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TORQUE COMMAND STEERING LAW
FOR
DOUBLE-GIMBALED CONTROL MOMENT GYROS
APPLIED TO
ROTOR ENERGY STORAGE

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INTRODUCTION

As space vehicles become larger and mission times become longer double-gimbaled control moment gyros (CMGs) will be used more and more for attitude control, angular momentum storage and energy storage. Therefore, a need exists for a simple, but universally applicable CMG steering law which takes the control torque command and generates gimbal rate commands in such a way that, in spite of the highly nonlinear CMG behavior, the resulting torque on the vehicle is ideally equal to the commanded one. Furthermore, this steering law should allow the use of any number of CMGs, which allows the tailoring to the angular momentum requirement while at the same time making the failure accommodation a built-in feature. It should also allow the mixing of CMGs of different momentum magnitudes to the extent that some or all CMGs could be used for energy storage. It also should allow adding energy storage fly-wheels to the mix.

A steering law is presented here, which has all these features, assuming the CMG outer gimbal freedom is unlimited (however, the range of the outer gimbal angle readout should be: $-\pi \geq \beta_i > +\pi$; the inner gimbal angle should be hardware limited to $\pm\pi/2$). The reason is the idea of mounting all the outer gimbal axes of the CMGs parallel to each other. This allows the decomposition of the steering law problem into a linear one for the inner gimbal angle rates and a planar one for the outer gimbal angle rates. The inner gimbal angle rates are calculated first, since they are not affected by the outer gimbal angle rates. For the calculation of the outer rates, the inner rates are then known quantities. An outer gimbal angle distribution function (to avoid singularities internal to the total angular momentum envelope) generates distribution rates next, and finally the pseudoinverse method is used to insure that the desired total torque is delivered.

This paper is a revision of Reference 1.

PARALLEL MOUNTING ARRANGEMENT

The proposed mounting, with all outer gimbal axes parallel, is shown in Figure 1 (for convenience only, the outer gimbal axes are also shown colinear). This mounting arrangement has many advantages. The mounting interfaces can be identical, i.e., the mounting brackets and hardware, cable harnesses, etc. There is no need to individually identify the CMGs, and the on-board computer can assign an arbitrary label to any CMG, which could be changed from one computation cycle to the next. This simplifies the steering law and the redundancy management. The parallel mounting in conjunction with a steering law that accepts any number of CMGs makes failure accommodation a built-in feature. However, if increasing angular momentum requirements during the design of a space vehicle demand it (and the moments of inertia always tend to increase), additional CMGs can be added with minimum impact on the hardware and almost none on the software. The parallel mounting also makes the visualization of the system operation exceedingly simple (especially when compared with the momentum envelopes and saturation surfaces of skewed mounted single-gimbaled CMGs).

