

FOUR NEW CAPABILITIES IN NASTRAN FOR  
 DYNAMIC AND AEROELASTIC ANALYSES OF  
 ROTATING CYCLIC STRUCTURES

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SUMMARY

Static aerothermoelastic design/analysis of axial-flow compressors, modal flutter analysis of axial-flow turbomachines, forced vibration analysis of rotating cyclic structures and modal flutter analysis of advanced turbopropellers with highly swept blades are four new capabilities developed and implemented in NASTRAN Level 17.7. The purpose of this paper is to briefly discuss the contents, applicability and usefulness of these capabilities which were developed and documented under the sponsorship of NASA's Lewis Research Center. Overall flowcharts and selected examples are also presented.

INTRODUCTION

Impellers, propellers, fans and bladed discs of turbomachines are some examples of structures that exhibit rotational cyclic symmetry in their geometric, material and constraint properties. The problem of statics and dynamics including aeroelastic analyses of such structures can be collectively and generally stated by the following equations of motion:

$$\begin{aligned}
 [M^n]\{\ddot{u}^n\} + \left[ [B^n] + 2\Omega [B_1^n] \right] \{\dot{u}^n\} \\
 + \left[ [K^e] + [K^d] - \Omega^2 [M_1^n] \right] \{u^n\} - [Q^n] \{u^n\} \\
 = \{P^n\}^{aero.} + \{P^n\}^{non-aero.} - [M_2^n] \{\ddot{r}_0\}, \quad (1)
 \end{aligned}$$

$$u_{side\ 2}^n = u_{side\ 1}^{n+1}, \quad (2)$$

where  $n = 1, 2, \dots, N$ .

The retention and interpretation of the terms of the above equations vary with the specific analysis being considered, and, as such, are discussed further under appropriate sections. A generic statement of the equations of motion is used to illustrate a logical approach to the solution of the problems of rotating cyclic structures. References 1 through 7 present extensive details of all the analyses discussed in this paper.

All capabilities described in this paper address tuned cyclic structures, i.e., structures composed of cyclic sectors identical in mass, stiffness, damping and constraint properties.

#### SYMBOLS

B	viscous damping matrix
$B_1$	Coriolis acceleration coefficient matrix
K	stiffness matrix
k	circumferential harmonic index
M	mass matrix
$M_1$	centripetal acceleration coefficient matrix
$M_2$	base acceleration coefficient matrix
N	number of rotationally cyclic sectors in complete structure
P	load vector
Q	aerodynamic matrix
$\ddot{R}_0$	base acceleration vector
u	displacement vector
$\Omega$	rotational velocity
Superscripts:	
d	differential
e	elastic
n	cyclic sector number

## STATIC AEROTHERMOELASTIC "DESIGN/ANALYSIS"

### OF AXIAL-FLOW COMPRESSORS

#### Problem Definition

At any operating point under steady-state conditions, the rotors and stators of axial-flow compressors are subjected to aerodynamic pressure and temperature loads. The rotors, in addition, also experience centrifugal loads. These loads result in deformation of the elastic structure, which, in turn influences the aerodynamic loads. These interactive loads and responses arise fundamentally from the elasticity of the structure and determine the performance of the "flexible" turbomachine. For a given flow rate and rotational speed, the elastic deformation implies a change in the operating point pressure ratio.

The process of arriving at an "as manufactured" blade shape to produce a desired (design point) pressure ratio (given the flow rate and rotational speed) is herein termed the "design" problem of axial-flow compressors. The subsequent process of analyzing the performance of "as manufactured" geometry at off-design operating conditions including the effects of flexibility is termed the "analysis" problem of axial-flow compressors.

The capability also determines:

- 1) the steady-state response of the structure (displacements, stresses, reactions, etc.), and
- 2) a differential stiffness matrix for use in subsequent modal, flutter and dynamic response analyses.

#### Formulation

Referring to equation 1, the degrees of freedom,  $u$ , are the steady-state displacements expressed in body-fixed global coordinate systems. The steady-state aerodynamic pressure and thermal loads,  $p^{aero}$ , are computed using a three-dimensional aerodynamic theory for axial-flow compressors (Ref. 8).  $K^e$ ,  $K^d$  and  $p^{non-aero}$  are the other terms retained in the analysis.

All cyclic sectors of the structure are assumed to respond identically, implying a zeroth circumferential harmonic distribution. Therefore only one rotationally cyclic sector is modelled and analyzed (Figure 1), with the intersegment boundary conditions (equation 2) imposed via MPC equations.

## NASTRAN Implementation

A new rigid format, DISP APP RF 16, has been developed for the solution of "design/analysis" problems of axial-flow compressors. The rigid format features new functional modules, bulk data cards and parameters. The computer code of Reference 8, with minor changes, has been adapted for NASTRAN in a new functional module ALG (Aerodynamic Load Generator). The NASTRAN Static Analysis with Differential Stiffness rigid format, DISP APP RF 4, has been modified to include the interactive effects of aerodynamic loads along with the effects due to centrifugal loads.

A simplified flowchart of the rigid format is shown in Figure 2.

## MODAL FLUTTER ANALYSIS OF AXIAL-FLOW TURBOMACHINES

### Problem Definition

Unstalled flutter boundaries of axial-flow turbomachines (compressors and turbines) can be determined using this capability. The stability of a given operating point of a given stage of the turbomachine is investigated in terms of modal families of several circumferential harmonic indices considered one at a time.

### Formulation

Considering the degrees of freedom,  $u$ , in equation 1 to represent the vibratory displacements superposed on the steady-state deformed shape of the rotor or stator, the natural modes and frequencies of the tuned cyclic structure can be grouped in terms of several uncoupled sets, with each set corresponding to a permissible circumferential harmonic index,  $k$ . Except for  $k = 0$  and  $N/2$  (even  $N$ ), the cyclic modes can further be separated into cosine and sine component modes (Ref. 9). For tuned cyclic structures, the modal flutter problem can be posed in terms of either cosine or sine modes with identical results (Ref. 2). For  $k = 0$  and  $N/2$ , only cosine modes are defined. In the present capability, this selection of mode type is provided as a user option.

$B_1$ ,  $M_1$  and the right hand side terms from equation 1 are omitted for this flutter capability.

For the computation of the generalized aerodynamic loads matrix,  $Q$ , two two-dimensional cascade unsteady subsonic and supersonic aerodynamic theories of References 10 and 11 are used in a strip theory manner from the blade root to the tip as shown in Figure 1. Based on the relative flow Mach number at a given streamline, either the subsonic or the supersonic theory is used. For the user specified transonic Mach number range, the aerodynamic matrix terms are interpolated from adjacent streamline values.

