N85-32439

STRESS AND BUCKLING ANALYSIS

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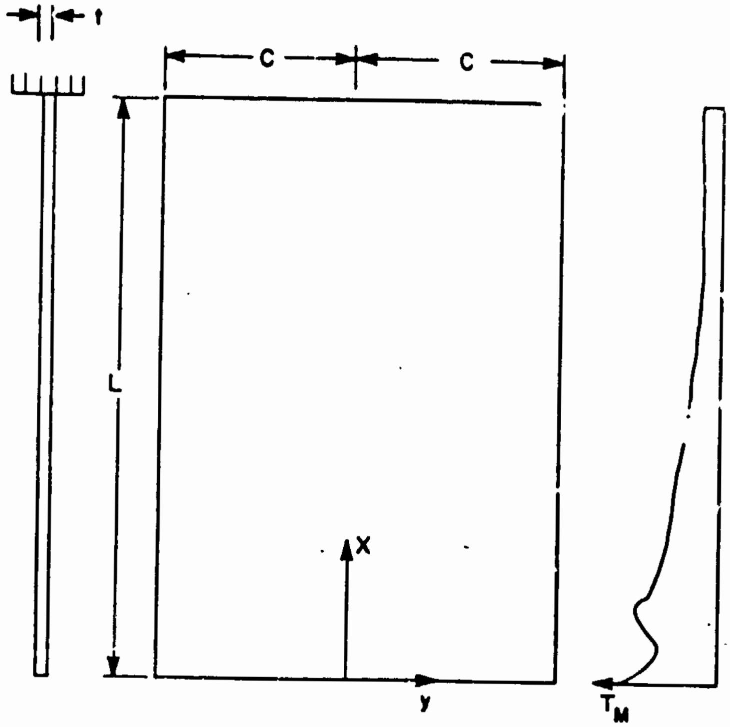
TECHNOLOGY STRESS AND BUCKLING ANALYSIS	**REPORT DATE** 10/3/84
APPROACH MODEL MATERIAL BEHAVIOR MODEL BUCKLING DUE TO THERMAL STRESSES **CONTRACTOR** UNIVERSITY OF KENTUCKY **GOALS** • PROVIDE GUIDANCE BASED ON ANALYSIS FOR THERMAL PROFILES FOR REDUCING STRESSES AND IMPROVING FLATNESS IN WIDE RIBBON. • HAVE RESULTS BE APPLICABLE TO ALL SHEET GROWTH SYSTEMS.	**STATUS** • ELASTIC STRESS AND BUCKLING ANALYSIS COMPLETED FOR CONSTANT MATERIAL PROPERTIES. • CRITICAL SHEET THICKNESS VS. SHEET WIDE COMPARED FOR FOUR THERMAL PROFILES. • RESULTS ARE REASONABLY CONSISTENT WITH EXPERIMENT. SURVEY OF THE MECHANICAL PROPERTIES OF SILICON PRE-BUCKLED STRESSES IN PLASTIC RANGE

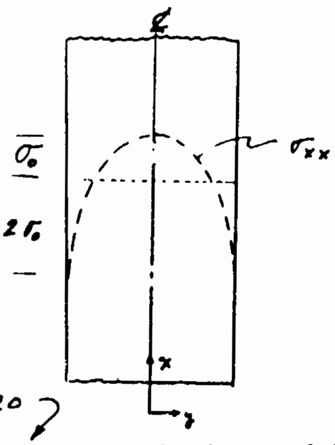
SILICON SHEET



@ FLORIDA (1983)

ASSUME: $F = (c^2 - y^2)\, f(x)$ (pre buckled)

ELASTIC BUCKLING



WAS ZERO — NEW

$$w = W_0(x) + y\,W_1(x) + y^2\,W_2(x)$$

TWIST "curling"

