

# FINITE ELEMENT PREDICTION OF ACOUSTIC SCATTERING AND RADIATION FROM SUBMERGED ELASTIC STRUCTURES

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## ABSTRACT

A finite element formulation is derived for the scattering and radiation of acoustic waves from submerged elastic structures. The formulation uses as fundamental unknowns the displacement in the structure and a velocity potential in the fluid. Symmetric coefficient matrices result. The outer boundary of the fluid region is terminated with an approximate local wave-absorbing boundary condition which assumes that outgoing waves are locally planar. The finite element model is capable of predicting only the near-field acoustic pressures. Far-field sound pressure levels may be determined by integrating the surface pressures and velocities over the wet boundary of the structure using the Helmholtz integral. Comparison of finite element results with analytic results show excellent agreement. The coupled fluid-structure problem may be solved with general purpose finite element codes by using an analogy between the equations of elasticity and the wave equation of linear acoustics.

## INTRODUCTION

There is a variety of practical engineering problems which cannot be addressed using only the separate disciplines of structural analysis and acoustics; they must instead be treated by formulating a coupled structural-acoustic problem. For example, for aircraft (Ref. 1), automobiles (Ref. 2), and railroad cars (Ref. 3), coupled analyses are being investigated to understand better the interior noise problem so that such noise can be reduced. In aerospace vehicles, the vibrations of fluid-filled tanks are of interest (Ref. 4). In aircraft hydraulic systems (Ref. 5) and shipboard piping systems (Ref. 6), the dynamic behavior of (and the transmission of sound in) fluid-filled piping systems has been analyzed. Other important naval problems involve the vibration of underwater structures such as rudders and propellers (Ref. 7-8), the shock response of submerged structures (Ref. 9-13), and the scattering of sound waves from underwater elastic structures (Ref. 14-17).

The commonality among all these problems is the mathematical model. The structure, if it can be assumed to remain elastic, behaves according to the classical theory of elasticity (Ref. 18) and the various approximate engineering theories for beams, plates, and shells. The fluid is generally treated as an acoustic medium (Ref. 19-21), a fluid whose pressure  $p$  satisfies the scalar wave equation

$$\nabla^2 p = \ddot{p}/c^2 \quad (1)$$

where  $c$  is the speed of sound in the fluid. The boundary condition at a fluid-structure interface can be obtained from momentum and continuity considerations:

$$\partial p / \partial n = -\rho \ddot{u}_n \quad (2)$$

where  $n$  is the normal at the interface,  $\rho$  is the mass density of the fluid, and  $\ddot{u}_n$  is the normal component of fluid particle acceleration.

Dynamics problems involving the interaction between an elastic structure and an acoustic fluid have been formulated for finite element solution (Ref. 22) by using either pressure (Ref. 19,23,24) or fluid particle displacement (Ref. 14,25-27) as the fundamental unknown in the fluid region. In three dimensions, the pressure and displacement formulations result in, respectively, one and three degrees of freedom per finite element mesh point. Thus the pressure approach has the advantage of fewer unknowns and a smaller overall matrix profile or bandwidth. On the other hand, the displacement approach results in symmetric coefficient matrices (in contrast to the pressure formulation, for which the matrices are nonsymmetric) and a fluid-structure interface condition which is easier to implement with general purpose finite element computer programs. However, the displacement approach also suffers from the presence of spurious resonances (Ref. 27), a situation which can be bothersome in time-harmonic problems, either forced or unforced. Recently it was shown (Ref. 28) that the principal disadvantage of the pressure formulation, nonsymmetric coefficient matrices, can be removed merely by reformulating the pressure solution approach so that a velocity potential rather than pressure is used as the fundamental unknown in the fluid region. For some situations, particularly steady-state problems involving damped systems and time-dependent problems, significant computational advantages result.

The principal goal of this paper is to present in detail the symmetric velocity potential formulation for application to the specific problem of acoustic scattering from submerged elastic structures. Previously (Ref. 28), the symmetric potential formulation was described only in general terms for a wider class of fluid-structure interaction problems with no details concerning specific types of applications (such as vibrations, shock response, or acoustic scattering).

The scattering approach described here has advantages over the displacement formulations for the reasons already given. Finite element modeling of exterior fluid regions also has advantages over the use of approximate theories such as the doubly asymptotic approximation (DAA) (Ref. 15). In this case, the primary trade-off is between the large, banded matrices which finite element models generate and the smaller, densely-populated matrices which the DAA generates. This trade-off often favors the finite element approach for long structures like ships which are "naturally banded." Similar trade-offs arise if the finite element approaches are compared with T-matrix methods (Ref. 16). T-matrix approaches, however, are not yet available in general purpose codes capable of handling arbitrary three-dimensional geometries.

