N85-31201

# ON THE STABILITY OF COLLOCATED CONTROLLERS IN THE PRESENCE OF UNCERTAIN NONLINEARITIES AND OTHER PERILS

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## ABSTRACT

Robustness properties are investigated for two types of controllers for large flexible space structures, which use collocated sensors and actuators. The first type is an attitude controller which uses negative definite feedback of measured attitude and rate, while the second type is a damping enhancement controller which uses only velocity (rate) feedback. It is proved that collocated attitude controllers preserve closed-loop global asymptotic stability when linear actuator/sensor dynamics satisfying certain phase conditions are present, or monotonic increasing nonlinearities are present. For velocity feedback controllers, the global asymptotic stability is proved under much weaker conditions. In particular, they have 90° phase margin and can tolerate nonlinearities belonging to the $[0,\infty)$ sector in the actuator/sensor characteristics. The results significantly enhance the viability of both types of collocated controllers, especially when the available information about the large space structure (LSS) parameters is inadequate or inaccurate.

## INTRODUCTION

Large flexible space structures are infinite-dimensional systems with very small inherent energy dissipation (damping). Because of practical limitations, only finite-dimensional controllers and point actuators and sensors must be used for controlling large space structures (LSS). In addition, considerable uncertainty exists in the knowledge of the parameters. For these reasons, the design of a stable controller for a large space structure (LSS) is a challenging problem.

A class of controllers, termed "collocated controllers" [1], represents an attractive controller because of its guaranteed stability properties in the presence of plant uncertainties. Collocated attitude (CA) controllers are designed to control the rigid-body attitude as well as the structural modes, while collocated direct velocity feedback (CDVFB) controllers are designed only for enhancement of structural damping. Both types of collocated controllers guarantee stability regardless of the number of modes in the LSS model and uncertainties in the knowledge of the parameters [1], [2]. A CA controller basically consists of compatible sensor/actuator pairs placed at the same

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locations, and utilizes negative definite feedback of position and velocity (e.g., LSS attitude and attitude rate). A CDVFB controller [3] is a special case of the CA controller where only rate feedback is used for damping enhancement without affecting the rigid-body modes. It has been proved in references [1], [2], [3] that, the closed-loop system is always stable in the sense of Lyapunov, and is also asymptotically stable (AS) under certain additional conditions.

Although collocated controllers have attractive stability properties with perfect (i.e., linear, instantaneous) sensors and actuators, the sensors and actuators available in practice tend to have nonlinearities and phase lags associated with them. In order to be useful in practical applications, the controller should be tolerant to nonlinearities (e.g., saturation, relays, deadzones, etc.), and to phase shifts (e.g., actuator dynamics and/or computational delays). Uncertainties usually exist in the knowledge of the nonlinearities and the phase lags. For these reasons, this paper investigates the closed-loop stability of collocated controllers in the presence of unmodeled sensor/actuator dynamics and nonlinearities. The situation is mathematically described by including an operator $\mathcal{H}$ in the feedback path. The actual input $u(t)$ is given by:

$$u(t) = \mathcal{H} u_c(t) \qquad (1)$$

where $u_c$ is the ideal (desired) input, $\mathcal{H}$ is a nonanticipative, linear or nonlinear, time-varying or invariant operator. For CA controllers, it is proved that the closed-loop system is globally asymptotically stable if

1) $\mathcal{H}$ is linear, time-invariant (LTI) and stable with a rational transfer matrix H(s) which satisfies certain frequency-domain conditions, or

2) If $\mathcal{H}$ consists of time-invariant, strictly monotonic increasing nonlinearities belonging to the $[0, \infty)$ sector. (A function $\phi(\sigma)$ is said to belong to the $[k, h)$ sector if $\phi(0) = 0$ and $k\sigma^2 \leq \sigma\phi(\sigma) < h\sigma^2$ for all $\sigma \neq 0$).

For CDVFB controllers, it is proved that global asymptotic stability is preserved when

1) $\mathcal{H}$ is a stable nonlinear dynamic operator and satisfies certain passivity conditions, or

2) $\mathcal{H}$ is a stable LTI operator with phase within $\pm 90^\circ$

3) $\mathcal{H}$ consists of nonlinear gains belonging to the $[0, \infty)$ sector.

These analytical results significantly enhance the stability and robustness properties of collocated controllers, and therefore increase their practical applicability.