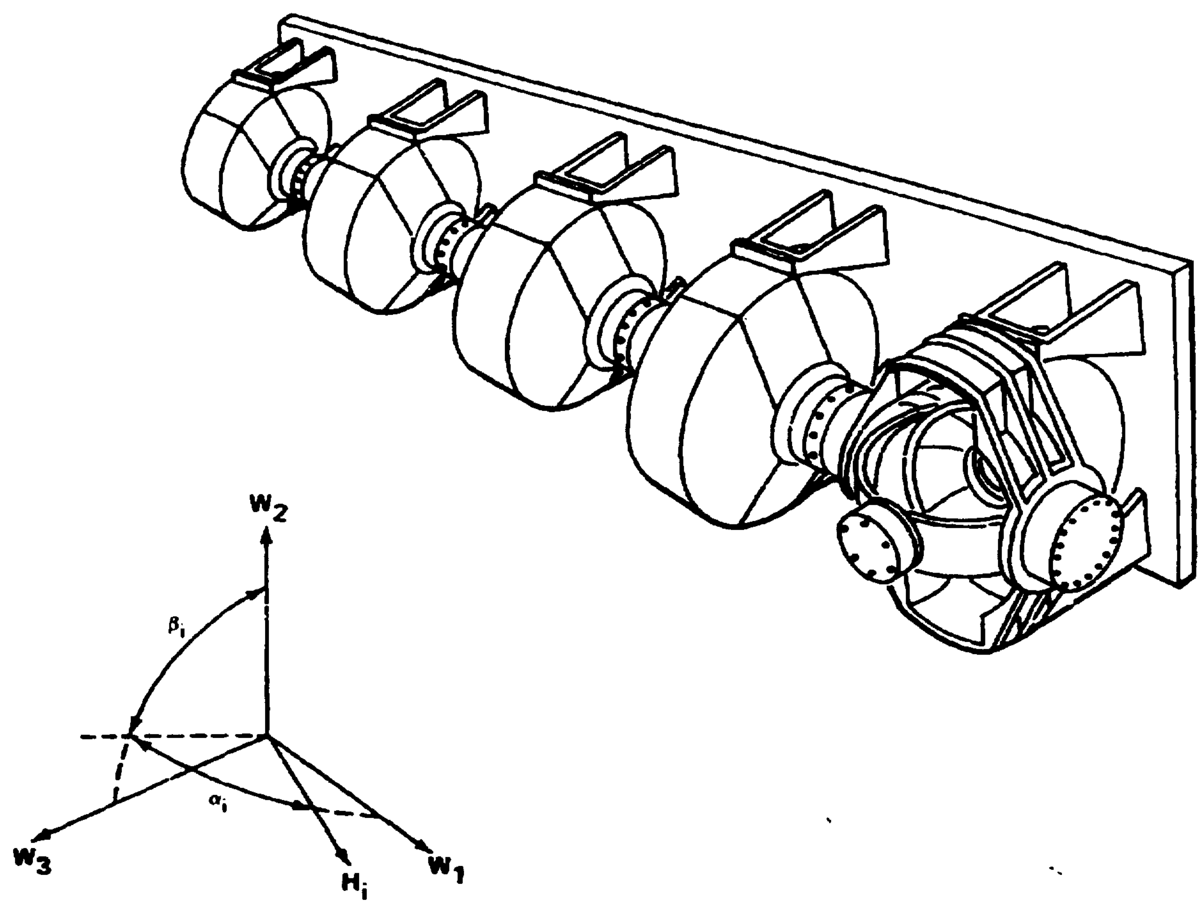


Figure 1.-Proposed mounting.

TOTAL ANGULAR MOMENTUM AND TORQUE COMMAND

Given a CMG coordinate system W, Figure 1 shows the inner and outer gimbal angles α_i and β_i of the ith CMG. With this definition, the angular momentum of n CMGs is (i ranges from 1 to n, H_i is the momentum magnitude):

$$\underline{H}_G = \begin{bmatrix} H_{G1} \\ H_{G2} \\ H_{G3} \end{bmatrix} = \begin{bmatrix} \sum_i H_i \, s\alpha_i \\ \sum_i H_i \, c\alpha_i \, c\beta_i \\ \sum_i H_i \, c\alpha_i \, s\beta_i \end{bmatrix} \tag{1}$$

The CMG angular momentum change is

$$\dot{\underline{H}}_G = \begin{bmatrix} \sum_i H_i \, c\alpha_i \, \dot{\alpha}_i \\ \sum_i (-H_i \, s\alpha_i \, c\beta_i \, \dot{\alpha}_i - H_i \, c\alpha_i \, s\beta_i \, \dot{\beta}_i) \\ \sum_i (-H_i \, s\alpha_i \, s\beta_i \, \dot{\alpha}_i + H_i \, c\alpha_i \, c\beta_i \, \dot{\beta}_i) \end{bmatrix} + \begin{bmatrix} \sum_i \dot{H}_i \, s\alpha_i \\ \sum_i \dot{H}_i \, c\alpha_i \, c\beta_i \\ \sum_i \dot{H}_i \, c\alpha_i \, s\beta_i \end{bmatrix} \tag{2}$$

We can assume that the $\dot{H}$ terms are known (from power demand or the difference between present and past value of $\underline{H}$) and it is accounted for in the torque command:

$$\underline{T} = \dot{\underline{H}}_G - \dot{\underline{H}} = \begin{bmatrix} \sum_i H_i \, c\alpha_i \, \dot{\alpha}_i \\ \sum_i (-H_i \, s\alpha_i \, c\beta_i \, \dot{\alpha}_i - H_i \, c\alpha_i \, s\beta_i \, \dot{\beta}_i) \\ \sum_i (-H_i \, s\alpha_i \, s\beta_i \, \dot{\alpha}_i + H_i \, c\alpha_i \, c\beta_i \, \dot{\beta}_i) \end{bmatrix} \tag{3}$$

The task is now to find a set of gimbal angle rate commands $\dot{\underline{\alpha}}$ and $\dot{\underline{\beta}}$ which when inserted into equation (3) will satisfy the torque command, while at the same time utilizing the excess degrees of freedom to distribute the gimbal angles such that the momentum singularities are avoided.