From an engineering point of view, it is convenient to be able to make use of existing general purpose finite element codes (such as NASTRAN, among others), because of their wide availability, versatility, reliability, consultative support, and abundance of pre- and postprocessors. Thus the next section summarizes an analogy between the equations of elasticity and the common field equations of classical mathematical physics (including the wave equation). This analogy allows the coupled structural-acoustic problem to be solved with standard finite element codes.

Subsequent sections of the paper will develop the formulation of the scattering problem for elastic obstacles with and without fluid inside. Examples will be shown for both cases.

## STRUCTURAL ANALOGIES FOR SCALAR FIELD PROBLEMS

Since we wish to solve the coupled structural-acoustic problem using standard finite element codes (which were developed principally for structural analysis), we summarize here the application of such codes to various nonstructural field problems (Ref. 24,29).

Many linear problems in mathematical physics involve the solution of an equation obtained by specializing the general form

$$\nabla^2\phi + g = a \ddot{\phi} + b \dot{\phi} \quad (3)$$

where  $\nabla^2$  is the Laplacian operator; dots denote partial time differentiation; the functions  $g$ ,  $a$ , and  $b$  are, in general, position-dependent; and the unknown scalar function  $\phi$  depends on both position and time.

Special cases of Eq. (3) arise in such diverse applications as heat conduction, acoustics, electrical and magnetic potential problems, torsion of prismatic bars, potential fluid flow, and seepage through porous media. Several common special cases are listed here:

$$\text{Laplace's equation:} \quad \nabla^2\phi = 0 \quad (4)$$

$$\text{Poisson's equation:} \quad \nabla^2\phi + g = 0 \quad (5)$$

$$\text{wave equation:} \quad \nabla^2\phi = \ddot{\phi}/c^2 \quad (6)$$

$$\text{heat equation:} \quad k\nabla^2\phi + q = \rho c \dot{\phi} \quad (7)$$

$$\text{telegraph equation:} \quad \partial^2\phi/\partial x^2 = LC\ddot{\phi} + RC\dot{\phi} \quad (8)$$

$$\text{Helmholtz equation:} \quad \nabla^2\phi + k^2\phi = 0 \quad (9)$$

Most boundary conditions likely to be encountered in connection with Eq. (3) will probably be special cases of the general form

$$a_1 \partial\phi/\partial n + a_2 \phi + a_3 \dot{\phi} + a_4 \ddot{\phi} + a_5 = 0 \quad (10)$$

where  $n$  is the outward normal at the boundary. For example, in heat conduction problems, a boundary with a prescribed temperature function satisfies the Dirichlet condition

$$\phi = \phi_0 \quad (11)$$

and a perfectly insulated boundary has the Neumann condition

$$\partial\phi/\partial n = 0 \quad (12)$$

In free surface flow problems, the linearized free surface condition on the velocity potential is (Ref. 30)

$$\ddot{\phi} + g_0 \phi_{,z} = 0 \quad (13)$$

where  $g_0$  is the acceleration due to gravity, the free surface is the plane  $z = \text{constant}$ , and commas denote partial differentiation. In one-dimensional radiation problems, the plane wave radiation condition that the velocity potential must satisfy at a non-reflecting boundary is

$$\phi_{,n} + \dot{\phi}/c = 0 \quad (14)$$

where  $c$  is the wave speed.

An example of a boundary condition not of the general form of Eq. (10) is the condition (2) which must be satisfied at an accelerating boundary of a fluid.

According to the analogy (Ref. 29) between Eq. (3) and the Navier equations of classical elasticity, Eqs. (3) and (10) can be solved with elastic finite elements using the following procedure:

1. Select one of the three Cartesian components of displacement (or the  $z$ -component in cylindrical coordinates) to represent the scalar field variable  $\phi$ . Constrain all other displacement components everywhere in the field.

2. Model the domain of interest (either 2-D or 3-D) with finite elements having material constants satisfying

$$E_e = \alpha G_e, \quad \rho_e = a G_e \quad (15)$$

where "a" is the variable appearing in Eq. (3), and  $E_e$ ,  $G_e$ , and  $\rho_e$  denote the Young's modulus, shear modulus, and mass density assigned on the material card to the finite elements. The subscript "e" has been added to emphasize that these constants are merely numbers assigned to the elements and may bear no resemblance to any actual material properties associated with a particular application. The dimensionless constant  $\alpha$  in Eq. (15) should, for 3-D problems, be chosen large enough to make  $\alpha+1$  numerically indistinguishable from  $\alpha$ . For 2-D problems,  $\alpha$  should be small, but not

so small that  $1+\alpha$  is numerically indistinguishable from unity. Thus, on most computers,

$$\alpha = \begin{cases} 10^{-5} & (2\text{-D}) \\ 10^{20} & (3\text{-D}) \end{cases} \quad (16)$$

The shear modulus  $G_e$  can be selected arbitrarily. The finite elements eligible for use in the model are those derived from classical elasticity theory rather than from the engineering theories involving beams, plates, or shells. Thus, for 2-D problems, the plane stress membrane elements are appropriate. For 3-D problems, the solid elements (e.g., the isoparametric or the axisymmetric solids) should be used.

