ROBUST CONTROL DESIGN FOR LARGE SPACE STRUCTURES

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INTRODUCTION

The control design problem for the class of future spacecraft referred to as large space structures (LSS) is by now well known [1-3]. The issue is the reduced order control of a very high-order, lightly damped system with uncertain system parameters, particularly in the high frequency modes. This paper presents a design methodology which incorporates robustness considerations as part of the design process. Combining pertinent results from multivariable systems theory and optimal control and estimation, LQG eigenstructure assignment [4] and LQG frequency-shaping, [5-7] were used to improve singular value robustness measures in the presence of control and observation spillover.

The design technique is summarized as follows. A low order LQG compensator is synthesized using the technique of recursive eigenstructure assignment to place closed-loop eigenvalues where desired. This design is evaluated for singular value performance margin and for singular value gain margin with respect to plant uncertainties (e.g., modeled dynamics). The compensator is then resynthesized using frequency-shaping concepts to improve the singular value robustness measures. The recursive eigenstructure assignment technique allows regulator close-loop eigenvalue placement at the desired locations for the plant and as required for frequency-shaping. Furthermore, the frequency-

shaped compensator eigenvalues can also be assigned, thus assuring LQG compensator stability, as well as estimator stability.

This procedure using robust frequency-shaped compensation was applied to the design of the controller for a representative large space structure. Results are presented as singular value Bode plots. Comparisons are made to a recent study[8] utilizing the same large space structure model.

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LQG CONTROL DESIGN FOR LSS

Control design plant modelling for LSS utilizes a high-order structural model, typically obtained by finite-element programs such as NASTRAN. The limitations of computer implementation require that the finite-element model be reduced to a design model. One approach is to truncate the high-order model into primary and residual modes, where the primary modes are to be used for control design. The modal truncation can be based on engineering judgement or on a selection criterion such as modal cost analysis [9].

The system model has the form

$$\dot{x}_p = A_p \, x_p + B_p u$$
$$\dot{x}_R = A_R \, x_R + B_R u \tag{1}$$
$$y = C_p \, x_p + C_R \, x_R$$

where x_p are the primary modes and x_R are the residual modes. An observer-based control design for the primary modes then has the form

$$\dot{\hat{x}}_p = A_p \hat{x}_p + B_p u + G(y - C_p \hat{x}_p) \tag{2}$$
$$u = -K\hat{x}_p$$

Using LQG design, the gains (K, G) are selected to minimize quadratic performance indices. The terms $B_R u$ and $C_R x_R$ were identified by Balas [3] as control spillover and observation spillover respectively. These terms have the potential for interacting through the observer (2) to produce instability.

LQG theory guarantees that the reduced-order closed loop system is stable with eigenvalues of (A_p-B_pK) and (A_p-GC_p). However, no such guarantee holds for the compensator,

$$u = Hy \tag{3}$$

which has the eigenvalues of $(A_p-B_pK-GC_p)$. This fact can be fatal for LSS reduced-order control, unless measures are taken to ensure system robustness.

ROBUSTNESS MEASURES FOR LSS

For multivariable feedback systems the emerging singular value robustness theory can be used to develop measures for stability and performance. Kosut, et al,[8] applied this theory to the large space structure control design problem, treating the residual dynamics as a perturbation. For a system with a

stable nominal feedback system (based on the reduced model) and stable pertur-
bations (due to the residual dynamics), sufficient conditions for stability
are obtained when the singular value stability measures exceed the maximum
perturbation due to model uncertainty. Fig. 1 defines the terminology for a
large space structure control system. For an additive perturbation, Fig. 2a,
the sufficient conditions for stability are

$$SM_1 = \underline{\sigma} [I + H(j\omega) \; G_C \; (y\omega)] > \overline{\sigma} [H(j\omega) \; G_R \; (j\omega)]$$