COMBINE

USE ANALYTICAL METHODS TO OBTAIN CRITICAL THICKNESS

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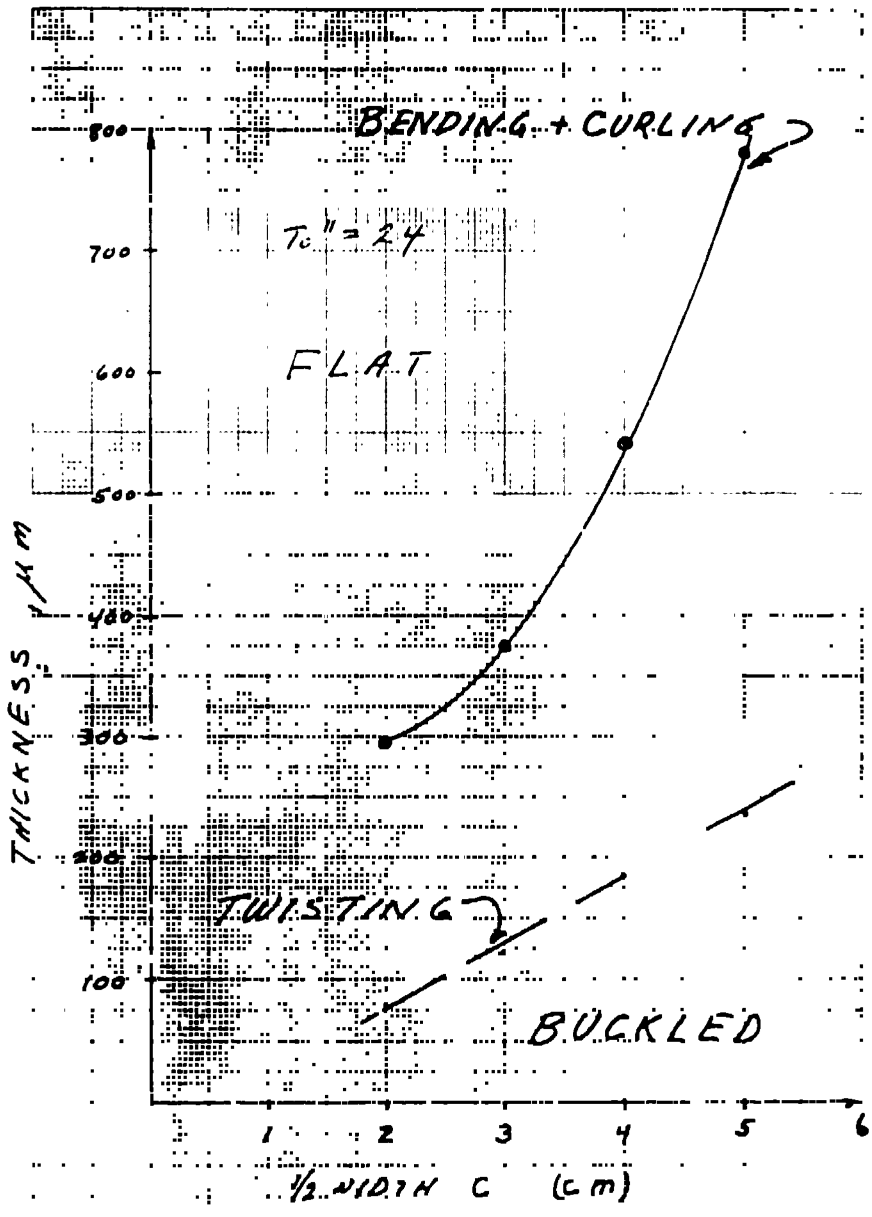
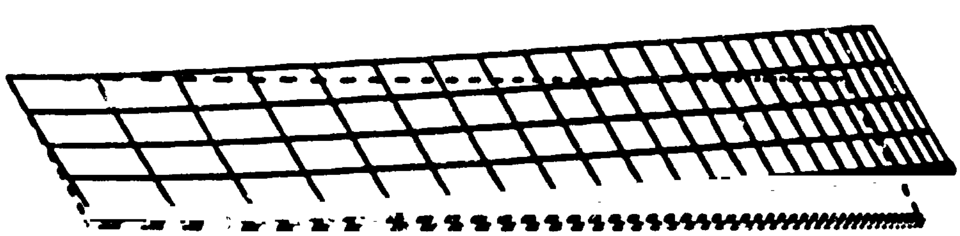


