Group-Kinetic Scaling as a Basis for Modeling Large Scale Turbulence

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I. NEW TRENDS IN TURBULENCE

In the NASA Workshop on Turbulence Modeling, at NASA Headquarters, Dec. 13-14, 1983, the trends in turbulence has been heavily debated. The recommendation was the need for new concepts and methods to develop a statistical theory of turbulence. The treatment should detach from the conventional concepts and should incorporate new insights from the modern statistical physics. Among them, the "catastrophe theory" was mentioned.

A nonlinear dynamical system can take the various forms of (i) algebric equations, (ii) ordinary differential equations, and (iii) partial differential equations. By the strong perturbations and the inherent nonlinearity, the system can become unstable, develop collective interactions, and reach a state of turbulence. The transition from the laminar motion to turbulence, and the characterization of those processes through which a nonlinear dynamical system undergoes before reaching turbulence are the subject matter of "chaos and universality". The interest on this topic is partially due to its easy availability to numerical computations, at least in one dimension and in the dynamical forms (i) and (ii). We must notice the important gap between the dynamical development of chaos and their statistical treatment. It is with the purpose of devising a mathematical tool for transforming the dynamical system into a statistical framework that we develop the group-kinetic method.

II. GROUP-KINETIC METHOD FOR THE TRANSFORMATION OF THE DYNAMICAL SYSTEM INTO A KINETIC SYSTEM AS THE BASIS FOR A STATISTICAL TREATMENT OF TURBULENCE

In our group method, the stochastic and nonlinear differential system, as governing the compressible turbulence and incompressible turbulence, undergoes three successive transformations. As the first step, we transform the differential system from the configurational space into the phase space. The resulting master equation becomes homogeneous, has lesser nonlinearity and emcompasses all the equations (for density, velocity and energy) of the dynamical system. Secondly, we note that the master equation and its Fourier decomposition contain too many minute details. For the statistical treatment a coarse-graining procedure by group-scaling is necessary in analogy with the group-renormalization. To this end, we use the scaling operators A^o, A', A'' to decompose a fluctuating quantity into the macro-group, the micro-group and the submicro-group of decreasing coherence, representative of the three transport processes of spectral evolution, eddy transport property and relaxation. By formulating the relaxation as a functional of the transport coefficient, we obtain a closure. The relaxation is investigated by a path-perturbation theory. This involves the evolution-operator and the derivation of the equation for the probability of retrograde transition. The kinetic equation thus obtained has an eddy collision coefficient in the form of an integral operator of memory. Thirdly, we transform the kinetic equation back into the configurational space by taking the moments. Hence we derive the equation of spectral flow for determining the spectral structure. The group-scaling has the important advantage of enabling the determination of the spectral function from the one-point distribution function alone, without the knowledge of the two-point distribution function as is required in the conventional methods of many-body statistical mechanics.

III. SPECTRAL STRUCTURE AS A BASIS FOR TURBULENCE MODELING

A. Shear turbulence and geostrophic turbulence

By the group-kinetic method, we find the direct and reverse cascades for the transfer across the spectrum. We derive the spectral distributions in inertia turbulence ($k^{-5/3}$ law), shear turbulence (k^{-1} law) and geostrophic turbulence. The geostrophic turbulence without driving force has a k^{-3} spectrum and the geostrophic turbulence with a random driving force has a k^{-4} spectrum.

B. Atmospheric boundary layer

For the convective turbulence in the stable atmospheric boundary layer, we find a spectral structure for the three subranges. The horizontal velocity components start with a buoyancy subrange of k^{-3} law, to be followed by a spectral gap, a shear subrange of k^{-1} law and an inertia subrange of $k^{-5/3}$ law at the tail of the spectrum. The vertical component has a depressed spectrum at small wavenumbers before it resumes the inertia tail.

C. Anisotropy

The self-consistent forces (pressure and buoyancy) act differently among the different components of the velocity spectral distribution. In the past their effects were assumed. Our group-kinetic method develops a homogeneous master equation that lumps these forces into the advection in the phase space, so that the path perturbations can be readily analyzed.

D. New concepts in the group-modeling of turbulence

For the prediction of profiles (mean wind velocity, temperature and humidity), a macroscopic version of our transport theory is needed. The closure of the transport hierarchy is obtained by a non-stationary theory of transport for the eddy stress and the pressure-strain correlation. The transport coefficients are integral operators of memory and are functionals of the spectral distributions, the mean flow parameters, and other dimensionless parameters, such as the Richardson number and the Rossby number. We summarize the following transport equations in the modeling: (i) the equation of spectral flow, (ii) the equations for the evolution of the transport coefficients, (iii) the transport equations of mean profiles, and (iv) the transport equations for the eddy stress and the pressure-strain correlation. The group-modeling is superior, because the conventional modelings have unknown scales and unknown length parametrization.

IV. ACCOMPLISHMENTS

Eight manuscripts have been completed by C. M. Tchen, and were compiled under two NASA Contractor Reports, entitled "Theory and Modeling of Atmospheric Turbulence".

Volume One contains the following manuscripts:

1. Kinetic basis of cascade transfer in turbulence 43 pages
2. Kinetic theory of turbulent transport with double memory-loss, 36 pages
3. Group-scaling theory for the enstrophy turbulence in two dimensions, 31 pages
4. Group-kinetic theory of turbulent collective collision, 44 pages.

Volume Two contains the following manuscripts:

1. Group-kinetic theory of two-dimensional geostrophic turbulence, 32 pages
2. Equivalent methods for describing the quasilinear turbulent trajectory, 29 pages

3. A new kinetic description for turbulent collisions, including mode-coupling, 29 pages
4. Spectral structure of turbulence in the stable atmospheric boundary layer, 14 pages.

V. PLANS FOR CONTINUING RESEARCH

Our group-kinetic method will be extended to the atmospheric processes with shear, stratification and Coriolis force. In particular, we investigate the following:

1. Eddy transport theory for the derivation of anisotropic transport properties
2. Kinetic theory of pressure-strain correlation
3. Spectral structure of anisotropic turbulence
4. Upper atmospheric boundary layer and inversion layer
5. Modeling of turbulence as based on the new concept of group-modeling, as outlined in Subsection IIID.

VI. RECOMMENDATIONS FOR NEW RESEARCH: MODULATIONAL INSTABILITY AND SOLITON TURBULENCE IN THE BAROCLINIC ATMOSPHERE

In 1926 Madelung observed a formal equivalence between a fluid system and the Schrödinger wave equation. He recommended this transformation for non-barotropic fluids preferably. The nonlinear Schrödinger equation is now of intense physical interest and has received a great attention for applications to plasma physics, astrophysics and ocean dynamics. Unfortunately the advantage of the correspondence between the concepts of fluid dynamics and those of wave mechanics has not been fully exploited in researches on the atmospheric wave processes.

The nonlinearity can be local, as in the cubic nonlinear Schrödinger equation. It is made non-local in the Zakharov equations. Zakharov added to the Schrödinger equation an acoustic equation for the non-local modulation. We recommend a modification of the Zakharov system by adding new scattering functions and new parameters representative of the baroclinic and rotational properties of the atmosphere. Not much progress has been reported on such modulational instabilities and soliton turbulence. The slow progress is due to the lack of a suitable mathematical tool. We are convinced that the group-kinetic method will help in overcoming this difficulty.