3. Apply to the unconstrained degree of freedom (DOF) at each grid point in the region a "force" given by

$$F = G_e g V \quad (17)$$

where  $V$  is the volume assigned to the point and  $g$  is the function appearing in Eq. (3). For problems for which the function  $g$  in Eq. (3) is independent of position (as, for example, in the classical St. Venant torsion problem), this load may be specified conveniently by applying to the "structure" a gravitational field for which the acceleration due to gravity  $g_0$  satisfies

$$\rho_e g_0 = G_e g \quad (18)$$

4. Connect between ground and the unconstrained DOF at each grid point in the region a scalar dashpot whose damping constant (the ratio of damping force to velocity) is  $G_e bV$ , where  $b$  is the function appearing in Eq. (3) and  $V$  is the volume assigned to the point.

5. Enforce the boundary condition (10) by applying to the unconstrained DOF at each grid point on the boundary of the region a "force" given by

$$F = -G_e A (a_2 \phi + a_3 \dot{\phi} + a_4 \ddot{\phi} + a_5)/a_1, \quad a_1 \neq 0 \quad (19)$$

where  $A$  is the area assigned to the point. (In general, the outward normal derivative  $\partial\phi/\partial n$  is enforced at a boundary point by applying a "force" to the unconstrained DOF at that point equal to  $G_e A \partial\phi/\partial n$ . A positive force corresponds to a positive outward normal derivative.) In Eq. (19), the  $a_2$  term is analogous to a scalar spring of constant  $G_e A a_2/a_1$  connected between the point and ground. The  $a_3$  term is analogous to a scalar dashpot of constant  $G_e A a_3/a_1$  connected between the point and ground. The  $a_4$  term is analogous to an added mass of value  $G_e A a_4/a_1$  attached to the point. (Here, one should probably use a consistent, rather than lumped, formulation since Zarda and Marcus (Ref. 30) showed that the differences between the two are not

insignificant for free surface flow problems.) The  $a_5$  term is a time-independent force given by  $-G_e A a_5 / a_1$ . As expected, the special case of the Neumann boundary condition ( $\phi_{,n} = 0$ ) corresponds to the traction-free boundary in elasticity and hence is a natural boundary condition. The Dirichlet condition ( $\phi = \phi_0$ ) is implemented merely by enforcing the desired value as a "displacement" boundary condition.

### SCATTERING FROM ELASTIC BODIES

In the scattering problem (as shown in Fig. 1), a submerged elastic body is subjected to a plane wave time-harmonic acoustic incident loading of circular frequency  $\omega$ . Without loss in generality, we can assume that the waves propagate in the  $-x$  direction. The speed of such propagation is  $c$ , the speed of sound in the fluid (usually water).

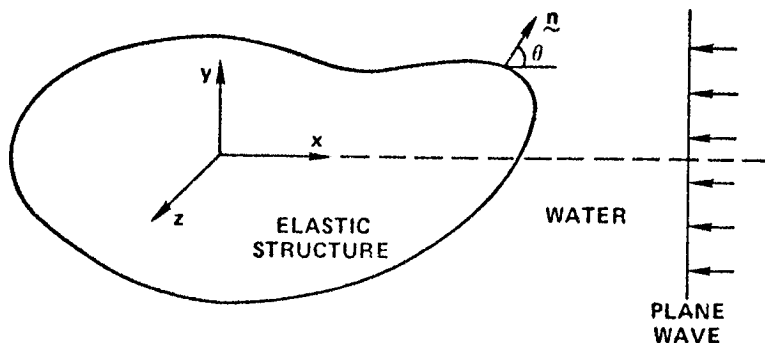


Figure 1 - The Scattering Problem

Within the fluid region, the total fluid pressure  $p$  satisfies the wave equation, Eq. (1). Since the free-field incident pressure  $p_i$  is known and is given by

$$p_i(x,t) = p_o e^{i(kx + \omega t)} \quad (20)$$

(where  $k = \omega/c$ ) it is frequently convenient to decompose the total pressure  $p$  into the sum of incident and scattered pressures

$$p = p_i + p_s \quad (21)$$

each of which satisfies the wave equation.

The solution of this scattering problem has been approached in various ways, including the T-matrix method (Ref. 16), numerical approaches using approximate fluid loading schemes (Ref. 15), and finite element schemes (Ref. 14). The latter approach models with finite elements both the structure and a portion of the infinite fluid, which is terminated with a simple radiation boundary condition to absorb the outgoing waves as much as possible. Kalinowski's finite element approach (Ref. 14) uses the total fluid particle displacement as the fundamental fluid unknown.

The approach presented here is most similar to Kalinowski's approach (of those mentioned above), except that we use as the fundamental fluid unknown the scattered component of a fluid velocity potential rather than the total fluid displacement. Both approaches yield symmetric matrix equations, but the use of velocity potential rather than displacement as fluid unknown results, in 3-D, in fluid matrices of one-third the order and one-third the matrix bandwidth.

