

ADAPTIVE CONTROL—ACTUAL STATUS AND TRENDS

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ABSTRACT

Important progress in research and application of Adaptive Control Systems has been achieved in the last ten years. The techniques which are currently used in applications will be reviewed. Theoretical aspects currently under investigation and which are related to the application of adaptive control techniques in various fields will be briefly discussed. Applications in various areas will be briefly reviewed. The use of adaptive techniques for vibrations monitoring and active vibration control will be emphasized.

I. INTRODUCTION

The use of adaptive control techniques is motivated by the need of automatically adjusting the parameters of the controller when plant parameters and disturbances are unknown or change with time, in order to achieve (or to maintain) a certain index of performance for the controlled system. While this problem can be reformulated as a nonlinear stochastic control problem (the unknown parameters are considered as auxiliary states) the resulting solutions are extremely complicated. Therefore, in order to obtain something useful, it is necessary to make approximations. Adaptive control techniques can be viewed as approximations for nonlinear stochastic control problems. Model Reference Adaptive Controllers (MRAC) and Self-Tuning Regulators (STR) can be considered as two approximations among other possible approximations. These two approaches to adaptive control problems have been extensively studied and they are well understood. These approaches have been proven to be usable in practice and an important number of successful applications have been reported. However, some important theoretical problems still need further investigation and more experience utilizing these techniques in practice should be gained.

As mentioned earlier the MRAC and STR approaches can be considered as possible approximations for the solutions of some nonlinear stochastic control problems. However, when making approximations, some hypothesis should be considered which can justify these approximations. The basic hypothesis for MRAC and STR is of an algebraic nature: for any possible values of the plant (and disturbance) parameters, there exists a linear controller with a fixed complexity such that the plant plus the controller has the pre-specified characteristics. The adaptive control loop

will only search for the values of the tuned parameters of a controller whose structure has been fixed using a standard control design technique.

The MRAC and STR techniques have been initially developed independently. Subsequently, connections between these two techniques have been investigated and emphasized. See Egardt (1980), Landau (1981), Landau (1982), Astrom (1983). For certain classes of problems these two approaches are equivalent. It is important to note that the development of these two adaptive control techniques is largely based on the deep understanding of certain types of linear algebraic control design techniques and of an appropriate interpretation of the controller design strategy.

A brief review of the underlying concepts and configurations used for MRAC and STR is given in Section II. The linear tracking and regulation problem is reviewed in Section III and this allows the definition of the structure of the controller. The structures of various adaptive control schemes are presented in Section IV. The parameter adaptation algorithms are discussed in Section V. Applications are listed in Section VI. Current research trends are indicated in Section VII.

II. MODEL REFERENCE ADAPTIVE CONTROLLERS AND SELF-TUNING REGULATORS - BASIC PRINCIPLES.

Figure 2.1 illustrates the basic philosophy for designing a linear controller. The desired performance is specified in terms of the characteristics of a dynamic system which is a "realization" of the desired input-output behavior of the closed loop control system. The controller is designed such that the closed loop control system is characterized by the same parameters as those of the "desired" dynamic system.

Since desired performance corresponds in fact to the output of the "desired" dynamic system which is pre-specified, the design problem can be recast as in Fig. 2.2. The objective is now to design a controller such that the error between the output of the plant and the output of the reference model (the dynamic system which has the desired characteristics) is identically null for identical initial conditions and such that an eventual initial error will vanish with a certain dynamics.

These two interpretations of the linear control design in the case of a plant with unknown or varying parameters lead to two adaptive control schemes, shown in Figs. 2.3 and 2.4. Figure 2.3 is an extension of the scheme given in Fig. 2.2 and is called (explicit) MRAC. The difference between the output of the plant and the output of the reference model is a measure of the difference between the real performance and the desired one. This information is used through an "adaptation mechanism" (parameter adaptation algorithm) to directly adjust the parameters of the controller. This is a "direct" adaptive control scheme.

