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RITZ PROCEDURE FOR COSMIC/NASTRAN

R. L. Citerley and P. J. Woytowitz
Anamet Laboratories, Inc., San Carlos, CA 94070

SUMMARY

An analysis procedure has been developed and incorporated into COSMIC/NASTRAN that permits large dynamic degree of freedom models to be processed accurately with little or no extra effort required by the user. The method employs existing capabilities without the need for approximate Guyan reduction techniques. Comparisons to existing solution procedures presently within NASTRAN are discussed.

INTRODUCTION

The search for an effective metho for performing a dynamic analysis of complex structures has been under continuing study for many decades. In order to reduce the number of governing equations, the usual approach was to use modal coordinates. These modal coordinates are developed by employing classical eigenvalue extraction procedures. With this approach as the accepted analysis procedure, a concerted effort starting in the 1950's was made in developing various eigenvalue extraction methods. Several of these options presently exist in NASTRAN to develop these dynamic properties.

In pursuit of efficient and accurate eigenvalue methods, Jennings (Ref. 1) gives a rather brief but complete review of methods for solving dynamic equations to determine characteristic responses. The Lanczos method was reported as one of the more efficient methods for the solution of structural eigenvalue problems. Further investigation by Nour-Omid et al. (Ref. 2), illustrated that the Lanczos method had tremendous advantages over other classical eigenvalue extraction methods. With all the various eigenvalue procedures available, the choice of which numerical procedure should be used for solving a dynamic response problem depends upon the problem characteristics and solution requirements.

The solution process for the dynamic analysis developed in NASTRAN is separated into three phases: assembly, solution, and response recovery. As pointed out in Section 4.0 of the NASTRAN Theoretical Manual, as problem size increases, the cost of the first and third phases increases linearly; whereas the second increases cubically with the number of degrees of freedom. The eigenvalue solution phase is usually considered the most costly.

Wilson (Ref. 3) demonstrated how the eigenvalue procedure could be circumvented. He classified it as the "Ritz" method. Arnold et al. (Ref. 4), demonstrated how the Ritz procedure could be incorporated into MSC/NASTRAN and showed that it would be less costly and more efficient than previous techniques. Reference 4 also discusses the relationship between Lanczos and Ritz.

Other methods using the basic concept of Lanczos have been suggested by Gupta (Ref. 5) using a Lanczos-Householder algorithm, Newman and Flanagan (Ref. 6) using FEER, and by Newman and Mann (Ref. 7) using complex FEER methods for COSMIC/NASTRAN. These methods differ from the present case in the selection process of the starting vectors. Also, the method of obtaining the additional vectors is different.

TECHNICAL DISCUSSION

Although the use of Ritz vectors can be applied to the dynamic analysis of any linear system, it is particularly well suited to application in the field of dynamic structural analysis. The nature of certain dynamic loadings, or rather the analytical representation of the loadings, allows the Ritz approach to almost assuredly use fewer modes for the same accuracy as a conventional analysis using natural frequencies and mode shapes. The use of a static load vector in the dynamic excitation direction in deriving the starting Ritz vector eliminates the need for a static correction due to higher order modes.

As mentioned earlier, the solution of the eigenvalue problem for large systems is the major computational task involved in dynamic analysis. For very large systems, this task is often not even practical, and instead is supplemented with an additional step of static condensation and Guyan reduction, or by use of substructuring techniques. All these techniques are acceptable and are in standard practice with COSMIC/NASTRAN today; however, all involve the solution of large eigenvalue problems even if the total problem has been reduced into workable substructures. The Ritz algorithm, to be defined in detail later, allows the analyst to bypass the solution of a large eigenvalue problem, and instead solve a smaller eigenvalue problem involving only the Ritz modes of significance to the analysis. Nour-Omid and Clough (Ref. 8) have shown how even this step could be eliminated. Also presented here is a method of using an error norm to define the number of Ritz vectors required to obtain accurate dynamic analysis results.

Derivation of Ritz Vectors

Given a physical system whose mass and stiffness properties have been discretized to N degrees of freedom given by the structure mass matrix, M, and structure stiffness matrix, K, the initial step in the algorithm is to factor K such that

$$K = LDL^T \tag{1}$$

The starting vector x_1^* can then be obtained by solving

$$Kx_1^* = f \tag{2}$$

where f is a static load vector which represents the spatial distribution of the dynamic loading.

Once the first Ritz vector x_1^* is obtained, an iterative process is begun for the solution of a full L set of Ritz vectors. This process can be divided into three steps which are given below.