DESIRABLE INNER GIMBAL ANGLE DISTRIBUTION

The n double-gimbaled CMGs have 2n degrees of freedom. Three are needed to satisfy the torque command; the excess of 2n-3 degrees of freedom is utilized to achieve a desirable gimbal angle distribution. Before one can decide on a desirable distribution, the characteristics of a double-gimbaled CMG have to be considered.

The inner gimbal angle rate needed to produce a given torque perpendicular to the inner gimbal axis is independent of the inner and the outer gimbal angles. However, the outer gimbal angle rate needed to produce a given torque perpendicular to the outer gimbal axis is inversely proportional to the cosine of the inner gimbal angle. Therefore, it is desirable to keep the cosine of the inner gimbal angle large, i.e., it is desirable to minimize the largest inner gimbal angle. This then reduces to outer gimbal angle rate requirements. The largest inner gimbal angle is minimized when all inner gimbal angles are equal to the inner gimbal reference angle, cf. equation (1),

$$\alpha_R = \sin^{-1} \left\{ \left(\sum_i H_i \, s\alpha_i \right) \Big/ \left(\sum_i H_i \right) \right\} \tag{4}$$

Inner gimbal angle redistribution should be made without resulting in a torque along the W_1 axis. This implies [cf. equation (3)]

$$T_{D1} = \sum_i H_i \, c\alpha_i \, \dot{\alpha}_{Di} = 0$$

For simplicity we select the distribution rate as $\dot{\alpha}_{Di} = K_a \, (\alpha_o - \alpha_i)$. The W_1 torque is now

$$T_{D1} = \sum_i H_i \, c\alpha_i \, K_a \, (\alpha_o - \alpha_i)$$

Setting this torque to zero results in

$$\alpha_o = \left(\sum_i H_i \, c\alpha_i \, \alpha_i \right) \Big/ \left(\sum_i H_i \, c\alpha_i \right) \tag{5}$$

When all $\alpha_i = \alpha_o$, α_o has converged to its ideal value of equation (4). However, a torque does result in the W_2-W_3 plane and it will be treated in conjunction with the outer gimbal angle distribution.

DESIRABLE OUTER GIMBAL ANGLE DISTRIBUTION

The situation is not as clear-cut with respect to the desirable outer gimbal angle distribution. However, for double-gimbaled CMGs a singular condition inside the total angular momentum envelope can only occur when some of the momentum vectors (at least one) are antiparallel and the rest parallel to their resultant total angular momentum vector. Maintaining adequate and more or less equal spacing between the vectors will therefore eliminate the possibility of a singularity. Many distribution functions are possible and the goal is to find one which minimizes the software requirements. The selected distribution function requires no transcendental functions or divisions and allows CMG failures or resurrections any time. The distribution function uses repulsion between all possible CMG pairs, i.e., proportional to the product of the angular momentum magnitudes and proportional to the supplement of the outer gimbal angle differences. Therefore, the repulsion ranges from a maximum (for equal outer gimbal angles) to zero (for antiparallel momentum vector components). Since the repulsion is generated by pairs (and all possible pairs are treated equally), no sorting is necessary. However, due to the stronger repulsion of the immediate neighbors, the outer gimbal angles act as if sorting had been done. A failed CMG is simply ignored (its angular momentum magnitude is set to zero) since it does not generate a repulsion. Any number of CMGs can be failed or resurrected at the beginning of any computation cycle. The rates due to this distribution function are (with $-\pi \leq \beta_i < +\pi$)

$$\dot{\beta}_{Di} = -K_b \sum_j \{H_i H_j [\beta_i - \beta_j - \pi \operatorname{sgn} (\beta_i - \beta_j)] (1 - \delta_{ij})\} \quad (6)$$

Without any momentum constraint the CMG outer gimbals would come to rest when they are separated by an angle of $2\pi/n$, assuming identical angular momentum magnitudes. (This would not make any sense for two CMGs since they would be antiparallel. However, two CMGs have only one excess degree of freedom and it is taken up by the inner distribution, and no distribution is possible for the outer gimbals.)