## NASTRAN Implementation

A new rigid format, AERO APP RF 9, has been developed for the cyclic modal flutter analysis of axial-flow turbomachines. The rigid format integrates the cyclic modal computations for a given circumferential harmonic index with currently available flutter solution techniques in NASTRAN. The unsteady aerodynamic theories have been incorporated in the existing functional module AMG (Aerodynamic Matrix Generator). New bulk data cards have been designed to meet specific needs of this flutter capability.

A flowchart outlining the rigid format is shown in Figure 3.

## FORCED VIBRATION ANALYSIS OF ROTATING CYCLIC STRUCTURES

### Problem Definition

Figure 4 illustrates the problem by considering a 12-bladed disc as an example. The bladed disc consists of twelve identical  $30^\circ$  segments. The disc rotates about its axis of symmetry at a constant angular velocity. The axis of rotation itself is permitted to oscillate translationally in any given inertial reference, thus introducing inertial loads. In addition, the bladed disc is allowed to be loaded with sinusoidal or general periodic loads moving with the structure. Under these conditions, it is desired to determine the dynamic response (displacements, accelerations, stresses, etc.) of the bladed disc.

### Formulation

The degrees of freedom,  $u$ , in equation 1 define the vibratory displacements due to the vibratory excitation provided by the directly applied loads and the inertial loads due to the acceleration of the axis of rotation ("base" acceleration). These displacements are measured from the steady-state deformed shape of the rotating structure, and are expressed in body-fixed global coordinate systems. The non-aerodynamic loads,  $P^{\text{non-aero}}$ , can either be sinusoidal loads specified as functions of frequency, or general periodic loads specified as functions of time. Physical loads on various segments or their circumferential harmonic components can be specified. The base acceleration,  $\ddot{R}_0$ , is noted as a function of frequency. All but  $Q$  and  $P^{\text{aero}}$  terms are retained in the analysis.

Based on the circumferential harmonic content of the excitation, the user can specify a range of such harmonic indices,  $k_{\text{min}}$  to  $k_{\text{max}}$ , for solution. Although the user models only one cyclic sector, results can be obtained for the complete structure.

## NASTRAN Implementation

The Direct Frequency and Random Response rigid format, DISP APP RF 8, and the Static Analysis with Cyclic Symmetry rigid format, DISP APP RF 14, have been suitably merged with extensive modifications, in the form of a package of ALTERs to the former rigid format. New functional modules for Coriolis, centripetal and base acceleration terms, bulk data parameters, and varied use of existing functional modules are some of the features of this alter package. Figure 5 presents a schematic flowchart of this forced vibration analysis capability for rotating cyclic structures.

### Illustrative Example

This example illustrates the out-of-plane displacement response of grid points 8 and 18 of the 12-bladed disc of Figure 4, when the disc, rotating at 600 rps, is simultaneously subjected to lateral base accelerations of  $\ddot{Y}_{inertial} = 1000 \cos 2\pi ft$  in/sec<sup>2</sup> and  $\ddot{Z}_{inertial} = 500 \cos 2\pi ft$  in/sec<sup>2</sup>,  $1700 \leq f \leq 1920$  Hz. Details of the bladed disc are given in Table 1. Table 2 lists the first few natural frequencies of the bladed disc for  $k=0,1$  and 2. Although the frequency band of input base acceleration is 1700-1920 Hz., the rotation of the disc at 600 Hz. splits the input bandwidth into two effective bandwidths: ( 1700-600 ) = 1100 to ( 1920-600 ) = 1320 Hz., and ( 1700+600 ) = 2300 to ( 1920+600 ) = 2520 Hz. Since the lateral base acceleration excites only  $k = 1$  modes, the only  $k = 1$  mode in the effective bandwidths is the first torsional mode of the blade, with the disc practically stationary ( 2460 Hz.,  $k=1$ , Table 2 ). This is shown by the out-of-plane displacement magnitudes of grid points 18 (blade) and 8 (disc) in Figure 6. For brevity, only the magnitude of the cosine component of the  $k = 1$  response is shown.

## MODAL FLUTTER ANALYSIS OF ADVANCED TURBOPROPELLERS

### Problem Definition

Advanced turbopropellers are multi-bladed propellers with thin blades of low aspect ratio and varying sweep ( Figure 7 ). The problem of determining the unstalled flutter boundaries of such propellers is identical to that discussed earlier for the axial-flow turbomachines with the exception that the effects of blade sweep and its spanwise variation are taken into account in computing the generalized unsteady aerodynamic loads. From a structural viewpoint, if the propeller hub is considered to be relatively much stiffer than the blades, the blades can be treated independently, and only the  $k = 0$  modes need be considered for flutter analysis.

## Formulation

This is the same as that for the axial-flow turbomachines, except that the subsonic unsteady aerodynamic theory of Ref. 10 has been modified to include the effects of blade sweep and its radial variability ( Ref. 6 ).

## NASTRAN Implementation

The functional module AMG has been modified to include the subsonic unsteady aerodynamic theory with sweep effects. This option can be invoked by including the NASTRAN System ( 76 ) = 1 card in front of the Executive Control Deck for the AERO APP RF 9. The STREAML2 bulk data card developed for turbomachine flutter analysis has been modified to also accept turboprop aerodynamic data.

## Illustrative Example

A comparison of the predicted flutter boundary using this NASTRAN capability and that obtained from NASA Lewis Research Center's wind tunnel test results is shown in Figure 8. The first six  $k = 0$  modes were included for flutter analysis of the 10-bladed advanced turboprop. The hub of the propeller was assumed to be rigid compared to its flexible blades.

## CONCLUDING REMARKS

A brief account of four new capabilities developed and implemented in NASTRAN Level 17.7 has been given in terms of problem definition, formulation, NASTRAN implementation and some selected examples. Details of all of these capabilities can be found in References 1 through 7.

## REFERENCES





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TABLE 1. GEOMETRIC DETAILS OF 12-BLADED DISC

Diameter at blade tip	=	19.4 in.
Diameter at blade root	=	14.2 in.
Shaft diameter	=	4.0 in.
Disc thickness	=	0.25 in.
Blade thickness	=	0.125 in.
Young's modulus	=	$30.0 \times 10^6$ lbf/in <sup>2</sup>
Poisson's ratio	=	0.3
Material density	=	$7.4 \times 10^{-4}$ lbf-sec <sup>2</sup> /in <sup>4</sup>
Uniform structural damping (g)	=	0.02

TABLE 2: BLADED-DISC NATURAL FREQUENCIES

Frequency (Mode No.), Hz.			Mode Description
* k = 0	k = 1	k = 2	
214 (1)	208 (1)	242 (1)	
591 (2)	594 (2)	622 (2)	
1577 (3)	1633 (3)	1814 (3)	
2468 (5)**	2460 (4)	2433 (4)	

\* k is the circumferential harmonic index

\*\* Mode No. 4 for k = 0 at 1994 Hz represents an in-plane shear mode not excited by the applied forces.