Step 1: Solution for x_i^* is made using the factored stiffness matrix from Equation (1)

$$Kx_i^* = Mx_{i-1} \tag{3}$$

Step 2: $i = i + 1$, if $i < L$ go to 1, otherwise proceed

Step 3: Orthogonalize x_i^* with respect to previous i - 1 vectors

$$c_j = x_j^T Mx_i^*, \quad j = 1, \ldots i - 1 \tag{4}$$

$$x_i^{**} = x_i^* - \sum_{j=1}^{i-1} c_j x_j \tag{5}$$

and normalize with respect to M to find x_i

$$x_i^T Mx_i = 1 \tag{6}$$

The result of this iterative process is an N by L matrix of vectors, X, which is mass orthogonal, but still must be orthogonalized with respect to the stiffness matrix. The final set of Ritz vectors must be mass and stiffness orthogonal to uncouple the equations of motion, and this orthogonalization can be accomplished by solving the following eigenvalue problem for z_i,

$$[K^* - \omega_i^2 M^*]z_i = 0 \tag{7}$$

where $$K^* = X^T KX \tag{8}$$

$$M^* = X^T MX = I \tag{9}$$

The final matrix of Ritz vectors, ${}^0X$, is then obtained using the matrix of z_i vectors, Z, as a transformation for X,

$${}^0X = XZ \tag{10}$$

Error Norm Definition

The definition of an error norm is a key element in application of the Ritz algorithm. Wilson originally devised an error norm defined as

$$e = \frac{f^T(f - \sum_j p_j M\,{}^0x_j)}{f^T f} \tag{11}$$

where

$$p_j = {}^0x_j^T f = \text{participation factor} \tag{12}$$

and the vectors ${}^0x_j$ are the final mass and stiffness orthogonal Ritz vectors obtained from Equation (10). This error norm varies from e = 1.0 if no vectors are used, to e = 0.0 if all vectors are used. Arnold, Citerley, et al., used this same definition of error, but applied it using the non-stiffness-orthogonalized Ritz vectors x_j, rather than ${}^0x_j$. This definition then permits error estimation prior to eigenvalue extraction performed on Equation (7). In addition, they established another error norm to quantify the influence of the input excitation frequency content on the adequacy of the selected Ritz vectors. Examination of this type of error norm will be required for high frequency input excitation.

IMPLEMENTATION INTO NASTRAN AND NUMERICAL RESULTS

The Ritz procedure was implemented into NASTRAN using the DMAP capabilities. A flow chart of a typical DMAP alter is shown in Figure 1. The procedure has been incorporated into SOL 3, 11, and 12. Although details of the DMAP alter are slightly different for each rigid format, the flow chart of Figure 1 outlines the general procedure.

Block 1 of Figure 1 processes the static loads. As many subcases as necessary are input by the user to generate the static loads which are to represent the spatial distribution of the dynamic loads. Modules SSG1, SSG2, and SSG3 are used for the processing and subsequent set reduction.

Block 2 of Figure 1 uses the lower triangular factor of the stiffness matrix (L of Equation (1)) to solve for the first set of vectors. Block 3 is a loop, which calculates additional vectors described previously.

Block 4 uses a dummy module, MODB, to orthonormalize the generated vectors (see Equations (4-6)). Block 5 then calculates the generalized mass and stiffness matrices of Equations (8) and (9). Block 6 uses the READ module to perform the eigenvalue/eigenvector analysis of Equation (7) and form the Ritz vectors using Equation (10). Finally, Block 7 equivalences the Ritz vectors to the regular A set eigenvectors (PHIA) for subsequent use in the DMAP rigid format.

A dummy module, MODB, was written to perform several tasks depending upon the input parameters. It is used in Block 3 to append the newly calculated vectors to the previous vectors. Additionally, although not shown, it was found necessary to normalize the newly computed vectors in Block 3 after each pass through the loop in order to prevent numerical round-off problems. This was also performed by the dummy module. The dummy module's final job was to orthonormalize the vectors in Block 4, using the Gram-Schmidt orthonormalization procedure.

The Ritz procedure was previously implemented into the MacNeal-Schwendler version of NASTRAN. Comparisons of the Ritz procedure to MSC/NASTRAN's Generalized Dynamic Procedure have been reported (Ref. 3). The Ritz procedure was also compared to COSMIC/NASTRAN's FEER method. Table 1 presents frequency comparisons and total CPU time using the FEER method and the Ritz method on a relatively small problem. As can be seen, both methods yield similar results, the Ritz method being slightly faster.

However, it is not proper to directly compare the FEER method with Ritz, since they are intended to serve different purposes. The FEER (or any other eigen-extraction) routine is designed to find all the eigenvalues and associated eigenvectors below a certain user specified number. These modes may or may not be good representations for use in a forced dynamic response. Often it is found that only certain modes participate due to the nature of the forcing function. The Ritz method uses static representations of the dynamic forces applied as a basis for its "eigenvectors". The only "modes" calculated correspond to those excited by the forcing function. Therefore, fewer unnecessary modes are used; additionally, as pointed out previously, static corrections (such as Modal Acceleration) techniques are not necessary, since the Ritz vectors necessarily accurately represent the total force applied to the structure.

CONCLUSIONS

The Ritz procedure for dynamic analysis has been presented. The procedure is related to other popular methods of eigenvalue/eigenvector analysis; however, it does not attempt to obtain all the modes of the system. Instead, only those "modes" which are excited by the dynamic loading are obtained.