We now formulate the problem for finite element solution. The finite element modeling of the elastic structure results in the matrix equation

$$M \ddot{u} + B \dot{u} + K u = -A p = -A (p_i + p_s) \quad (22)$$

where  $u$  is the vector of displacement components in the structure;  $M$ ,  $K$ , and  $B$  are the structural mass, stiffness, and damping matrices, respectively;  $p$  is the vector of fluid pressures at the nodes of the fluid region; and  $A$  is the area matrix which converts fluid pressures at interface points to structural loads.

A finite element model of the fluid region results in a matrix equation of the form

$$Q \ddot{p}_s + H p_s = 0 \quad (23)$$

where  $p_s$  is the vector of scattered fluid pressures at the nodes of the fluid region, and  $Q$  and  $H$  are the fluid "inertia" and "stiffness" matrices, respectively. According to the analogies described in the preceding section, the same finite element code may be used to model both the structural and fluid regions. From Eq. (15), material constants assigned to the elastic elements used to model the fluid satisfy

$$E_e = \alpha G_e, \quad \rho_e = G_e/c^2 \quad (24)$$

where  $\alpha$  is given by Eq. (16).

As is, Eq. (23) does not account for either the fluid-structure interface condition (2) or a wave-absorbing boundary condition. As in Kalinowski's work (Ref. 14), we will use the simple plane wave absorbing condition (Ref. 31)

$$\partial p / \partial n = -\dot{p} / c \quad (25)$$

Other possibilities are discussed by, for example, Engquist and Majda (Ref. 32), Bayliss, Gunzburger, and Turkel (Ref. 33), and Israeli and Orszag (Ref. 34). However, these are not of the general form of Eq. (10). Kalinowski showed (Ref. 14, 26,35,36), and our Example 1 verifies, that the plane wave absorbing condition is satisfactory if the outer boundary is far enough away from the structure.

Both boundary conditions (2) and (25) can be handled using Eq. (19). At the fluid-structure interface, from Eqs. (2) and (21),

$$\partial p_s / \partial n = -\partial p_i / \partial n - \rho \ddot{u}_n = \rho (\ddot{u}_{ni} - \ddot{u}_n) \quad (26)$$

where  $n$  is the outward unit normal from the structure (into the fluid), and  $\ddot{u}_n$  and  $\ddot{u}_{ni}$  are, respectively, the total and incident outward components of fluid particle acceleration at the interface. Thus, using Eq. (19), we impose the condition (26) by applying a load to each interface fluid point given by

$$F^{(p)} = -G_e A \rho (\ddot{u}_{ni} - \ddot{u}_n) \quad (27)$$

Similarly, the radiation condition (25) is enforced by applying a load to each fluid point on the outer boundary given by

$$F^{(p)} = -(G_e A / c) \dot{p}_s \quad (28)$$

That is, a dashpot of constant  $G_e A / c$  is connected between each boundary point and ground.

The overall matrix system describing the coupled problem is obtained by combining Eqs. (22), (23), (27), and (28):

$$\begin{bmatrix} M & 0 \\ -G_e \rho A^T & Q \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{p}_s \end{Bmatrix} + \begin{bmatrix} B & 0 \\ 0 & C \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{p}_s \end{Bmatrix} + \begin{bmatrix} K & A \\ 0 & H \end{bmatrix} \begin{Bmatrix} u \\ p_s \end{Bmatrix} = \begin{Bmatrix} -A p_i \\ -G_e \rho A \ddot{u}_{ni} \end{Bmatrix} \quad (29)$$

This system, which is nonsymmetric, can be symmetrized (Ref. 28) by reformulating the problem to use a new fluid unknown  $q$  such that

$$p_s = \dot{q} \quad (30)$$

If the second partition of Eq. (29) is integrated in time, if  $p_s$  is replaced by  $\dot{q}$ , and if the fluid element "shear modulus"  $G_e$  is chosen as

$$G_e = -1/\rho \quad (31)$$

the system (29) is transformed into

$$\begin{bmatrix} M & 0 \\ 0 & Q \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{q} \end{Bmatrix} + \begin{bmatrix} B & A \\ A^T & C \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{q} \end{Bmatrix} + \begin{bmatrix} K & 0 \\ 0 & H \end{bmatrix} \begin{Bmatrix} u \\ q \end{Bmatrix} = \begin{Bmatrix} -A p_i \\ A v_{ni} \end{Bmatrix} \quad (32)$$



where  $v_{ni} = \dot{u}_{ni}$ . This is the form of the equations which we will use to solve the scattering problem.