$$(4)$$

$$SM_2 = \underline{\sigma} [I + G_C \; (j\omega) \; H(j\omega)] > \overline{\sigma} [G_R \; (j\omega) \; H(j\omega)]$$

where $\overline{\sigma}$ $(\cdot)$ indicates the maximum singular value and $\underline{\sigma}$ $(\cdot)$ indicates the mini-
mum singular value. (Singular values of the complex matrix A are the positive
square roots of the eigenvalues of A^*A, where $(\cdot)^*$ indicates conjugate
tran-spose.) If $G_C(s)$ is minimum phase and invertible, a multiplicative per-
turbation can be formed, Fig. 2b, and the sufficient conditions for stability
are then

$$SM_1 = \underline{\sigma} [I + (HG_C)^{-1}] > \overline{\sigma} [G_C^{-1}G_R]$$

$$(5)$$

$$SM_2 = \underline{\sigma} [I + (G_CH)^{-1}] > \overline{\sigma} [G_RG_C^{-1}]$$

where the $j\omega$ arguments have been supressed. Good performance within the oper-
ating frequency region (i.e., the "control bandwidth") is provided when the
performance measure

$$PM = \underline{\sigma} \ [I + G_cH] \tag{5}$$

is large. The stability measures (4) are generalizations of Nyquist polar
plot analysis; the measures (5) are generalizations of Nyquist inverse polar
plot analysis. The need for large performance measure (6) is a generalization
of the desirability of large loop gains.

ROBUST COMPENSATION DESIGN

The stability and performance measures presented above require stability of
the nominal feedback system. In a previous work [4], the authors presented a
recursive design procedure which assigns the closed-loop eigenstructure in
linear quadratic regulators. At each stage, the required solution for the
steady state Riccati matrix which shifts a pole or pole pair to specified
values is obtained. For pole pair placement, a free parameter in the solution
permits selection of closed-loop eigenvectors. This design procedure is sum-
marized in Appendix 1.

Using duality, the procedure also applies to estimator design. By extension,
the procedure can be used to design stable compensators by considering the
closed-loop regulator dynamics matrix (A-BK) as the open-loop system and pick-
ing the estimator gain to place the compensator eigenstructure of (A-BK-GC).

Compensator robustness can be enhanced through the use of frequency-shaped control and estimation [5,6]. In frequency-shaped estimation, a frequency-domain performance index is considered,

$$J = E \frac{1}{2\pi} \int_{-\infty}^{\infty} [w'Q(j\omega)w + v' R(j\omega)v] d\omega \qquad (7)$$

where w is the disturbance and v is the sensor noise. Sensor noise frequency-shaping is realized by treating v as an auto correlated noise source of the form

$$v(j\omega) = R^{1/2}(j\omega) v'(j\omega) \qquad (8)$$

where v'(jω) is a white noise process. In the approach used here, Q(jω) is determined by pole placement, equivalent to injecting fictitious process noise. $R^{1/2}(j\omega)$ must be proper (not strictly proper) to maintain sensor noise weighting over the entire spectrum. Then define a pseudo-measurement

$$z' = R^{-1/2} (j\omega) z = R^{-1/2}(j\omega) Cx(j\omega) + v' (j\omega) \qquad (9)$$

$R^{-1/2} (j\omega)$ can be realized in state space as

$$\dot{x}_v = A_v x_v + B_v Cx$$
$$y = C_v x_v + D_v Cx \qquad (10)$$
$$z' = C_v x_v + D_v Cx + v'$$

68

This dynamic model is appended to the system dynamics to form the frequency-shaped estimator,

$$\dot{\hat{x}} = A\hat{x} + G(z' - C_v\hat{X}_v - D_v C\hat{x}) + B_c u$$

$$\dot{\hat{x}}_v = A_v\hat{x}_v + B_v C\hat{x} + G_v (z' - C_v\hat{X}_v - D_v C\hat{x})$$ (11)

where z' is obtained from (10). The gains G and G_v can be picked to place the eigenvalues of (11) at those of the frequency-shaping filter (10) and the others as required for performance. A dual result can be used to develop frequency-shaped gains for the regulator.

