MATRIX TRANSFER FUNCTION DESIGN FOR FLEXIBLE STRUCTURES—AN APPLICATION

T. J. Brennan, A. V. Compito, A. L. Doran, C. L. Gustafson, and C. L. Wong
Aerospace Corporation
Los Angeles, CA 90009

ABSTRACT

The application of matrix transfer function design techniques to the problem of disturbance rejection on a flexible space structure is demonstrated. The design approach is based on parameterizing a class of stabilizing compensators for the plant and formulating the design specifications as a constrained minimization problem in terms of these parameters. The solution yields a matrix transfer function representation of the compensator. A state space realization of the compensator is constructed to investigate performance and stability on the nominal and perturbed models. The application is made to the ACOSS (Active Control of Space Structures) optical structure.

I. INTRODUCTION

The problem of flexible space structure control has motivated a great deal of research for theoreticians and practitioners of multivariable control design. In spite of the efforts directed in this area there still remains a significant gap between the multivariable theory and the control design implementation. This gap stems from two sources. The first difficulty is one of problem specification. Translation of complex system requirements and constraints into the specific mathematical cost functionals required by most design methods may be impossible in many cases. Free parameters in the chosen design methodology may not be traceable to the parameters which describe the

system in terms of desired performance, plant uncertainty, hardware
limitations, etc. A second roadblock to the implementation of modern control
design techniques is the lack of reliable algorithms and software to perform
the sophisticated mathematical manipulations required by these techniques.
Recent years have shown very considerable advances in this field (see [1]) but
much remains to be done.

Most of the MIMO (multi-input/multi-output) compensators which have
actually left the textbook and been calculated in computers are based on state
space methods, and, in particular, LQG (Linear-Quadratic-Guassian) design
theory. This is due in part to the long history of development of these
design techniques as well as the availability of reliable algorithms to solve
matrix Riccati equations and the ease of performing most state space
manipulations. Frequency domain techniques for calculating MIMO feedback
systems have been avoided. The extensions of classical frequency domain
concepts to MIMO systems have not been totally satisfying and calculations
involving matrices of transfer functions present an entirely new set of
problems. Nonetheless, frequency domain design is still appealing and certain
feedback notions cannot be adequately expressed without reference to transfer
functions.

We have carried through a compensator design for a flexible structure
based on transfer function parameterization techniques. General theories of
feedback control system parameterization have been developed by several
authors ([2], [3], and [4]). The goal of a parametric approach is the
selection of a set of numerical quantities, along with an acceptable range of
values, which span a class of possibly acceptable compensators and, wi ˉ
which, one is able to adequately express the system requirements in terms of
costs and constraints. A particularly simple parameterization for stable
plants was introduced by Zames, [4], and exploited for the unity feedback
configuration of Figure 1 bv Desoer and Chen [5]. This is the
parameterization we will implement here. The details are in section IV.
Previous examples of this design approach can be found in [6] and [7].

II. ACOSS STRUCTURE

The ACOSS optical structure was developed by the Charles Stark Draper Laboratories as a control design test specimen to evaluate the design approaches developed for the DARPA ACOSS program, [8]. It was designed to exhibit the closely spaced, low frequency mode distribution expected on some future space systems. The structure is provided as a finite element model having 84 dynamic degrees of freedom (see Figure 2). In addition to the nominal structure, two perturbed structures were defined to represent plant uncertainty. The perturbed models represent mass and stiffness variations of approximately 10%. The nominal model is denoted PO, the perturbed models are P2 and P4.

The performance goal is expressed in terms of a line of sight error on a focal plane on the lower section of the truss as shown in Figure 2. The error has two angular components and a defocus component resulting from deviations in the optical path due to structural vibrations. Three rigid mirrors determine the optical path. These are assumed to be rigidly mounted to the structure. Two disturbances are defined on the structure as shown in the figure. For our design problem we are only considering the disturbance propagating from the equipment panel and we assume it has a flat PSD out to 5 Hz. The equipment panel is isolated from the structure by a spring-damper system. The residual disturbance propagation through this isolation system into the line of sight is still unacceptably high. The control problem is to further reduce this residual with active structural control.