INNER GIMBAL RATE COMMANDS

The change of the W_1 angular momentum component is not a function of the outer gimbal rates [equation (3)],

$$\dot{H}_{G1} = T_{G1} = \sum_i H_i \, c\alpha_i \, \dot{\alpha}_i$$

Since all inner gimbal angles should be equal (enforced by the inner gimbal distribution), an inner gimbal rate command for all CMGs of

$$\dot{\alpha}_C = T_{C1} \Big/ \sum_i H_i \, c\alpha_i \tag{7}$$

will result in $T_{G1} = T_{C1}$, if it is assumed that the actual and the commanded gimbal rates are equal. For the total inner gimbal angle rate command we have then [cf. equation (5)],

$$\dot{\alpha}_i = K_a (\alpha_o - \alpha_i) + T_{C1} \Big/ \sum_i H_i \, c\alpha_i \tag{8}$$

where it should be remembered that the distribution rates are non-torque-producing with respect to the W_1 axis. The effect of the total inner gimbal angle rates on the W_2 and W_3 momentum components is treated later. Gimbal rate limiting will be discussed after the outer gimbal rate commands have been treated.

OUTER GIMBAL RATE COMMANDS

The outer distribution rates of equation (6) as well as the total inner gimbal angle rate commands of equation (8) generate a torque in the W_2-W_3 plane. This torque is

$$\begin{bmatrix} T_2' \\ T_3' \end{bmatrix} = \begin{bmatrix} -\sum_i H_i \ (c\alpha_i \ s\beta_i \ \dot{\beta}_{Di} + s\alpha_i \ c\beta_i \ \dot{\alpha}_i) \\ \sum_i H_i \ (c\alpha_i \ c\beta_i \ \dot{\beta}_{Di} - s\alpha_i \ s\beta_i \ \dot{\alpha}_i) \end{bmatrix}$$

and the W_2-W_3 torques must be modified as follows:

$$\begin{bmatrix} T_{CM2} \\ T_{CM3} \end{bmatrix} = \begin{bmatrix} T_{C2} - T_2' \\ T_{C3} - T_3' \end{bmatrix}$$

To generate the modified torque, we now apply the pseduo-inverse method to get a unique set of outer gimbal rate commands

$$\begin{bmatrix} \dot{\beta}_{C1} \\ \dot{\beta}_{C2} \\ \vdots \\ \dot{\beta}_{Cn} \end{bmatrix} = [B]^T \{[B][B]^T\}^{-1} \begin{bmatrix} T_{CM2} \\ T_{CM3} \end{bmatrix}$$

where

$$[B] = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \end{bmatrix} \qquad \begin{pmatrix} b_{1i} = -H_i \ c\alpha_i \ s\beta_i \end{pmatrix} \quad \begin{pmatrix} b_{2i} = H_i \ c\alpha_i \ c\beta_i \end{pmatrix}$$

The total outer gimbal rate commands are then

$$\dot{\beta}_i = \dot{\beta}_{Di} + \dot{\beta}_{Ci}$$

PROPORTIONAL GIMBAL RATE LIMITING

The gimbal torques will have a definite torque limit, T_{LIM}. In order not to exceed this limit, we need to know the torque demand on the torquers due to the gimbal rate commands. Setting the outer gimbal angle to zero we get for the torque of the ith CMG [equation (3)],

$$\begin{bmatrix} T_{1i} \\ T_{2i} \\ T_{3i} \end{bmatrix} = H_i \begin{bmatrix} c\alpha_i \, \dot{\alpha}_i \\ -s\alpha_i \, \dot{\alpha}_i \\ c\alpha_i \, \dot{\beta}_i \end{bmatrix}$$

The torque T_{1i} is the torque load on the outer gimbal torquer and T_{3i} is the torque load on the inner gimbal torquer. Assuming the same torque limits for both inner and outer gimbal torquers, we get for the gimbal rate limits

$$\dot{\alpha}_{TLIM} = \dot{\beta}_{TLIM} = K_i / c\alpha_i$$

with $K_i = T_{LIMi}/H_i$ a CMG design constant, more or less the same for all double-gimbaled CMGs. There are also other rate limits ($\dot{\alpha}_{LIM}$, $\dot{\beta}_{LIM}$) due to hardware limits like gimbal rate tachometer limits, voltage limits, etc.