Because frequency-shaping adds states to the compensator, an efficient choice of the loops to be shaped is desirable. Kim [7] has developed a procedure for loop selection based on the singular vectors of the return ratio matrices $G_c H$ or HG_c. He conjectured that an input vector y in the direction of q_1, the singular vector corresponding to $\overline{\sigma}(A)$ will get the largest amplification by A. Similarly, a vector in the direction of q_n, the singular vector corresponding to $\underline{\sigma}(A)$ will get smallest amplification. Therefore, if the component of y in the direction which is closest to q_1, is reduced by a filter before it enters A, $\overline{\sigma}(A)$ is effectively reduced. $\underline{\sigma}(A)$ increased by increasing the component of y closet to q_n before it enters A. It can be shown that frequency-shaping introduces transmission zeros into the compensator transfer function.

DESIGN METHODOLOGY

The discussion which has been presented above suggests the following design methodology:

1. Compensator design for performance of the reduced order system.

2. Evaluation of the stability margins (4,5) against the perturbation due to the residual dynamics.

3. Selection of frequency-shaping filters to enhance stability robustness.

4. Synthesis of frequency-shaped compensator to incorporate performance and stability margins.

The recursive eigenstructure design algorithm can be used for the designs.

EXAMPLE.

The design methodology was applied to a control design for the ACOSS-1 model, also used in the comparison study [9]. The model is illustrated and the state-space data are listed in Appendix 2. As in the comparison study the first eight structural modes were retained. A regulator was designed with closed-loop poles at 20% damping; a compensator was designed with poles at critical damping. Fig. 3 illustrates stability measure (5) for the loop broken at the output. Performance is adequate at low frequencies but stability robustness is inadequate above 1 Hz.

To improve stability robustness, frequency-shapeo estimation was incorporated in all three output loops using second-order low-pass filters. Fig. 4 illustrates the recovery of stability robustness while still retaining good low frequency performance, Fig. 5.

DISCUSSION

In the comparative study by Kosut, et al [8], both LQG modal control and a frequency-shaped control were investigated (along with others). LQG control was found to have poor performance as well as poor stability robustness. Frequency-shaped control was found to have adequate stability robustness, but poor low frequency performance.

The methodology presented here addresses both of these issues. Performance is achieved by pole placement design of the compensator, achieving good loop gains at low frequency. Stability robustness is achieved by adding frequency-shaping without sacrificing low frequency performance, since the gain of the frequency-shaping filters is one at low frequencies.

CONCLUSIONS

A design meth dology for control systems for large space structures has been proposed which incorporates both performance and stability robustness concerns

as an integral part of the design process. Performance was achieved by plac-
ing the poles of the compensator. Stability robustness was achieved by fre-
quency-shaping the compensator to satisfy a frequency domain stability
robustness test.

An example was presented which applied the methodolgy to a system with the
loop broken at the output. A full design study would also require examination
of the system with the loop broken at the input, using regulator fre-
quency-shaping to enhance robustness.

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APPENDIX 1

Recursive Eigenstructure Design

The steady-state optimal control law for the linear, time-invariant, controllable system:

$$\dot{x} = Ax + Bu \tag{A.1}$$

which minimizes the quadratic performance index,

$$J = 1/2 \int_o^\infty [x^T Qx + u^T Ru] \, dt \tag{A.2}$$

is linear state feedback

$$u = Kx = - R^{-1} B^T Sx \tag{A.3}$$

where S is the solution of the steady-state Riccati equation,

$$-SA - A^T S + SBR^{-1} B^T S - Q = 0 \tag{A.4}$$

In this appendix we summarize an interactive design technique which solves (A.4) to provide specified eigenvalues of the closed-loop system dynamics matrix A+BK and which also permits some freedom in selecting closed-loop

eigenvectors. The method is reported elsewhere [4] in detail. It extends the procedure of Solheim [10] in which, for fixed R, the elements of Q providing the required pole placement are calculated directly.