The new variable  $q$  is, except for a multiplicative constant, the velocity potential  $\phi$  long used by fluid dynamicists, since

$$p = -\rho \dot{\phi} \quad (33)$$

Eq. (32) could also be recast in terms of  $\phi$  rather than  $q$  as the fundamental fluid unknown, but no particular advantage would result. In fact, the use of  $q$  rather than  $\phi$  has the slight practical advantage that the fluid pressure can be recovered directly from the finite element program as the time derivative (velocity) of the unknown  $q$ .

To summarize, both structural and fluid regions are modeled with finite elements. For the fluid region, the material constants assigned to the finite elements are

$$G_e = -1/\rho, \quad E_e = \alpha G_e, \quad \rho_e = -1/\rho c^2 \quad (34)$$

where  $\alpha$  is given by Eq. (16). The dashpots making up matrix  $C$  in Eq. (32) are applied at the outer fluid boundary with damping constant  $-A/\rho c$  at each grid point to which the area  $A$  has been assigned. At the fluid-structure interface, matrix  $A$  is entered using the areas (or areal direction cosines) assigned to each wet degree of freedom.

The right-hand side of Eq. (32) can be simplified further since, for plane waves propagating in the negative  $x$ -direction at speed  $c$ , the free-field incident pressure and incident fluid particle velocity in the  $x$ -direction are related by (Ref. 37)

$$p_i = -\rho c v_{xi} \quad (35)$$

Then, as in Fig. 1, if we define  $\theta$  as the angle between the normal  $n$  and the positive  $x$ -axis,

$$v_{ni} = v_{xi} \cos \theta \quad (36)$$

The  $x$ -component of the free-field fluid particle velocity  $v_{xi}$  is the same at all points in space except for a phase angle, which may be introduced into the analysis by means of the time delay between two points having different  $x$ -coordinates.

#### EXAMPLE 1: SCATTERING FROM INFINITE CYLINDRICAL SHELL

The formulation derived in the preceding section will be illustrated first on the two-dimensional (plane strain) problem of scattering from an infinitely long

cylindrical steel shell (Fig. 2) of radius 0.5 m and thickness 0.01 m. The material properties of steel are  $E = 19.5 \times 10^{10} \text{ N/m}^2$ ,  $\nu = 0.28$ , and  $\rho_s = 7700 \text{ kg/m}^3$ . The surrounding medium is seawater, for which  $c = 1500 \text{ m/sec}$  and  $\rho = 1026 \text{ kg/m}^3$ .

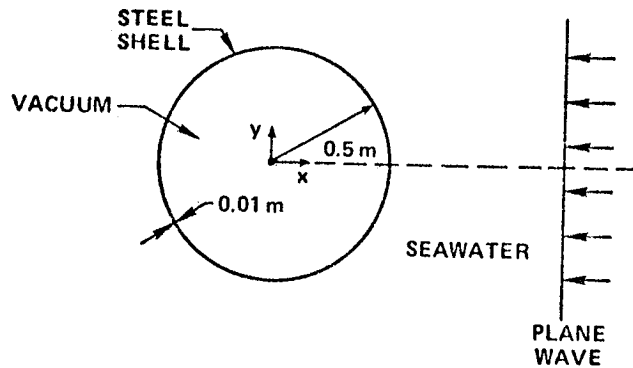


Figure 2 - Scattering from Infinite Cylindrical Shell

The shell is modeled with beam finite elements and the water with 2-D quadrilateral four-node plane stress membrane elements having properties assigned according to Eq. (34). The thickness in the  $z$ -direction for all elements is arbitrarily chosen as 1 m. By symmetry, only half of the problem (the half-plane  $y > 0$ ) needs to be modeled.

For a frequency of excitation of 4100 Hz. ( $ka = 8.6$ ), we modeled the region with 128 elements spanning the 180 degrees of circumference. The outer fluid boundary (a circle of radius 1.026 m) was 1.44 acoustic wavelengths away from the shell. According to Kalinowski (Ref. 14), this amount of fluid should provide excellent absorption of outgoing waves.

A typical result for this problem is shown in Fig. 3, which shows a comparison of the surface scattered pressures calculated both by finite elements and by using an analytic expression presented by Junger and Feit (Ref. 38). The agreement is clearly excellent. Although the Junger-Feit solution used for comparison is a series solution (and hence another numerical approach), it makes no approximation concerning the radiation boundary condition. The agreement in Fig. 3 thus confirms that spurious boundary reflections are not contaminating the finite element solution for this problem.

Additional results for this problem (at frequencies both above and below that shown here) have been presented by Henderson (Ref. 17).

#### SCATTERING FROM FREE-FLOODED ELASTIC SHELL WITH RIGID INNER CORE

Here we derive the finite element formulation for scattering from a submerged elastic thin shell which contains water on the inside (as well as on the outside) of the shell and also contains a rigid inner core (Fig. 4).

The formulation of this problem proceeds along lines similar to that of the previous problem except that the fundamental fluid unknown in the inner fluid region is the total velocity potential rather than the scattered component of velocity potential as in the outer fluid region.