## PROBLEM FORMULATION

The linearized equations of motion of a large flexible space structure (using torque actuators) are given by:

$$A\ddot{x} + B\dot{x} + Cx = \sum_{i=1}^{m} \Gamma_i^T u_i \tag{2}$$

where

$$x = (\phi_s, \theta_s, \psi_s, q_1, q_2, \ldots, q_{nq})^T \tag{3}$$

$$A = \text{diag}\,(I_s, I_{nq \times nq}) \tag{4}$$

$$B = \text{diag}\,(0_{3 \times 3}, D) \tag{5}$$

$$C = \text{diag}\,(0_{3 \times 3}, \Lambda) \tag{6}$$

$$\Gamma_i = [I_{3 \times 3}, \Phi_i] \tag{7}$$

$$u_i = (u_{xi}, u_{yi}, u_{zi})^T \tag{8}$$

where $\phi_s$, $\theta_s$, $\psi_s$ denote the three rigid-body Euler angles, $n_q$ is the number of structural modes, $q_i$ denotes the modal amplitude of ith structural mode ($i = 1, 2, \ldots, n_q$), $I_s$ denotes the 3 x 3 moment of inertia matrix, $\Psi_i$ is the 3 x $n_q$ mode-slope matrix at the ith (3-axis) actuator location. It is assumed that m, 3-axis torque actuators are used. $I_{\ell \times \ell}$ denotes the $\ell$ x $\ell$ identity matrix, and diag( ) denotes a block-diagonal matrix. D is a symmetric positive definite or semidefinite matrix which represents the inherent structural damping. Since some damping, no matter how small, is always present, we assume $D > 0$ throughout this paper. $\Lambda$ is an $n_q$ x $n_q$ diagonal matrix of squared structural frequencies

$$\Lambda = \text{diag}\,(\omega_1^2, \omega_2^2, \ldots, \omega_{nq}^2) \tag{9}$$

Assuming that m, 3-axis attitude and rate sensors (e.g., star trackers and rate gyros) are placed at the locations of the actuators, the measured 3-axis attitude $y_{ai}$ and rate $y_{ri}$ at actuator location i (ignoring noise) are given by:

$$y_{ai} = \Gamma_i x \tag{10}$$

$$y_{ri} = \Gamma_i \dot{x} \tag{11}$$

denoting

$$u = [u_1^T, u_2^T, \ldots, u_m^T]^T \tag{12}$$

$$\Gamma^T = [\Gamma_1^T, \Gamma_2^T, \ldots, \Gamma_m^T] \tag{13}$$

$$y_a = [y_{a1}^T, y_{a2}^T, \ldots, y_{am}^T]^T \tag{14}$$

$$y_r = [y_{r1}^T, y_{r2}^T, \ldots, y_{rm}^T]^T \tag{15}$$

where u, $y_a$, $y_r$ are 3m x 1 vectors, and $\Gamma$ is a 3m x ($n_q$ + 3) matrix. The control law for the collocated attitude controller is given by:

$$u_c = u_{cp} + u_{cr} \tag{16}$$

$$u_{cp} = -G_p y_a \tag{17}$$

$$u_{cr} = -G_r y_r \tag{18}$$

where $u_c$ represents the command input, $u_{cp}$ and $u_{cr}$ represent command attitude and rate inputs, and $G_p$, $G_r$ are 3m x 3m feedback gain matrices.

For CDVFB controllers, the rigid-body rates are removed from the feedback signal by subtracting attitude rates at two locations. Consequently, the model used for damping enhancement has the form:

$$\ddot{q} + D\dot{q} + \Lambda q = \tilde{\Phi}^T u \tag{19}$$

where $\tilde{\Phi}$ consists of appropriate differences between the mode-slopes. The control law is given by:

$$u_c = -G_r \tilde{y}_r \tag{20}$$

where

$$\tilde{y}_r = \tilde{\Phi}\dot{q} \tag{21}$$

The control laws given above for CA and CDVFB controllers have very attractive robustness properties. It was shown in [1], [2] that, if D > 0, $G_p = G_p^T > 0$, and $G_r = G_r^T > 0$, then the closed-system is asymptotically stable (AS). The stability result holds regardless of the number of modes in the model, and regardless of inaccuracy in the knowledge of the parameters. In real life, however, nonlinearities and phase lags exist in the sensors and actuators, which invalidate these robust stability properties. The real problem then is to investigate the closed-loop stability for the case where the actual input is given by Eq. (1), where $\mathcal{H}$ is a nonanticipative, linear or nonlinear, time-varying or invariant operator. The situation is shown in Figure 1. Our approach is to make use of input-output stability concepts and Lyapunov methods. We assume throughout the paper that the problem is well-posed, and that a unique solution exists. We start by defining the terminology and the concepts, which are adopted from [4].

## MATHEMATICAL PRELIMINARIES

Consider the linear vector space $L_n^2$ of real square-integrable n-vector functions of time t, defined as:

$$L_n^2 = \{g: R_+ \to R^n \mid \int_0^\infty g^T(t)g(t)dt < \infty\} \tag{22}$$

where $R^n$ is the linear space of ordered n-tuples of real numbers, and $R_+$ denotes the interval $0 \leq t < \infty$. The scalar product is defined as

$$\langle g_1, g_2 \rangle = \int_0^\infty g_1^T(t)g_2(t)dt \tag{23}$$

For $g \epsilon L_n^2$, its norm is defined as

$$\|g\| = \langle g,g \rangle^{1/2} \tag{24}$$

Define the truncation operator $P_T$ such that

$$g_T(t) \triangleq P_T g(t) = \begin{cases} g(t) & 0 \leq t \leq T \\ 0 & t > T \end{cases} \tag{25}$$

Define the extended space $L_{ne}^2$:

$$L_{ne}^2 = \{g: [R_+ \to R^n \mid g_T \; \epsilon \; L_n^2 \; \forall T \geq 0\} \tag{26}$$

Thus $L_{ne}^2$ is a linear vector space of functions of t whose truncations are square-integrable on [0,T) for all $T < \infty$. For $g_1$, $g_2$ $\epsilon$ $L_{ne}^2$, define the truncated inner product

$$\langle g_1, g_2 \rangle_T = \langle g_{1T}, g_{2T} \rangle = \int_0^T g_1^T(t)g_2(t)dt \tag{27}$$

The truncated norm is defined by: $\|g\|_T = \langle g,g \rangle_T^{1/2}$ .