The design technique is recursive; at each stage, the system dynamics matrix A in (A.1) incorporates previous state feedback. We then implement the following eigenstructure calculation:

$$X^{-1}[\Lambda - H\bar{S}]X = \bar{\Lambda} \tag{A.5}$$

where $\Lambda = T^{-1} AT$ is block diagonal, T is the real eigenvector matrix of A, and $H = T^{-1}BR^{-1}B^TT^{-T}$ is symmetric and positive semi-definite. $\bar{\Lambda}$ is identical to Λ except for a block of shifted poles. X is the transformation from open-loop eigenvectors to closed-loop eigenvectors; it is defined as the "stage" eigenvector matrix. $\bar{S}$ is the Riccati matrix in the open-loop diagonalized coordinate system; $\bar{S}$ is chosen to shift a single pole or a pair of poles. The corresponding gain matrices, K, determined for each stage are subsequently added to obtain a final gain which achieves the same closed-loop pole locations.

To provide the required pole shift, the only non-zero elements of S correspond to the entries of Λ which are to be shifted. With this choice of S, the characteristic equation factors into the product of terms for the unshifted poles and a term for the desired shifted poles. Thus,

$$|sI - \Lambda| = D(s) \prod_{i \in I} (s - \lambda_i) \tag{A.6}$$

75

where I is the index set for the unshifted poles, and D(s) contains explicit elements of S, H, and the block of Λ which is to be shifted. Matching the coefficients of powers of s in D(s) to the equivalent terms in the closed-loop characteristic equation provides a set of equations in the required elements of $\bar{S}$. For the single pole shift $\Lambda_{jj} = \lambda$ to κ, the only non-zero element of $\bar{S}$ satisfies

$$\bar{S}_{jj} = \frac{\lambda - \kappa}{H_{jj}} \; .$$

For double pole placement it can be shown that the three required elements of $\bar{S}$ lie on the intersection of two quadric surface in a mathematical space having the three $\bar{S}$ elements as coordinates. (It can also shown that a direct solution for Q has a similar geometric interpretation.) If the corresponding submatrix of H is positive definite, the surfaces are a plane and a hyperboloid of one or two sheets; the intersection, if it exists, is always an ellipse. If the relevant submatrix of H is singular, the surfaces are planes, and the intersection is a line. The different points comprising the solution all provide the desired eigenvalue placement, but with different eigenvectors.

In ref. 4 a solution for $\bar{S}$ is presented which takes advantage of the quadric surface geometry to define a free parameter that allows design freedom in the choice of closed-loop eigenvectors. The solution for the stage eigenvector X partitions into two sets of equations. The first is a homogeneous Lyapunov equation for the submatrix corresponding to the shifted pole block in Λ. For a role pair shift, the submatrix is 2x2. Hence, depending upon the nature of

76

the closed-loop poles (real or complex), one or two elements of the submatrix may be chosen arbitrarily; the remaining elements then depend on the choice of elements of $\bar{S}$. The other equation is a non-homogeneous Lyapunov equation in the remaining elements of the columns of X containing the 2x2 submatrix; its solution depends upon the 2x2 submatrix, the elements of $\bar{S}$ and $\bar{\Lambda}$, and certain elements of H.

The closed-loop system eigenvector matrix is then $T_{CL} = TX$. The solution of X depends upon $\bar{S}$, which varies with the choice of the free parameter. Therefore, by recursively shifting pole pairs, design freedom exists to select closed-loop eigenvectors while providing required pole placements.

The procedure outlined above lends itself to a recursive procedure for practical multivariable regulator design. The steps in the procedure are as follows:

1. System (A.1) is placed in modal form.

2. The designer selects the control weighting matrix R, then H is calculated.

3. The designer selects a real pole or pair of poles to be shifted and their desired location; a pair, he also selects the free parameter which determines the closed-loop eigenvectors.

