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OPTIMAL SENSOR LOCATIONS FOR STRUCTURAL IDENTIFICATION

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SUMMARY

The optimum sensor location problem, OSLP, may be thought of in terms of the set of systems, S, the class of input time functions, I, and the identification algorithm (estimator) used, E. Thus, for a given time history of input, the technique of determining the OSL requires, in general, the solution of the optimization and the identification problems simultaneously. However, this paper introduces a technique which uncouples the two problems. This is done by means of the concept of an efficient estimator for which the covariance of the parameter estimates is inversely proportional to the Fisher Information Matrix.

INTRODUCTION

The problem of structural identification in structural engineering is one which has received considerable attention from several researchers in the recent past (Refs. 1-4). Though various methods have been developed for identifying the different parameters that characterize a structure from records obtained in them under various loading conditions, few investigators, if any, have looked at the question of where to locate sensors in a structure to acquire data for "best" parametric identification (Ref. 5). The problem of optimally locating sensors in a structural system arises from considerations of (1) minimizing the cost of instrumentation; and (2) efficiently detecting structural changes in the system with a view to acquiring improved assessment of structural integrity.

The problem addressed in this paper can be stated as follows: Given m sensors, where should they be located in a structure so that records obtained from those locations yield the "best" estimates of the unknown parameters?

In the past, the optimal sensor location problem (OSLP) was solved by positioning the given number of sensors in the system, using the records obtained at those locations with a specific estimator, and repeating the procedure for different sensor locations. The set of locations which yield the "best" parameter estimates would then be selected as optimal. The estimates obtained, of course, would naturally depend upon the type of estimator used. Thus the optimal locations are estimator dependent, and an exhaustive search needs to be performed for each specific estimator. Such a procedure, besides being highly computationally intensive, suffers from the major drawback of not yielding any physical insight into why certain locations are preferable to others.

Recently, work on the solution of the OSLP was done by Shah and Udwadia (Ref. 5). In brief, they used a linear relationship between small perturbations in a finite dimensional representation of the system parameters and a finite sample of observations of the system time response. The error in the parameter estimates are minimized, yielding the optimal locations. In this paper, we develop a more direct approach to the problem which is both computationally superior, and throws considerable light on the rationale behind the optimal selection process.

We uncouple the optimization problem from the identification problem using the concept of an efficient estimator (e.g., the maximum likelihood estimator as time becomes very large). For such an estimator the covariance of the parameter estimates is a minimum. Using this technique and motivated by heuristic arguments, a rigorous formulation and solution of the OSLP is presented. The method is applied to a building structure modelled as a general linear dynamic system. For the N degree of freedom system considered, the methodology for selecting $m (m < N)$ of the nodal displacements for purposes of measurement is presented.

Sample calculations are made for a simple building structure modelled as a two-degree-of-freedom system subjected to base excitations. The optimal sensor location for the identification of: (a) the mass ratio; and (b) the stiffness ratio is investigated.

The results indicate that the OSLP depends on:

1) the class of systems, S, to which the structure belongs;

2) the type of excitation;

3) the actual system parameters involved; and

4) the parameters to be identified.

THEORY

Consider a system modelled by the equation

$$M\ddot{X} + C\dot{X} + KX = F(t) \tag{1}$$

where M, C, and K are the (NXN) mass, damping and stiffness matrices, F(t) is an (NX1) vector containing inertial forces and externally applied loads and X is the N-vector of nodal displacements. Let θ_M, θ_C and θ_K be vectors containing the various parameters related to the mass matrix, the damping matrix and the stiffness matrix, respectively, which need to be identified. For convenience, we collect these quantities in the parameter vector, θ, defined as

$$\theta^T = \left[\theta_M^T | \theta_C^T | \theta_K^T\right]$$

where the superscript T indicate matrix transpose. If the M, C and K are symmetric each of the three subvectors has a maximum dimension of $N(N+1)/2$.