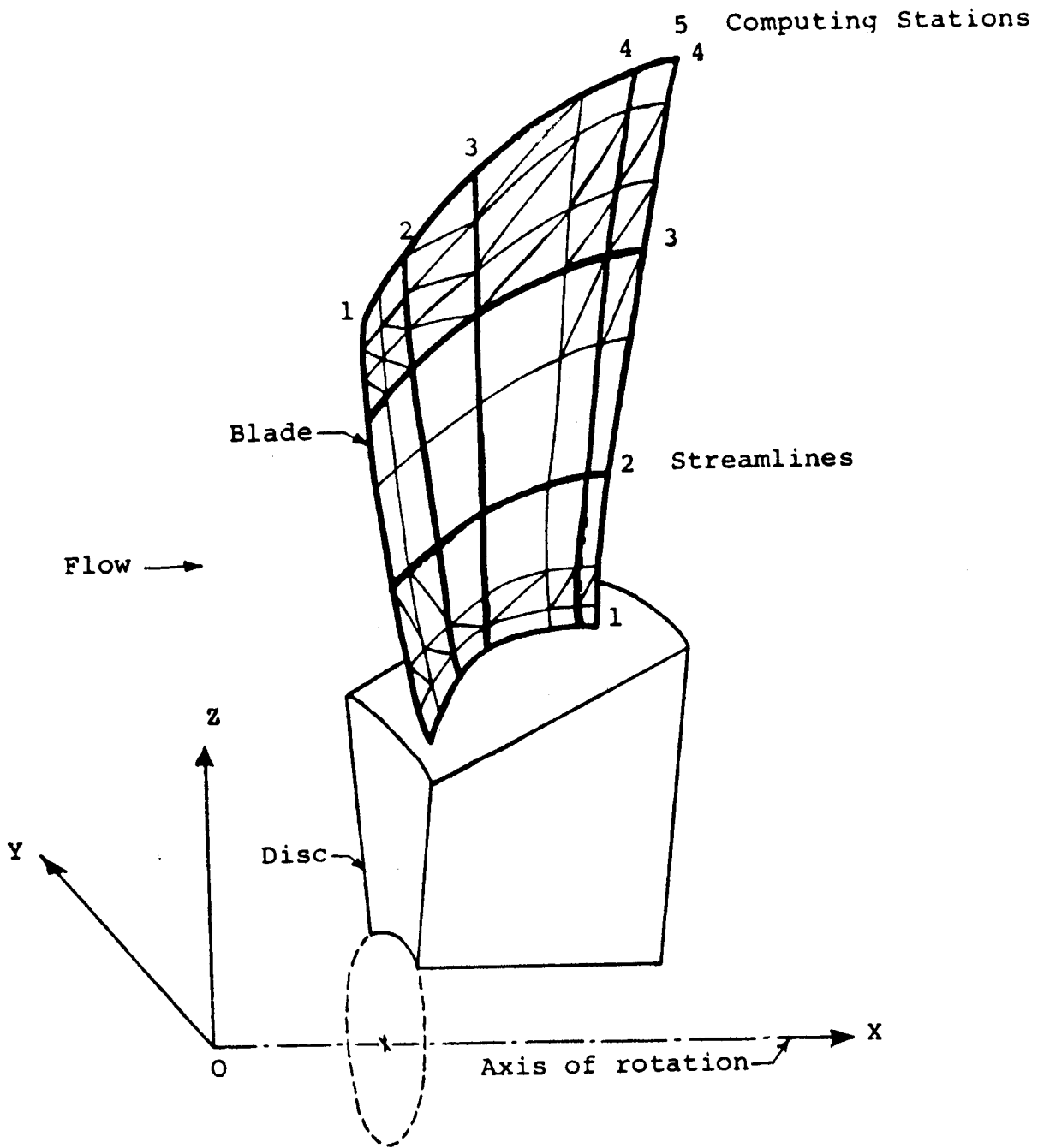


FIGURE 1. ROTATIONAL CYCLIC SECTOR OF A BLADED DISC

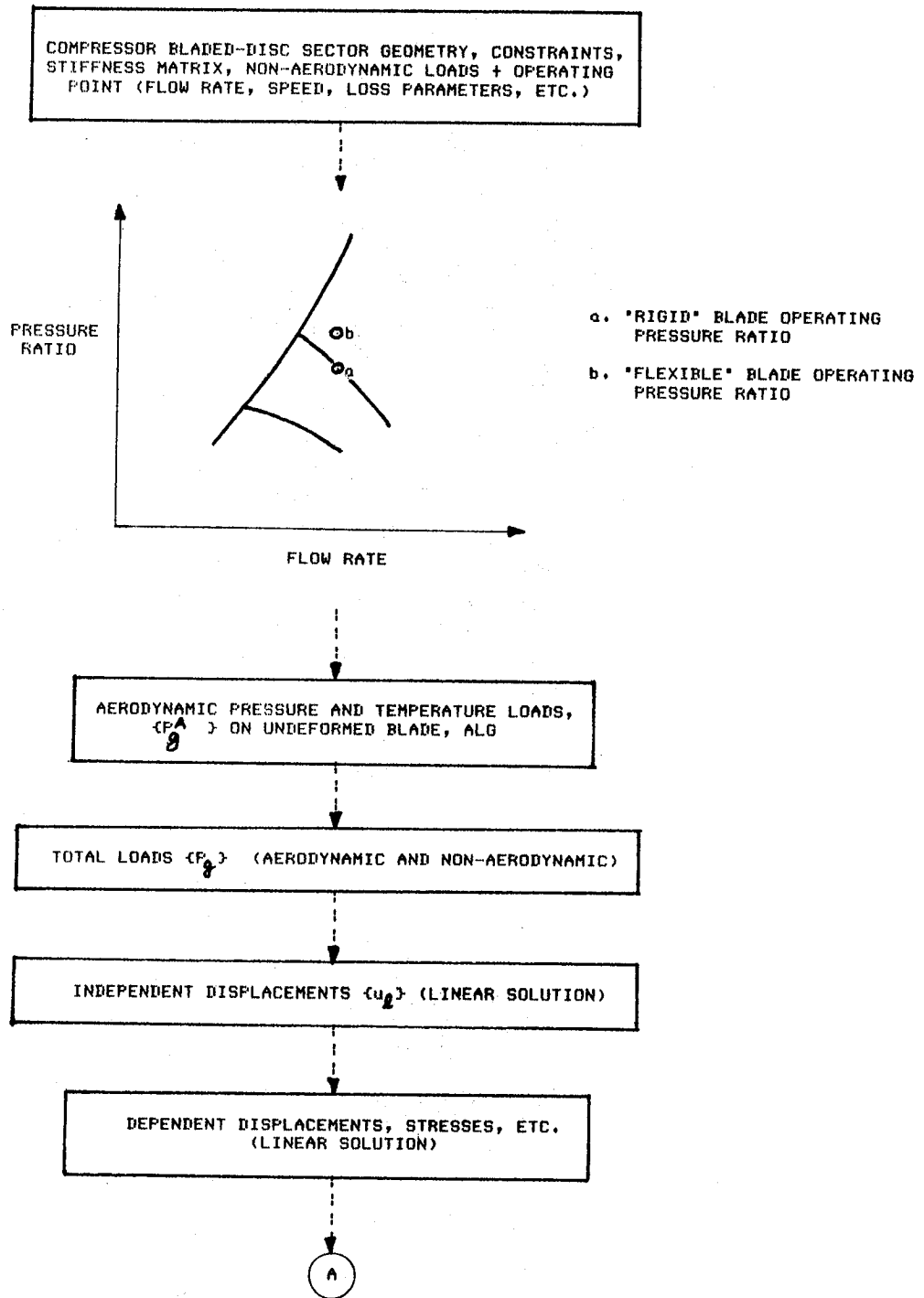