Figure 2.3 is an extension of the scheme considered in Fig. 2.1 in the sense that a suitable controller can be designed if a plant model is estimated on-line based on the current input-output data available. This scheme is called STR and it is inspired by the separation theorem

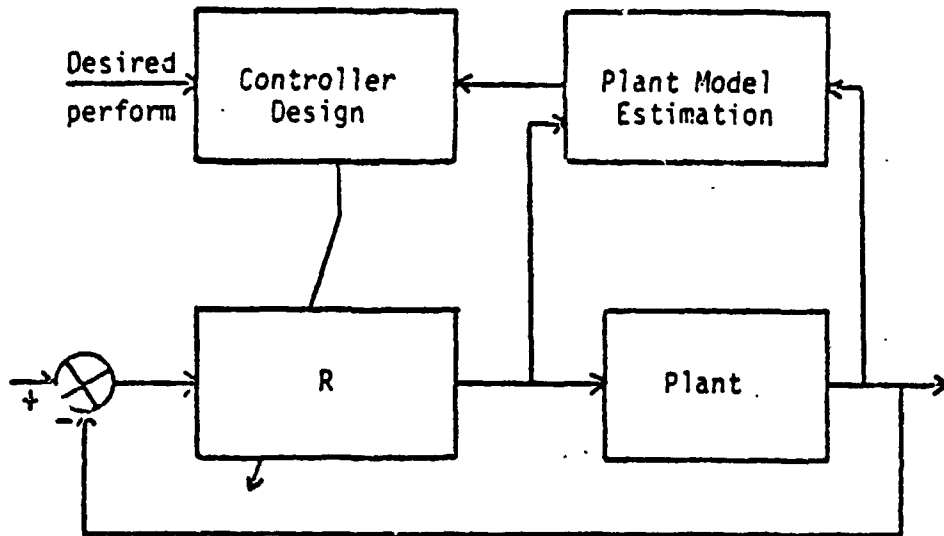


Figure 2.3

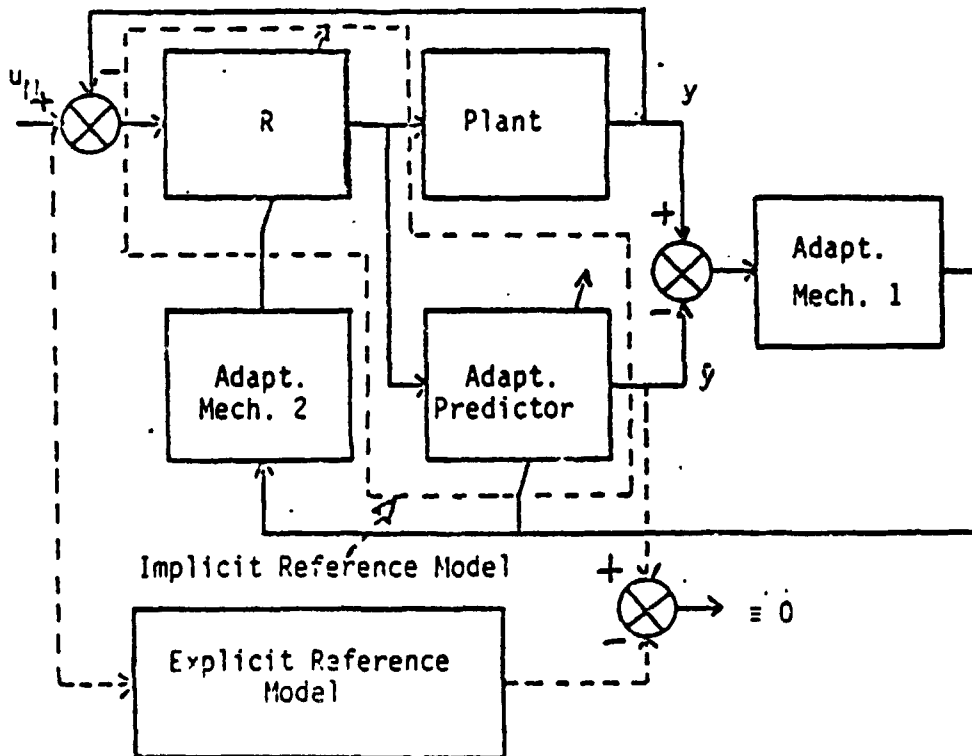


Figure 2.4

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A. Minimum Phase Plants

Consider the S.I.S.O. discrete linear time invariant plant described by:

a) deterministic environment:

$$A(q^{-1})y(k+d) = B(q^{-1})u(k), \quad d > 0, \quad y(0) \neq 0 \quad (3.1)$$

b) stochastic environment:

$$A(q^{-1})y(k+d) = B(q^{-1})u(k) + C(q^{-1})\omega(k+d) \quad (3.2)$$

where:

$$\begin{aligned} A(q^{-1}) &= 1 + a_1 q^{-1} + \dots + a_n q^{-n} \\ B(q^{-1}) &= b_0 + b_1 q^{-1} + \dots + b_m q^{-m} \quad b_0 \neq 0 \end{aligned} \quad (3.3)$$

$$\begin{aligned} C(q^{-1}) &= 1 + c_1 q^{-1} + \dots + c_n q^{-n} \\ C_R(q^{-1}) y(k+1) &= 0 \end{aligned} \quad (3.4)$$

where:

$$C_R(q^{-1}) = 1 + c_1^R q^{-1} + \dots + c_n^R q^{-n} \quad (3.5)$$

is an asymptotically stable polynomial.

In order to design the controller, we will consider two strategies, one using an explicit reference model as part of the control system and the other using a 1-step ahead predictor of the plant output which together with the controller will form an implicit reference model.

Strategy 1: Explicit Reference Model

One considers an explicit reference model given by:

$$C_T(q^{-1}) y^M(k+1) = D(q^{-1}) u^M(k) \quad (3.6)$$

where $y^M(k)$ is the output of the explicit reference model. The design objective is:

$$C_R(q^{-1}) \varepsilon(k+1) = 0 \quad k \geq 0 \quad (3.7)$$

where

$$\varepsilon(k) = y(k) - y^M(k) \quad (3.8)$$

is the plant model error. It is obvious that Eq. (3.7) includes the regulation objective specified by Eq. (3.4) (for $u_M(k) \equiv 0$, $\varepsilon(k) = y(k)$). Equation (3.1) with $d = 1$ can be rewritten as:

$$\begin{aligned} C_R(q^{-1}) y(k+1) &= [C_R(q^{-1}) - A(q^{-1})] y(k+1) + B(q^{-1}) u(k) \\ &= R(q^{-1}) y(k) + b_o u(k) + B^*(q^{-1}) u(k) \end{aligned} \quad (3.9)$$

where

$$R(q^{-1}) = C_R(q^{-1}) - A(q^{-1}) = \sum_{i=1}^n (c_i^R - a_i) q^{-i+1} = r_1 + r_2 q^{-1} \dots r_n q^{-n+1} \quad (3.10)$$

$$B^*(q^{-1}) = B(q^{-1}) - b_o \quad (3.11)$$

and Eq. (3.7) becomes:

$$C_R(q^{-1}) \varepsilon(k+1) = R(q^{-1}) y(k) + b_o u(k) + B^*(q^{-1}) u(k) - C_R(q^{-1}) y_M(k+1) = 0 \quad (3.12)$$

which yields the desired control

$$u(k) = \frac{C_R(q^{-1}) y^M(k+1) - R(q^{-1}) y(k) - B^*(q^{-1}) y(k)}{b_o} \quad (3.13)$$

Introducing the notation:

$$\phi_0^T(k) = [u(k-1) \dots u(k-m), y(k) \dots y(k-n+1)] \quad (3.14)$$

$$\theta_0^T = [b_1 \dots b_m, r_1 \dots r_n] \quad (3.15)$$

Equation (3.15) can be written:

$$u(k) = \frac{C_R(q^{-1}) y^M(k+1) - \theta_0^T \phi_0(k)}{b_o} \quad (3.16)$$

or in an equivalent form:

$$C_R(q^{-1}) y^M(k+1) = \theta^T \phi(k) \quad (3.17)$$

where:

$$\phi(k)^T = [u(k), \phi_0^T(k)] \quad (3.18)$$

$$\theta^T = [b_o, \theta_0^T] \quad (3.19)$$

The resulting control scheme is given in Fig. 3.1.

