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A STUDY ON THE CONTROL OF THIRD GENERATION SPACECRAFT*

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ABSTRACT

An overview of some studies which have recently been carried out in [1]-[3] on the control of third-generation spacecraft, as modelled by the MSAT space vehicle configuration, is made. This spacecraft is highly non-symmetrical and has appendages which cannot in general be assumed to be rigid. In particular, it is desired to design a controller for MSAT which stabilizes the system and satisfies certain attitude control, shape control, and possibly station-keeping requirements; in addition, it is desired that the resultant controller should be robust and avoid any undesirable "spill-over effects". In addition, the controller obtained should have minimum complexity.

The method of solution adopted to solve this class of problems is to formulate the problem as a robust servomechanism problem [5]-[7], and thence to obtain existence conditions and a controller characterization to solve the problem.

The final controller obtained for MSAT has a distributed control configuration and appears to be quite satisfactory.

INTRODUCTION

This paper summarizes studies carried out in [1]-[3] on control system structures known as third-generation spacecraft. Such spacecraft have:

(1) Large mass
(2) High power
(3) Large non-symmetric flexible appendages
(4) Precise communication RF beam control requirements.

In particular, the class of spacecraft represented by the Mobile Communications Satellite (MSAT) is used as a reference for these studies. This spacecraft has non-symmetric appendages which cannot be assumed to be rigid (see Figure 1).

There are a number of control problems associated with the attitude-control, shape-control and possibly station-keeping control for such third generation spacecraft (referred to as LFSS), which may be listed as follows:

A. <u>The LFSS Control Problem</u>

Problem 1: Lightly Damped, Oscillatory Plant

A LFSS has eigenvalues either at the origin or approximately distributed along the imaginary axis. One of the basic objectives that a controller must accomplish in this case is to stabilize the rigid body modes of the LFSS, and at the same time to stabilize the elastic modes of the LFSS. This is called the LFSS stabilization problem.

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Problem 2: Modelling

In modelling a LFSS, experience has shown that dynamic analysis may provide a framework for the modelling of the low frequency elastic modes of the LFSS in a reasonably accurate way, but that the high frequency elastic modes cannot be expected to be determined accurately, i.e. there will always be errors present in modelling the high frequency elastic modes of the LFSS. In addition, the calculation of dampening effects on the LFSS can only be done with great uncertainty.

Problem 3: The Infinite Dimensional Plant - The "Spill-Over Problem"

The classical modelling of elastic structures as continua results in the well known "infinite dimensional" system representation of a LFSS. Whether or not one adopts this infinite dimensionality representation seriously from an engineering standpoint, there is no question that the number of system elastic modes present in a LFSS is always larger than the number which any design model of a LFSS can accommodate. In trying to control the modelled rigid and elastic modes, it is essential that the controller should not cause these unmodelled high frequency elastic modes to become unstable. This is called the "Spill-Over Problem".

Problem 4: The Sensor/Actuator Placement Problem

The LFSS is intrinsically distributed, and the configuration of control hardware is not in general specified. Thus, unlike many conventional control problems, part of the LFSS control problem is in determining the number and location of sensor/actuators on the LFSS.

Problem 5: Requirement for Multivariable Control Theory

The concept of "third generation" spacecraft, unlike the first and some second generation spacecraft, precludes single-input, single-output control design. Some type of multivariable control design method is mandatory to deal with the severe interaction occurring in the system.

Problem 6: Minimization of Number of Sensors/Actuators

There is a practical limiation on the quantity of hardware that can be distributed over the LFSS vehicle. This implies in particular that one cannot assume full state feedback is available, and that the number of actuators/sensors used must be limited, i.e. one must minimize any unnecessary sensor/actuators required for LFSS control.

The following problem definition is now given:

B. The LFSS Robust Servomechanism Problem

Assume that a LFSS can be exactly described by the following finite dimensional linear time invariant model:

$$\begin{aligned} \dot{x} &= Ax + Bu + E\omega \\ y &= Cx + F\omega \\ y_m &= C_m x + F_m \omega_m \end{aligned} \qquad (1)$$

where $x \epsilon R^n$ is the state, $u \epsilon R^m$ is the control (actuator inputs), $y_m \epsilon R^{r_m}$ are the measured (sensor) outputs, and $y \epsilon R^r$ are the outputs to be regulated. Here $\omega \epsilon R^{\Omega}$ are assumed to be constant unmeasurable disturbances applied to the structure, $\omega_m \epsilon R^{\Omega_m}$ are assumed to be constant unknown measurement errors and $e \stackrel{\Delta}{=} y - y_{ref}$ is the error in the system where y_{ref} is a constant set-point. Thus, it is assumed that (1) may include an arbitrarily large number of elastic modes (but not infinite).

Assume now that an approximate model of (1), called the design model for (1), is given by:

$$\begin{aligned} \dot{\bar{x}} &= \bar{A}\bar{x} + \bar{B}u + \bar{E}\omega \\ y &= \bar{C}\bar{x} + \bar{F}\omega \\ y_m &= \bar{C}_m\bar{x} + \bar{F}_m\omega_m \end{aligned} \tag{2}$$

where $\bar{x} \epsilon R^{\bar{n}}$ is the state of the design model, and where $\bar{n} << n$. It is desired now to find a controller based on the design model (2), such that when it is applied to (1), the system is asymptotically stable, i.e. no spill-over occurs, and such that:

$$\lim_{t \to \infty} e(t) = 0 \quad , \quad \forall x(0) \epsilon R^n, \quad \forall \omega \epsilon R^{\Omega}, \quad \forall \omega_m \epsilon R^{\Omega_m} \tag{3}$$

This is called the LFSS Robust Servomechanism Problem, which includes the following subproblems:

(1) Stabilization
(2) Station-keeping
(3) Attitude control
(4) Shape control.

THE MSAT CONTROL PROBLEM

The MSAT spacecraft is illustrated in Figure 1. It consists of four components, one of wich is rigid (the bus) and three of which are flexible (the solar array, the tower, and the reflector). The tower-reflector-hub hinge point is assumed to have a gimbal (see Figure 2).

The coordinates assumed for each of these substructures are as follows:

(1) Bus - three rigid rotations $(\theta_x, \theta_y, \theta_z)'$

(2) Tower - relative displacement of tower tip to tower root $(f^{-1}\delta_1, f^{-1}\delta_2, f^{-1}\delta_3)'$
- relative angular displacement of reflector with respect to frame fixed at tower root (with zero gimbal angles) $(\alpha_1, \alpha_2, \alpha_3)'$

(3) Reflector - two gimbal angles at tower-reflector-hub hinge point $(\beta_1, \beta_2)'$

The actuators which are assumed to be available are as follows:

(1) Eight thrusters f_i, $i=1,2,\ldots,8$, four from thrusters on the bus and four from thrusters at the reflector hinge point, aligned as shown in Figure 2.