Consider an operator $\mathcal{H} : L_{ne}^2 \to L_{ne}^2$. $\mathcal{H}$ is said to be strictly passive if there exist finite constants $\beta$ and $\delta > 0$ such that

$$\langle \mathcal{H} g, g \rangle_T \geq \beta + \delta \|g\|_T^2 \qquad \forall \; T \geq 0, \; \forall g \; \epsilon \; L_{ne}^2 \tag{28}$$

$\mathcal{H}$ is passive if $\delta = 0$ in (28).

## Stability With Dynamic Operator in the Loop

We consider the case where the operator $\mathcal{H}$ is linear and time-invariant (LTI), and has a finite-dimensional state-space representation. We denote $\mathcal{H}g$ by $\mathcal{H}(z_0;\ g)$ where $z_0$ is the initial state vector of $\mathcal{H}$, and assume $m = 1$ for simplicity (i.e., one 3-axis actuator).

Theorem 1. Suppose $\mathcal{H}$ is a non-anticipative, strictly stable, completely observable, LTI operator whose transfer matrix is $H(s) = \varepsilon I+\hat{H}(s)$, where $\varepsilon > 0$ and $\hat{H}(s)$ is a proper, minimum-phase, rational matrix. Under these conditions, the closed-loop system given by Eqs. (1), (2), (10), (11), (16)-(18) is asymptotically stable (AS) if

$$\hat{H}(j\omega)\ (\omega G_r - jG_p) + (\omega G_r + jG_p)\ \hat{H}^*(j\omega) \geq 0 \text{ for all real } \omega. \tag{29}$$

where * denotes the conjugate transpose.

Proof - Define the function

$$V(t) = x^T Cx + \dot{x}^T A\ \dot{x} \tag{30}$$

Since $C \geq 0$, $A > 0$, $V(t) \geq 0$ for all $t \geq 0$. Differentiating V with respect to t, and using (1), (10), (11), (16)-(18),

$$\dot{V} = -\ 2\dot{x}^T B\dot{x} - 2u_{cr}^T\ G_r^{-1}\ \mathcal{H}[z_0;\ u_c] \tag{31}$$

where $\mathcal{H}$ also depends on its initial state $z_0$. Since $\mathcal{H}$ is linear,

$$\mathcal{H}[z_0;\ u_c] = h_o(t) + \mathcal{H}[0;\ u_c] \tag{32}$$

where $h_0(t)$ is the unforced response of $\mathcal{H}$ due to nonzero initial state. Since $\mathcal{H}$ is strictly stable, $\|h\|_0$ is finite for any finite $z_0$.

Substituting (32) in (31) and integrating from 0 to T, since $V(T) \geq 0$,

$$0 \leq V(T) = V(0) - 2\langle\dot{x},\ B\dot{x}\rangle_T - 2\ \langle u_{cr},\ G_r^{-1} h_o\rangle_T$$

$$-\ 2\langle u_{cr},\ G_r^{-1}\ \mathcal{H}_p u_{cp}\rangle_T \tag{33}$$

where

$$\mathcal{H}_p u_{cp} = \mathcal{H}[0;\ (G_p + sG_r)\ u_{cp}] \tag{34}$$

In (34), "s" denotes the derivative operator. ("s" is technically noncausal; however, this difficulty can be overcome by defining the derivative of a truncation at T to be equal to that of the untruncated function.) Using Parseval's theorem,

$$\langle u_{cr}, G_r^{-1} \mathcal{H}_p u_{cp} \rangle_T = \frac{1}{2\pi} \int_{-\infty}^{\infty} U^*_{cr_T}(j\omega)\, G_r^{-1} H(j\omega)\, [G_p + j\omega G_r]\, U_{cp_T}(j\omega) d\omega$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} U^*_{cr_T}(j\omega)\, G_r^{-1} H(j\omega)\, [\frac{G_p}{j\omega} + G_r]\, U_{cr_T}(j\omega) d\omega$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} U^*_{cr_T}(j\omega)\, [G_r^{-1} H(j\omega) (\frac{G_p}{j\omega} + G_r)$$

$$+ (\frac{G_p}{-j\omega} + G_r) H^*(j\omega)\, G_r^{-1}]\, U_{cr_T}(j\omega) d\omega$$