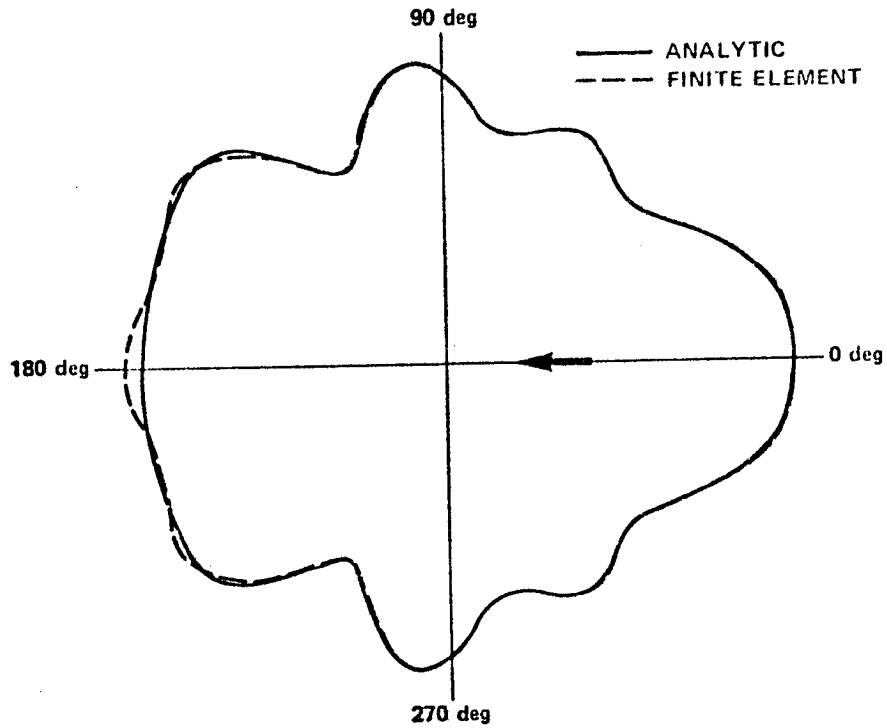


Figure 3 - Scattered Pressures on the Surface of an Infinite Cylinder ( $ka = 8.6$ )

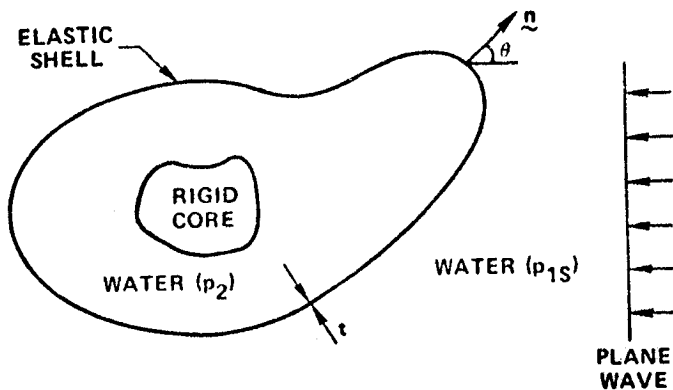


Figure 4 - Scattering from Free-Flooded Elastic Thin Shell with Rigid Inner Core

Therefore, let  $p_{1s}$  denote the scattered pressure in the outer fluid region and  $p_2$  denote the total pressure in the inner fluid region. Since the elastic shell is thin, the same shell variables interact with both inner and outer fluid regions. The main difference between the treatment of the two interfaces is that the normal  $n$  in Fig. 4 is an inward normal for the outer fluid and an outward normal for the inner fluid.

The generalization of Eq. (29) can then be written immediately as

$$\begin{aligned}
 & \begin{bmatrix} Q_2 & -A^T & 0 \\ 0 & M & 0 \\ 0 & A^T & Q_1 \end{bmatrix} \begin{Bmatrix} \ddot{p}_2 \\ \ddot{u} \\ \ddot{p}_{1s} \end{Bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{bmatrix} \begin{Bmatrix} \dot{p}_2 \\ \dot{u} \\ \dot{p}_{1s} \end{Bmatrix} \\
 & + \begin{bmatrix} H_2 & 0 & 0 \\ -A & K & A \\ 0 & 0 & H_1 \end{bmatrix} \begin{Bmatrix} p_2 \\ u \\ p_{1s} \end{Bmatrix} = \begin{Bmatrix} 0 \\ -Ap_i \\ A\ddot{u}_{ni} \end{Bmatrix} \quad (37)
 \end{aligned}$$

where  $G_e$  has been specified as in Eq. (31), and we have assumed that the same area matrix  $A$  is used on both interfaces to convert pressures to forces.