FIGURE 2. SIMPLIFIED PROBLEM FLOW FOR STATIC AEROTHERMOELASTIC "DESIGN/ANALYSIS" RIGID FORMAT FOR AXIAL FLOW COMPRESSORS

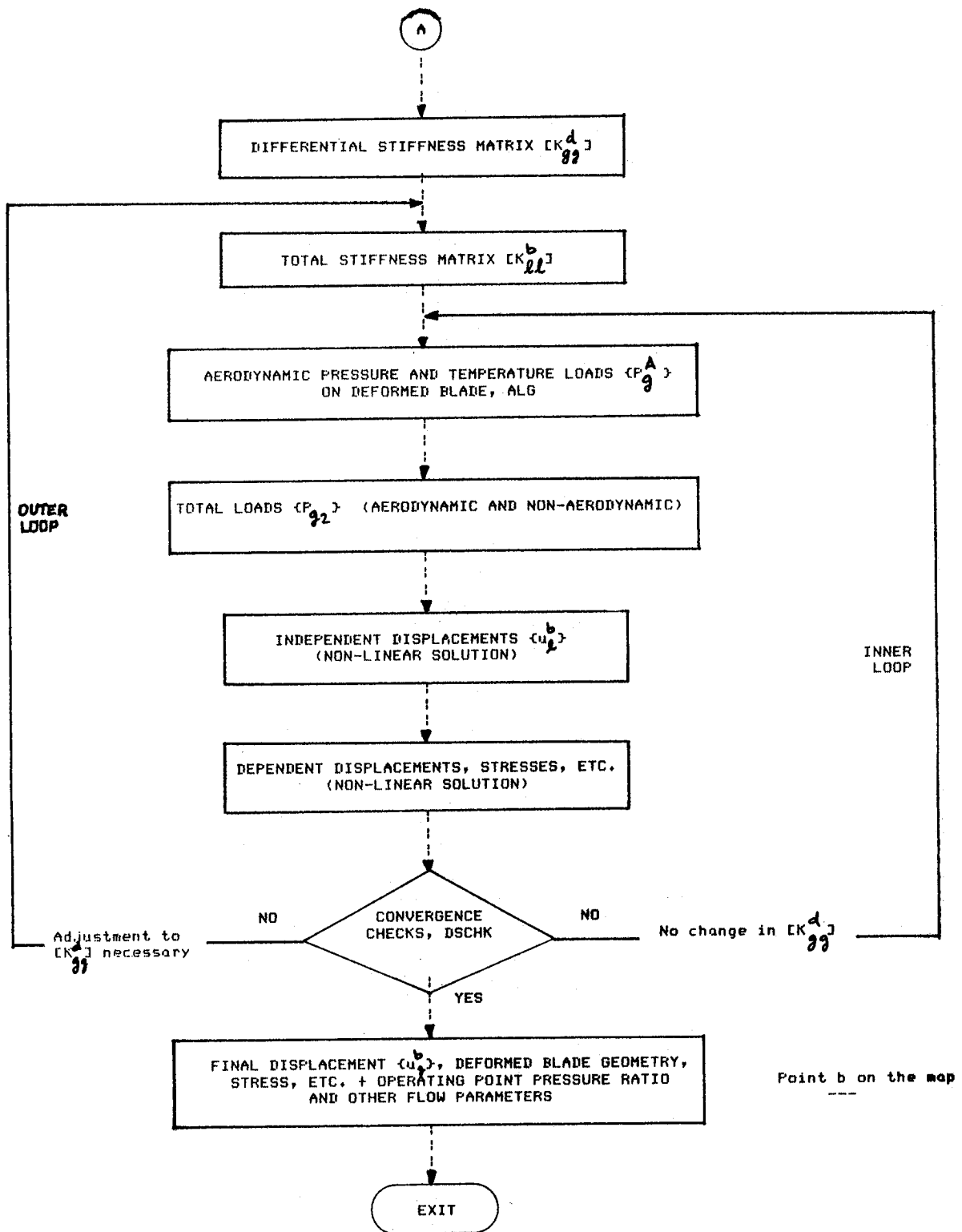


FIGURE 2. (Concluded)