Given m sensors $(m < N)$, we then need to find where to locate them so that the covariance of the estimate, $\hat{\theta}$, is a minimum. Assume further that the measurement vector Z(t) can be expressed as

$$Z_i(t) = g_i[X(\theta,t)] + N_i(t) \quad , \quad i = 1,2,\ldots,N \tag{2}$$

where Z_i is the ith component of $Z(t)$, and the functionals g_i represent the "measurement process". The dependence of the response X on the parameter vector θ is explicitly noted The measurement noise $N_i(t)$ is taken as non-stationary Gaussian White noise with a variance of $\psi^2(t)$. Therefore,

$$E\left[N_i(t_1)N_j(t_2)\right] = \psi^2(t_1)\delta_K(i-j)\delta_D(t_1-t_2) \quad . \tag{3}$$

where δ_K and δ_D stand for the kroneker and the dirac-delta functions, respectively. A total of m out of N responses need to be selected so that they contain the most information about the system parameters and are maximally sensitive to any changes in the parameter values. This "selection" process can be represented by an m-dimensional vector Y such that

$$Y(t) = SZ(t) \tag{4}$$

where S is the $(m \times N)$ upper triangular selection matrix with each row containing null elements except for one which is unity. The m different components of Z selected to be measured are so ordered in vector Y, that if the element in the i-th row and k-th column of S is unity, the (i+1)-ith row has unity in its ℓ-th column with $\ell > k$. The matrix S has the property that $P = S^TS$ in an (NXN) diagonal matrix with unity in its i-th row if, and only if, Z_i is selected to be measured. The elements of P are otherwise zero. Hence, one can write

$$Y(t) = Sg[X(\theta,t)] + SN(t) \tag{5A}$$

$$\triangleq H[X(\theta,t)] + V(t) \tag{5B}$$

If g_i is linearly related to the response X_j, in general, then

$$H[X(\theta,t)] = SRX \tag{6}$$

where $R(t)$ can be thought of as a dynamic gain matrix. In the case that g_i is related to the response X_i only, then matrix R will reduce to a diagonal matrix, $\left[\diagdown \rho_j \diagdown\right]$.

The problem of locating sensors in an optimal manner then reduces to determining the selection matrix S, or alternatively, finding the m locations in P that should be unity. These locations must be so chosen as to obtain the "best" parameter estimates.

SOME MOTIVATING THOUGHTS AND THE FISHER INFORMATION MATRIX

Consider a case in which one tries to estimate only one parameter, θ_1 (to be identified) involved in a dynamic system model with only one sensor provided. Therefore, one wants to ideally choose a location i (out of N possible such locations) such that the measurement $y_i(t)$, $i\varepsilon[1,N]$, $t\varepsilon(0,T)$ at location i yields the best estimate of the parameter θ_1. Heuristically, one should place the sensor at such a location that the time history of measurements obtained at that location is most sensitive to any changes in the parameter θ_1. Hence, in equation (5B) it is really the slope of $H[X(\theta_1,t)]$ with respect to θ_1 that needs to be maximized. However, since only the absolute magnitude of this slope is of interest, it is logical to want to find i (or equivalently determine the selection matrix S described previously) such as to maximize $(\partial H/\partial\theta_1)^2$ over the interval (0,T) during which the response is to be measured. This leads to maximizing the following integral:

$$q_i = \int_0^T \left(\frac{\partial H}{\partial \theta_1}\right)^2 dt \quad . \tag{7}$$

When there is more than one parameter to be estimated, and the number of sensors is greater than unity, this intuitive approach needs to be extended in a more rigorous manner. In such cases recourse to mathematical treatment is necessary, and we shall see that such treatment will be in agreement with our heuristic solution outlined above.

To further understand the problem, let us look at it from another angle, namely, the concept of an efficient unbiased estimator. For such as estimator the covariance of the estimates is a minimum. Furthermore, it can be shown that for any unbiased estimator of θ,

$$E\left[(\theta-\hat{\theta})(\theta-\hat{\theta})^T\right] \geq \left[\int_0^T \left(\frac{\partial H}{\partial \theta}\right)^T \left(\frac{\partial H}{\partial \theta}\right) / \psi^2(t)dt\right]^{-1} \tag{8}$$

where $\hat{\theta}$ is the estimate of θ and the matrix $[\partial H/\partial\theta]_{ij} \triangleq \partial H_i/\partial\theta_j$. If the estimator is "efficient", the above inequality becomes an equality. This means that the left-hand side of inequality (8) takes its lowest value (minimum covariance). Hence,

$$E\left[(\theta-\hat{\theta})(\theta-\hat{\theta})^T\right] = \left[\int_0^T \left(\frac{\partial H}{\partial \theta}\right)^T \left(\frac{\partial H}{\partial \theta}\right) / \psi^2(t)dt\right]^{-1} \tag{9}$$

The term inside the bracket on the right-hand side of the equation (9) is known as the Fisher Information Matrix, Q(T). Thus, maximizing Q(T) would indeed lead to a minimization of the covariance of the estimate, $\hat{\theta}$.