Calculated Buckled Web Shape



Sumino

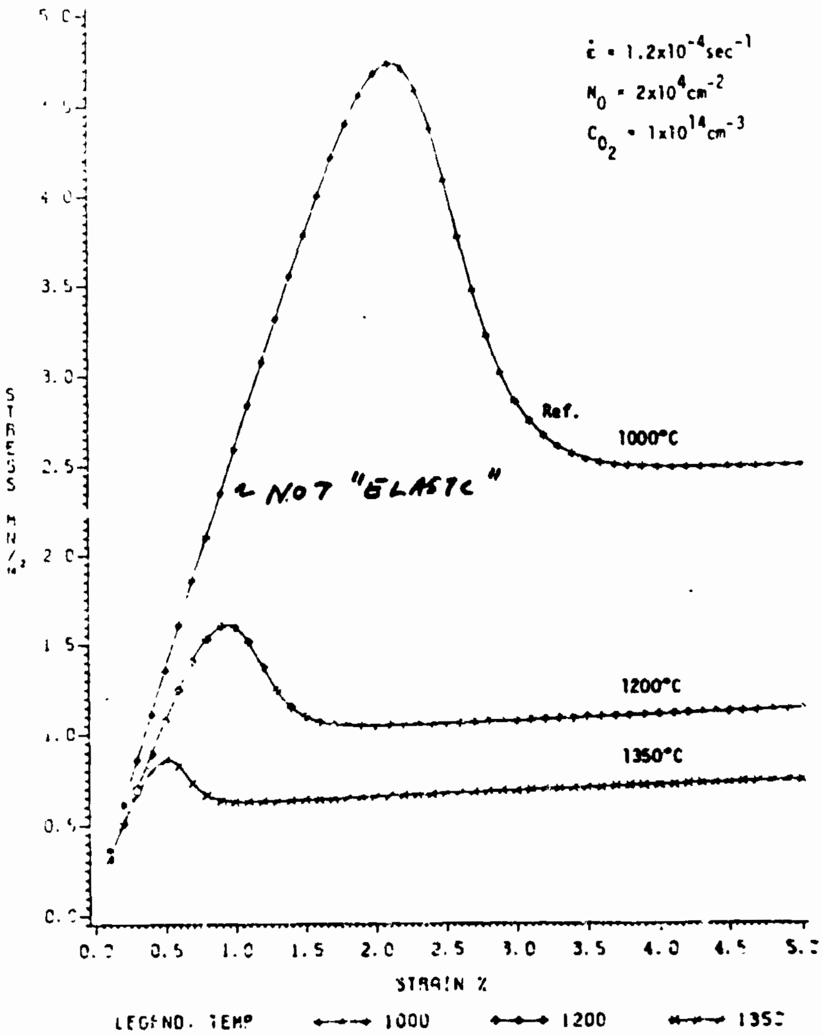
$$\dot{\epsilon}^{PL} = \frac{bB}{\tau_0^m} N_m(\tau - D\sqrt{N_m})^m e^{-Q/RT}$$

MOBILE DISLOCATION DENSITY — BACK STRESS

$$\dot{N}_m = K_1 N_m(\tau - D\sqrt{N_m})^{\lambda+m} e^{-\,)/RT}$$

DENSITY CHANGES

Stress vs Strain for Si (Temperature in °C)