(2) Two torquers at the reflector hub, one about each gimbal axis $(g_{\beta_1}, g_{\beta_2})'$ (see Figure 2).

In this case, a design model and an evaluation model was developed in [4], in which the design model has 18 states consisting of 5 rigid body modes (corresponding to the three rigid rotations of the bus and two gimbal angles of the reflector) together with 4 elastic modes, and the evaluation model has 32 states consisting of 5 rigid body modes and 11 elastic modes. Table 1 gives the eigenvalues of the open loop system for the two models. The models used in this study included the effect of dampening terms D, D_E (see Table 1).

TABLE 1: Open Loop Eigenvalues of MSAT Vehicle

	Standard Design Model		Evaluation Model	
	With Damping Term D Excluded	With Damping Term D Included	With Damping Term D_E Excluded	With Damping Term D_E Included
Rigid Body Modes	0 (repeated 10 times)	0 (repeated 10 times)	0 (repeated 10 times)	0 (repeated 10 times)
Elastic Body Modes	0±j0.124 0±j0.239 0±j0.556 0±j0.780	-0.000923±j0.124 -0.00170 ±j0.240 -0.00856 ±j0.556 -0.0211 ±j0.779	0±j0.124 0±j0.151 0±j0.239 0±j0.556 0±j0.690 0±j0.780 0±j1.55 0±j3.14 0±j3.96 0±j9.95 0±j14.0	-0.000923±j0.124 -0.000853±j0.151 -0.00171 ±j0.239 -0.00856 ±j0.556 -0.00553 ±j0.690 -0.0211 ±j0.780 -0.0751 ±j1.55 -0.0280 ±j3.14 -0.0528 ±j3.96 -0.524 ±j10.1 -1.17 ±j13.8

It may be noted that the elastic modes of the evaluation model interweave with the elastic modes of the design model.

A. Description of Problem to be Solved

In this case it is desired to solve the LFSS Robust Servomechanism Problem for the MSAT vehicle. In particular, there are two separate requirements for the controller to be designed for the MSAT vehicle:

Requirement I

Find a controller, based on the MSAT design model, which solves the following problems:

- Stability: stabilize the 5 rigid body modes and the 4 elastic modes of the system.
- Attitude control: regulate θ_x, θ_y, θ_z to desired constant set points θ_x^{ref}, θ_y^{ref}, θ_z^{ref} respectively, in the presence of unknown constant disturbances.

- Shape control: regulate $\beta_1+\alpha_1$, $\beta_2+\alpha_2$, $f^{-1}\delta_1$, $f^{-1}\delta_2$, $f^{-1}\delta_3$, α_3 to zero, in the presence of unknown constant disturbances.
- Spill-over problem: it is desired that the controller should satisfy the above requirements, and not cause any instability to occur with respect to any of the vehicle's elastic modes which are not included in the design model.
- Controller complexity: it is desired to minimize the number of sensors and actuators which are required to solve the problem.
- Discrete controller implementation: it is desired that the controller, when implemented digitally, should not require an excessively large sampling rate to maintain stability.

Requirement II

Apply the controller obtained, based on the MSAT design model, to the MSAT evaluation model, and verify that all objectives above are satisfied.

The outputs to be regulated in this case are given by:

$$y \triangleq (\theta_x,\theta_y,\theta_z,\beta_1+\alpha_1,\beta_2+\alpha_2,f^{-1}\delta_1,f^{-1}\delta_2,f^{-1}\delta_3,\alpha_3)' \tag{4}$$

B. Assumptions Made in Problem Formulation

In this problem, it is assumed that there is no requirement for controlling the ω_x, ω_y, ω_z rigid body modes. (Note: this assumption is not essential, e.g. [2], [3] also deals with the case of station-keeping.) It is also assumed that there is no need to include any gyroscopic terms in the design and evaluation models.

METHOD OF SOLUTION ADOPTED TO OBTAIN A CONTROLLER TO SOLVE PROBLEM

The method of approach adopted to solve this problem was based on using the results of the "robust servomechanism problem" [5]-[7], in conjunction with a parameter optimization method [8] to determine the controller's parameters, e.g. see [9] which solves a special case of the above problem when the sensors and actuators are collocated, using a decentralized control configuration. In this case, existence conditions for a solution to the problem were obtained, and a necessary controller structure developed. In particular, it was found that any controller which solves the MSAT problem specifications must consist of a "servo-compensator" [5] (unique), together with a stabilizing compensator (non-unique). In this study, the simplest possible stabilizing compensator, i.e. a stabilizing compensator consisting of only proportional and rate feedback terms, was used.

In this case, in order to satisfy the existence conditions obtained for a solution to exist to the problem, it was necessary to choose the following inputs (actuators) and measurable outputs (sensors) for the controller:

Outputs (sensors):

$$y_m \triangleq (\theta_x,\theta_y,\theta_z,\beta_1,\beta_2,\alpha_1,\alpha_2,f^{-1}\delta_1,f^{-1}\delta_2)' \tag{5}$$

Inputs (actuators):

$$u \triangleq (g^*_{c_3}, g_{\beta_1}, g_{\beta_2}, f^*_1, f^*_2, f^*_5, f^*_6)' \tag{6}$$

where $g^*_{c_3}$, f^*_1, f^*_2, f^*_5, f^*_6 correspond to various combinations of the thrusters $f_1, f_2, \ldots, f_7, f_8$ (see Figure 2), as described in Appendix I.

FINAL CONTROLLER CONFIGURATION OBTAINED

In this case, the following distributed controller was obtained as a solution to the MSAT robust servomechanism problem, based on the MSAT design model:

$$\begin{pmatrix} g^*_{c_3} \\ g_{\beta_1} \\ g_{\beta_2} \\ f^*_1 \\ f^*_2 \end{pmatrix} = -K_1 \begin{pmatrix} \theta_x - \tilde\theta_x \\ \theta_y - \tilde\theta_y \\ \theta_z - \tilde\theta_z \\ \beta_1 \\ \beta_2 \end{pmatrix} - K_2 s \begin{pmatrix} \theta_x \\ \theta_y \\ \theta_z \\ \beta_1 \\ \beta_2 \end{pmatrix} - \frac{K_3}{s} \begin{pmatrix} \theta_x - \tilde\theta_x \\ \theta_y - \tilde\theta_y \\ \theta_z - \tilde\theta_z \\ \beta_1 + \alpha_1 \\ \beta_2 + \alpha_2 \end{pmatrix} - \frac{K_4}{s} \begin{pmatrix} f^{-1}\delta_1 \\ f^{-1}\delta_2 \end{pmatrix} \tag{7}$$