We note then that the m sensor locations need to be so chosen that a suitable norm of the matrix Q(T) given by

$$Q(T) = \int_0^T \left(\frac{\partial H}{\partial \theta}\right)^T \left(\frac{\partial H}{\partial \theta}\right) / \psi^2(t)dt \tag{10}$$

is maximized. This constitutes an extension of equation (7), which we heuristically derived earlier for the scalar case, to the vector situation. Introducing equation (6) in equation (10) one may write

$$Q(T) = \int_0^T \frac{X_\theta^T R^T P R X_\theta}{\psi^2(t)}\, dt, \tag{11}$$

where the ij element of X_θ can be written as:

$$\left[X_\theta\right]_{ij} = \frac{\partial x_i}{\partial \theta_j}, \quad i \in [1,N],\ j \in (1,m)$$

where $X = \{x_i\}_N$ and $\theta = \{\theta_i\}_L$. We note that the Fisher Matrix is symmetric and is dependent on the length of the record available, as well as the locations of the sensors as determined by the matrix P.

If the m locations where the sensors are to be placed are denoted by s_k, $k = 1,2,\ldots,m$, then

$$P = \sum_{k=1}^{m} I_{s_k} \tag{12}$$

where the $(N \times N)$ diagonal matrix I_{s_k} has all its elements equal to zero except the element of the s_k row, which is unity. Noting that P is a diagonal matrix, equation (11) can be simplified to yield

$$Q[T;s_1,s_2,\ldots,s_m;S,\theta;I] = \sum_{k=1}^{m} \int_0^T \frac{X_\theta^T r_{s_k}^T r_{s_k} X_\theta dt}{\psi^2(t)} \tag{13}$$

where r_{s_k} is the s_k row of the matrix R. Also in eq. (13) explicit mention is made of the dependence of the Fisher Matrix on the time length T of the available data, the system S, the parameter vector θ, and the time-variant input I. If the matrix R is diagonal, with diagonal elements $\rho_1,\ldots,\rho_N$, then the ij element of the matrix Q, after some manipulation, reduces to

$$Q_{ij}\left[T;s_1,s_2,\ldots,s_m;S,\theta;I\right] = \sum_{k=1}^{m} \int_0^T \left[\frac{\partial x_{s_k}}{\partial \theta_i}\,\frac{\partial_{s_k}}{\partial \theta_j}\left(\frac{\rho(t)_{s_k}}{\psi(t)}\right)^2\right] dt. \tag{14}$$

Each element of Q_{ij} represents the cross-sensitivity of measurement with respect to the response x_{s_k} of node s_k.

The optimal sensor locations are then obtained by picking m locations s_k, $k = 1,2,\ldots,m$, out of a possible N, so that a suitable norm of the matrix Q is maximized (e.g., the trace norm, etc...). This may be specified by the condition

$$\max_{s_k \, \varepsilon(1,N)} \left|\left|Q\left[T;s_1,s_2,\ldots,s_m;S,\theta;I\right]\right|\right| . \tag{15}$$

Although there are several matrix norms which could be used, perhaps the most useful and physically meaningful in this context is the trace norm. In order not to detract the reader from the basic methodology we defer an exhaustive treatment of suitable matrix norms to a later communication.

The methodology presented up to this point is valid for both linear and non-linear systems since the criterion developed in equation (13) was derived using only equations (5) and (9). We will now indicate its application to linear multi-degree-of-freedom systems.

APPLICATION TO LINEAR DYNAMIC SYSTEMS

Consider the N-degree-of-freedom dynamic system whose governing differential equation of motion is given by eq. (1), together with $X(t_0) = X_0$, $\mathring{X}(t_0) = \mathring{X}_0$, where X_0 and $\mathring{X}_0$ are the given initial conditions for the system. Assume the system to be classically damped. Introducing

$$X(t) = \Phi\eta(t) \tag{16}$$

where Φ is the $(N \times N)$ weighted modal matrix and $\eta(t)$ is the N-vector of generalized coordinates we get

$$\ddot{\eta} + 2\xi_N\omega_N\dot{\eta} + \Lambda\eta = \Phi^T F(t), \quad \eta(t_0) = \phi^T M X_0, \; \mathring{\eta}(t_0) = \phi^T M \mathring{X}_0, \tag{17}$$

where the $(N \times N)$ diagonal matrix Λ is given by

$$\left[\diagdown\Lambda\diagdown\right] = \Phi^T K \Phi = \left[\diagdown\omega_N^2\diagdown\right], \text{ and } \xi_N = \left[\diagdown\xi_i\diagdown\right].$$