To reduce the magnitude of the actual torque only, but keep the same direction as the commanded torque, a proportional scaling of all gimbal rates by dividing by DIV is done, where:

$$DIV = MAX \left\{ 1, \frac{|\dot{\alpha}_1|}{\dot{\alpha}_L}, \frac{|\dot{\alpha}_2|}{\dot{\alpha}_L}, \ldots, \frac{|\dot{\alpha}_n|}{\dot{\alpha}_L}, \frac{|\dot{\beta}_1|}{\dot{\beta}_L}, \frac{|\dot{\beta}_2|}{\dot{\beta}_L}, \ldots, \frac{|\dot{\beta}_n|}{\dot{\beta}_L} \right\}$$

with

$$\dot{\alpha}_L = MIN\ (\dot{\alpha}_{TLIM}, \dot{\alpha}_{LIM}) \qquad \dot{\beta}_L = MIN\ (\dot{\beta}_{TLIM}, \dot{\beta}_{LIM})$$

PERFORMANCE DISCUSSION

As implied by the discussion of the desirable gimbal angle distribution, there is no need for a strict adherence to the ideal distribution. For the inner distribution gain K_a this means that its value can be chosen over a wide range up to $1/\Delta t$ where Δt is the computer cycle time, especially since the torque producing portion of the inner gimbal rate command tends to keep the inner gimbal angles on their distribution. To select the outer gimbal distribution gain we have to remember that the maximum magnitude of the outer distribution rate can be as large as [cf. equation (6)],

$$\dot{\beta}_{DMAX} = K_b \pi H_i \sum_j H_j (1 - \delta_{ij})$$

For a desired maximum outer distribution rate of 0.02 rad/s and with 5 CMGs of an angular momentum magnitude of 3000 Nms we get for example,

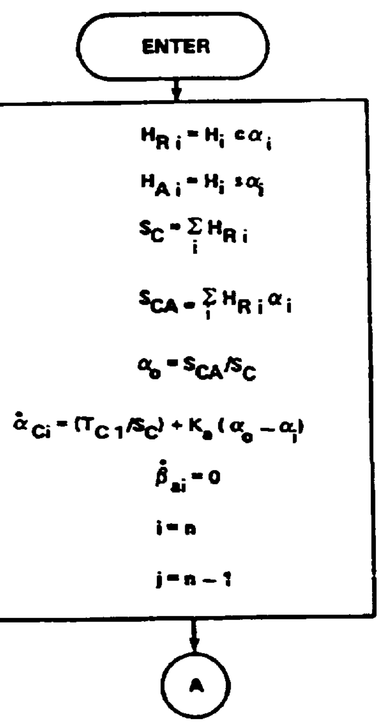
$$K_b \approx 1.8E-10$$

No special effort has to be made to "unhook" the CMG momentum vectors when, after bunching up due to saturation, the torque command is such that the momentum magnitude is reduced. In the real world, the gimbal angle readouts are noisy enough to introduce unhooking. Therefore, if a simulation is too ideal and problems are encountered, some noise should be added to the gimbal angles. The same considerations apply for starting up from an internal singularity (the distribution functions prevent any later internal singularities from occurring).

While any number of double-gimbaled CMGs can be accommodated (two CMGs are the minimum), the cycle time might become a problem for very large numbers of CMGs. Since any angular momentum magnitude is allowed, there is then the possibility to group several CMGs, add their angular momentum, and consider the group as one CMG as far as the steering law is concerned. The group is then commanded to its combined outer and inner gimbal angle. Setting the angular momentum magnitude of any CMG to zero is a convenient way of signaling that the CMG has failed.

The general logic flow, including the input and output parameters for the steering law is shown in Figure 2. The implementation for the high level language APL is shown in Figure 3 and a detailed flow in Figure 4.