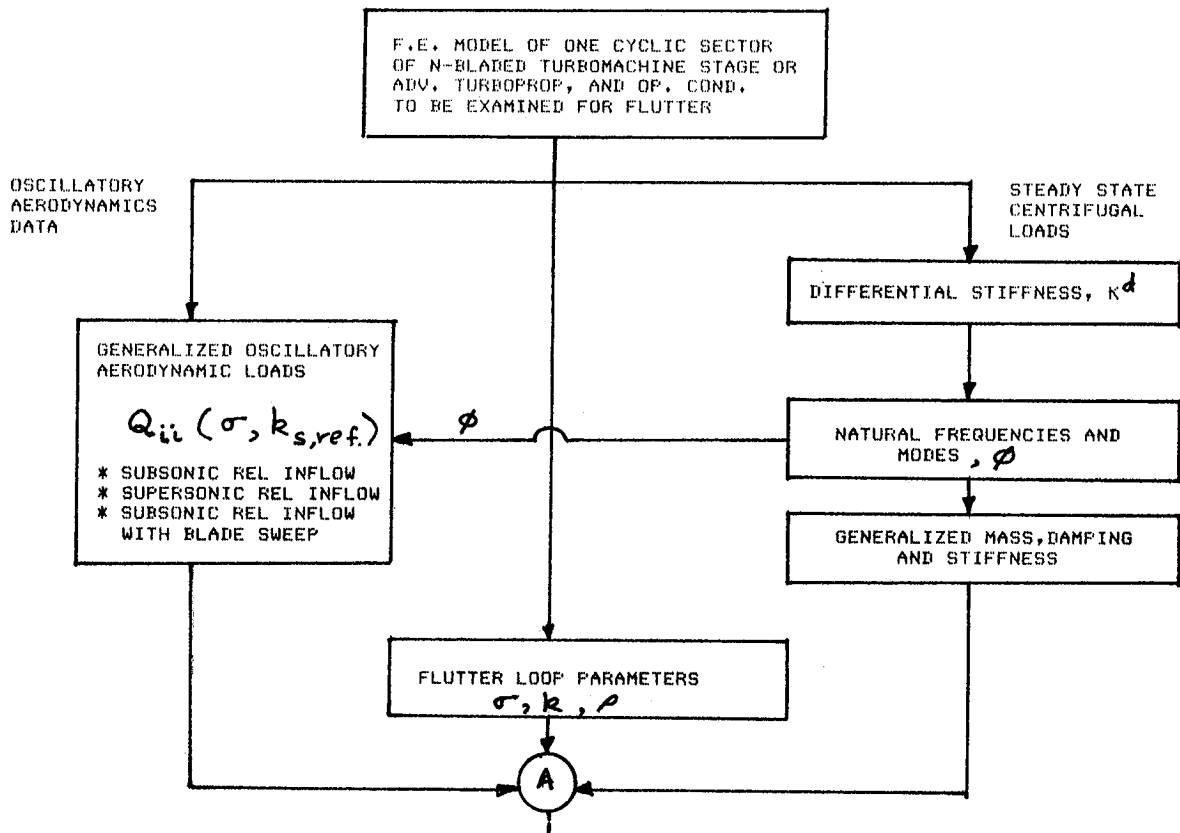


FIGURE 3. OVERALL FLOWCHART OF CYCLIC MODAL FLUTTER ANALYSIS RIGID FORMAT FOR