The solution of equation (17) is given as

$$\eta_i(t) = \eta_{0_i} u_i(t-t_0) + \dot{\eta}_{0_i} v_i(t-t_0) + \int_{t_0}^{t} h_i(t-\tau)p_i(\tau)d\tau \tag{18}$$

where η_{0_i} and $\dot{\eta}_{0_i}$ are initial conditions and

$$u_i(t) = EXP\left(-\xi_i\omega_i t\right)\left[Cos\omega_{d_i} t + \frac{\xi\omega_i}{\omega_{d_i}} Sin\omega_{d_i} t\right] ,$$

$$v_i(t) = \frac{1}{\omega_{d_i}} EXP\left(-\xi_i\omega_i t\right) Sin\omega_{d_i} t,$$

$$h_i(t) = v_i(t),$$

$$\omega_{d_i} = \omega_i\sqrt{1 - \xi_i^2} , \quad \text{and}$$

$$p_i(t) = \phi^T F(t), \quad i = 1,2,\ldots,N.$$

Also, differentiating equation (1) with respect to θ, yields

$$M\ddot{X}_\theta + C\dot{X}_\theta + KX_\theta = F_\theta(t) - \left(\widehat{M_\theta\ddot{X}} + \widehat{C_\theta\dot{X}} + \widehat{K_\theta X}\right); \; X_\theta(0) = 0, \; \dot{X}_\theta(0) = 0$$

where

$$\left[X_\theta\right]_{ij} = \frac{\partial x_i}{\partial \theta_j} , \qquad \text{with}$$

$$\widehat{M_\theta\ddot{X}} = [M_{\theta_1}\ddot{X} \vdots M_{\theta_2}\ddot{X} \vdots M_{\theta_j}\ddot{X} \vdots M_{\theta_L}\dot{X}]$$

$$i = 1,\ldots,N, \quad \text{and} \quad j = 1,\ldots,L.$$

Introducing

$$X_\theta = \Phi z \tag{19}$$

yields

$$\ddot{z} + 2\xi_N\omega_N\dot{z} + \Lambda z = G(t) \tag{20}$$

where

$$G(t) = \Phi^T\left[F_\theta - \left(\widehat{M_\theta\ddot{X}} + \widehat{C_\theta\dot{X}} + \widehat{K_\theta X}\right)\right] . \tag{21}$$

Equation (21) can further be simplified to give

$$G(t) = \phi^T\left[F_\theta - \left(\widehat{M_\theta\Phi\ddot{\eta}} + \widehat{C_\theta\Phi\dot{\eta}} + \widehat{K_\theta\Phi\eta}\right)\right] \tag{22}$$

where $\dot{\eta}$ and $\ddot{\eta}$ can be obtained by differentiation of eq. (18). This may be shown as follows

$$\dot{\eta}_i(t) = \eta_{0_i} W_i\left(t-t_0\right)+\dot{\eta}_{0_i} Y_i\left(t-t_0\right) + \int_{t_0}^{t} \bar{h}_i(t-\tau)p_i(\tau)d\tau \tag{23}$$

where

$$W_i(t) = -\mathrm{EXP}\left(-\xi_i\omega_i t\right) \left[\omega_{d_i} + \frac{\left(\xi_i\omega_i\right)^2}{\omega_{d_i}}\right] \mathrm{Sin}\omega_{d_i} t \quad ,$$

$$Y_i(t) = \mathrm{EXP}\left(-\xi_i\omega_i t\right) \left[\mathrm{Cos}\omega_{d_i} t - \left(\frac{\xi_i\omega_i}{\omega_{d_i}}\right) \mathrm{Sin}\omega_{d_i} t\right] \quad ,$$