4. The stage gain is calculated and the closed-loop system is placed in modal form.

77

5. Steps 3 and 4 are repeated for other poles until the designer is satisfied.

6. The total system gain is obtained by adding the stage gains.

Clearly by duality, the same process can be applied to estimator design, permitting the development of multivariable compensators.

The ACOSS-1 flexible spacecraft model was developed by the Charles Stark
Draper Laboratory.[10] It is representative of many radar and optical control
problems, but is small enough to be tenable for research studies. The struc-
ture is a tetrahedral truss supported by three right-angle bipods. The truss
members are flexible in the axial direction only. The model has 12 modes; for
control design, only eight are assumed to be known.

LOS

CSOL SPACE STRUCTURE I
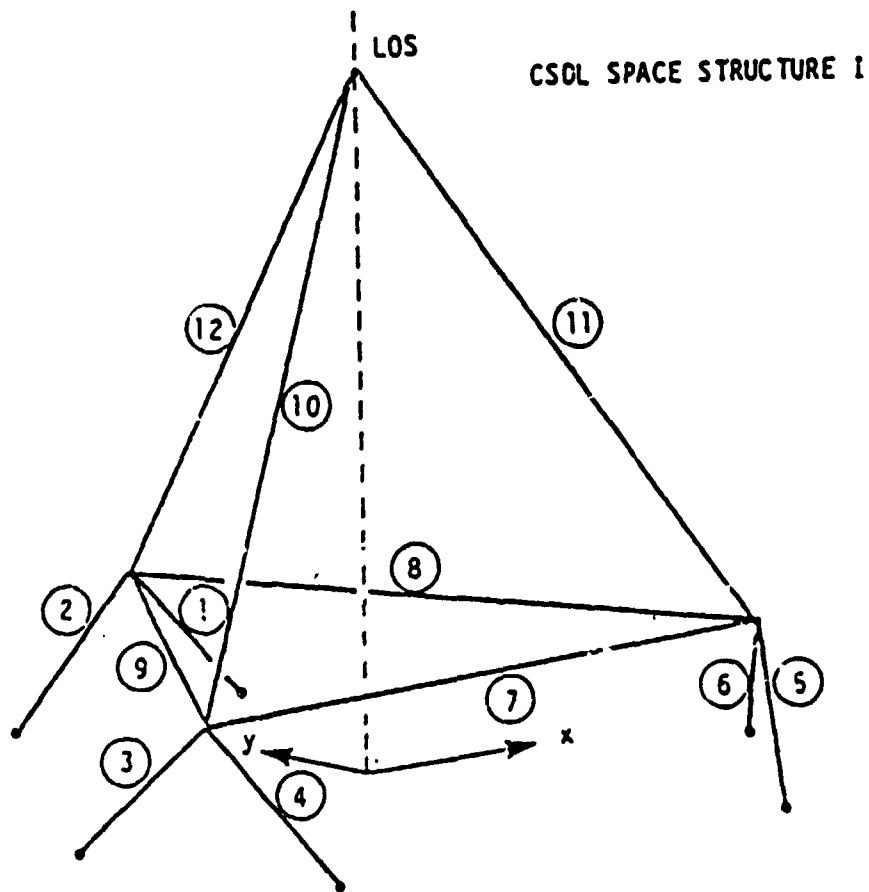
Residual Dynamics
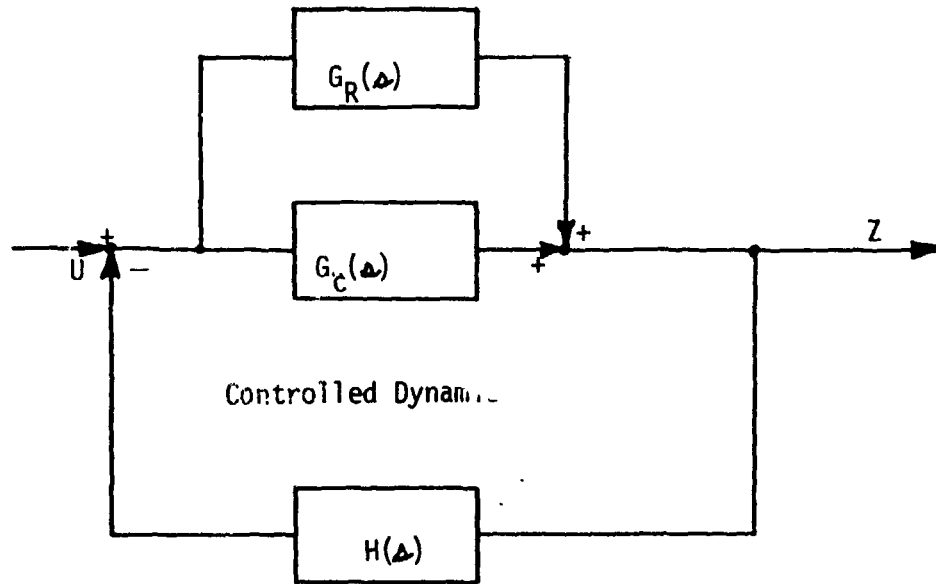
Compensator