TURBOMACHINES AND ADVANCED TURBOPROPELLERS

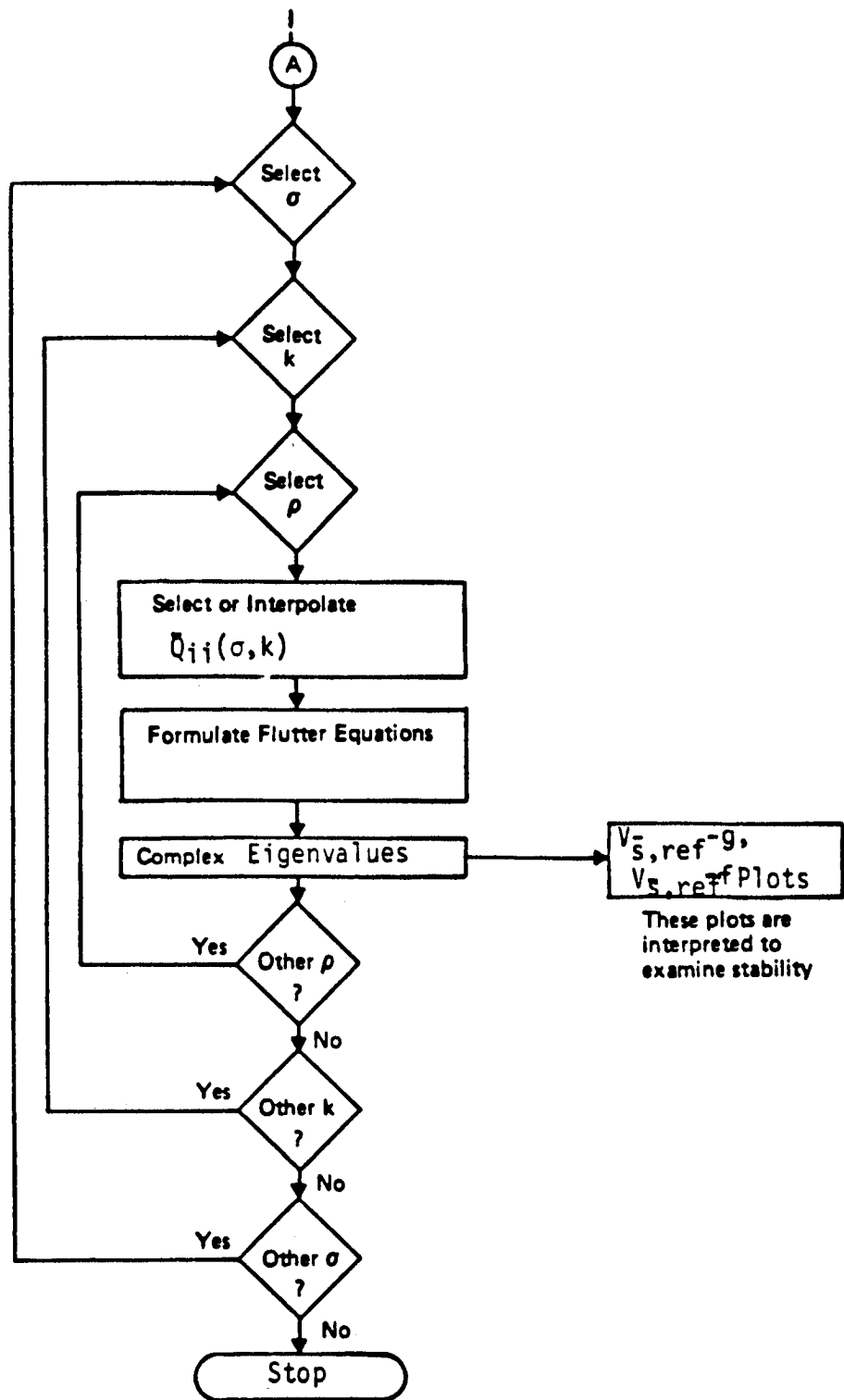
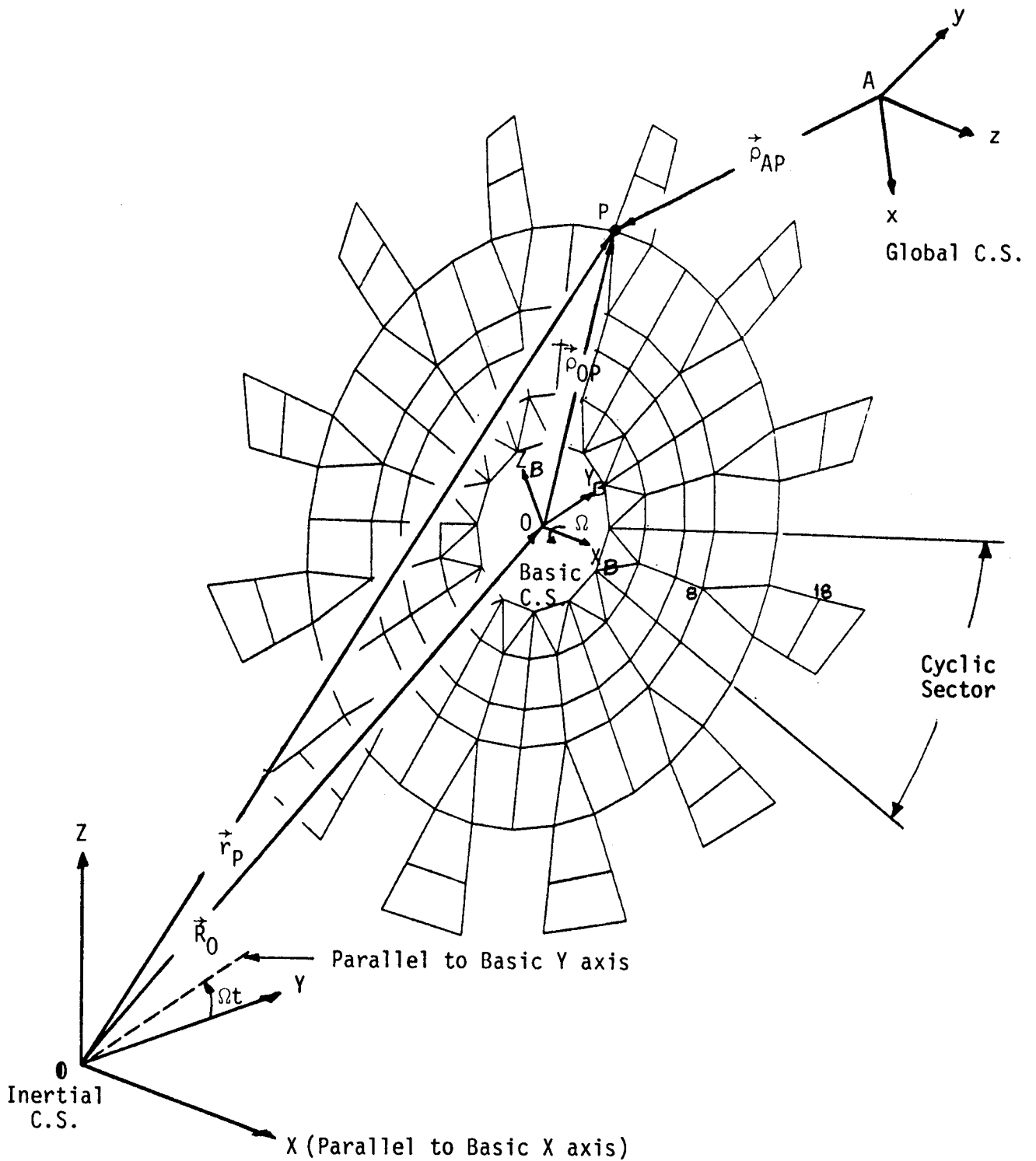