$\bar{h}_i(t) = Y_i(t)$, and

$p_i(t) = \Phi^T F(t)$, $\quad i = 1,2,\ldots,N.$

Also

$$\ddot{\eta}_i(t) = \eta_{0_i} \bar{W}_i\left(t-t_0\right)+\dot{\eta}_{0_i} \bar{Y}_i\left(t-t_0\right) + \int_{t_0}^{t} \bar{\bar{h}}_i(t-\tau)p_i(\tau)d\tau \tag{24}$$

where

$$W_i(t) = \mathrm{EXP}\left(-\xi_i\omega_i t\right)\left\{\left[\frac{\left(\xi_i\omega_i\right)^3}{\omega_{d_i}} + \omega_{d_i} \left(\xi_i\omega_i\right)\right] \mathrm{Sin}\omega_{d_i} t - \left[\omega_{d_i}^2 + \left(\xi_i\omega_i\right)^2\right] \mathrm{Cos}\omega_{d_i} t \right\} \quad ,$$

$$\bar{Y}_i(t) = \text{EXP}\left(-\xi_i\omega_i t\right)\left\{\left[\frac{\left(\xi_i\omega_i\right)^2}{\omega_{d_i}} - \omega_{d_i}\right] \text{Sin}\omega_{d_i} t - 2\xi_i\omega_i \text{Cos}\omega_{d_i} t\right\} ,$$

$$\bar{\bar{h}}_i(t) = \bar{Y}_i(t),$$

$$p_i(t) = \phi^T F(t), \qquad i = 1,2,\ldots,N.$$

Therefore, substituting equations (23) and (24) into equation (22) gives G(t). Consequently the solution of equation (20) can be written as:

$$z_{ij}(t) = \int_{t_0}^{t} h_i(t-\tau)G_{ij}(\tau)d\tau \tag{25}$$

where $h_i(t)$ is the same as that of eq. (18). Notice that the initial conditions in eq. (20) are zero. This is due to the fact that the initial conditions of (18) are known constants.

If we assume that [C] is expressed as a linear combination of [K] and [M], then eq. (22) can further be simplified. Namely,

$$C = 2\alpha K + 2\beta M, \tag{26}$$

where α and β are known constants. Hence in equation (17), the percentage of damping, ξ_N, can be expressed as:

$$\xi_i = \alpha\omega_i + \frac{\beta}{\omega i} , \quad i = 1,2,\ldots,N \tag{27}$$

To further simplify equation (22) under this assumption, let us consider the following three cases:

1) The vector θ contains only θ_M, i.e., only estimation of mass parameters is undertaken. Then

$$G(t) = \phi^T [F_\theta - \widehat{M_\theta \phi(\ddot{\eta}+2\beta\dot{\eta})}] . \tag{28A}$$

2) The vector θ contains only the subvector θ_K. Then

$$G(t) = \phi^T[F_\theta - \widehat{K_\theta \phi(\eta + 2\alpha\dot{\eta})}] \tag{28B}$$

3) Finally if the vector $\theta = [\alpha\ \beta]^T$,

$$G(t) = \langle \phi^T F_\alpha - 2\Lambda\dot{\eta},\ \phi^T F_\beta - 2I\dot{\eta} \rangle^T \tag{29}$$

If the input F(t) is not a function of θ, then F_θ would be omitted all through this discussion. Once the solution of equation (25) is obtained, the Fisher Matrices may be obtained as in equation (13). Hence

$$Q = \sum_{k=1}^{m} \int_0^T \frac{z^T \Phi^T r_{s_k}^T r_{s_k} \Phi z}{\psi^2(t)}\, dt \tag{30}$$

We note that the summation form of relation (30) is particularly amenable to the maximization of the trace norm of Q.

EXAMPLE

To illustrate some of the ideas of the previous section, consider the problem of finding the optimal sensor location (OSL) in a structural system modelled by the two-degree-of-freedom system (shown in Figure 1) which is subjected to the base excitation of f(t).

The governing differential equation of motion can be expressed as

$$\ddot{M} + C\dot{X} + KX = -W\ f(t) \tag{31}$$

where $X = \langle x_1\ x_2 \rangle^T$, $C = \alpha K$, $W = \langle Am\ m \rangle^T$ and the matrices M and K are

$$M = \begin{bmatrix} A & 0 \\ 0 & 1 \end{bmatrix} m\ , \text{ and } K = \begin{bmatrix} B+1 & -1 \\ -1 & 1 \end{bmatrix} k\ .$$

A case study for locating sensors to best identify (1) the mass ratio, A, of the first to the second floor and (2) the stiffness ratio, B, of the first to the second floor, will be presented.

Let s_1 denote the lower mass location and s_2 the upper mass location. The selection between the locations can be equated to determining the one non-zero element of the [1x2] selection matrix, S, with the measurement H(t) defined by

$$H(t) = SX + V(t)\ ,$$

where, V(t) is Stationary Gaussian White Noise (S G W N) with $\psi(t) = \psi_o$.