Fig. 1 - LSS Control System
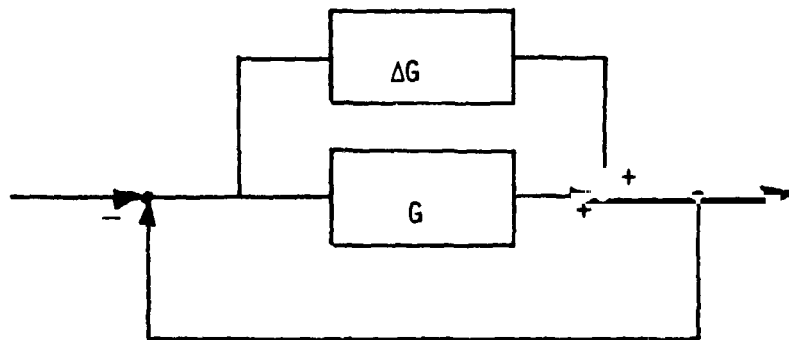
Fig. 2a - Additive Perturbation
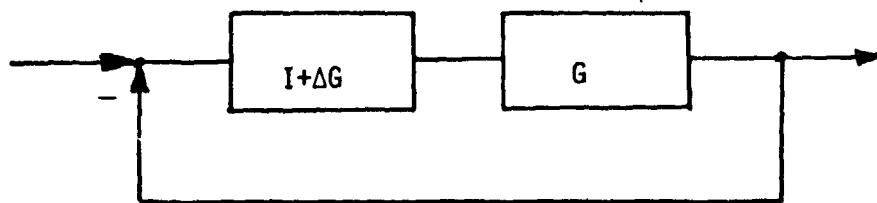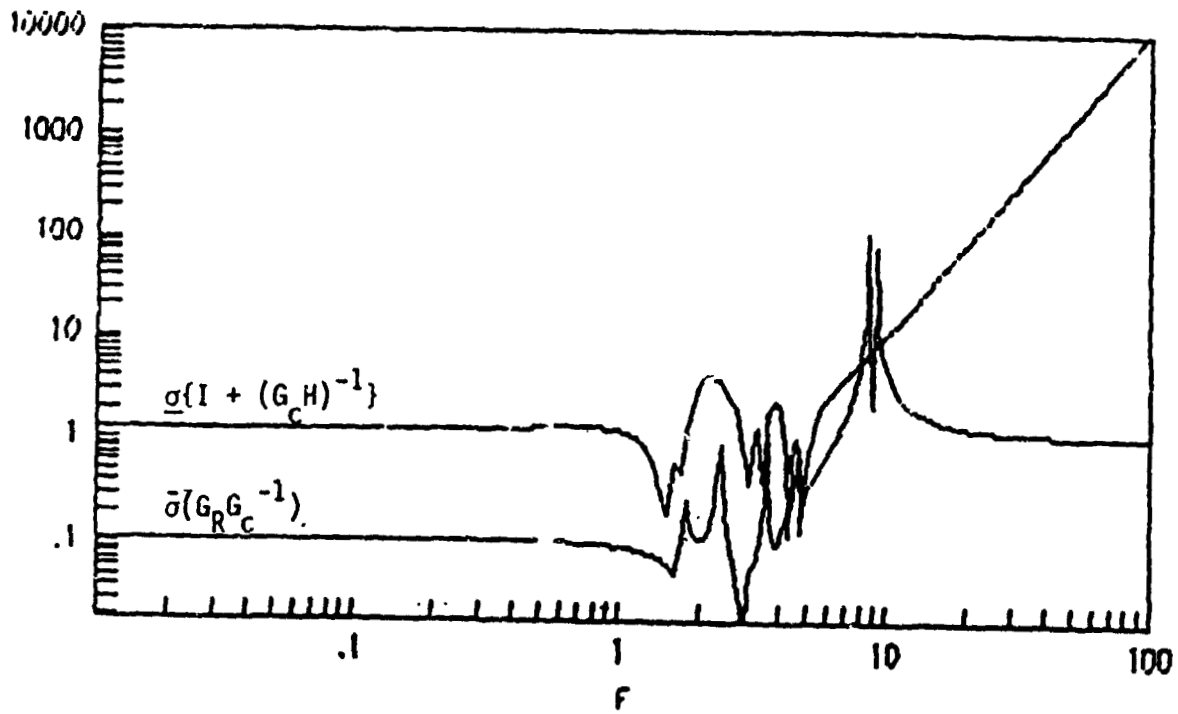