INPUTS	$\underline{T}_C$	Nm	TORQUE COMMAND (T_{C1}, T_{C2}, T_{C3})
	α_i	rad	INNER GIMBAL ANGLES
	β_i	rad	OUTER GIMBAL ANGLES
	$s\alpha_i$, $c\alpha_i$		SINES & COSINES OF THE α_i'S
	$s\beta_i$, $c\beta_i$		SINES & COSINES OF THE β_i'S
	H_i	Nms	CMG ANGULAR MOMENTUM MAGNITUDES
OUTPUTS	$\dot{\alpha}_{Ci}$	rad/s	INNER GIMBAL RATE COMMANDS
	$\dot{\beta}_{Ci}$	rad/s	OUTER GIMBAL RATE COMMANDS



n = MAXIMUM NUMBER OF CMGs

K_a = INNER GIMBAL ANGLE DISTRIBUTION GAIN

ALL i's RANGE FROM 1 to n

Figure 2.-Steering law logic flows.

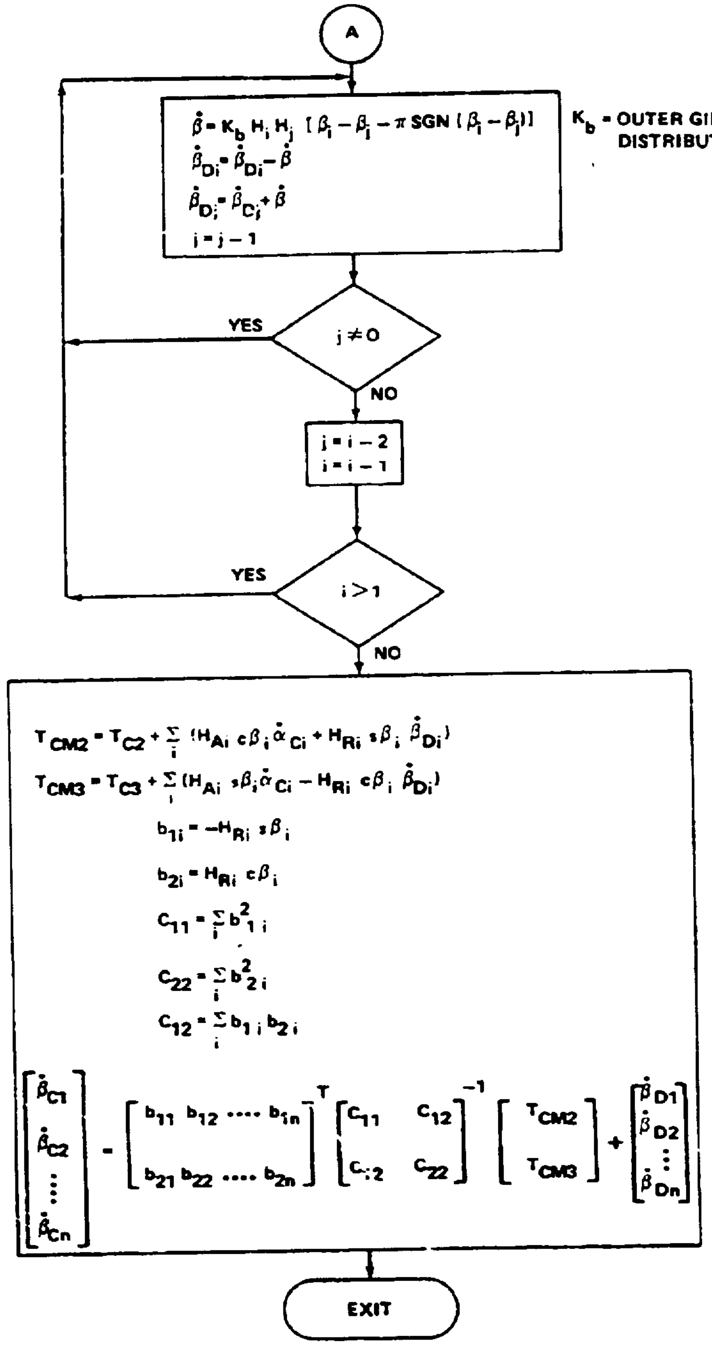


Figure 2.-Concluded.