FIGURE 3. (Concluded)



- $\hat{i}, \hat{j}, \hat{k}$  Unit vectors along Inertial XYZ axes
- $\hat{i}_B, \hat{j}_B, \hat{k}_B$  Unit vectors along Basic  $X_B Y_B Z_B$  axes
- $\hat{i}, \hat{j}, \hat{k}$  Unit vectors along Global xyz axes

FIGURE 4. BLADED DISC EXAMPLE FOR FORCED VIBRATION ANALYSIS OF ROTATING CYCLIC STRUCTURES

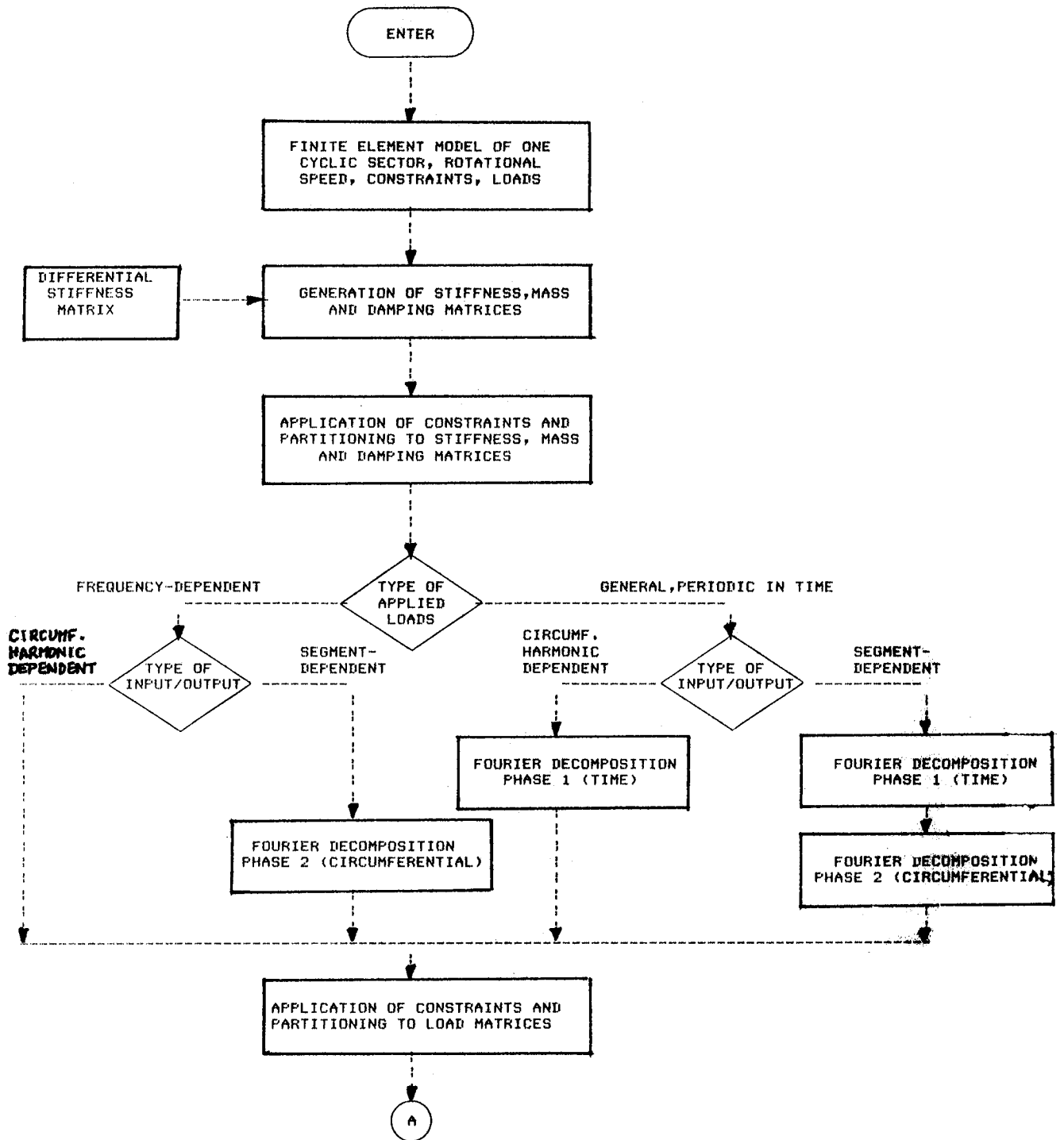


FIGURE 5 - OVERALL FLOWCHART OF FORCED VIBRATION ANALYSIS OF ROTATING CYCLIC STRUCTURES.



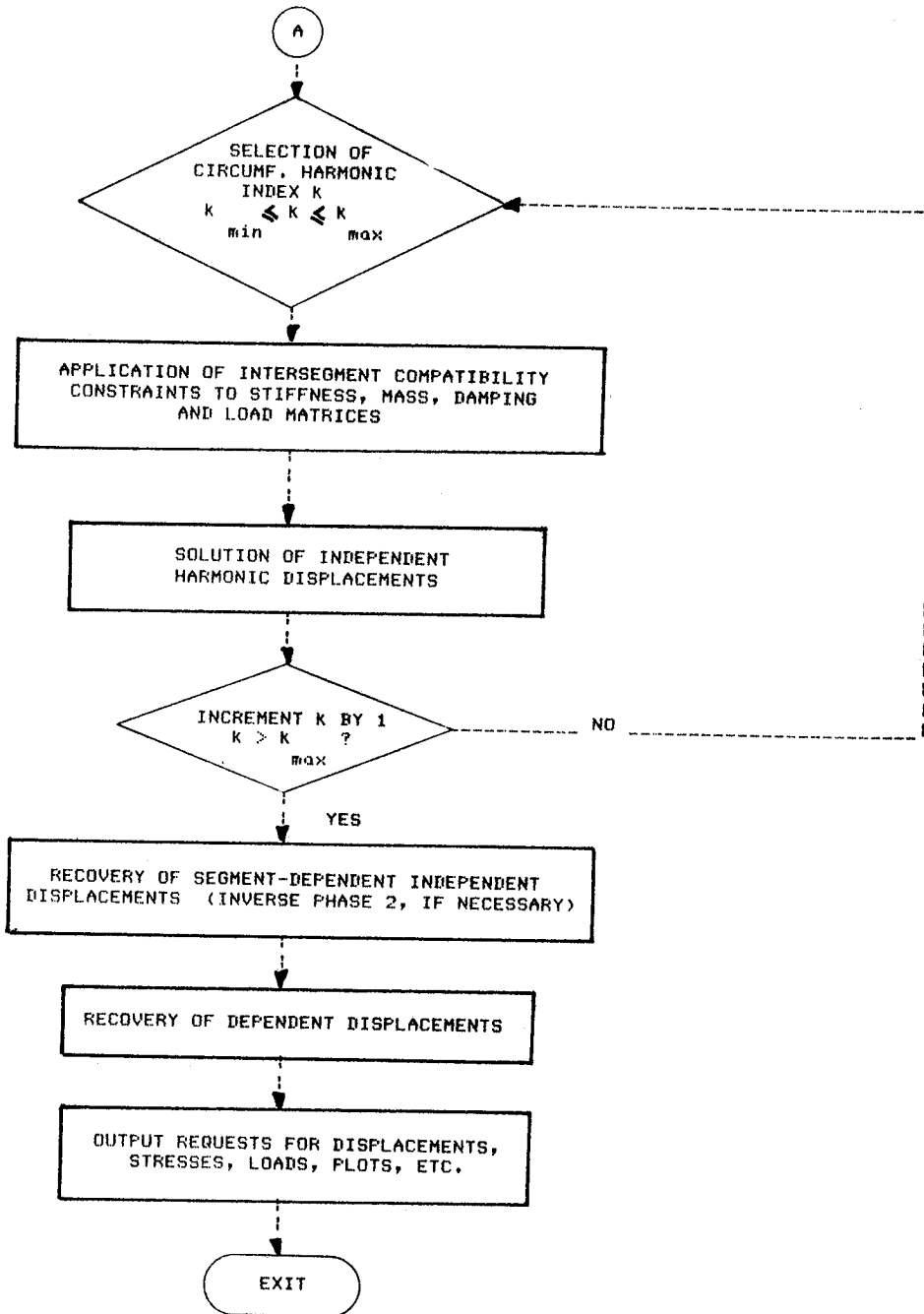


FIGURE 5 - (Concluded)