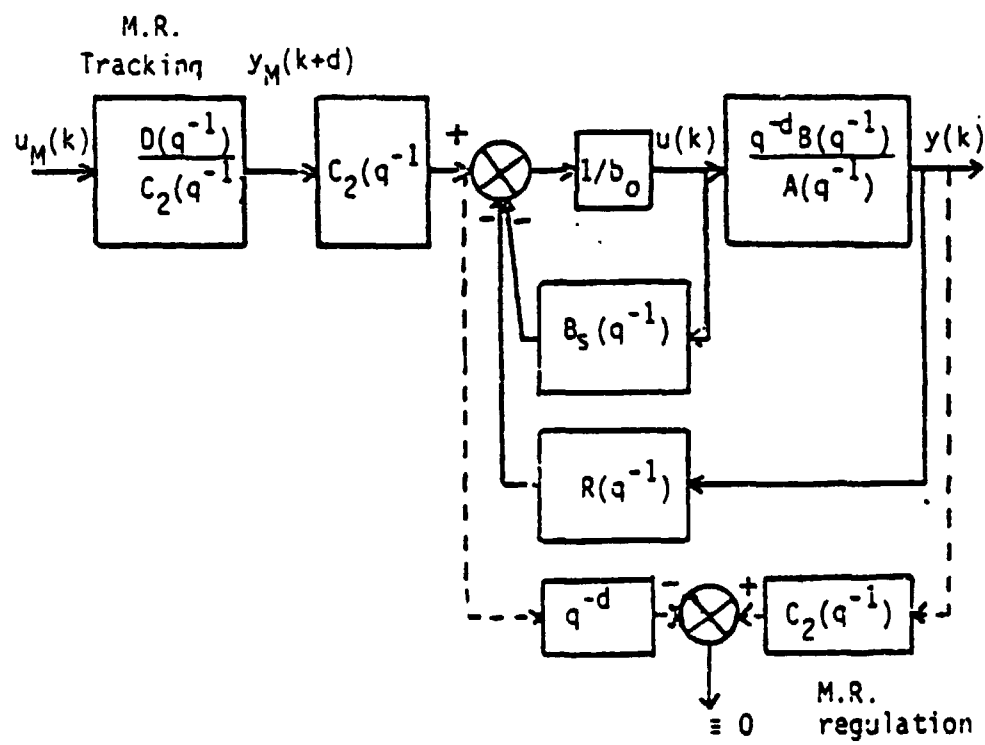


Figure 3.1

Strategy 2: Implicit Reference Model.

This strategy is directly inspired by the separation theorem: one first designs an appropriate predictor for the plant output, and then a control will be computed such that the output of the predictor behaves as the desired output in tracking.

First step: (predictor design). The predictor will be designed such that the 1-step ahead prediction error $\hat{\epsilon}(k+1)$ is defined by:

$$\hat{\epsilon}(k+1) = y(k+1) - \hat{y}(k+1) \quad (3.20)$$

where $\hat{y}(k+1)$ is the predictor output and will vanish according to:

$$C_R(q^{-1}) \hat{\epsilon}(k+1) = 0 ; k \geq 0 \quad (3.21)$$

Using Eq. (3.9), one obtains from Eq. (3.21) that the 1-step ahead predictor is characterized by:

$$C_R(q^{-1}) \hat{y}(k+1) = b_0 u(k) + R(q^{-1}) y(k) + b^*(q^{-1}) u(k) = \theta^T \phi(k) \quad (3.22)$$

where $R(q^{-1})$, $b^*(q^{-1})$, θ , $\phi(k)$ are given by Eqs. (3.10), (3.11), (3.18), and (3.19) respectively.

Second step: (computation of the control). The control is computed such that $\hat{y}(k+1) = y^M(k+1)$; where $y^M(k+1)$ is the desired output given by Eq. (3.6). One finally obtains:

$$C_R(q^{-1}) \hat{y}(k+1) = C_R(q^{-1}) y^M(k+1) = \theta^T \phi(k) \quad (3.23)$$

and the control is given by Eq. (3.17) as expected.

Because of the output of the predictor is equal to the output of the explicit reference model, the predictor plus the control will form an "implicit reference model."