The matrix in the brackets is positive (from Eq. 29), and we have

$$\langle u_{cr}, G_r^{-1} \mathcal{H}_p u_{cp} \rangle_T \geq \epsilon \, \| u_{cr} \|_T^2 \tag{35}$$

which yields (from (33)

$$0 \leq V(o) - 2 \langle \dot{q}, D\dot{q} \rangle_T - 2\, \epsilon \, \| u_{cr} \|_T^2 - 2 \langle u_{cr}, G_r^{-1} h_o \rangle_T \tag{36}$$

wherein we have used the fact that $\dot{x}^T B\dot{x} = \dot{q}\, D\dot{q}$. Therefore,

$$\lambda_m(D) \, \|\dot{q}\|_T^2 + \epsilon \| u_{cr} \|_T^2 \leq V(o)/2 + \| u_{cr} \|_T \| G_r^{-1} \|_s \| h_o \| \tag{37}$$

where $\| \ \|_s$ denotes the spectral norm of a matrix, and $\lambda_m$ denotes the smallest eigenvalue. Eq. (37) can be written as

$$\lambda_m(D) \, \|\dot{q}\|_T^2 + (c_1 \| u_{cr} \|_T - \frac{c_2}{2c_1})^2 \leq V(o)/2 + c_2^2/4c_1 \tag{38}$$

where $c_1 = \sqrt{\epsilon}$ and $c_2 = \| h_o \|$ Therefore, $\lim_{t\to\infty} \dot{q}\,(t) = 0$, and $\lim_{t\to\infty} u_{cr}(t) = 0$. Denoting the rigid-body attitude $\alpha = (\phi_s, \theta_s, \psi_s)^T$, this implies that $\lim_{t\to\infty} \alpha\,(t) = 0$. Taking the limit of the closed-loop equation as $t\to\infty$,

$$\begin{bmatrix} 0 \\ \Lambda\bar{q} \end{bmatrix} = \begin{bmatrix} I \\ \phi^T \end{bmatrix} \quad \overline{\mathcal{H}}\, u_{cp} \tag{39}$$

where the overhead bar denotes the limit as $t \to \infty$. From (39), $\overline{\mathcal{H} u_{cp}} = 0$ and $\bar{q} = 0$, which yields $\bar{\alpha} = 0$. Since $\mathcal{H}$ is observable and its output tends to zero, its state vector tends to zero as $t \to \infty$, and the system is asymptotically stable.

The following corollary essentially states that, for diagonal $G_p$, $G_r$, and H, it is sufficient that the phase lag of $\hat{H}(j\omega)$ is less than the phase lead introduced by the controller.

Corollary 1.1. Suppose $G_p$, $G_r$ and H are diagonal and satisfy the assumptions of Theorem 1. Then the closed-loop system is globally asymptotically stable if

$$-\tan^{-1} \frac{\omega G_{ri}}{G_{pi}} \leq \operatorname{Arg} \{\hat{H}_i(j\omega)\} \leq 180° - \tan^{-1} \frac{\omega G_{ri}}{G_{pi}} \quad \text{for all real } \omega \tag{40}$$

where Arg( ) denotes the phase angle of a complex variable.

For the case where $H_{ii}(s) = k_i/(s + a_i)$, with $k_i, a_i > 0$, condition (40) becomes

$$\frac{G_{ri}}{G_{pi}} \geq 1/a_i \tag{41}$$

Thus, for the case of first-order sensor/actuator dynamics, the system is asymptotically stable if the ratio of rate-to-proportional gain is at least equal to the magnitude of the actuator pole.

In Theorem 1 and Corollary 1.1, the transfer function of $\mathcal{H}$ was assumed to be of the form: $H(s) = \varepsilon I + \hat{H}(s)$, where $\varepsilon > 0$. That is, a direct transmission term, no matter how small, was present. From Theorem 1, the closed-loop system is AS for any $\varepsilon > 0$. Therefore, the closed-loop eigenvalues are all in the open left half-plane (OLHP). Because of continuity, it is obvious that, when $\varepsilon = 0$, the eigenvalues will not cross the imaginary axis. That is, the eigenvalues will be in the closed left half-plane (CLHP). Theorem 2 given below considers the case when $\varepsilon = 0$. It essentially shows that, if the closed-loop system with no elastic modes is AS with $\mathcal{H}$ in the loop, then so is the system with elastic modes, provided that (29) is satisfied with H replacing $\hat{H}$.

Theorem 2. Suppose $\mathcal{H}$ is a non-anticipative, strictly stable, completely observable, LTI operator with rational transfer matrix H(s) which is proper and minimum-phase. If the closed-loop system for the rigid body model alone (i.e., Eqs. (1), (2), (10), (11), (16)-(18) with $n_q = 0$) is AS, then the entire closed loop system (i.e., with $n_q \neq 0$) is AS provided that

$$H(j\omega)(\omega G_r - jG_p) + (\omega G_r + jG_p) H^*(j\omega) \geq 0 \text{ for all real } \omega \tag{42}$$