Fig. 2b - Multiplicative Perturbation
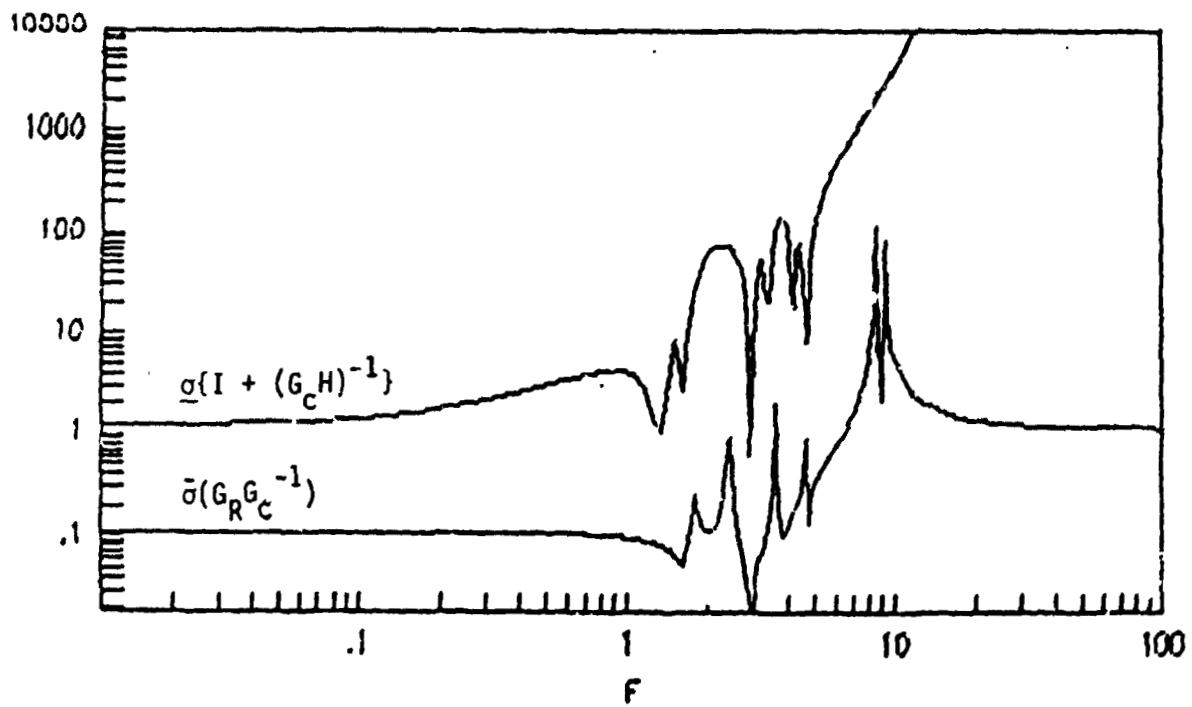
Fig. 3 Reduced - Order Control Stability Margin