$$\begin{pmatrix} f^*_5 \\ f^*_6 \end{pmatrix} = -\frac{K_5}{s} \begin{pmatrix} f^{-1}\delta_1 \\ f^{-1}\delta_2 \end{pmatrix}$$

where s denotes the Laplace Transform operator, where

$$\begin{pmatrix} \tilde\theta_x \\ \tilde\theta_y \\ \tilde\theta_z \end{pmatrix} \triangleq \left(\frac{\gamma}{s+\gamma}\right)^2 \begin{pmatrix} \theta^{ref}_x \\ \theta^{ref}_y \\ \theta^{ref}_z \end{pmatrix} \tag{8}$$

and where K_1, K_2, K_3, K_4, K_5, γ are given as follows:

$$K_1 = \begin{bmatrix} 1.43 & 0.500 & 24.7 & 1.34 & -0.0460 \\ 0.0255 & 4.64 & 1.12 & 15.6 & -0.000439 \\ -6.81 & -0.000957 & 0.00981 & -0.000483 & 18.6 \\ 0.00326 & 38.0 & -0.231 & 14.5 & -0.00955 \\ 59.0 & -0.00916 & 0.0216 & 0.0127 & -2.40 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} 28.5 & 10.0 & 494 & 26.7 & -0.920 \\ 0.510 & 92.8 & 22.3 & 312 & -0.00877 \\ -136 & -0.0191 & 0.196 & -0.00965 & 372.2 \\ 0.0653 & 760 & -4.63 & 290 & -0.191 \\ 1180 & -0.183 & 0.432 & 0.254 & -48.1 \end{bmatrix}$$

$$K_3 = \begin{bmatrix} 7.14\times10^{-4} & 2.50\times10^{-4} & 1.24\times10^{-2} & 6.68\times10^{-4} & -2.30\times10^{-5} \\ 1.28\times10^{-5} & 2.32\times10^{-3} & 5.58\times10^{-4} & 7.80\times10^{-3} & -2.19\times10^{-7} \\ -3.41\times10^{-3} & -4.79\times10^{-7} & 4.90\times10^{-6} & -2.41\times10^{-7} & 9.31\times10^{-3} \\ 1.63\times10^{-6} & 1.90\times10^{-2} & -1.16\times10^{-4} & 7.26\times10^{-3} & -4.78\times10^{-6} \\ 2.95\times10^{-2} & -4.58\times10^{-6} & 1.08\times10^{-5} & 6.34\times10^{-6} & -1.20\times10^{-3} \end{bmatrix}$$

$$K_4 = \begin{bmatrix} -0.464 & 0.0433 \\ -0.0144 & 0.0536 \\ -0.00438 & 0.201 \\ 0.136 & 0.0461 \\ -0.000753 & 1.13 \\ 0.0268 & 0.225 \\ 0.0273 & -0.226 \end{bmatrix}$$

$$K_5 = \begin{bmatrix} 0.0268 & 0.225 \\ 0.0273 & -0.226 \end{bmatrix}$$

$$\gamma = 2.0\times10^{-3}$$

This controller is just a multivariable generalization of the classical three term controller used in classical control. The controller has minimal complexity in the sense that it has minimum order feedback dynamics and has the minimum number of actuators/sensors required in order to solve the problem. It is to be noted that no a priori assumption on the distributed structure of (7) was made — the distributed structure of the controller (7) arose from the analysis automatically.

PROPERTIES OF PROPOSED CONTROLLER

The main features of the proposed controller when applied to the MSAT design model and evaluation model will now be described. The main features of interest are:

(1) The stabilization properties of the proposed controller.
(2) The steady state regulation properties of the proposed controller.

The following results are obtained:

A. Eigenvalues of Closed Loop System Using Proposed Controller

Table 2 gives a listing of all eigenvalues obtained by applying the proposed controller (7) to the MSAT design model and evaluation models.

TABLE 2: Listing of Closed Loop Eigenvalues Using Proposed Controller (7) When Applied to MSAT Design and Evaluation Models

Standard Design Model		Evaluation Model	
$-0.00047 \pm j0.0085$ $-0.0024 \pm j0.016$ $-0.0051 \pm j0.022$ $-0.0097 \pm j0.030$ $-0.010 \pm j0.031$	rigid body modes	$-0.00047 \pm j0.0085$ $-0.0024 \pm j0.016$ $-0.0051 \pm j0.023$ $-0.0097 \pm j0.030$ $-0.010 \pm j0.031$	rigid body modes
$-0.00014 \pm j0.124$ $-0.0061 \pm j0.240$ $-0.017 \pm j0.557$ $-0.029 \pm j0.780$	elastic body modes	$-0.00014 \pm j0.124$ $-0.00020 \pm j0.151$ $-0.0061 \pm j0.240$ $-0.017 \pm j0.557$ $-0.0079 \pm j0.690$ $-0.0029 \pm j0.780$ $-0.129 \pm j1.35$ $-0.067 \pm j3.16$ $-0.069 \pm j3.95$ $-0.5 \pm j8.88$ $-0.51 \pm j11.3$	elastic body modes
-5.0×10^{-4} -5.0×10^{-4} -5.0×10^{-4} -5.0×10^{-4} -5.0×10^{-4} -5.0×10^{-4} -5.0×10^{-4}	servo-compensator modes	-1.7×10^{-3} -5.0×10^{-4} -5.0×10^{-4} -5.0×10^{-4} -5.0×10^{-4} -5.0×10^{-4} -5.0×10^{-4}	servo-compensator modes
-2.0×10^{-3} -2.0×10^{-3} -2.0×10^{-3} -2.0×10^{-3} -2.0×10^{-3} -2.0×10^{-3}	feedforward controller modes	-2.0×10^{-3} -2.0×10^{-3} -2.0×10^{-3} -2.0×10^{-3} -2.0×10^{-3} -2.0×10^{-3}	feedforward controller modes

It is observed that the resultant closed loop system is asymptotically stable for both the design and evaluation models, i.e. no undesirable spill-over effects occur. It is also observed that the dominant time constant of the system is mainly associated with the servo-compensator modes. This implies that one would expect for the case of tracking, that the dominant time response of the system would be associated with the feedforward controller modes, i.e. $TC_{dom} \doteqdot 500$ sec $\doteqdot 8$ min., and for the case of disturbance rejection, that the dominant time of the system would be associated with the servo-compensator modes, i.e. $TC_{dom} \doteqdot 2000$ sec $\doteqdot 0.6$ hrs. This result is verified in the simulation studies to follow.

B. Steady-State Values of Outputs Using Proposed Controller: Tracking Case

Table 3 gives a summary of results obtained for the case of unit step function tracking, when the proposed controller (7) is applied to the MSAT design and evaluation model. It is observed that all 9 outputs of the system are asymptotically regulated to their correct values as desired.