If S = [1 0] the lower mass is selected for measurement; if S = [0 1] the upper mass is selected. The location s_1 would then be preferred over the location s_2 for identifying the parameter A, if $Q[T,s_1] > Q[T,s_2]$, where T is the time that the measurement is taken,

$$Q_1[T] \triangleq Q_1[T,s_1] = \frac{1}{\psi_o^2} \int_0^T \left(\frac{\partial x_1}{\partial A}\ \frac{\partial x_2}{\partial A} \right) \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \frac{\partial x_1}{\partial A} \\ \frac{\partial x_2}{\partial A} \end{Bmatrix} dt$$

$$= \frac{1}{\psi_o^2} \int_0^T \left(\frac{\partial x_1}{\partial A} \right)^2 dt, \tag{32A}$$

and

$$Q_2(T) \triangleq Q[T,s_2] = \frac{1}{\psi_o^2} \int_0^T \begin{pmatrix} \frac{\partial x_1}{\partial A} & \frac{\partial x_2}{\partial A} \end{pmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} \frac{\partial x_1}{\partial A} \\ \frac{\partial x_2}{\partial A} \end{Bmatrix} dt$$

$$= \frac{1}{\psi_o^2} \int_0^T \left(\frac{\partial x_2}{\partial A}\right)^2 dt \; . \qquad (32B)$$

Since only one parameter is being estimated the Fisher matrices reduce to scalars.

The dependence of the OSL on various types of the base excitations can be studied now. Let us for this presentation consider ground acceleration in the form of a delta function, i.e., $f(t) = \delta(t)$.

In this case, closed form solutions for Q_1 and Q_2 can be obtained.

For the OSL problem for the "best" (minimum covariance) identification of the parameter A (given the parameters B and α) using an impulsive base excitation, Figure 2-A shows the plots of the ratio of the information matrices $Q_1(T)/Q_2(T)$, for T = 50 secs, for various values of the parameters A (which is to be identified) and $\alpha^* \triangleq \alpha\omega_o$, where $\omega_o \triangleq \sqrt{k/m}$. Points on the graph with ordinates greater than unity indicate the optimal location to be the lower mass level and vice versa. The graphs indicate that the optimal location in most cases, for the range of A considered, is the upper mass level. However, we observe that for some small values of A and α^* the OSL is the lower level. We note, interestingly enough, that the optimal sensor location for identification of A actually depends not only on the actual values of B and α which are presumably known, but also on the value of the parameter A itself which is to be identified! Thus to be able to ascertain the optimal sensor location some a priori assessment of A is necessary.

Figure 2-B shows that the optimal location for identification of the parameter B (given A and α), using an impulsive base input, is again the upper mass level for the range of B values considered. For larger B values, however, and $\alpha^*>0.05$, the trend appears to be more and more in favor of the upper mass. This seems intuitively correct, for as B becomes larger, the lower part of the system becomes stiffer and the OSL would be the upper mass level.

Figure 2-C is associated with the OSLP for estimating the parameter B using a sinusoidal base excitation, $f(t) = a \sin \omega t$. The figure shows that as the normalized driving frequency $\gamma = \omega/\omega_o$ varies, the OSL changes. For this example the Fisher Matrices can be computed in closed form. For the estimation of B, (given A and $\alpha^* = 0$) the dimensionless driving frequency $\gamma = \sqrt{1+1/A}$ yields no information on B from records at either of the two mass levels. The responses at the two mass levels yield identical amounts of information on B at $\gamma = 0$ and $\gamma = \sqrt{2}$ for $A \neq 1$, as indicated by the values of $Q_1/Q_2 = 1$ at these frequencies. The value of $Q_1/Q_2 = 0$ at $\gamma = 1$ is indicative of the fact that the upper mass level is a far better location for a sensor when estimating B with $\alpha^* = 0$. Figure 2-D shows the mean value of the ratio Q_1/Q_2 for a random Gaussian white noise base excitation together with the 1-σ band. The OSL appears to be at the upper mass level for identification of A.

CONCLUSIONS AND DISCUSSION

This paper presents a general methodology for determining the optimal sensor locations in dynamic systems for obtaining records which would enable the "best" (minimum covariance) identification of a given set of unknown parameters in the system. The technique utilizes the concept of an efficient estimator to uncouple the identification from the optimization problem. In order to present the basic idea in as clear a fashion as possible, we have restricted the discussion in this sequel to linear systems.