III. MODEL SELECTION AND ACTUATOR PLACEMENT

For the current design problem we chose a 5 mode model of the structure, selecting those modes having most significant influence between disturbance and line of sight. The modal influence was determined based on ideas from internally balanced coordinates. For a description of internally balanced

coordinates see [9] and for an application to modal coordinates see [10].
Given a second order model description,

$$\ddot{q}_i + 2 \zeta_i \omega_i \dot{q}_i + \omega_i^2 q_i = g_i^T \underline{w} , \qquad i = 1, \ldots, n \qquad (1)$$

$$\underline{y} = \sum_{i=1}^{n} \underline{h}_i q_i \qquad (2)$$

with natural damping ζ_i, frequency ω_i, inputs $\underline{w}$, and outputs $\underline{y}$, an index
for
ranking the modes can be calculated as the approximate "second
order modes," ([10]) by

$$\sigma_i^2 = \frac{\sqrt{(g_i^T g_i)(\underline{h}_i^T \underline{h}_i)}}{4 \zeta_i \omega_i} \qquad (3)$$

Using the modal disturbance influence matrix for the g_i's and the line
of sight measurement matrix for the $\underline{h}_i$'s the 5 highest rank modes are
tabulated in Table 1. Agreeing with our intuition, these turn out to be two
isolator rotations, two isolator translations, and the first bending mode of
the upper truss. A description of the modes of the structure can be found in
[8].

Mode	7	8	12	13	21
Frequency (Hz)	.15	.26	.58	.58	2.3

Table 1. Design Modes

The line of sight measurement matrix is a function of 21 nodal degrees
of freedom. From among these 21 degrees of freedom we chose to locate three
force actuators (assumed to be of the momentum exchange or proof mass type) to
control the three line of sight measurements. To make this selection an
appeal is again made to the approximate second order modes of equation (3).

50

If the forcing function on the right of (1) is $g_{ij}u$ where g_{ij} is the influence of the jth actuator, $j = 1,\ldots,21$ on the ith mode, $i = 1,\ldots,5$, then we denote the corresponding second order mode by σ_{ij} and define

$$\alpha_j^2 = \sum_{i=1}^{5} \sigma_{ij}^2 \qquad (4)$$

Here, α_j is a measure of the influence of the jth actuator on the line of sight for the selected 5 mode model. We chose three actuators whose force directions span the three spatial directions and have large α_j with respect to the total 21 possible actuators. Two of the actuators selected are located on the corners of the primary mirror and the third is on the lower truss.

To complete the description of the design plant we assumed the availability of direct measurements of the line of sight. No other sensors were used for the control design. We now have a state space description of the design plant in modal coordinates,

$$\dot{x} = Fx + Gu + Dd \qquad (5)$$

$$y = Hx \qquad (6)$$

where u is the actuator command and d is the disturbance input.

For calculation of the compensator we need a transfer function representation of the design plant. The convenient representation for constructing state space realizations of the compensator is a polynomial matrix coprime factorization [11,12], that is, $P = ND^{-1}$ where N and D are coprime polynomial matrices. An algorithm to construct a coprime factorization from a state space description can be found in [13].

51

IV. DESIGN PROBLEM

The feedback configuration used for the design is shown in Figure 1. The closed loop system is referred to as $\mathscr{A}$. P is the open loop design plant, a 3x3 transfer function given from the state space equations by $H(sI-F)^{-1}G$. The inputs are u_1 and u_2 with the reference input, u_1, identically zero. The outputs are y_1 and y_2 with the line of sight represented by y_2. The disturbance propagates into the line of sight through the transfer function $\widetilde{P} = H(sI-F)^{-1}D$ and may thus be represented as an additive disturbance, $\widetilde{P}d$, at the plant output.

The closed loop system transfer function is defined to be

$$H_{yu} : \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \to \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}. \tag{7}$$

Stability of H_{yu} can be taken to be closed loop stability. H_{yu} may be expressed in a simple parameterized form as

$$H_{yu} = \begin{bmatrix} Q & -QP \\ \\ PQ & P(I-QP) \end{bmatrix} \tag{8}$$

where Q is referred to as the Zames parameterization, [4], with

$$Q = C(I + PC)^{-1}. \tag{9}$$

We state here the fundamental result from [5] which is the basis of this design approach.