TABLE 3: Steady-State Values of Outputs Using Proposed Controller (7) When Applied to Design and Evaluation Model - Tracking Case

	$\theta_x^{ref}=1$	$\theta_y^{ref}=1$	$\theta_z^{ref}=1$
θ_x	1	0	0
θ_y	0	1	0
θ_z	0	0	1
$\beta_1+\alpha_1$	0	0	0
$\beta_2+\alpha_2$	0	0	0
$f^{-1}\delta_1$	0	0	0
$f^{-1}\delta_2$	0	0	0
$f^{-1}\delta_3$	0	0	0
α_3	0	0	0

Note: Any $|\text{number}|<10^{-16}$ is assumed to be zero.

C. Steady-State Values of Outputs Using Proposed Controller: Disturbance Rejection Case

Tables 4 and 5 give a summary of all results obtained for the case of disturbance rejection, when the proposed controller is applied to the MSAT design and evaluation models respectively. In this case, it is assumed that a unit step function change occurs for different disturbances corresponding to $\bar{g}_{c_1},\bar{g}_{c_2},\ldots,\bar{f}_0,\bar{f}_9$ defined in Table 6. It is observed that the first 7 outputs of the system are asymptotically regulated to zero, and that the remaining two outputs are approximately equal to zero in all cases, as is desired.

D. Sampling Rate Requirements for Digital Implementation of Proposed Controller

If it is assumed that the proposed controller (7) is to be implemented digitally, then it is necessary that the sensor outputs and actuator signals be updated at a fast enough rate so as to guarantee closed loop stability, when the the controller is applied to the evaluation model. In this case, on assuming that the sensor and actuator signals are updated at the same rate, it was found that a sampling rate of at least 0.1 Hz must be used to implement the proposed controller. This requirement is not demanding.

TABLE 4: Steady-State Values of Outputs Using Proposed Controller (7) When Applied to MSAT Design Model - Disturbance Rejection Case

	$\bar{g}_{c_1}=1$	$\bar{g}_{c_2}=1$	$\bar{g}_{c_3}=1$	$\bar{g}_{\beta_1}=1$	$\bar{g}_{\beta_2}=1$	$\bar{f}_1=1$	$\bar{f}_2=1$	$\bar{f}_5=1$	$\bar{f}_6=1$	$\bar{f}_0=1$	$\bar{f}_9=1$
θ_x	0	0	0	0	0	0	0	0	0	0	0
θ_y	0	0	0	0	0	0	0	0	0	0	0
θ_z	0	0	0	0	0	0	0	0	0	0	0
$\beta_1+\alpha_1$	0	0	0	0	0	0	0	0	0	0	0
$\beta_2+\alpha_2$	0	0	0	0	0	0	0	0	0	0	0
$f^{-1}\delta_1$	0	0	0	0	0	0	0	0	0	0	0
$f^{-1}\delta_2$	0	0	0	0	0	0	0	0	0	0	0
$f^{-1}\delta_3$	2×10^{-10}	8×10^{-8}	0	0	0	3×10^{-10}	-3×10^{-10}	0	0	3×10^{-9}	-8×10^{-9}
α_3	-2×10^{-8}	8×10^{-6}	0	0	0	-1×10^{-7}	1×10^{-7}	0	0	-2×10^{-6}	3×10^{-6}

Note: Any $|\text{number}|<10^{-16}$ is assumed to be zero.

TABLE 5: Steady-State Values of Outputs Using Proposed Controller (7) When Applied to MSAT Evaluation Model - Disturbance Rejection Case

	$\bar{g}_{c_1}=1$	$\bar{g}_{c_2}=1$	$\bar{g}_{c_3}=1$	$\bar{g}_{\beta_1}=1$	$\bar{g}_{\beta_2}=1$	$\bar{f}_1=1$	$\bar{f}_2=1$	$\bar{f}_5=1$	$\bar{f}_6=1$	$\bar{f}_0=1$	$\bar{f}_9=1$
θ_x	0	0	0	0	0	0	0	0	0	0	0
θ_y	0	0	0	0	0	0	0	0	0	0	0
θ_z	0	0	0	0	0	0	0	0	0	0	0
$\beta_1+\alpha_1$	0	0	0	0	0	0	0	0	0	0	0
$\beta_2+\alpha_2$	0	0	0	0	0	0	0	0	0	0	0
$f^{-1}\delta_1$	0	0	0	0	0	0	0	0	0	0	0
$f^{-1}\delta_2$	0	0	0	0	0	0	0	0	0	0	0
$f^{-1}\delta_3$	4×10^{-5}	-3×10^{-8}	0	0	0	4×10^{-9}	-4×10^{-9}	0	0	-3×10^{-6}	-8×10^{-4}
α_3	-3×10^{-7}	5×10^{-5}	0	0	0	-7×10^{-8}	7×10^{-8}	0	0	1×10^{-6}	3×10^{-6}

Note: Any $|\text{number}|<10^{-16}$ is assumed to be zero.

SIMULATIONS OBTAINED USING PROPOSED CONTROLLER TO SOLVE MSAT PROBLEM

This section gives some typical simulations of the closed loop system obtained by using the proposed controller (7) applied to the MSAT design and evaluation models. Additional simulation studies are given in [3].

A. Example No. 1 (Attitude Control: $\theta_x^{ref}=1$)

In this example, it is assumed that the system has zero initial conditions, that there are no disturbances present, and that a unit step function change of +1 occurs in the set point for θ_x at t=0, i.e. $\theta_x^{ref}=1$, $\theta_y^{ref}=0$, $\theta_z^{ref}=0$.

Figure 3 gives a plot of all 9 output variables y given by (4) when the controller is applied to both the design and evaluation model in this case. It is observed that the system's response is almost decoupled, i.e. the output θ_x is approximately equal to its desired value of +1 at t ≑ 50 min, and that all other 8 outputs are barely excited.

Figure 4 gives a plot of the 7 control variables u given by (5) for this example.

B. Example No. 2 (Disturbance Rejection: $\bar{f}_5=1$)

In this example, it is assumed that the system has zero initial conditions, that all set points are identically equal to zero, and that a unit step function change of +1 occurs at t=0 corresponding to a disturbance thrust $\bar{f}_5=1$, where $\bar{f}_5$ is defined in Table 6. This example would correspond to a misaligned thruster associated with the proposed controller.

Figure 5 gives a plot of all 9 output variables y when the controller is applied to both the design and evaluation model in this case. It is observed that the elastic modes of the vehicle are now excited, and that the output variables are asymptotically regulated to zero in approximately 2.7 hours, which is consistent with the closed loop eigenvalues of the system given in Table 2.