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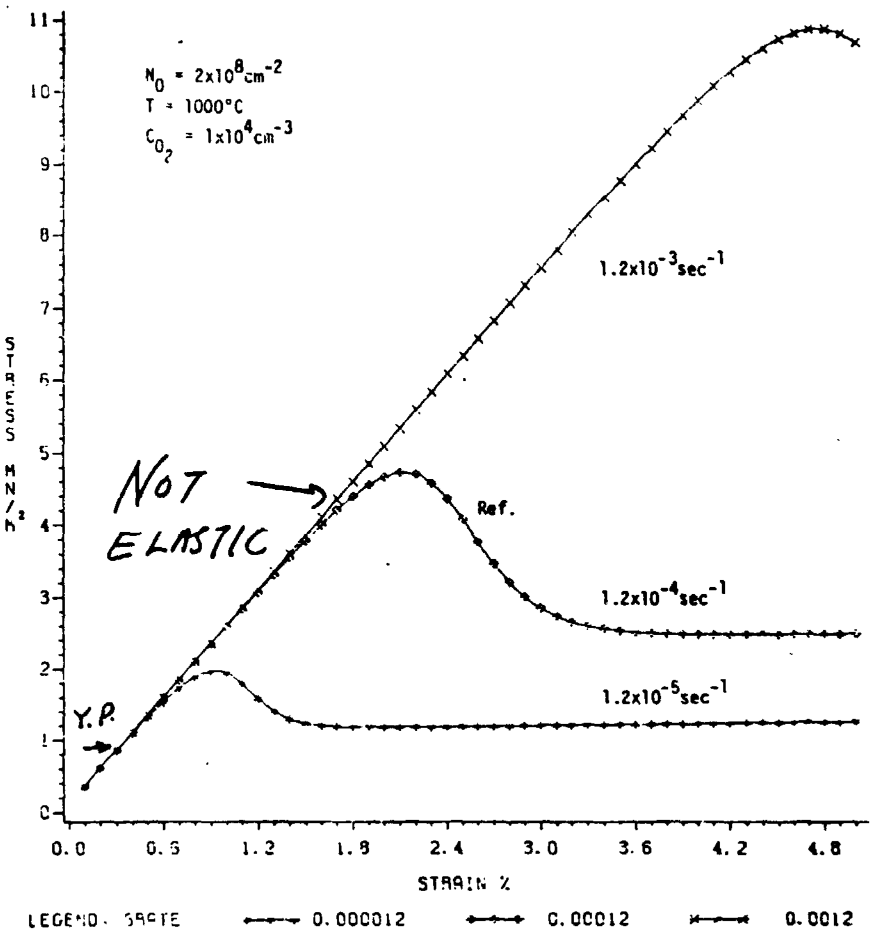
Stress vs Strain for Si (Strain Rate 1/sec)