This nonsymmetric system can also be symmetrized by reformulating the equations in terms of two new fluid variables  $q_1$  and  $q_2$  such that

$$\dot{q}_1 = p_{1s}, \quad \dot{q}_2 = p_2 \quad (38)$$

As before, the nonsymmetric terms can be moved to the damping matrix by integrating the first and third partitions of Eq. (37) in time:

$$\begin{aligned}
 & \begin{bmatrix} Q_2 & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & Q_1 \end{bmatrix} \begin{Bmatrix} \ddot{q}_2 \\ \ddot{u} \\ \ddot{q}_1 \end{Bmatrix} + \begin{bmatrix} 0 & -A^T & 0 \\ -A & B & A \\ 0 & A^T & C \end{bmatrix} \begin{Bmatrix} \dot{q}_2 \\ \dot{u} \\ \dot{q}_1 \end{Bmatrix} \\
 & + \begin{bmatrix} H_2 & 0 & 0 \\ 0 & K & 0 \\ 0 & 0 & H_1 \end{bmatrix} \begin{Bmatrix} q_2 \\ u \\ q_1 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -Ap_i \\ Av_{ni} \end{Bmatrix} \quad (39)
 \end{aligned}$$

where, for both fluid regions, the material constants assigned to the finite elements are given by Eq. (34).

EXAMPLE 2: SCATTERING FROM INFINITE CYLINDRICAL SHELL ENCLOSING A FLUID  
BOUNDED BY A CONCENTRIC RIGID CYLINDER

The formulation derived in the preceding section will be illustrated on the two-dimensional (plane strain) problem of scattering from an infinitely long cylindrical steel shell which contains on the inside both fluid and a concentric rigid cylinder (Fig. 5). The rigid inner core has a radius of 0.254 m. The shell is identical to that used in Example 1. Seawater floods the region between the shell and the rigid inner core.

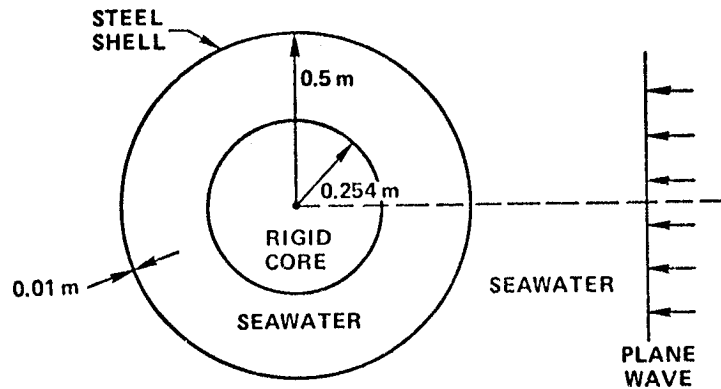


Figure 5 - Scattering from Infinite Cylindrical Shell Enclosing a Fluid Bounded by a Concentric Rigid Cylinder

A finite element model was prepared for this problem for excitation at 2100 Hz. ( $ka = 4.4$ ). Since this frequency is lower than that used in Example 1, a coarser mesh (having 96 elements spanning the 180 degrees of circumference) was used. Here, the outer fluid boundary was a circle located 1.32 m away from the shell, a distance equal to 1.85 wavelengths of the incident free-field acoustic wave.

For this problem, no analytic results were readily available for comparison, so instead a comparison was made to the corresponding problem with the inside of the shell evacuated (as in Example 1, but at a different frequency). See Fig. 6. Clearly, the presence of the contained fluid and the inner core affect the solution significantly.

Although the two cylinders in this problem were chosen to be concentric, the formulation derived in the preceding section is clearly general enough to handle arbitrary geometry. In addition, the formulation applies to both 2-D and 3-D problems.

#### RADIATED PRESSURES

A finite element model of an exterior fluid-structure interaction problem is capable of predicting only near-field acoustic pressures because of the approximate nature of the radiation boundary condition on the fluid. However, given fluid pressures and normal velocities on the fluid-structure interface, the fluid pressure at any point in the exterior field can be calculated by a numerical quadrature.