Figure 6 gives a plot of the 7 control variables u for this example.

C. Example No. 3 (Disturbance Rejection: $\bar{f}_9=1$)

This example is similar to Example No. 2 except that it is assumed that a unit step function of +1 occurs at t=0 corresponding to a disturbance thrust $\bar{f}_9=1$, where $\bar{f}_9$ is defined in Table 6. This disturbance is representative of an arbitrary constant disturbance which may affect the system.

Figure 7 gives a plot of all 9 output variables y when the controller is applied to both the design and evaluation models in this case. It is observed that the elastic modes of the vehicle are now also excited as they were in Example No. 2, and that the output variables are satisfactorily asymptotically regulated with the same time constant as in Example No. 2.

Figure 8 gives a plot of the 7 control variables u for this example.

TABLE 6: Definition of Disturbances Assumed

$\bar{f}_1,\bar{f}_2,\bar{f}_5,\bar{f}_6,\bar{f}_0,\bar{f}_9$	Disturbance forces corresponding to the thrusters f_1,f_2,f_5,f_6,f_0,f_9 respectively of Figure 2
$\bar{g}_{\beta_1},\bar{g}_{\beta_2}$	Disturbance torques corresponding to g_{β_1},g_{β_2} respectively about the gimbal axis β_1,β_2
$\bar{g}_{c_1},\bar{g}_{c_2},\bar{g}_{c_3}$	Disturbance torques in the bus about the x,y,z axis respectively

ROBUST PROPERTIES OF CONTROLLER DESIGN METHOD

A study of the robustness properties of the proposed controller design method was carried out [3]. This was done by comparing the controller designs obtained using the proposed method to different design models of MSAT. It was concluded that the proposed design method appears to be quite insensitive to the type of design model used, e.g. all controllers obtained, when based on MSAT design models which had at least two dominant elastic body modes included, produced stable closed loop systems and give satisfactory tracking/regulation, when applied to the MSAT evaluation model. Other studies showed that the controller is robust with respect to evaluation models of arbitrary complexity.

CONCLUSIONS

This paper gives a brief summary of the work performed in [1]-[3]. In these studies, the control system design of a third-generation spacecraft, as modelled by the MSAT space configuration is studied. This spacecraft is highly non-symmetrical and has appendages which cannot, in general, be assumed to be rigid; the elasticity of these appendages makes the control system design particularly demanding. In particular, it is desired to design a controller for MSAT which stabilizes the system and satisfies certain attitude control, shape control and possibly station-keeping requirements. In addition, it is desired that the resultant controller should be robust and avoid any "spill-over effects", i.e. it should satisfy the problems' specifications based on only an approximate design model for MSAT being available. In addition, the controller obtained should have minimum complexity, i.e. a minimum number of sensors/actuators should be used.

The method of solution adopted to solve this class of problems was to formulate the problem as a robust servomechanism problem and thence to obtain existence conditions and a controller characterization to solve the problem. In this case, the controller obtained must contain a servo-compensator together with a stabilizing compensator.

The final controller obtained for MSAT has a distributed control configuration, and appears to be quite satisfactory, i.e. extensive testing of the controller shows that the controller is indeed robust with respect to the choice of the design model, and that it satisfies all specifications of the problem statement.

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APPENDIX I

Definitions of $g_{c_3}^*, f_1^*, f_2^*, f_5^*, f_6^*$

$g_{c_3}^*$ is defined in terms of thrusters f_1, f_2, f_3, f_4 as follows:

$$\begin{pmatrix} f_1 \\ f_3 \end{pmatrix} = \begin{pmatrix} 8.66 \\ 8.66 \end{pmatrix} g_{c_3}^* \qquad \text{if } g_{c_3}^* \geq 0$$

$$\begin{pmatrix} f_2 \\ f_4 \end{pmatrix} = \begin{pmatrix} 8.66 \\ 8.66 \end{pmatrix} g_{c_3}^* \qquad \text{if } g_{c_3}^* < 0$$

f_5^*, f_6^* are defined in terms of thrusters f_5, f_6, f_7, f_8 as follows:

$$\begin{pmatrix} f_5 \\ f_7 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} f_5^* \quad \text{if } f_5^* \geq 0 \quad ; \quad \begin{pmatrix} f_6 \\ f_8 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} f_6^* \quad \text{if } f_6^* \geq 0$$

$$\begin{pmatrix} f_5 \\ f_7 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} f_5^* \quad \text{if } f_5^* < 0 \quad ; \quad \begin{pmatrix} f_6 \\ f_8 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} f_6^* \quad \text{if } f_6^* < 0$$

f_1^*, f_2^* are defined in terms of thrusters f_1, f_2, f_3, f_4 as follows:

$$\begin{pmatrix} f_1 \\ f_2 \\ f_4 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} f_1^* \\ f_2^* \end{pmatrix} \qquad \text{if } f_1^* \geq 0 \text{ and } f_2^* \geq 0$$

$$\begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} f_1^* \\ f_2^* \end{pmatrix} \qquad \text{if } f_1^* \geq 0 \text{ and } f_2^* < 0$$

$$\begin{pmatrix} f_3 \\ f_4 \\ f_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} f_1^* \\ f_2^* \end{pmatrix} \qquad \text{if } f_1^* < 0 \text{ and } f_2^* \geq 0$$

$$\begin{pmatrix} f_2 \\ f_3 \\ f_4 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} f_1^* \\ f_2^* \end{pmatrix} \qquad \text{if } f_1^* < 0 \text{ and } f_2^* < 0$$