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∇ STLAW;HA;HR;SC;T ;BD;B;M
⍝
⍝
⍝
⍝
⍝            *****   STEERING LAW FOR N DOUBLE   *****
⍝            ***** GIMBALED CONTROL MOMENT GYROS ******
⍝
⍝
⍝ INPUT_PARAMETERS:  TC  =  T_C1  T_C2  T_C3
⍝
⍝                    AL  =  α_1   α_2   ....  α_n
⍝
⍝                    BE  =  β_1   β_2   ....  β_n
⍝
⍝                    H   =  H_1   H_2   ....  H_n
⍝
⍝
⍝ LOCAL_VARIABLES:   HA, HR, SC, T, BD, B, M
⍝
⍝
ALD←(TC[1]+SC)-KA×AL-(+/HR×AL)÷SC←+/HR←H×2○AL
BD←KB×+/(M×⍉M←(N,N)⍴H)×B-PI××B←B-⍉B←(N,N)⍴BE
T←TC[2 3]+(+/(HA×ALD×2○BE)+HR×BD×1○BE),+/((HA←H×1○AL)×ALD×1○BE)-HR×BD×2○BE
BED←BD+(⍉B)+.×(⌹B+.×⍉B←(2,N)⍴(-HR×1○BE),HR×2○BE)+.×T
⍝
⍝
⍝ OUTPUT_PARAMETERS:  ALD  =  α̇_1  α̇_2  ....  α̇_n
⍝
⍝                     BED  =  β̇_1  β̇_2  ....  β̇_n
∇
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Figure 3.-Steering law for n double-gimbaled control moment gyros.