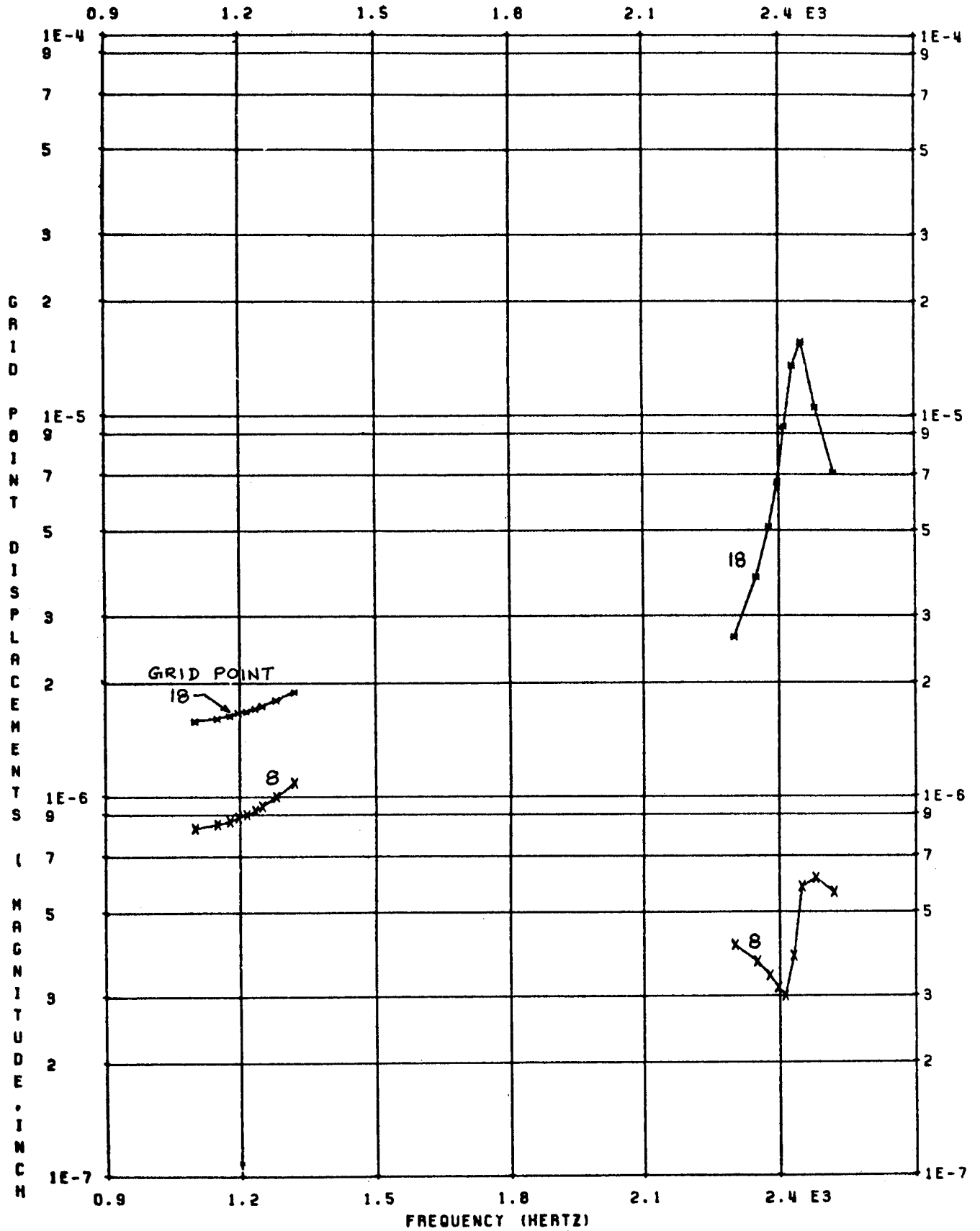


FIGURE 6.  $k=1c$  DISPLACEMENT RESPONSE OF 12-BLADED DISC TO LATERAL BASE ACCELERATION EXCITATION



FIGURE 7. AN ADVANCED TURBOPROPELLER

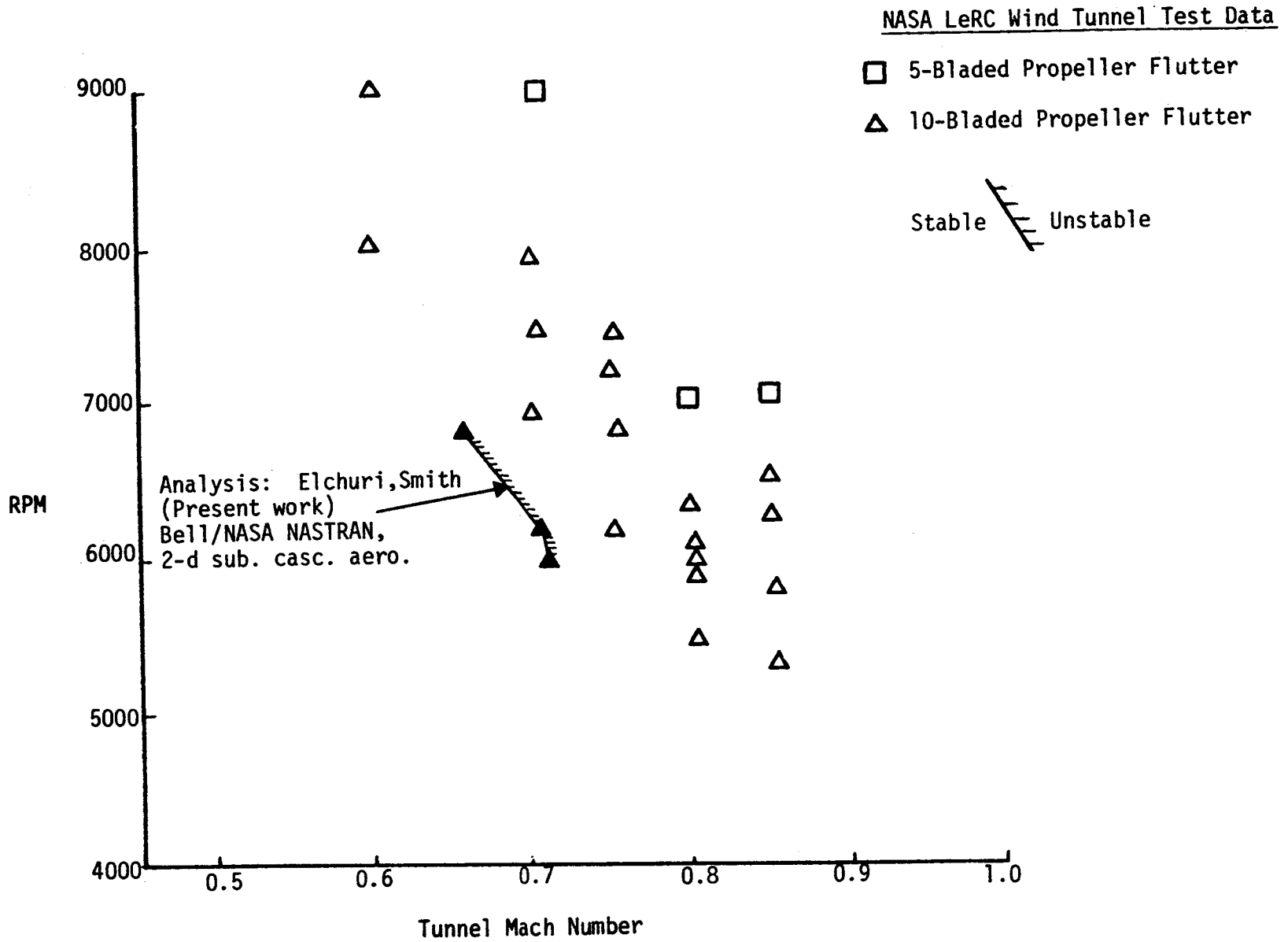


FIGURE 8. TURBOPROP FLUTTER SUMMARY