Proof. Considering the rigid-body equations,

$$I_s \ddot{\alpha} = \mathcal{H} u_c = \mathcal{H}(u_\alpha + u_q) \tag{43}$$

where $u_\alpha = -G_p\alpha - G_r\dot{\alpha}$ and $u_q = -G_p\Phi q - G_r\Phi\dot{q}$. Thus the transfer function from $\dot{q}$ to $\dot{\alpha}$ is given by

$$M(s) = [I + H(s)\{G_p + G_r s\}]^{-1} H(s)\{G_p + G_r s\}\Phi$$

Since the closed-loop rigid-body system is strictly stable by assumption, M(s) is strictly stable and finite-gain, which implies

$$\|\dot{\alpha}\|_T \leq \gamma\|\dot{q}\|_T + \|h_m\|_T \tag{44}$$

where $\gamma$ is the gain of M and $h_m$ is its free response. Proceeding as in the proof of Theorem 1, we can arrive at Eq. (37) wherein $\varepsilon = 0$ and $n_0$ is replaced by $h_m$. Since $u_{cr} = -G_r(\dot{\alpha} + \Phi\dot{q})$, we have from (44),

$$\|u_{cr}\|_T \leq c_1\|\dot{q}\|_T + c_2\|h_m\|_T \tag{45}$$

where $c_1$ and $c_2$ are positive constants. Completing squares as in (38) and noting that $\|h_m\|$ is finite, it can be proved that $\|\dot{q}\|_T$ is bounded for all $T > 0$, and that $\lim_{t\to\infty}\dot{q}(t)=0$. From (45), $u_{cr}$ also tends to zero as $t\to\infty$. The remainder of the proof is similar to that of Theorem 1.

Corollary 2.2 With the same assumptions as in Theorem 2, if $G_p$, $G_r$, and H are diagonal, then the closed-loop system is AS if (40) is satisfied with H replacing $\hat{H}$.

From Corollary 2.2, for the case where $H_{ii}(s) = k_i/(s + a_i)$ with $k_i$, $a_i > 0$, the closed-loop asymptotic stability is assured if $G_{pi} \leq a_i G_{ri}$ for $i = 1, 2, \ldots, m$.

The significance of the results of this section is that the stability can be assured by making the ratio of the rate-to-proportional gains sufficiently large. One has to know only the sensor/actuator characteristics, and the knowledge of the plant parameters is not required. This result is completely consistent with the result obtained in [5] for single-input, single-output systems, for small $G_p$ and $G_r$, using a root-locus argument.

The next section considers the case where nonlinearities are present in the loop.

## Stability in the Presence of Nonlinearities

Suppose Eq. (1) is replaced by

$$u = \psi(u_c) \tag{46}$$

where $\psi$ is an m-vector, one-to-one, time-invariant function, $\psi$: $R^m \to R^m$, as follows:

$$\psi(\sigma) = [\psi_1(\sigma_1), \psi_2(\sigma_2), \ldots, \psi_m(\sigma_m)] \tag{47}$$

For this case, the stability of the closed-loop system can be investigated using Lyapunov methods. A function $\phi(\nu)$: $R^1 \to R^1$ is said to belong to the $(0, \infty)$ sector if $\phi(0) = 0$ and $\nu\phi(\nu) > 0$ for $\nu \neq 0$. $\phi$ is said to belong to the $[0, \infty)$ sector if $\phi(0) = 0$ and $\nu\phi(\nu) \geq 0$ for $\nu \neq 0$. [Fig. 2] Many nonlinearities encountered in practice, such as saturation, relay, dead-zones, belong to the $[0, \infty)$ sector. As in the previous section, we assume that the problem is well-posed, and that a unique solution exists, and we consider the case with one 3-axis actuator for simplicity.

Theorem 3. Consider the closed-loop system given by Eqs. (2), (10), (11), (16)-(18), and (46), where $G_p$ and $G_r$ are positive definite and diagonal, and each $\psi_i$ is in the $(0, \infty)$ sector and is strictly monotonic increasing for $i = 1, 2, \ldots, m$. Then the closed-loop system is globally asymptotically stable.

Proof. Define

$$V(x, \dot{x}) = x^T Cx + \dot{x}^T A\dot{x} + 2\sum_{i=1}^{3} G_{pi}^{-1} \int_o^{u_{cpi}} \psi_i(\nu)d\nu \tag{48}$$

where $G_{pi}$ and $u_{cpi}$ denote the iith and ith elements of $G_p$ and $u_{cp}$, respectively. This form is the well-known "Lure'-type" Lyapunov function [6]. From Eqs. (4) and (6), $x^T Cx + \dot{x}^T A\dot{x} = 0$ only when $\dot{\alpha} = 0$, $q = \dot{q} = 0$. That is, this quantity can be zero when $\alpha \neq 0$. However, when $q = 0$, $u_{cpi} = G_{pi}\alpha$, which is nonzero when $\alpha \neq 0$. Thus the third term on the right hand side of (48) is positive (since $\psi_i$ is in the $(0, \infty)$ sector) for $\alpha \neq 0$. Therefore, V is positive definite. From (48), using (2), (46), (16)-(18),

$$\dot{V} = -2\dot{x}^T B\dot{x} - 2\sum_{i=1}^{3} u_{cri}G_{ri}^{-1} \psi_i(u_{cpi} \cdot u_{cri}) - G_{pi}^{-1}\psi_i(u_{cpi})\dot{u}_{cpi} \tag{49}$$