Fact: For P exponentially stable and strictly proper, Q is exponentially stable and proper if and only if

(i) C is proper and

(ii) H_{yu} is exponentially stable and proper.

When this is the case the compensator is given by

$$C = Q(I - PQ)^{-1} \qquad (10)$$

In other words designing stabilizing compensators for $\mathcal{A}$ is equivalent to specifying exponentially stable, proper Q.

From (8) we see that the I/O map, that is, the transfer function from u_1 to y_2 is

$$H = H_{y_2 u_1} = PQ \qquad (11)$$

Given an invertible plant transfer function, P, one can see from the relation (11) that a parametrization of the closed loop system by Q is equivalent to a parameterization by H. Moreover, for P exponentially stable, Q exponentially stable implies the same for H. But since

$$Q = P^{-1}H \qquad (12)$$

it becomes clear that exponential stability of H only implies exponential stability of Q when P has no unstable zeros. However, by imposing an additional condition on H, namely that H has the same right half plane zero structure as P, then parameterization by such H is equivalent to parameterization by exponentially stable Q. If a proper compensator is desired the additional constraint of properness of Q is required and will result in an excess pole over zero constraint on H which depends on P.

Parameterization by the I/O map, H, may simplify the design problem and allow the designer to more directly specify his design objective. For example, for a disturbance attenuation problem, the closed loop disturbance to output map, or sensitivity map, is simply given as (I - H). In addition, in some applications, a decoupled I/O map is desirable and one is directly able

53

to parameterize a diagonal H. This is the approach we take for this design. Calculating the transmission zeros of our design plant using the QZ algorithm [14] we find that there are no zeros in the right half plane so we may freely specify H as diag(h_i, i=1,2,3) with each h_i of the form

$$\frac{g \, p_n(s)}{p_{d_1}(s) \, p_{d_2}(s)} \tag{13}$$

where g is a gain and

$$p_n(s) = s^2 + 2\zeta_n \omega_n s + \omega_n^2 \tag{14}$$

$$p_{d_j}(s) = s^2 + 2\zeta_{d_j} \omega_{d_j} s + \omega_{d_j}^2 \tag{15}$$

This parameterization has 21 parameters consisting of the gains, and second order damping and frequency terms.

We set for ourselves a design goal of minimizing closed loop response to the disturbance over a low frequency band of 5 Hz. To achieve this we define a constrained optimization problem as follows:

Minimize

$$J = \|(I - H(j\omega)) \, \tilde{P}(j\omega)\|_2 \, , \quad \omega = 10\pi \tag{16}$$

subject to

$$0.01 < \zeta_n, \, \zeta_{d_j} \qquad \text{:Stability}$$

$$0.04 < \omega_n, \, \omega_{d_j} < \omega_b \qquad \text{:Bandwidth}$$

$$h_i(0) = 1 \qquad \text{:Low frequency noise rejection}$$

The matrix under the norm of J is diagonal so we simp'v take the Euclidean vector norm of the diagonal. The minimization of ae cost at 5 Hz and the DC unity gain constraint will result in disturbance rejection across the 5 Hz band. The $\widetilde{P}$ term in the cost weights the diagonal terms in $(I - H)$ according to the way the disturbance propagates through the structure.

In general $(I - H)$ is the ratio of the relative uncertainty in the I/O map to the relative uncertainty in the open loop plant. More precisely

$$(\Delta H)\hat{H}^{-1} = (I-H)(\Delta P)\hat{P}^{-1} \tag{17}$$

where

$$\Delta P = \hat{P} - P \tag{18}$$

$$\Delta H = \hat{H} - H \tag{19}$$

for a "perturbed" plant $\hat{P}$ which results in a perturbed I/O map $\hat{H}$. In effect, minimization of J reduces the impact of plant uncertainties on closed loop system performance.