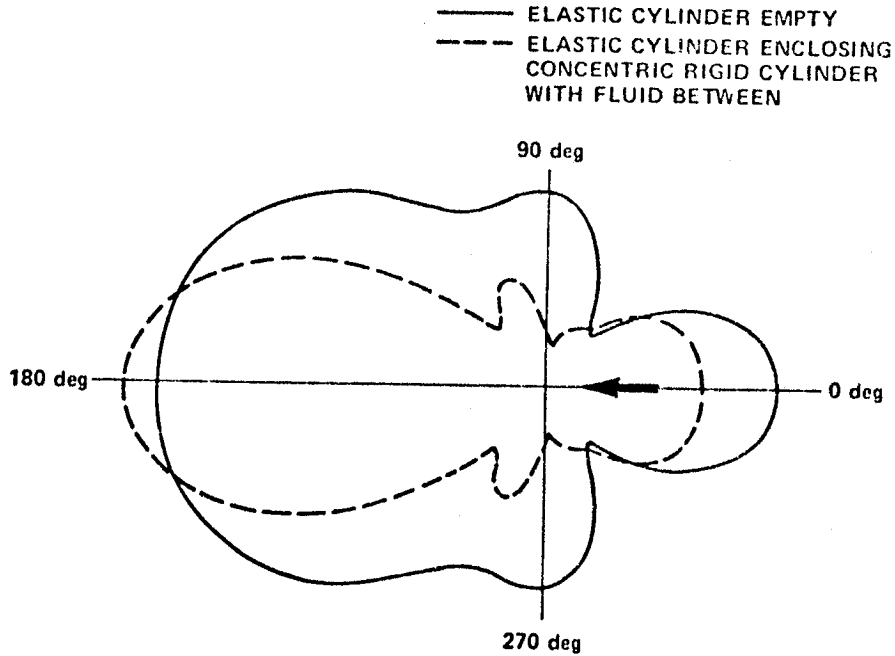


Figure 6 - Comparison of Surface Pressures Scattered by an Evacuated Cylinder with Pressures Scattered by a Cylinder Containing a Fluid and a Concentric Rigid Cylinder ( $ka = 4.4$ )

In Fig. 7, let  $\underline{z}$  be the position vector to an exterior field point P, and  $z = |\underline{z}|$ . Let  $\underline{x}$  be the position vector to a point on the fluid-structure interface (with  $x = |\underline{x}|$ ), let  $\underline{r} = \underline{z} - \underline{x}$  (with  $r = |\underline{r}|$ ), and let  $\underline{n}$  be the unit outward normal at the location  $\underline{x}$ . The pressure at  $\underline{z}$  is (Ref. 39)

$$p(\underline{z}) = - \int_S q(\underline{x})(e^{-ikr}/4\pi r) dS + \int_S p(\underline{x}) \partial/\partial n(e^{-ikr}/4\pi r) dS \quad (40)$$

where

$$q = \partial p/\partial n = -i\omega\rho v_n \quad (41)$$

$$k = \omega/c \quad (42)$$

and it is assumed that the harmonic time-dependence of the variables is  $\exp(i\omega t)$  rather than  $\exp(-i\omega t)$ .

Since

$$\partial/\partial n(e^{-ikr}/4\pi r) = (e^{-ikr}/4\pi r)(ik + r^{-1}) \cos \beta \quad (43)$$

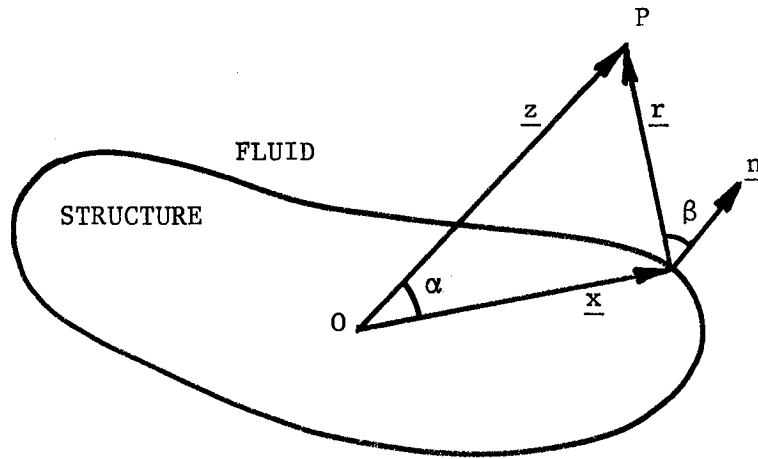


Figure 7 - Notation for Far-Field Radiated Pressure Calculations

it follows that

$$p(\underline{z}) = \int_S [i\omega\rho v_n(\underline{x}) + (ik + r^{-1}) p(\underline{x}) \cos \beta] (e^{-ikr}/4\pi r) dS \quad (44)$$

This expression is valid for any point in the exterior field.

Eq. (44) can be simplified if only far-field locations are of interest. As  $|\underline{z}| \rightarrow \infty$ ,  $ik + r^{-1} \rightarrow ik$ , and, from the law of cosines,  $r \rightarrow z - x \cos \alpha$ . Thus, at far-field locations,

$$p(\underline{z}) \cong (ik e^{-ikz}/4\pi z) \int_S [\rho c v_n(\underline{x}) + p(\underline{x}) \cos \beta] e^{ikx \cos \alpha} dS \quad (45)$$

where

$$\cos \beta \cong (\underline{z}/|\underline{z}|) \cdot \underline{n} \quad (46)$$

The numerical integrations in Eqs. (44) and (45) require, for each wet structural point, the knowledge of the location (coordinates), normal, area, pressure, and outward normal velocity. All these quantities can be obtained directly from NASTRAN using the OUTPUT2 utility module. The grid point coordinates and areas (or areal direction cosines) can be obtained from an abbreviated static analysis in which a unit outward pressure load is applied to the structure. The unit normal at a point is then the unit vector parallel to the area vector. The pressures and velocities are obtained from the frequency response analysis.

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