The method has been illustrated by application to a two degree of freedom system. Though the results presented here for the simple system chosen form only a first step towards acquiring a detailed understanding of the OSL problem, the following conclusions appear to be relevant at this time:

(1) The OSL for a given system heavily depends on the class of forcing functions used for obtaining response data. In this study, an impulsive base motion is considered.

(2) The OSL for linear dynamic systems is independent of the amplitude of the forcing function.

(3) The OSL depends in general on all the values of system parameters. For instance, the OSL for estimating A with minimum covariance depends not only on the actual parameter values B and α but on the value of A itself for the system! This implies that the OSL problem associated with identifying a given parameter (or a set of parameters) in a dynamic system necessitates the knowledge of some a priori estimates of the unknown parameter(s).

(4) The results of our simple example show that the OSL problem may yield solutions which may be difficult to predict on purely heuristic grounds. The OSL appears to depend, even for this relatively simple problem, in a rather complex manner on the actual parameter values of the system and the nature of the base excitation.

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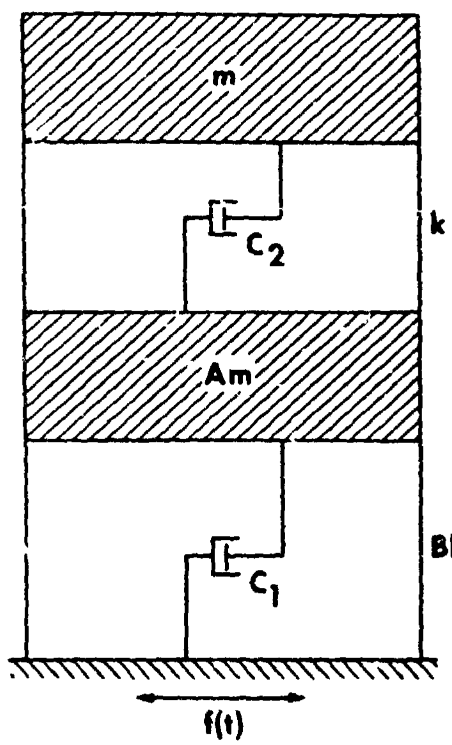

Figure 1. 2-degree-of-freedom generic structural system.

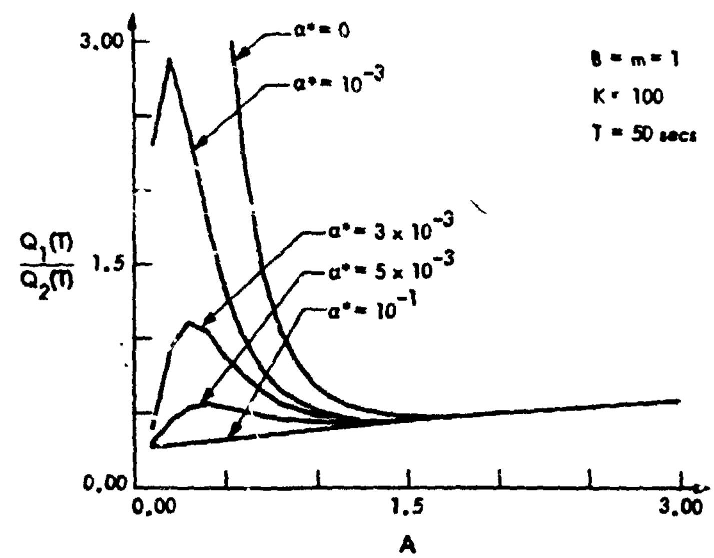

Figure 2-A. Variation of Q_1/Q_2 for various values of the parameter A. Q_1/Q_2 greater than unity indicates that optimal is at lower mass.

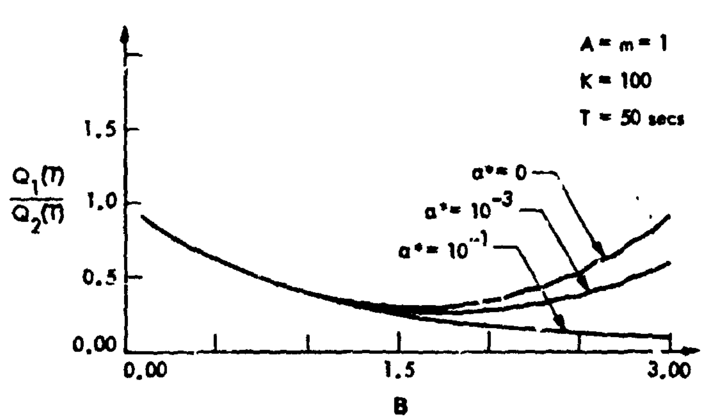

Figure 2-B. Variation of Q_1/Q_2 for various values of B.