Having specified the optimization problem one can use numerical or analytical means to solve it. Omitting the details, we calculated a local minimum to this problem analytically. The achievable performance is clearly dependent on the bandwidth, ω_b. For a given ω_b the local minimum satisfies

$$\omega_{d_j} = \omega_b \tag{20}$$

$$P_{d_1}(s) = P_{d_2}(s) \tag{21}$$

$$\frac{2\zeta_n}{\zeta_{d_j}} = \frac{\omega_n}{\omega_{d_j}} \tag{22}$$

55

$$g = \frac{\omega_{d_i}^4}{\omega_n^2} \qquad (23)$$

Given this solution we can adjust the bandwidth of each of the three loops to achieve a desired performance level. To achieve 0.04 reduction in each channel we have the following parameter values:

	ω_{d_i}	ζ_{d_i}	ω_n	ζ_n	g
h_1	300	1.5	10^4	100	81
h_2	600	.8	4×10^4	107	81
h_3	300	5	2×10^3	200	25

V. COMPENSATOR REALIZATION

Having arrived at parameter values we have specified the desired I/O map, H. The compensator which will produce this I/O map is

$$C = Q(I - PQ)^{-1} \qquad (24)$$

$$= P^{-1}H(I - H)^{-1} . \qquad (25)$$

Since the I/O map is given by $\mathrm{diag}(h_i)$ with each h_i of the form

$$h_i = \frac{n_i}{d_i} , \qquad (26)$$

the compensator becomes

$$C = P^{-1} \text{diag} \left(\frac{n_i}{d_i - n_i} \right). \tag{27}$$

We have already expressed P as a polynomial matrix coprime factorization, $P = ND^{-1}$. Thus (27) becomes

$$C = D \ N^{-1} \text{diag} \left(\frac{n_i}{d_i - n_i} \right). \tag{28}$$

Since the degree of $d_i - n_i$ is 4, we can factor this polynomial into two quadratics as

$$d_i - n_i = \tilde{d}_{i_1} \tilde{d}_{i_2}, \quad i = 1,2,3. \tag{29}$$

Hence (29) can be rewritten as

$$C = D \ \{ \ \text{diag}(\tilde{d}_{i_1})N \ \}^{-1} \ \text{diag} \left(\frac{n_i}{\tilde{d}_{i_2}} \right) \quad . \tag{30}$$

By inspection, $\text{diag}(\tilde{d}_{i_1})N$ is column-reduced [12], and has column degrees equaling those of D. Consequently $D \ \{\text{diag}(\tilde{d}_{i_1})N\}^{-1}$ is proper and has a state space realization [12, Sec. 6.4]. Now, since $\text{diag}(n_i/\tilde{d}_{i_2})$ also has a state space realization, the two realizations can be cascaded to yield a realization for C.

VI. RESULTS

Having computed a state space description of the compensator we are now able to determine closed loop stability for various versions of the plant simply by extracting the eigenvalues from the closed loop state equations derived from Figure 1. We find that for all three versions of the plant (P0, P2, and P4) the five mode description remains stable under feedback by our compensator.

To investigate the robustness of the design with respect to unmodeled dynamics we appended additional modes to the plant model and found that the closed loop system became unstable in almost all cases. Upon investigation of this problem we discovered that though the 5 mode design plant was minimum phase, the addition of almost any other mode or set of modes resulted in a nonminimum phase plant. Information about these unstable zeros was not available in the design plant so the resulting compensator tended to place closed loop poles at these zeros. Thus the stability problem experienced is one of modeling or model reduction. In general, any control design approach must have information about the right half plane zeros of the plant.

The performance of the closed loop system remained very consistent with the predictions made during the design stage. The steady state RSS response at 5 hz of the two angular components of the line of sight is given as a fraction of open loop response for the three models by:

P0	P2	P4
4.3×10^{-2}	5.1×10^{-2}	4.6×10^{-2}

The broadband disturbance attenuation is illustrated on the Bode plots of Figures 3 and 4 which compare open and closed loop response. Across a significant portion of the 5 Hz band the performance improvement is 3 to 4 orders of magnitude.

58

VII. CONCLUSIONS

We have demonstrated the applicability of a transfer function parameterization design approach for problems of broadband disturbance attenuation on flexible space structures. This methodology provides the control designer with a great deal of flexibility to meet system requirements by the choice of parameter set and selection of cost function and constraints. Although the implementation of this technique requires difficult numerical calculations involving matrix transfer functions, algorithms and software for these types of problems are already emerging. The success of this approach is dependent on an appropriate parameter selection in which to express the problem specifications. This suggests research, probably application specific, which addresses the issues of problem description and requirements interpretation in the control design process.