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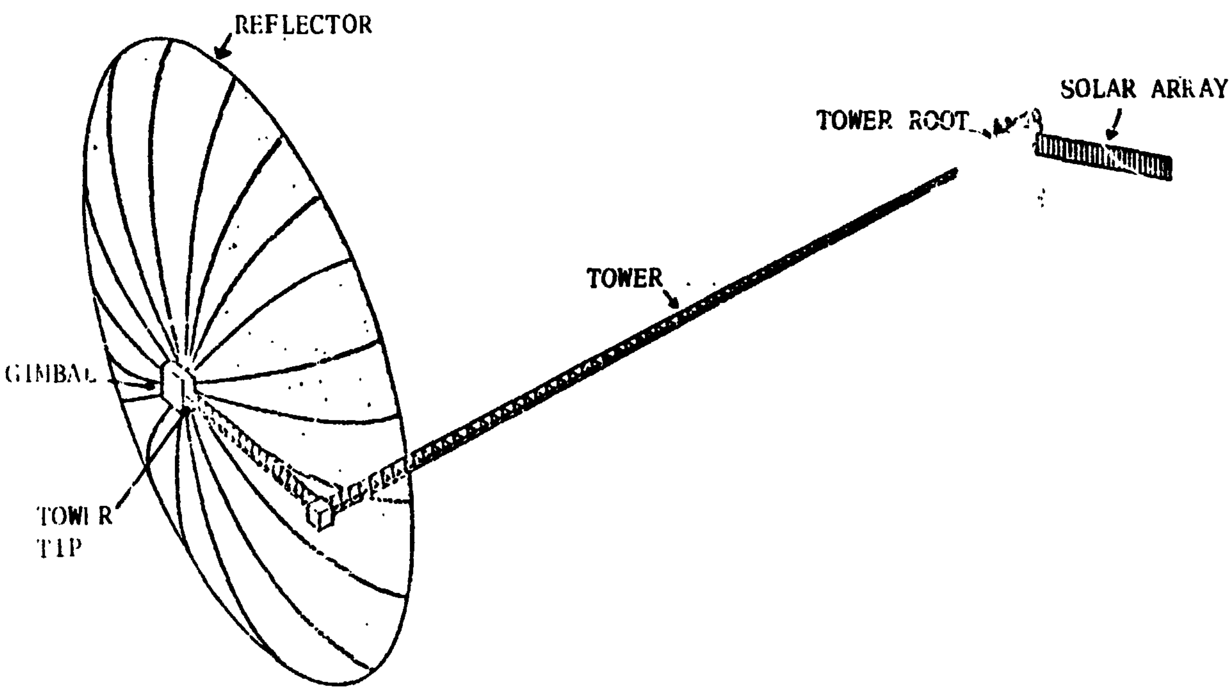

Figure 1: The MSAT configuration - a typical third generation spacecraft.

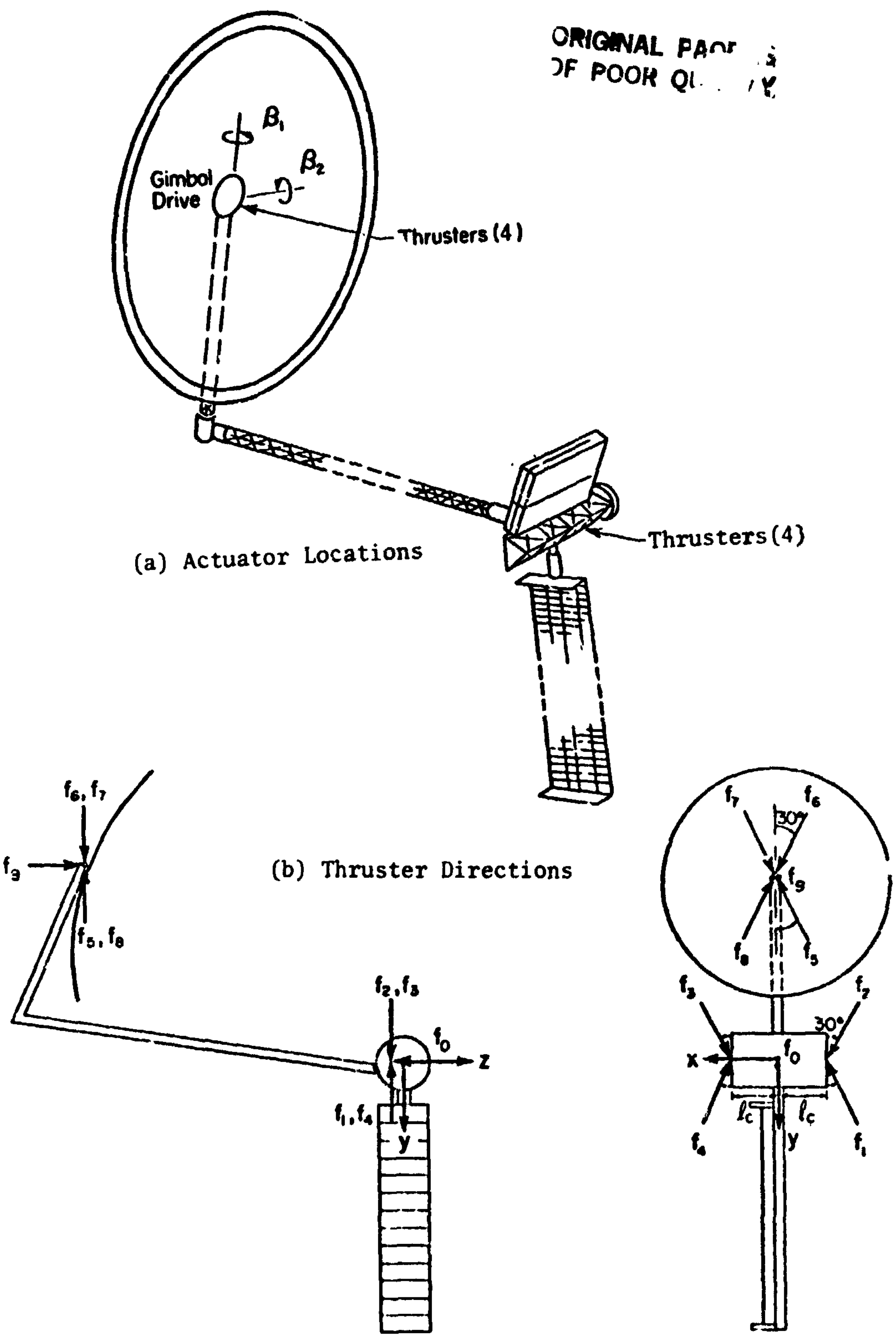

Figure 2: Assumed control inputs for MSAT spacecraft (taken from [4]).