Since $\dot{u}_{cpi} = G_{pi}G_{ri}^{-1}u_{cri}$, we have from (49):

$$\dot{V} = -2\dot{x}^T B\dot{x} - 2\sum_{i=1}^{3} u_{cri}G_{ri}^{-1} [\psi_i(u_{cpi} + u_{cri}) - \psi_i(u_{cpi})] \tag{50}$$

Since $\psi_i$ is strictly monotonic increasing,

$$\dot{V} \leq -2\dot{q}^T D\dot{q} \tag{51}$$

$\dot{V}=0$ only when $\dot{q}=0$ and $u_{cri} = 0$, which implies $\dot{\alpha} = 0$. Considering the closed-loop equation,

$$\begin{bmatrix} 0 \\ \Lambda q \end{bmatrix} = \begin{bmatrix} I \\ \Phi T \end{bmatrix} \psi(u_{cp}) \tag{52}$$

which yields $\psi_i\ (u_{cpi}) = 0$ and $\dot{q} = 0$. Since $\psi_i(v) = 0$ only at $v = 0$, this implies that $\alpha = 0$. Thus $\dot{V} \equiv 0$ only at the origin, and the system is globally asymptotically stable.

Thus the collocated controller is guaranteed to be globally asymptotically stable in the presence of monotonic increasing nonlinearities. This r ... of the nonlinearities is also called "incremental passivity." As se n t previous section, if the nonlinearities are replaced by dynamic operat s, mere incremental passivity is not sufficient for stability.

## ROBUSTNESS OF VELOCITY FEEDBACK CONTROLLERS

### Stability with Dynamic Operator in the Loop

Consider the case where a nonlinear dynamic operator $\mathcal{H}(z_o;\ v)$ is present in the loop. Suppose $\mathcal{H}$ is represented by the following state-space model:

$$\dot{z} = f(z,\ v,\ t),\ z(0) = z_o \tag{53}$$

$$w(t) = p(z,\ t) \tag{54}$$

where v and w are 3m x 1 vectors which are the input and the output of $\mathcal{H}$. Define the operator

$$\partial\mathcal{H}(z_o;\ g) = \mathcal{H}(z_o;\ g) - \mathcal{H}(z_o;\ 0) \tag{55}$$

We define $\mathcal{H}$ to be internally stable if $\|\mathcal{H}(z_o;\ 0)\|$ is finite for any finite $z_o$.

Theorem 4. Consider the system given by Eqs. (1), (19), (20) (21), where the operator $\mathcal{H}$ has the state-space representation given by (53), (54). Suppose $G_r^{-1}\partial\mathcal{H}$ is passive and $\mathcal{H}$ is uniformly observable, finite-gain, internally stable, continous operator. Then the closed-loop system is globally asymptotically stable.

Proof. Defining

$$V(t) = q^T\Lambda q + \dot{q}^T\dot{q} \tag{56}$$

$V(t) \geq 0$ for all $t \geq 0$. Differentiating V(t) with respect to t and using Eqs. (19), (20), (21) and (1),

$$\dot{V} = -2\dot{q}^T D\dot{q} - 2u_{cr}^T G_r^{-1}\ \mathcal{H}(z_o;\ u_{cr}) \tag{57}$$

Integrating from 0 to T , since $V \geq 0$,

$$0 \leq V(T) = V(0) - 2\langle \dot{q}, D\dot{q}\rangle_T - 2\langle u_{cr}, G_r^{-1} \mathcal{H}(z_o; u_{cr})\rangle_T \qquad (58)$$

which yields (after manipulation)

$$2\,\lambda_m(D)\, \|\dot{q}\|_T^2 \leq V(0) - \beta + 2\|\dot{q}\|_T \|\tilde{\Phi}\|_s \|\mathcal{H}(z_o; 0)\| \qquad (59)$$

where $\beta$ is a constant (see Eq. 28).

By using a procedure similar to that in the proof of Theorem 1, it can be proved that $\|\dot{q}\|$ is bounded, and that the system is globally asymptotically stable.

The following corollary is an immediate consequence of Theorem 3.

Corollary 4.1. If $\mathcal{H}$ is a strictly stable, completely observable, LTI operator with rational, mimimum-phase transfer matrix H(s), the closed-loop system of Eqs. (1), (19), (20), (21) is asymptotically stable provided that

$$H(j\omega)G_r + G_r H^*(j\omega) \geq 0 \text{ for all real } \omega \qquad (60)$$

Note that the above condition is equivalent to passivity of $G_r^{-1}\mathcal{H}$.

Corollary 4.2. Under the assumptions as in Corollary 1.1, if $G_r$ and are diagonal, the closed-loop system of Eqs. (1), (19), (20), (21) is asymptotically stable if

$$Re[H_i(j\omega)] \geq 0 \text{ for all real } \omega$$

As a result of Corollary 4.2, CDVFB controllers can tolerate stable first-order dynamics in the loop. If $H_i(s) = e^{-j\phi_i}$, we have $Re[H_i(j\omega)] \geq 0$ for $-90° \leq \phi_i \leq 90°$; therefore, CDVFB controllers have 90° phase margin.