REFERENCES

[1] A. J. Laub, "Numerical Aspects of Control Design Computations," Computer-Aided Design and Analysis of Digital Guidance and Control Systems, NATO AGARD-LS-128, July 1983.

[2] L. Pernebo, "An Algebraic Theory for the Design of Controllers Linear Multivariable Feedback Systems," IEEE Trans. Aut. Contr., Vol. AC-26, pp. 171-194, February 1981.

[3] D. C. Youla, H. A. Jabr, and J. J. Bongiorno, Jr., "Modern Wiener-Hopf Design of Optimal Controllers - Part II: The Multivariable Case," IEEE Trans. Aut Contr., Vol. AC-21, pp.319-338, June 1976.

[4] G. Zames, "Feedback and Optimal Sensitivity: Model Reference Transformations, Multiplicative Seminorms, and Approximate Inverses," IEEE Trans. Aut. Contr., Vol. AC-26, pp. 301-320, April 1981.

[5] C. A. Desoer and M. J. Chen, "Design of Multivariable Feedback Systems with Stable Plant," IEEE Trans. Aut. Contr., Vol. AC-26, pp. 408-415, April 1981.

[6] F. P. Fernandez, "Multivariable Controller Design Using Parameter Optimization," M.I.T. master's thesis, May 1983.

REFERENCES (Continued)

[7] C. L. Gustafson and C. A. Desoer, "Controller Design for Linear
      Multivariable Feedback Systems with Stable Plants, Using Optimization
      with Inequality Constraints," Int. J. Contr., Vol. 37, No. 5,
      pp. 881-907, 1983.

[8] Charles Stark Draper Laboratory, "ACOSS Six (Active Control of Space
      Structures)," Interim Report, RADC-TR-80-377, January 1981.

[9] B. C. Moore, "Principal Component Analysis in Linear Systems:
      Controllability, Observability, and Model Reduction," IEEE Trans. Aut.
      Contr., Vol. AC-26, pp. 17-32, February 1981.

[10] C. Z. Gregory, Jr., "Reduction of Large Flexible Spacecraft Models Using
      Internal Balancing Theory," Proceedings of the 1983 AIAA Guidance and
      Control Conference, Gatlinburg, Tennessee.

[11] F. M. Callier and C. A. Desoer, Multivariable Feedback Systems,
      Springer-Verlag: New York - Heidelberg - Berlin, 1982.

[12] T. Kailath, Linear Systems, Prentice Hall: Englewood Cliffs, N.J., 1980.

[13] R. V. Patel, "Computation of Matrix Fraction Descriptions of Linear
      Time-Invariant Systems," IEEE Trans. Aut. Contr., Vol. AC-26,
      pp. 148-161, February 1981.