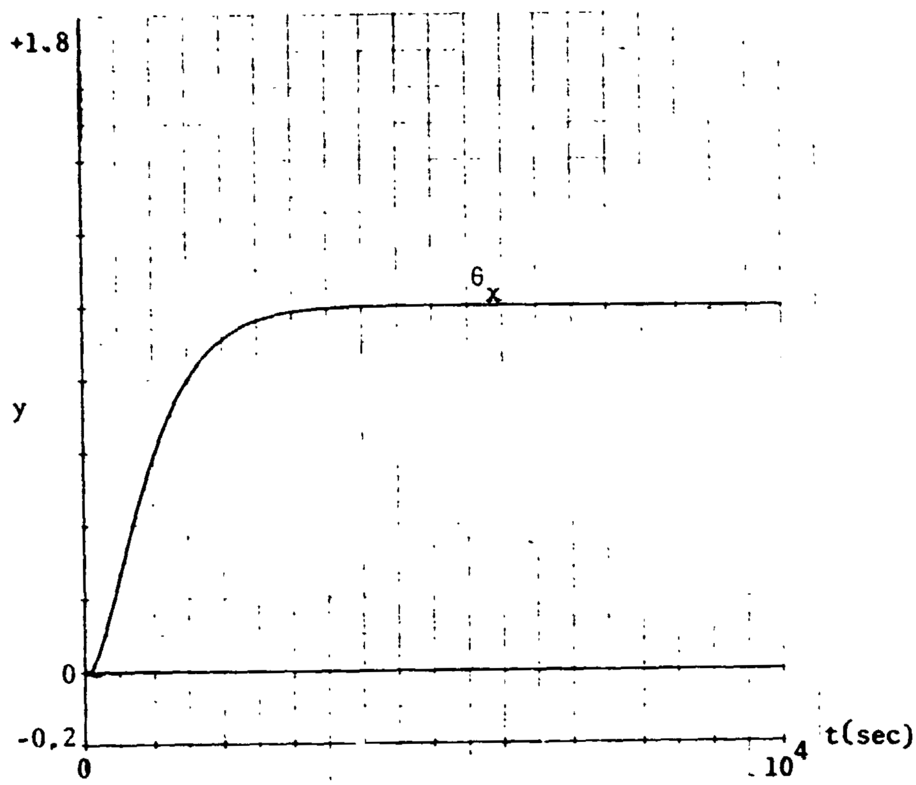

Figure 3: Plot of 9 regulated outputs y for example no. 1.

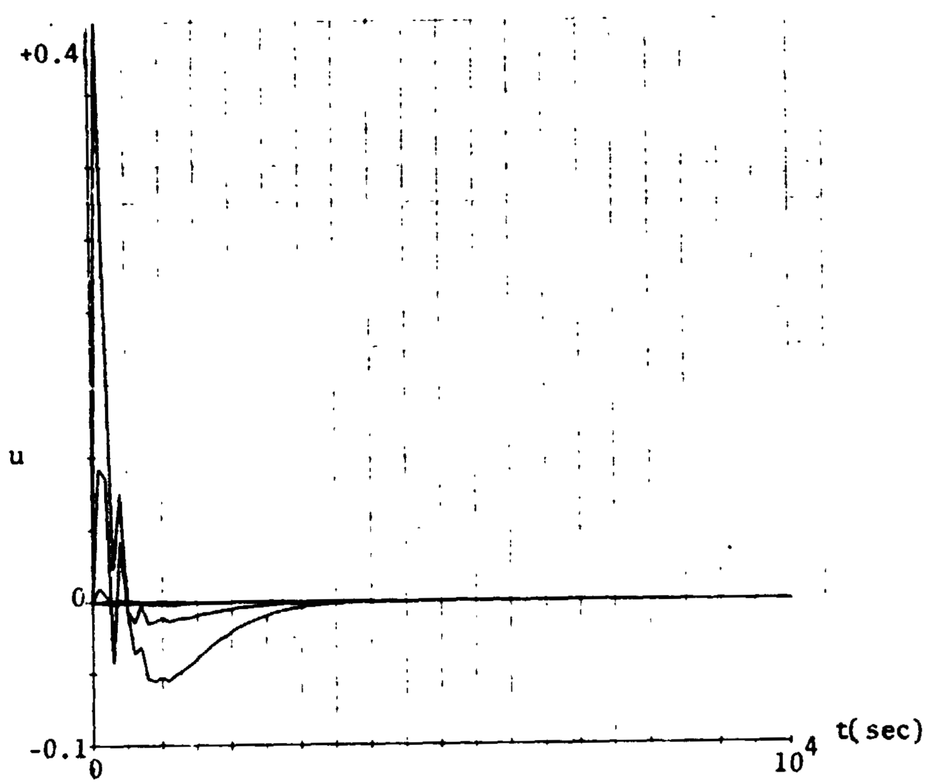

Figure 4: Plot of 7 control inputs u for example no. 1.

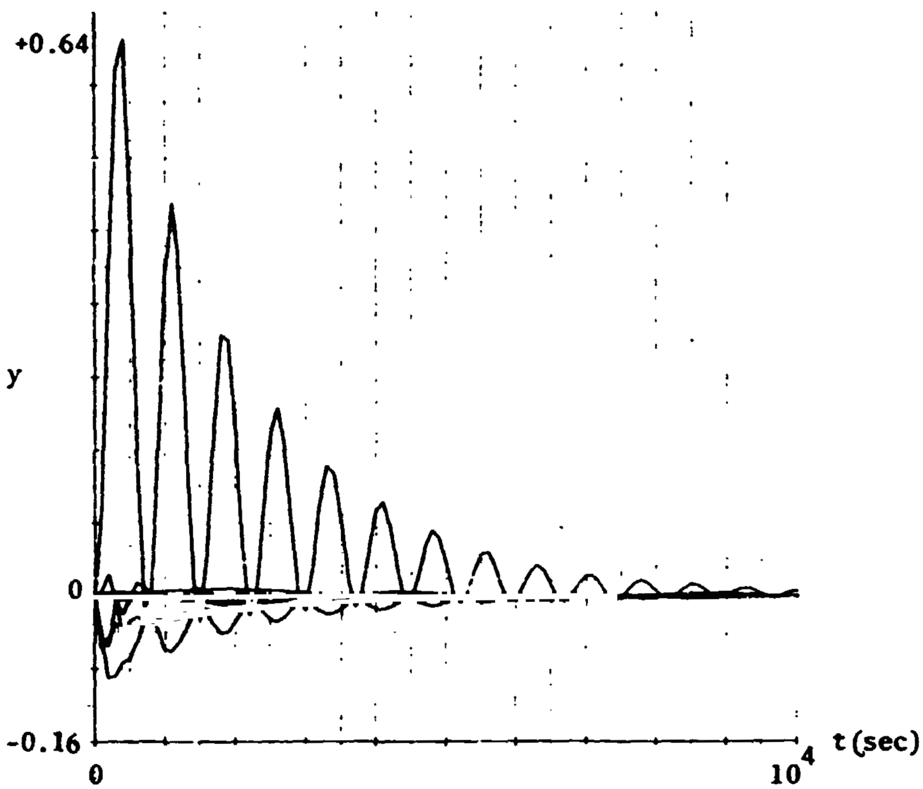

Figure 5: Plot of 9 regulated outputs y for example no. 2.