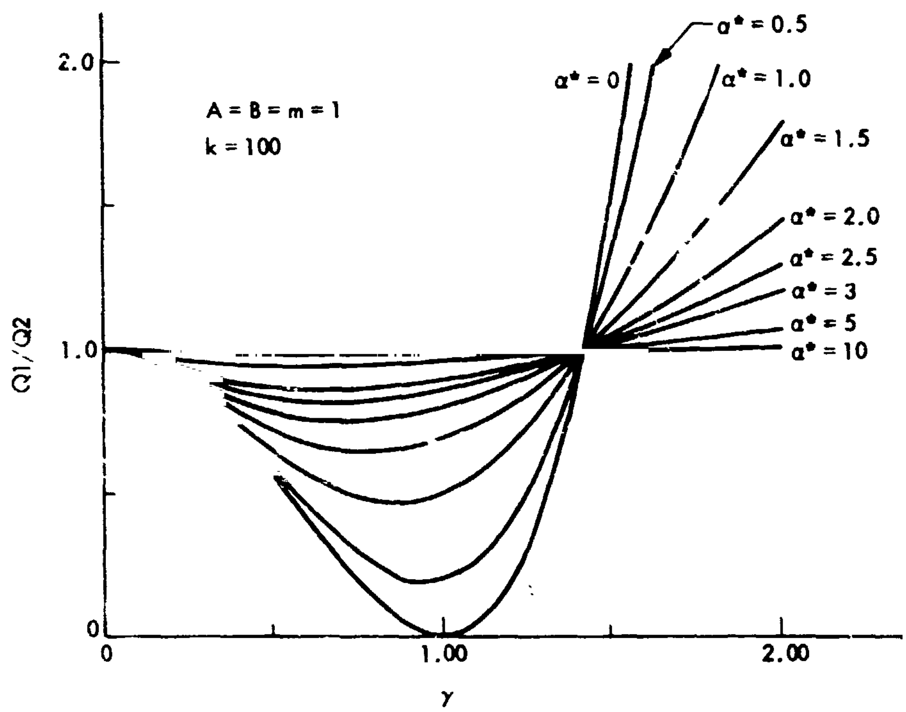

Figure 2-C. Variation of Q_1/Q_2 with $\gamma \triangleq \omega/\omega_0$ for different α^* given A=B=m=1, k=100 and f(t) = Sinωt.

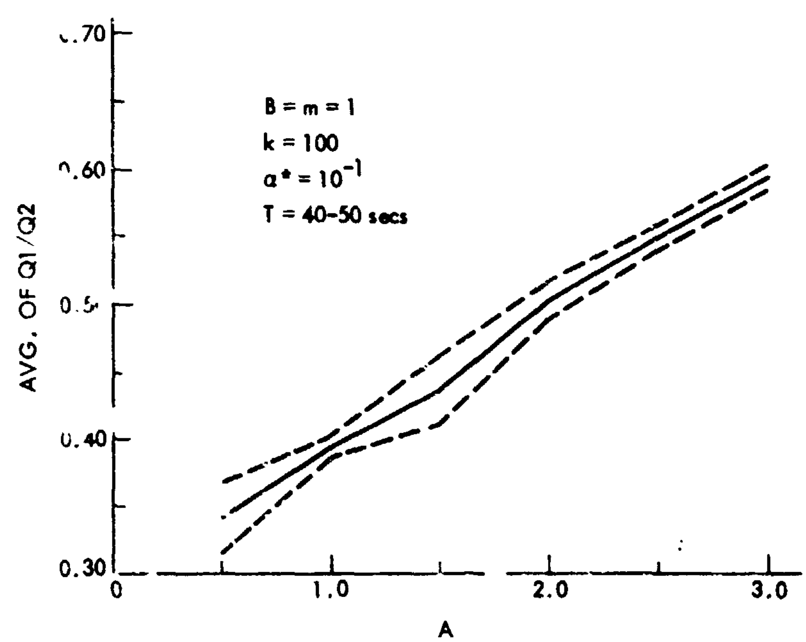

Figure 2-D. Variations in the mean value of $Q_1(T)/Q_2(T)$ and the 1-σ band with different values of A, when B=m=1, k=100, α^*=0.1. The input is Gaussian White Noise. Integration was done over a ten second period.