TEMPERATURE=1000 C

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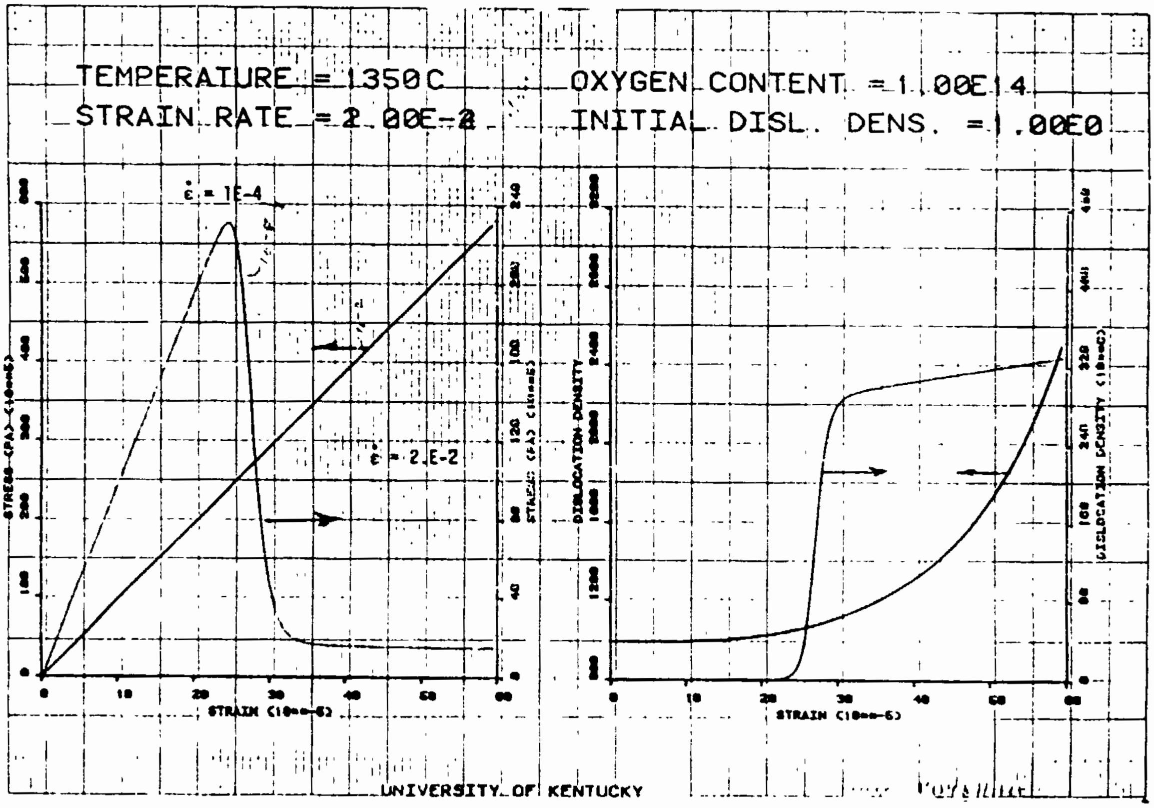
