaussian statistial models based on Stokes Wave expansion. The new statistical model indicates the existence of skewness which is the cause of the EM bias in the remote sensing application. Furthermore, the existence of the non-zero mean in Stokes expansion which contributed to the Skewness can also cause a dynamic bias. The magnitudes of both bias terms are estimated. For future altimeter applications, we should seriously consider the correction of these biases.

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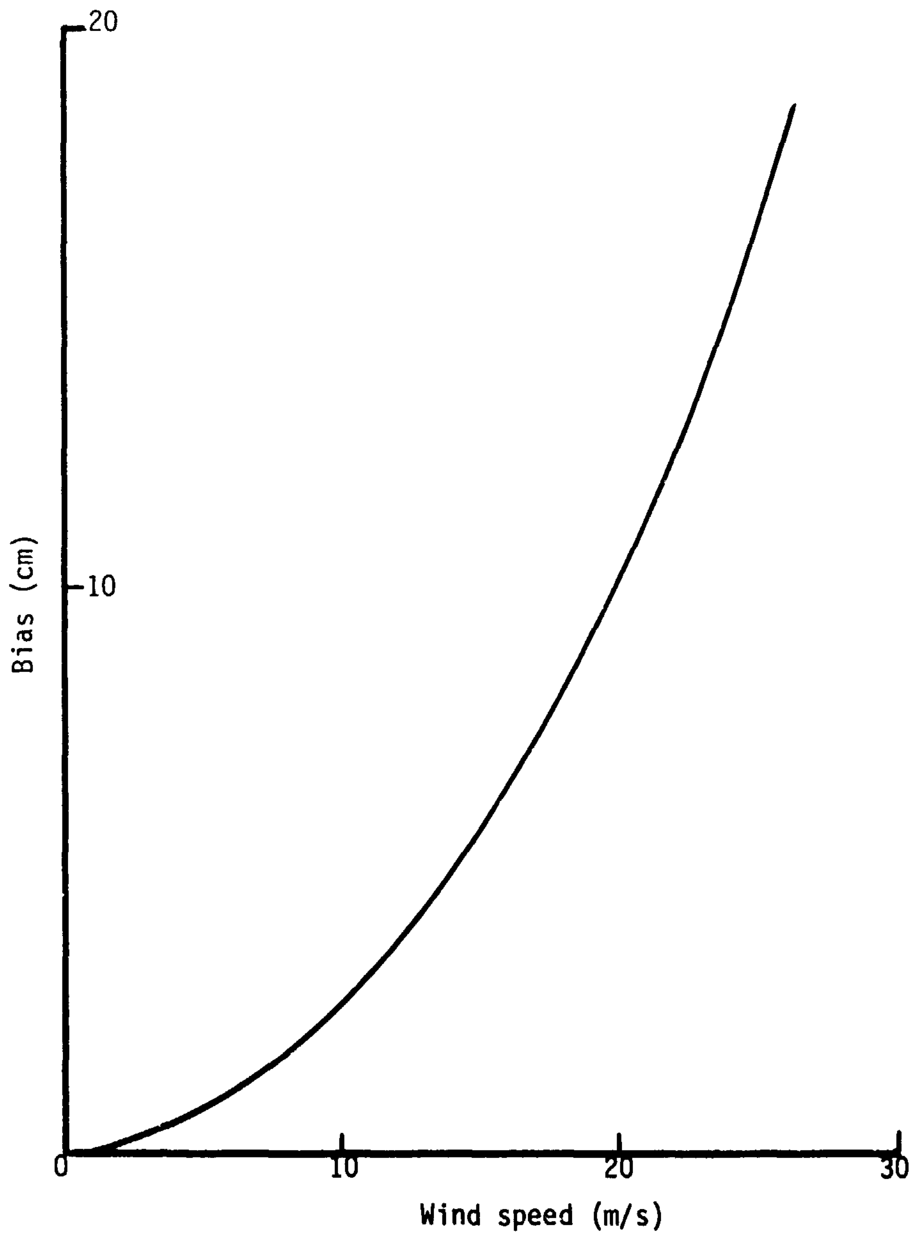

FIGURE 3.2 The Dynamic Bias as function of wind speed.

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