## Stability in the Presence of Nonlinearities

Suppose the operator $\mathcal{H}$ in (1) is replaced by an m-vector nonlinear function $\psi$ as in Eq. (47), except that $\psi$ is allowed to be time-varying. The following theorem gives sufficient conditions for global asymptotic stability.

Theorem 5. Consider the closed-loop system given by Eqs. (1), (19), (20), (21), where $G_r$ is diagonal and positive definite, and each $\psi_i$ belongs to the $[0, \infty)$ sector. Then the closed-loop system is globally asymptotically stable.

*Proof.* Starting with V as in Eq. (56),

$$\dot{V} = -2\dot{q}^T D\dot{q} - 2\sum_{i=1}^{3m} G_{ri}^{-1} u_{cri}\psi_i(u_{cri}, t) \tag{62}$$

Thus $\dot{V} < 0$, and $\dot{V} \equiv 0$ only if $\dot{q} \equiv 0$, which can happen (from the equations of motion) only when $q \equiv 0$. Therefore, the system is globally asymptotically stable.

The next theorem considers a special case when nonlinearities and first-order dynamics are simultaneously present in the loop, as shown in Fig. 3.

*Theorem 6.* Consider the closed-loop system given by Eqs. (1), (19), (20), (21), where $G_r > 0$ is diagonal. Suppose $\mathcal{H} = \text{diag}\ (\mathcal{H}_1, \mathcal{H}_2, \ldots \ldots, \mathcal{H}_m)$, where

$$\mathcal{H}_i g = \psi_i(\mathcal{G}_i g) \tag{63}$$

where each $\psi_i$: $R^1 \to R^1$ is a time-invariant, differentiable function belonging to the $[0, \infty)$ sector, and there exists a constant $K < \infty$ such that $|\psi_i'| < K$ over the interval $(-\infty, \infty)$. Suppose $\mathcal{G}_i$ is an LTI operator whose transfer function is: $G_i(s) = a_i(1 + p_i s)^{-1}$, $a_i > 0$, $p_i > 0$ for $i = 1, 2, \ldots, m$. Then the system is globally asymptotically stable.

*Proof.* Starting with V as in Eq. (56) and proceeding as in the proof of Theorem 4, we have

$$0 \leq V(0) - 2\langle\dot{q}, D\dot{q}\rangle_T - 2\sum_{i=1}^{3m} G_{ri}^{-1} \langle u_{cri}, \psi_i\{\mathcal{G}_i(0; u_{cri}) + g_{oi}\}\rangle_T \tag{64}$$

where $g_{oi}$ is the unforced response of $\mathcal{G}_i$ due to nonzero initial state. Using mean value theorem, Eq. (64) can be written as:

$$0 \leq V(0) - 2\langle\dot{q}, D\dot{q}\rangle_T - 2\sum_{i=1}^{3m} \langle u_{cri}, \psi_i\{\mathcal{G}_i(0; u_{cri})\}\rangle_T$$

$$+ \langle u_{cri}, \psi_i'(\hat{u})g_{oi}\rangle_T \tag{65}$$

where $\hat{u}$ lies in the interval bounded by $\mathcal{G}_i(0; u_{cri})$ and $\mathcal{G}_i(0; u_{cri}) + g_{oi}$. Noting that the operator $\psi_i\ \{\mathcal{G}_i(0; u_{cri})\}$ is passive [4], and simplifying, we have

$$\lambda_m(D)\|\dot{q}\|_T^2 \leq V(0)/2 + \|\bar{\Phi}\|_s K\|\dot{q}\|_T\|g_o\| \tag{65}$$

where

$$\|g_o\| = \sum_{i=1}^{3m} \|g_{ci}\| < \infty \tag{66}$$

The remainder of the proof is similar to that of Theorem 4.

## CONCLUDING REMARKS

Robustness properties were investigated for two types of controllers for large space structures, which use collocated sensors and actuators. The first type is the collocated attitude (CA) controller, which controls the rigid-body attitude and the elastic motion using negative definite feedback of measured-attitude and rate. The second type of controller is the collocated direct velocity feedback (CDVFB) controller for damping enhancement. Such controllers are known to provide closed-loop asymptotic stability regardless of the number of modes and parameter values, provided that the actuators and sensors are perfect. This robust stability property was extended further in this paper by proving that the global asymptotic stability is preserved even when sensors/actuators are not perfect. The CA controller preserves global asymptotic stablity when the sensors/actuators are represented by (i) linear, time-invariant dynamics which satisfy certain simple phase conditions, or (ii) time-invariant, monotonic increasing nonlinearities belonging to the $(0, \infty)$ sector. The CDVFB controller preserves global asymptotic stability under much weaker conditions. In particular, CDVFB controllers have 90° phase margin and are tolerant to time-varying nonlinearities in the $[0, \infty)$ sector. These global asymptotic stability results are valid regardless of the number of modes in the model and regardless of parameter values. Therefore, it can be concluded that these controllers offer viable methods for robust attitude control or damping enhancement, especially when the parameters are not accurately known. An important application of the collocated attitude controller would be during deployment or assembly of a large space structure, when the dynamic characteristics are changing, and during initial operating phase, when the dynamic characteristics are not known accurately. A robust collocated controller can provide stable interim control which can perhaps be replaced later by a high-performance controller designed using parameters estimated on orbit.