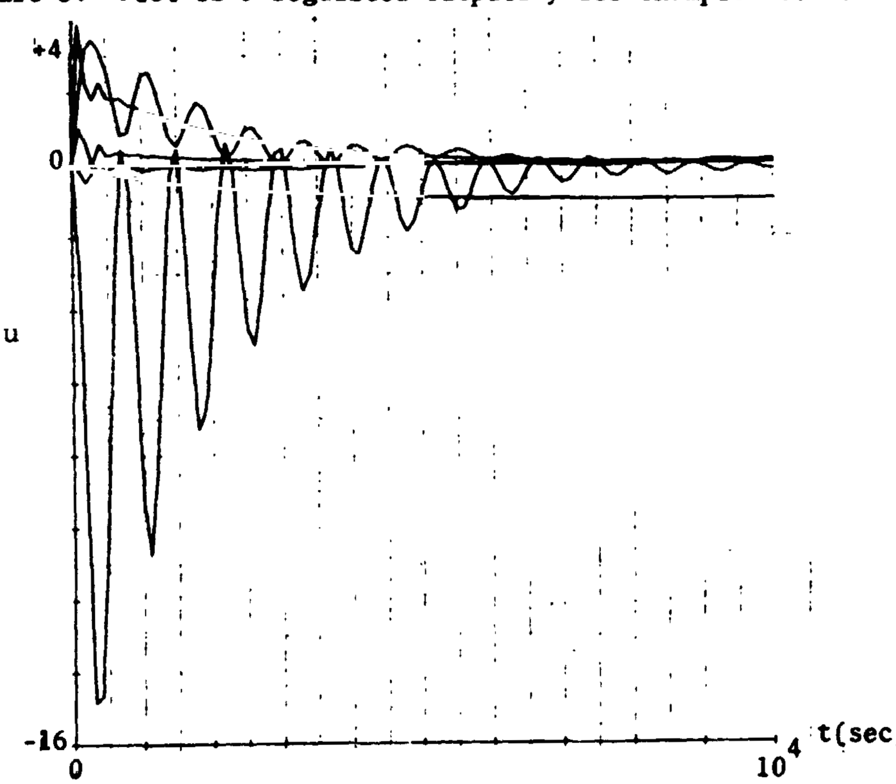

Figure 6: Plot of 7 control inputs u for example no. 2.

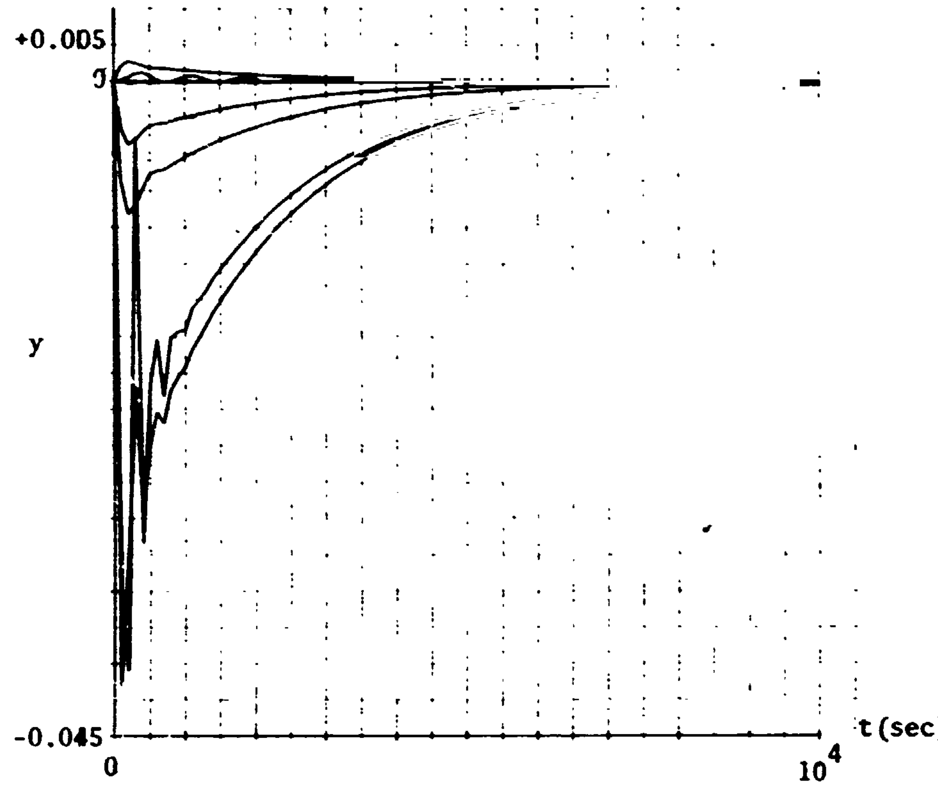

Figure 7: Plot of 9 regulated outputs y for example no. 3.

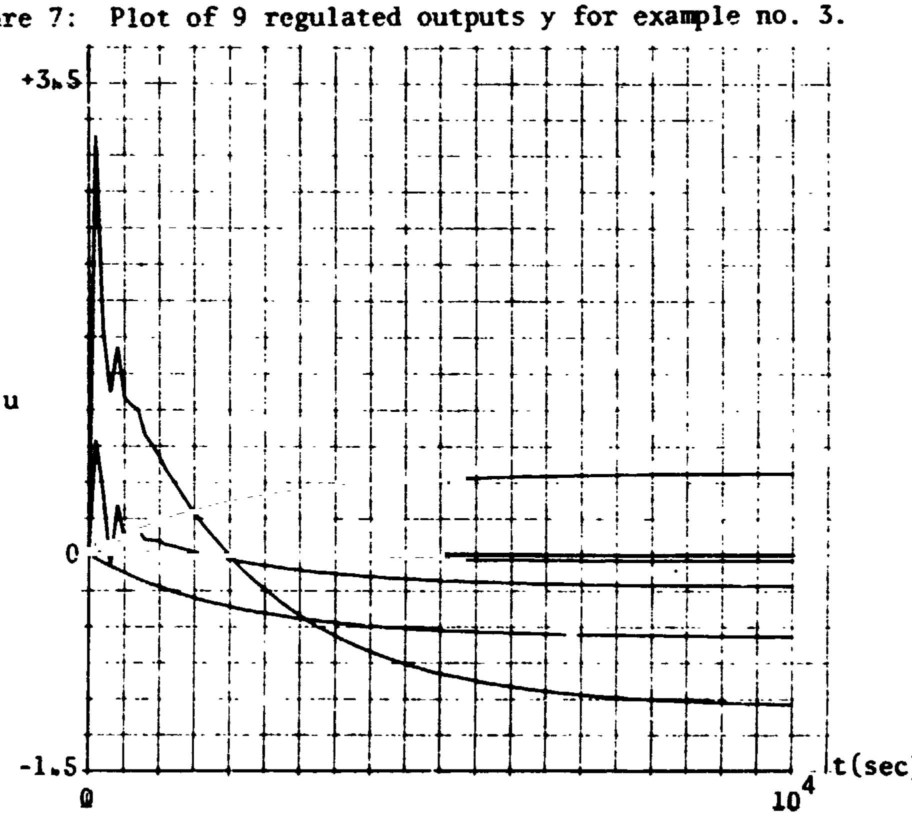

Figure 8: Plot of 7 control inputs u for example no. 3.