B. Tracking and Regulation in Stochastic Environment

We will examine first the behavior of the controller designed in the previous section when the plant is subject to a stochastic disturbance of the type considered in Eq. (3.2). For $d = 1$ Eq. (3.2) becomes:

$$A(q^{-1}) y(k+1) = B(q^{-1}) u(k) + C(q^{-1}) \omega(k+1) \quad (3.24)$$

Using the control given in Eq. (3.13) one obtains:

$$C_R(q^{-1}) y(k+1) = C_R(q^{-1}) y^M(k+1) + C(q^{-1}) \omega(k+1) \quad (3.25)$$

Neglecting the effect of the deterministic disturbance (which vanishes with the dynamics defined by $C_R(q^{-1})$) one can re-write Eq. (3.25) as:

$$y(k+1) = y^M(k+1) + \frac{C(q^{-1})}{C_R(q^{-1})} \omega(k+1)$$

$$= \frac{D(q^{-1})}{C_T(q^{-1})} u^M(k) + \frac{C(q^{-1})}{C_R(q^{-1})} \omega(k+1) \quad (3.26)$$

Equation (3.26) shows the presence of two reference models: a deterministic one for tracking by $\frac{D(q^{-1})}{C_T(q^{-1})}$ whose input is the reference signal $u^M(k)$ and a stochastic one for regulation defined by $\frac{C(q^{-1})}{C_T(q^{-1})}$ whose input is the white noise sequence $\omega(k+1)$.

In general the objective of the design in a stochastic environment is to obtain a minimum variance tracking and regulation, i.e.:

$$E \{ [y(k+1) - y_M(k+1)]^2 \} = \min \quad (3.27)$$

From Eq. (3.26) it results straightforwardly that the objective of Eq. (3.27) is achieved if one chooses:

$$C_R(q^{-1}) = C(q^{-1}) \quad (3.28)$$

which leads to:

$$E \{ [y(k+1) - y_M(k+1)]^2 \} = E \{ \omega^2(k+1) \} = \sigma^2 \quad (3.29)$$

For the case $d > 1$, the control can no longer be computed directly using the strategies given above since this will lead to a non-causal controller (future values of the output and input are involved for the computation of the control at the instant k). This problem can be avoided by using a polynomial identity which allows us always to express the output $y(k+d)$ in terms only of $y(k)$; $y(k-1)$... and $u(k)$, $u(k-1)$...

Consider the following polynomial identity:

$$C_R(q^{-1}) = A(q^{-1}) S(q^{-1}) + q^{-d} R(q^{-1}) \quad (3.30)$$

which has a unique solution for the polynomials $S(q^{-1})$ and $R(q^{-1})$ for $\deg S(q^{-1}) = d-1$ where

$$S(q^{-1}) = 1 + S_1 q^{-1} \dots S_{d-1} q^{-d+1} \quad (3.31)$$

$$R(q^{-1}) = r_1 + r_2 q^{-1} \dots r_n q^{-u+1} \quad (3.32)$$

Using the identity of Eq. (3.30) in Eq. (3.9) for $d > 1$ one obtains:

$$C_R(q^{-1})y(k+d) = R(q^{-1})y(k) + b_o u(k) + B_S(q^{-1})u(k) \quad (3.33)$$

where

$$B_S(q^{-1}) = B(q^{-1})S(q^{-1}) - b_o \quad (3.34)$$

Equation (3.7) for $d > 1$ becomes:

$$\begin{aligned} C_R(q^{-1})\varepsilon(k+d) &= R(q^{-1})y(k) + b_o u(k) + B_S(q^{-1})u(k) \\ &\quad - C_R(q^{-1})y_M(k+d) = 0 \end{aligned} \quad (3.35)$$

which yields the desired control

$$\begin{aligned} u(k) &= \frac{C_R(q^{-1})y^M(k+d) - R(q^{-1})y(k) - B_S(q^{-1})u(k)}{b_o} \\ &= \frac{C_R(q^{-1})y^M(k+d) - \theta_o^T \phi_o(k)}{b_o} \end{aligned} \quad (3.36)$$

The control has the same structure as for the case $d = 1$ except that the polynomials $R(q^{-1})$ and $B(q^{-1})$ are different, as well as θ_o and $\phi_o(k)$:

Note that the strategy presented above achieves a poles-zeros placement.

C. Non-minimum Phase Plants

In this case one can no longer assume that $B(z^{-1})$ is asymptotically stable and therefore the zeros of the plant transfer function can no longer be cancelled. The basic control strategy (algebraic approach) is the poles placement technique without zeros cancelling. The basic relation for the design of the controller is the Bezout identity:

$$A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1}) = C_R(q^{-1})$$

and the controller has the structure:

$$S(q^{-1}) u(k) = \frac{1}{\beta} C_R(q^{-1}) y_M(k+d) - R(q^{-1}) y(k)$$

$$\beta = \begin{cases} 1 & \text{if } B(1) = 0 \\ B(1) & \text{elsewhere} \end{cases}$$

For a survey of the control strategies for non-minimum phase plants, see Landau, M'Saad, Ortega (1973).