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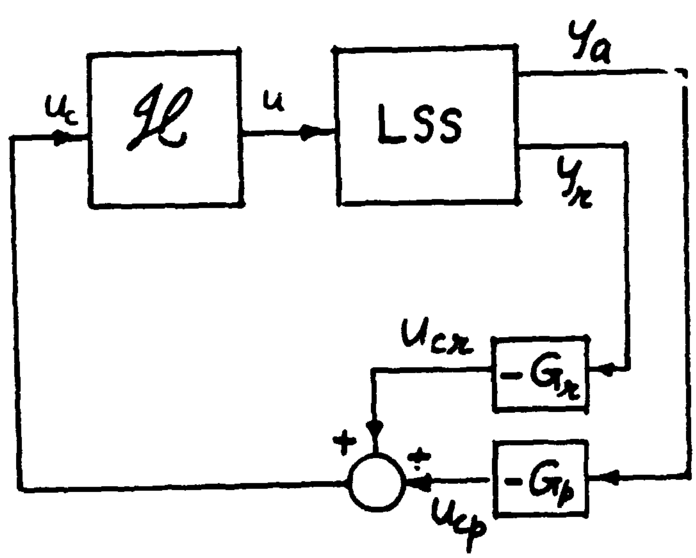


Figure 1. - Collocated Contoller

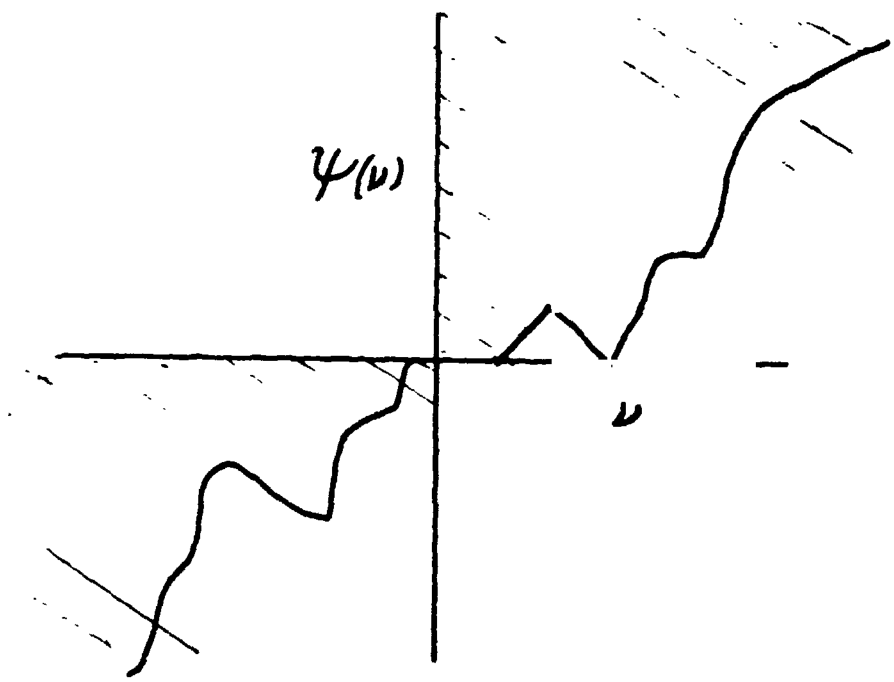


Figure 2.- Nonlinearity belonging to the $[0,\infty)$ sector

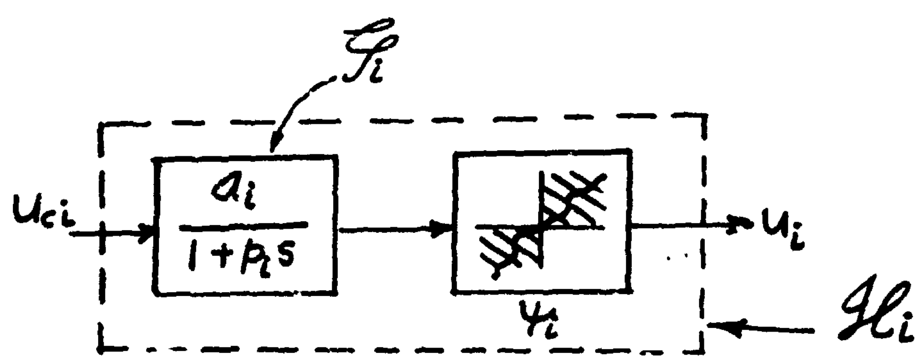


Figure 3.- Linear dynamics and nonlinearities simultaneously in the loops