[14] A. J. Laub and B. C. Moore, "Calculation of Transmission Zeros Using QZ
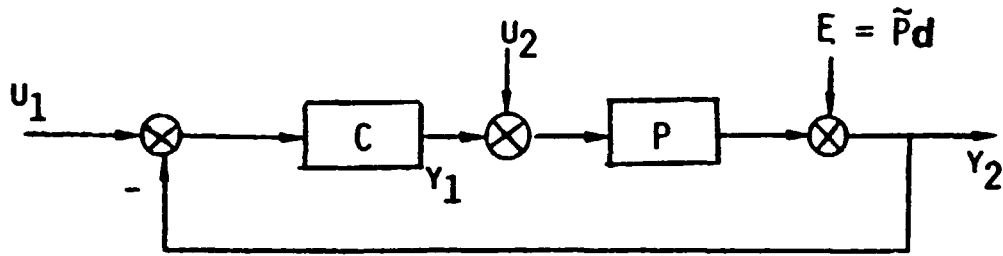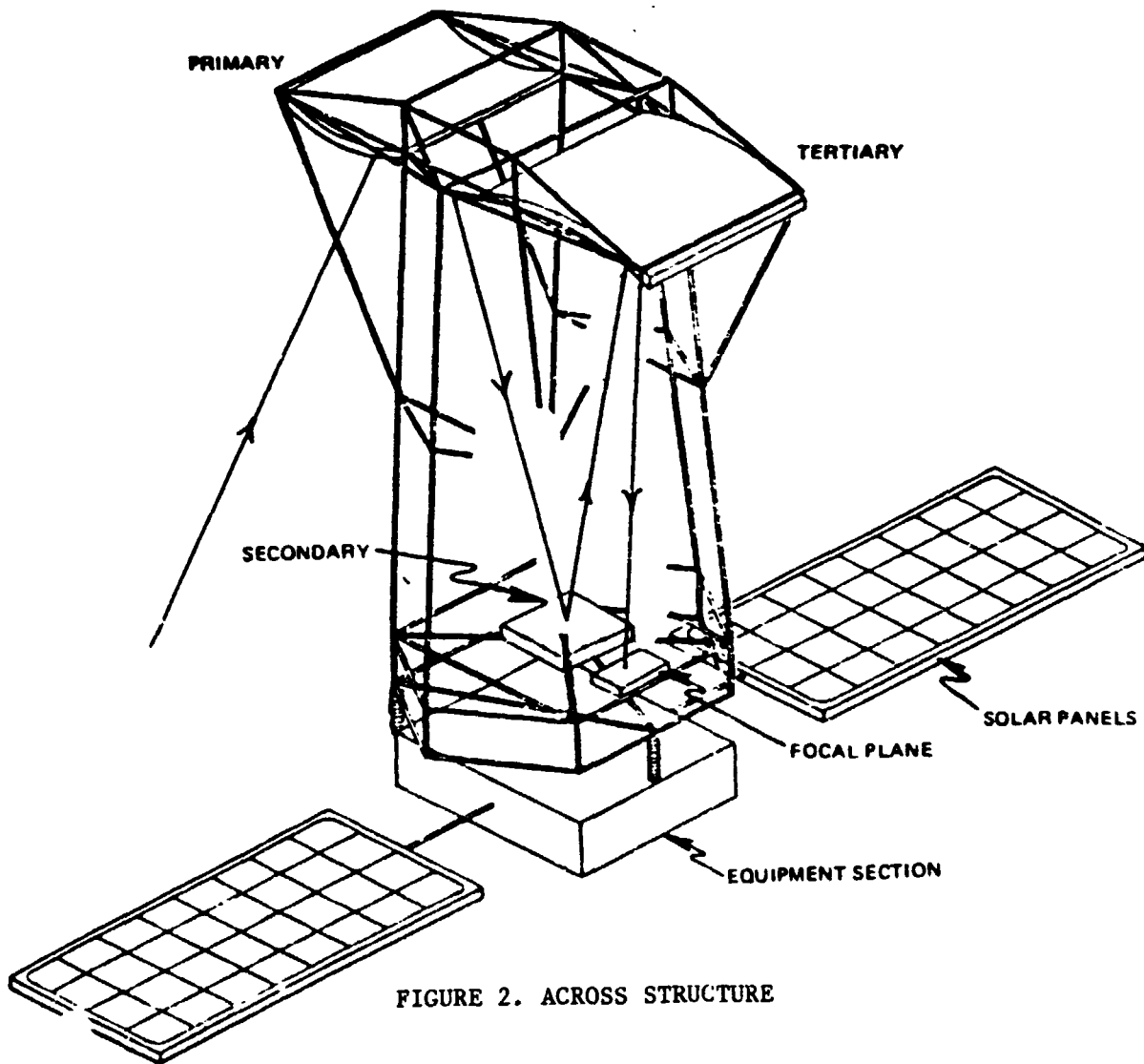   Techniques," Automatica, Vol 14, 1978, pp. 557-566.

FIGURE 1. BLOCK DIAGRAM OF $\mathscr{A}$

FIGURE 2. ACROSS STRUCTURE

MAGNITUDE (db)

FIGURE 3. P0 RESPONSE

FREQUENCY
(rad/sec)

MAGNITUDE (db)

FIGURE 4. P2 RESPONSE

FREQUENCY
(rad/sec)

62