IV. STRUCTURES OF ADAPTIVE CONTROL SYSTEMS

In adaptive control schemes the fixed controller designed for the case of known parameters is replaced by an adjustable controller having the same structure, i.e., the fixed parameter vector will be replaced by an adjustable parameter vector which for the case of the design considered for minimum phase plants is given by:

$$\hat{\theta}^T(k) = [\hat{b}_0(k), \hat{\theta}_0^T(k)] \quad (4.1)$$

and the corresponding control law will be given (either in deterministic or stochastic environment) by:

$$u(k) = \frac{C_R(q^{-1})y^M(k+1) - \hat{\theta}_0^T(k)\phi_0(k)}{\hat{b}_0(k)} \quad (4.2)$$

or:

$$\hat{\theta}^T(k) \phi(k) = C_R(q^{-1}) y^M(k+1) \quad (4.3)$$

See Fig. 4.1a.

Note that in the case of schemes using an implicit (prediction) reference model (STR) the plant predictor will be replaced by an adaptive predictor governed by:

$$C_2(q^{-1}) \hat{y}(k+1) = \hat{\theta}^T(k) \phi(k) \quad (4.4)$$

and the control will be computed according to the strategy in the linear case with known parameters which will lead to Eq. (4.3). See Fig. 4.1b.

V. PARAMETER ADAPTATION ALGORITHMS

Various approaches have been considered for the development of parameter adaptation algorithms (PAA). A fairly general structure for the PAA is given by:

$$\hat{\theta}(k+1) = \hat{\theta}(k) + F_k \phi(k) v(k+1) \quad (5.1)$$

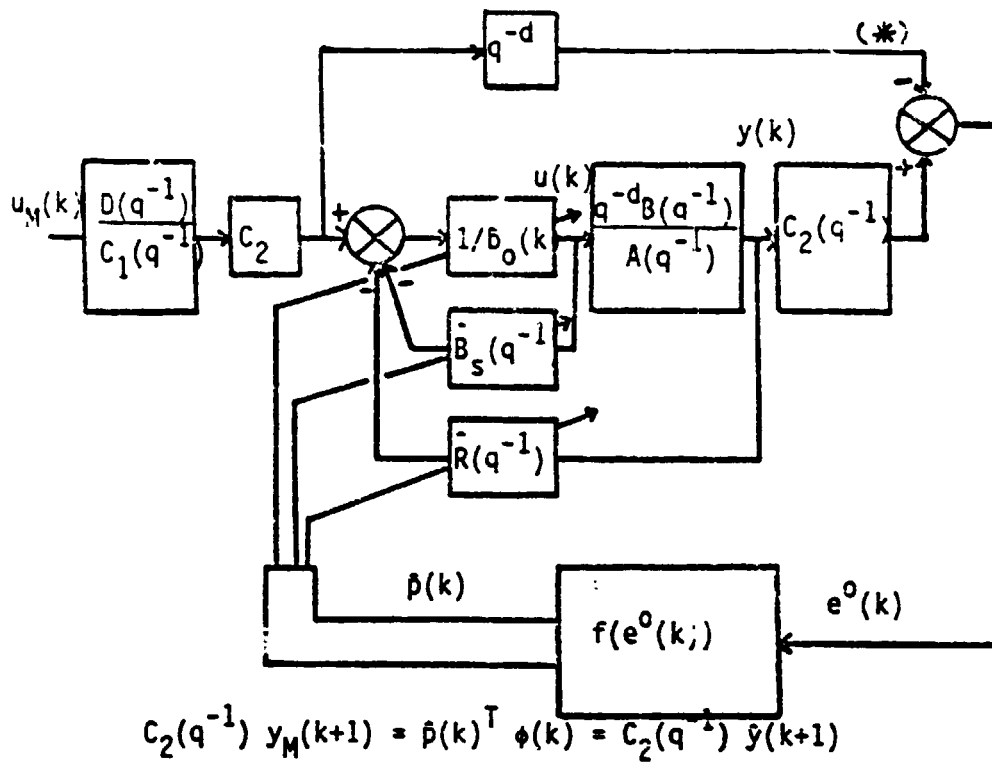


Figure 4.1a

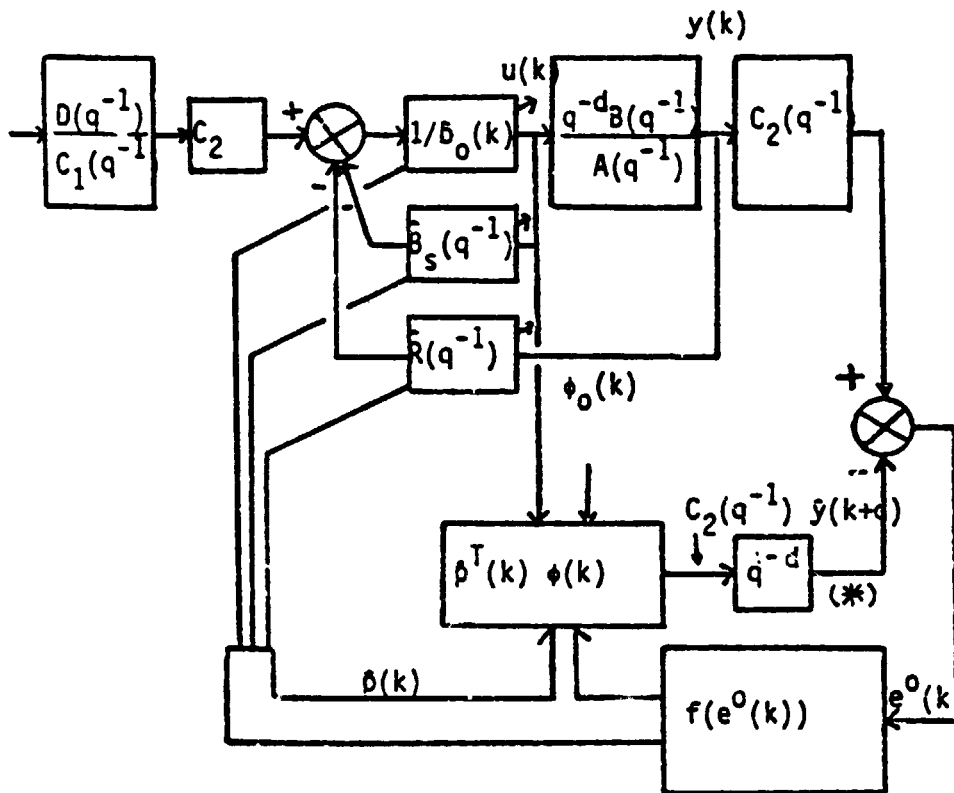


Figure 4.1b

$$v(k+1) = \frac{v_{k+1}^0}{1 + \phi(k)^T F_k \phi(k)} \quad (5.2)$$

$$F_{k+1}^{-1} = \lambda_1(k) F_k^{-1} + \lambda_2(k) \phi(k) \phi(k)^T \quad (5.3)$$

$$0 < \lambda_1(k) \leq 1 ; 0 \leq \lambda_2(k) < 2 ; F_0 > 0 \quad (5.4)$$

Using the matrix inversion lemma:

$$F_{k+1} = \frac{1}{\lambda_1(k)} \left[F_k - \frac{F_k \phi(k) \phi(k)^T F_k}{\lambda_1(k) + \phi(k)^T F_k \phi(k)} \right] \quad (5.5)$$

where $\hat{\theta}(k)$ is the adjustable parameter vector, $F(k)$ is the matrix adaptation gain, $\phi(k)$ is the measurement or the observation vector and $v^0(k+1)$ and $v(k+1)$ are the "a priori" and the "a posteriori" adaptation errors respectively. The "a priori" adaptation error is a measurable quantity which depends on $\hat{\theta}(i)$ up to the instant k , and the "a posteriori" adaptation error which enters in the adaptation algorithm is not directly measurable (it depends on $\hat{\theta}(k+1)$) but can be expressed in terms of the "a priori" adaptation error as indicated in eq. (5.2).