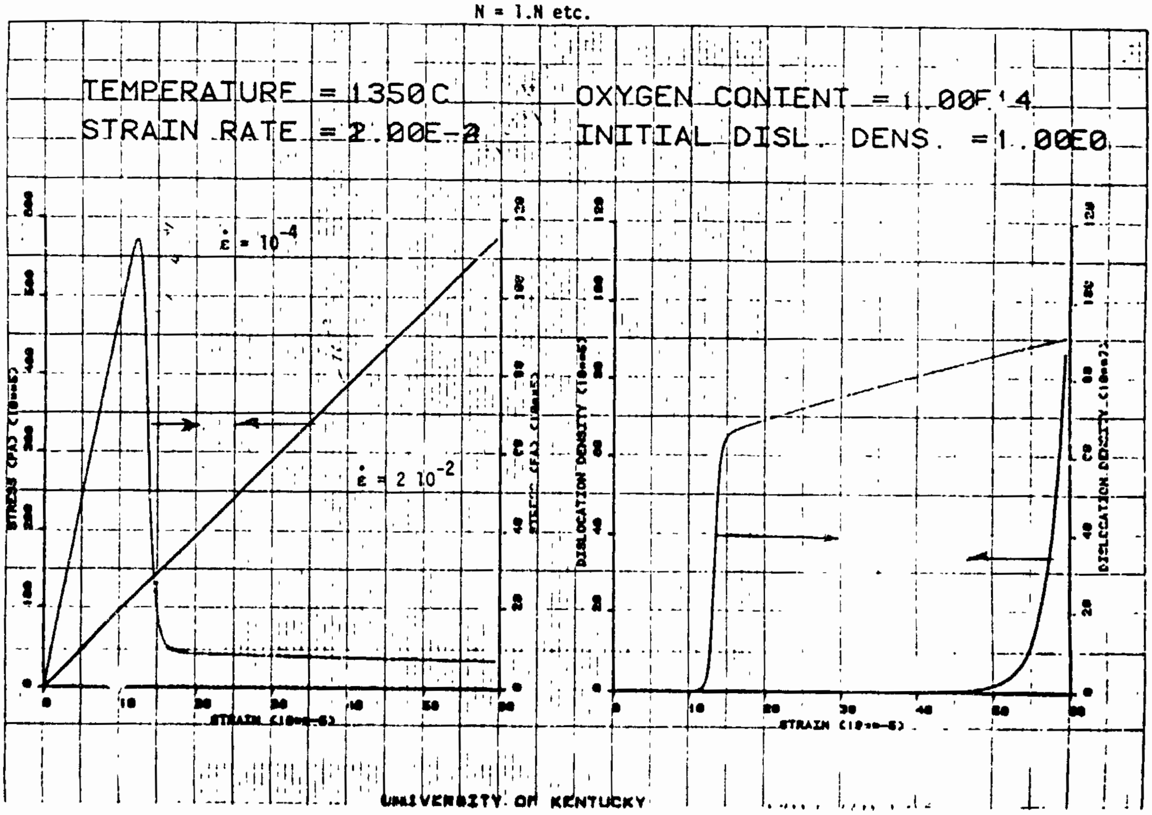
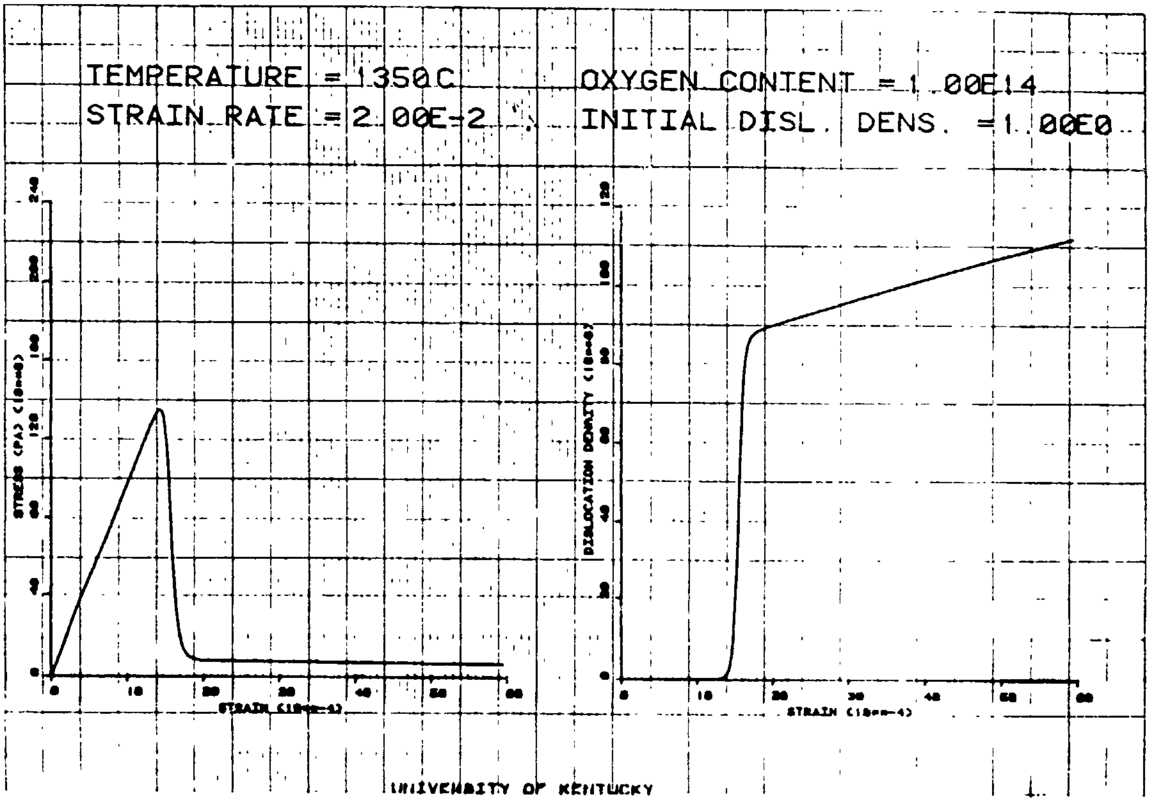
$\dot{N}$ = .1N etc.