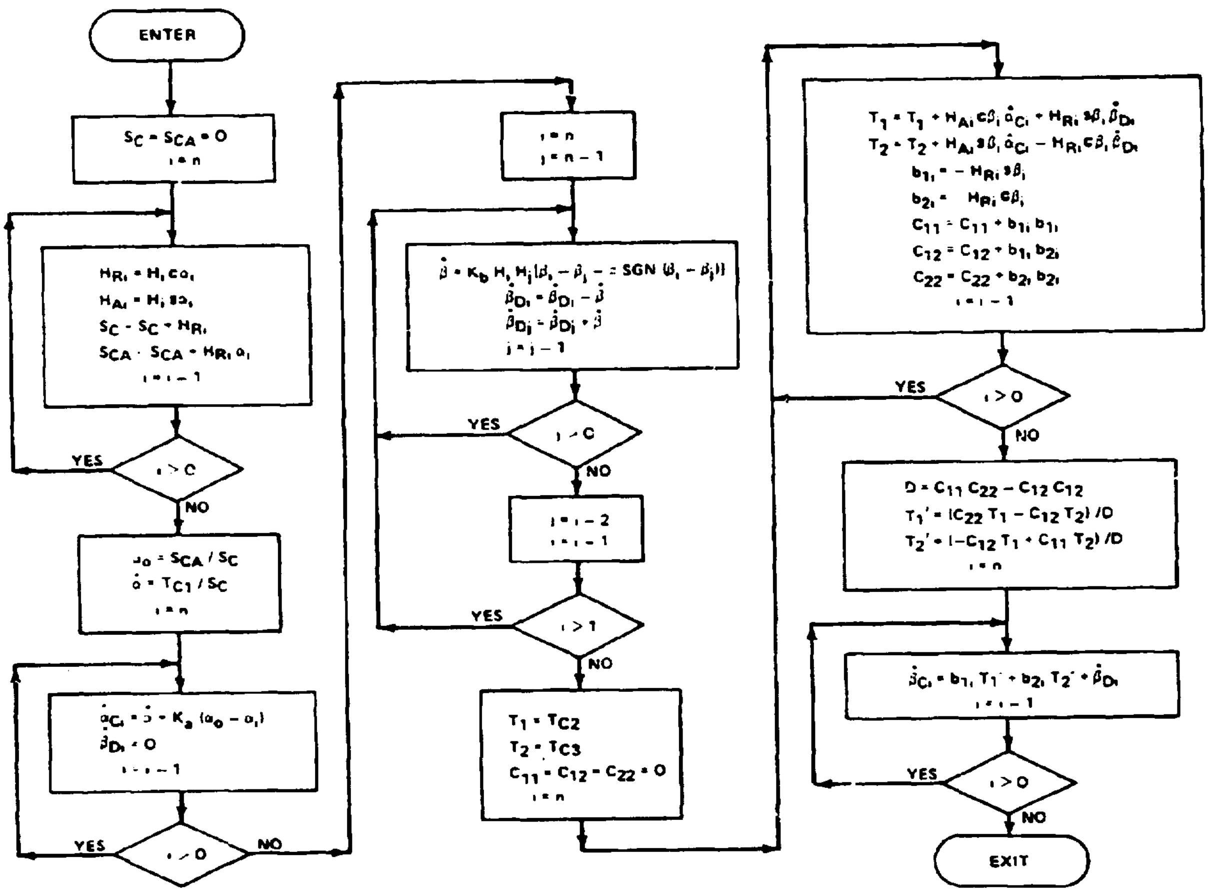

Figure 4.-Detailed logic flow.

DEFINITION OF SYMBOLS

Symbol	Definition
[B]	2 × n torque matrix (for outer rates); Nms
b_{ij}	components of [B]; i = 1,2; j = 1,2, ..., n; Nms
c	cos (before Greek symbol)
DIV	divisor for rate limiting
H_i	angular momentum magnitude of the i-th CMG; i = 1,2, ..., n; Nms
$\underline{H}_G$	total angular momentum of the CMG system; Nms
H_{G1}, H_{G2}, H_{G3}	components of $\underline{H}_G$; Nms
$\underline{\dot{H}}_P$	angular momentum change for power generation; Nm
i	index
j	index
K_a	inner distribution gain; 1/s
K_b	outer distribution gain; 1/s
K_i	T_{LIMi}/H_i; CMG torque constant; 1/s
n	number of double-gimbaled CMGs
s	sin (before Greek symbol)
$\underline{T}_C$	control torque command; Nm
T_{C1}, T_{C2}, T_{C3}	components of $\underline{T}_C$; Nm
T_{CM2}, T_{CM3}	modified torque commands; Nm
$\underline{T}_G$	total CMG torque on the vehicle; Nm
T_{G1}, T_{G2}, T_{G3}	components of $\underline{T}_G$; Nm
T_{LIM}	maximum CMG gimbal torquer torque; Nm
T_{1i}, T_{2i}, T_{3i}	torque components in the W-coordinate system (with $\beta_i = 0$); i = 1,2,3 ... n; Nm

DEFINITION OF SYMBOLS (Concluded)

Symbol	Definition
T_2', T_3'	torque components due to $\dot{\alpha}_i$'s and $\dot{\beta}_{Di}$'s; Nm
W_1, W_2, W_3	CMG coordinate system axes
α_o	inner gimbal reference angle; rad
α_i, β_i	inner and outer gimbal angle; i = 1,2, ..., n; rad
α_R	ideal inner gimbal reference angle; rad
$\dot{\alpha}_i$, $\dot{\beta}_i$	total inner and outer rates; i = 1,2, ..., n; rad/s
$\dot{\alpha}_{Di}$, $\dot{\beta}_{Di}$	inner and outer distribution rate; i = 1,2, ..., n; rad/s
$\dot{\alpha}_L$, $\dot{\beta}_L$	inner and outer rate limit; rad/s
$\dot{\alpha}_{LIM}$, $\dot{\beta}_{LIM}$	inner and outer rate limit due to hardware; rad/s
$\dot{\alpha}_{TLIM}$, $\dot{\beta}_{TLIM}$	inner and outer rate limit due to T_{LIM}; rad/s
$\dot{\underline{\beta}}_C$	outer rate command vector (due to T_{CM2}, T_{CM3}); rad/s
$\dot{\beta}_{Ci}$	components of $\dot{\underline{\beta}}_C$; i = 1,2, ..., n ; rad/s
$\dot{\beta}_{DMAX}$	maximum possible outer distribution rate; rad/s
δ_{ij}	$= 0$ for $i \neq j$, $= 1$ for $i = j$ } Kronecker delta
$\_$	a bar under a quantity denotes a vector
$[\ ]^T$	a superscript T denotes a transpose on a vector or a matrix
$\{\ \}^{-1}$	a superscript -1 denotes the matrix inverse
$\cdot$	a dot over a quantity indicates time derivative

REFERENCE

1. Kennel, Hans F.: Steering Law for Parallel Mounted Double-Gimbaled Control Moment Gyros - Revision A. NASA TM-82390, January 1981.