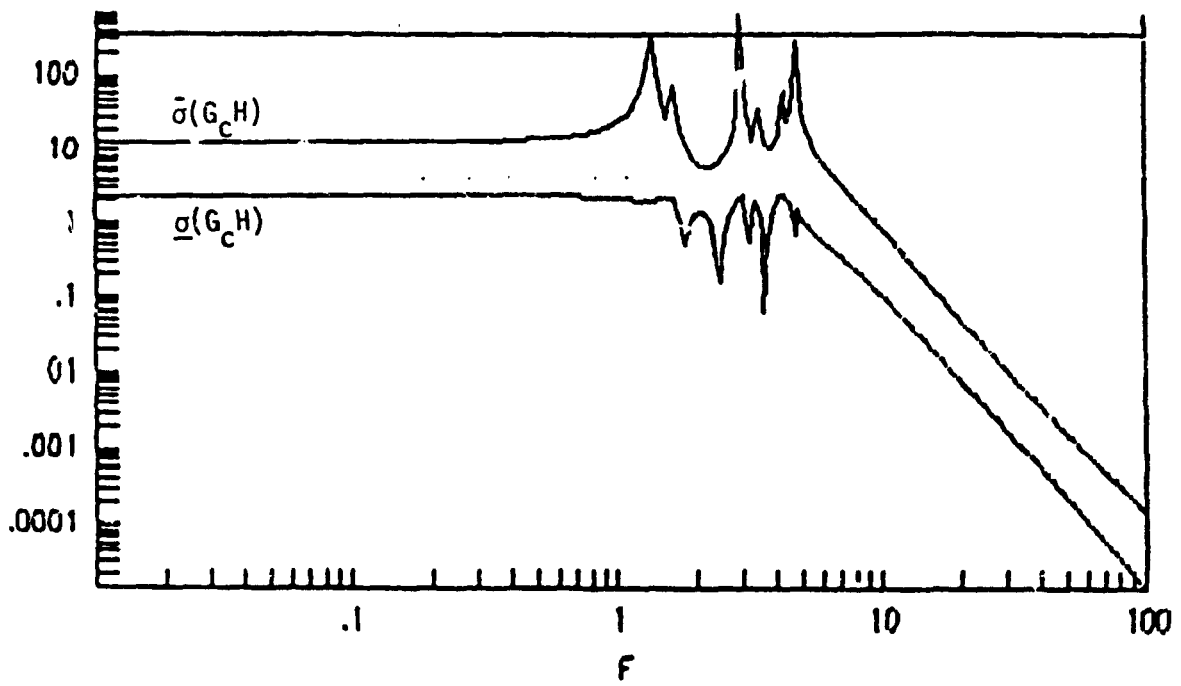

Fig. 4 Frequency - Shaped Control Stability Margin

Fig. 5 Loop Gains