A flow chart of the DMAP implementation has been presented. NASTRAN's DMAP capability, along with the use of a dummy module, made this implementation fairly simple. The method was compared to the FEER method for one problem. The predicted frequencies compared well, while the cost of the Ritz method was less. The same conclusions were shown by Arnold, Citerley, et al. with more dramatic cost savings for very large analysis problems. These results clearly demonstrate the impact that the Ritz algorithm can have on reducing analysis cost.

The benefits of the Ritz algorithm in dynamic analysis can therefore be summarized as:

1. Order of magnitude reduction in deriving eigenvalues and eigenvectors for mode superposition analysis.
2. Better accuracy for fewer vectors.
3. No static correction needed for higher order modes.
4. No static condensation or Guyan reduction required for large systems.
5. Error estimation can be made to determine the adequacy of selected Ritz vectors.

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TABLE 1

FREQUENCY AND TIME RESULTS FOR A SMALL MODEL, RITZ VERSUS FEER METHODS

MODE	CYCLIC FREQUENCY	
	RITZ (HZ)	FEER (HZ)
1	2.836471E+02	2.836471E+02
2	5.015972E+02	5.015972E+02
3	7.418652E+02	7.401528E+02
4	8.908441E+02	8.792196E+02
5	9.876880E+02	9.876871E+02
6	1.189213E+03	1.189210E+03
7	1.902981E+03	1.691335E+03
8	2.178580E+03	2.165648E+03
9	2.852047E+03	2.467984E+03
10	3.239252E+03	2.906139E+03
Total Run Time (cpu-sec)	218	270

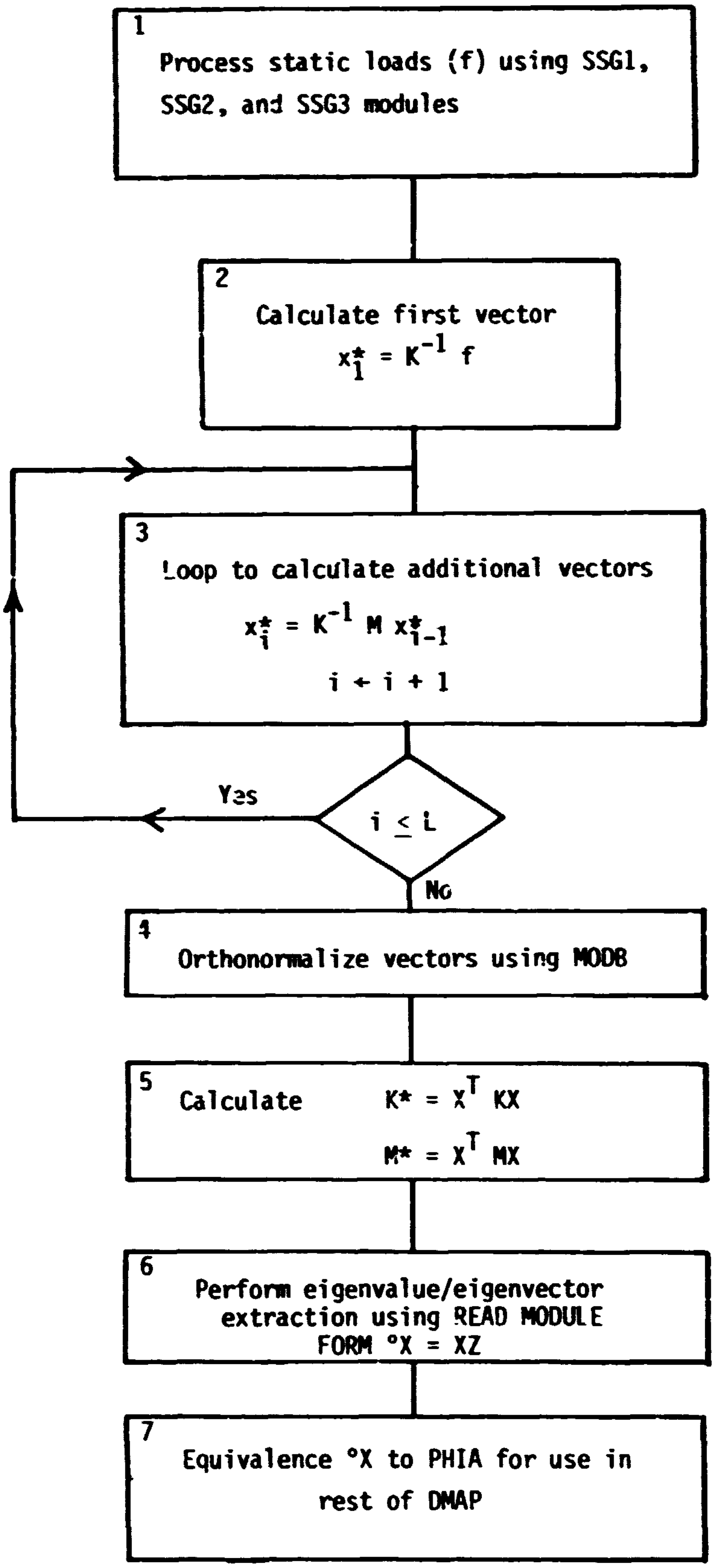

Figure 1 Flow Chart for Implementing Ritz Procedure into NASTRAN