Different choices for $\lambda_1(k)$ and $\lambda_2(k)$ are possible leading to different types of variations of the adaptation gains. The performances of the adaptive control systems in various situations depend upon the choices of these two parameters. For details see Landau, Lozano (1981) and Landau (1983).

VI. APPLICATIONS

There are already a significant number of applications of adaptive control systems as well as a few commercial products. For references, see Aström (1983), Landau (1981), Landau, Tomizuka, Auslander (1983), Narendra, Mcnopoly (1980), Unbehauen (1980).

The adaptive control schemes can be used in three modes of operation:

- 1) Auto-tuning of a linear controller in the case of plants with unknown but constant parameters.
- 2) Building a gain schedule for unknown plants with dynamics depending on operating points.
- 3) Adapting in real-time the controller for plants with unknown and time-varying parameters.

An important remark to be made is that adaptive control algorithms cannot be used in practice without a priori analysis of the control problem corresponding to each tentative application. This analysis should give answers to two categories of questions regarding (a) the need of adaptive control and (b) specific design requirements.

The main areas of applications are:

- Grinding
- Drying furnaces
- Cement mills
- Chemical reactors
- Distillation columns
- Diesel and explosion engines
- Heating and ventilation
- Paper machines
- Power systems
- Electrical drives
- Autopilots for ships
- Robotics
- Heat exchangers
- pH-control
- Active vibration control

An adaptive active vibrations control is described in Mote, Rahimi (1983). It uses first a recursive parameter estimation technique for estimating in real-time the parametric model of the composite vibration signal for circular plates (the vibrations frequencies). Then the parameters of the transfer from control heat to vibration frequency are estimated on-line and used for computing in real time the controller parameters.

VII. THEORY

The most complete theory is available today for the adaptive control of minimum phase plants achieving a poles-zeros placement. For this type of plant, tracking and regulation with independent objectives can be achieved both in deterministic and stochastic environments. Both MRAC and STR approaches lead in this case to "direct" adaptive control schemes.

The basic assumptions for the design of adaptive control systems for minimum-phase plants in deterministic and stochastic environments are summarized next.

- Exact knowledge of the plant delay (d).
- Knowledge of an upper bound for the degree of $A(q^{-1})$ which is the denominator of the plant transfer function.
- The zeros of the plant transfer function must lie within the unit circle.
- A lower bound of the magnitude of the leading coefficient of the plant transfer function should be known.
- The sign of the leading coefficient of the numerator plant transfer function is useful to be known (in order to avoid large adaptation transients).
- The stochastic disturbances are modeled by ARMA processes.
- Asymptotic type convergence is considered.

However, in practice some of these assumptions cannot be reasonably satisfied, in particular, the need for knowing an upper bound for the denominator degree (which in many cases simply does not exist) and the requirement that the disturbance is of ARMA type.

The use of reduced order models in adaptive control design is one of the main research topics today, and interesting results have been obtained leading to improved design techniques. See Ioannou (1983), Ortega, Lanuau (1983), Kosut (1983).

The case of disturbances which cannot be modeled by ARMA processes has also been considered. See, for example, Samson (1983), Peterson, Narendra (1982).

Another aspect is the extension of the adaptive control design for the multi inputs - multi outputs systems. Except for trivial cases, the extension raises important parameterization problems for MIMO plants. A survey of the various designs available can be found in Dion, Dugard (1983). More a priori knowledge on the plant structure than in the SISO case is required, and the research is directed towards the development of adaptive control schemes requiring less a priori structural information. The Hermite form of MIMO transfer matrix plays a key role in understanding the multi-variable case.

The case of adaptive control of non-minimum phase plants is more complicated both from the point of view of the complexity of the adaptive control schemes and of the analysis. A survey of the adaptive control techniques for this type of plant is given in Landau, M'Saad, Ortega (1983). Most of the schemes are of "indirect" type, and the major question to be answered in order to show the convergence of the system is whether the estimated plant model converges towards the model with satisfactory properties (stabilizable). Global convergence results have been obtained, but with the requirement of using an additional persistent excitation signal, see Goodwin, Teoh, Innis (1982). The robustness of the adaptive control designs for non-minimum phase plants with respect to model reduction and ill-modeled disturbances has also been studied, see, for example, Praly (1983).

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