$\dot{N}$ = 1.N etc.



Ṅ = .1N etc.


TEMPERATURE = 1350C
STRAIN RATE = 2.00E-2
OXYGEN CONTENT = 1.00E14
INITIAL DISL. DENS. = 1.00E0
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Pre-Buckled Stresses

Elastic

APPENDIX A

The equations governing the stress in a ribbon consist of those which reflect that the material is in equilibrium, that the deformations are compatable and that there is a material constitutive relation. Assuming that (a) one can neglect the three stresses σ_{iz}, (b) that the material is elastic and that Young's modulus and Poissons ratio are constant, the governing equations are

Equilibrium

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = 0 .$$

$$\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = 0 . \qquad \text{(A-1)}$$

Compatability

$$\frac{\partial^2 \epsilon_{xx}}{\partial y^2} + \frac{\partial^2 \epsilon_{yy}}{\partial x^2} = 2 \frac{\partial^2 \epsilon_{xy}}{\partial x \partial y} \qquad \text{(A-2)}$$

Constitutive relations

$$\epsilon_{xx} = \frac{\sigma_{xx}}{E} - \frac{\nu}{E} \sigma_{yy} + \alpha T \qquad \text{(A-3)}$$

$$\epsilon_{yy} = \frac{\sigma_{yy}}{E} - \frac{\nu}{E} \sigma_{xx} + \alpha T$$

$$\epsilon_{xy} = \frac{(1+\nu)\, \sigma_{xy}}{E} \qquad \text{(A-4)}$$

Equations (A-1) combine to yield

$$\frac{\partial^2 \sigma_{xx}}{\partial x^2} = \frac{\partial^2 \sigma_{yy}}{\partial y^2} \qquad \text{(A-5)}$$

While Eqs. (A-2) through (A-4) yield

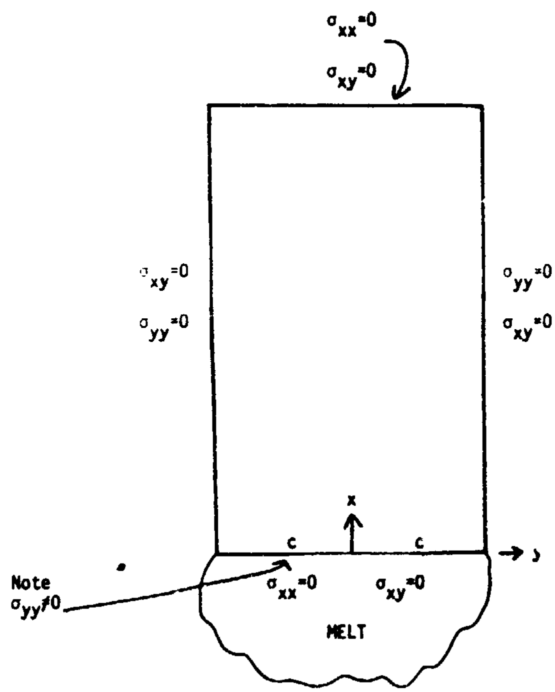
$$\frac{\partial^2 \sigma_{yy}}{\partial x^2} = - \frac{\partial^2 \sigma_{xx}}{\partial x^2} - \frac{\partial^2 \sigma_{yy}}{\partial y^2} - \frac{\partial^2 \sigma_{xx}}{\partial y^2} - \frac{\alpha E \partial^2 T}{\partial x^2} \qquad \text{(A-6)}$$

Second order central differences equivalents of these equations are:

$$\sigma_{xx}(I+1,J) = 2\sigma_{xx}(I,J) - \sigma_{xx}(I-1,J) + [\sigma_{yy}(I,J+1) - 2\sigma_{yy}(I,J) + \sigma_{yy}(I,J-1)]\,\Delta x^2/\Delta y^2 \qquad \text{(A-5a)}$$

and

$$\sigma_{yy}(I+1,J) = 2\sigma_{xx}(I,J) - \sigma_{yy}(I-1,J) - \sigma_{xx}(I+1,J) + 2\sigma_{xx}(I,J) - \sigma_{yx}(I-1,J) - \frac{\alpha E \partial^2 T}{\partial x^2}\Delta x^2 - \left(\frac{\Delta x}{\Delta y}\right)^2 [\sigma_{xx}(I,J+1) - 2\sigma_{xx}(I,J) + \sigma_{xx}(I,J-1) + \sigma_{yy}(I,J+1) - 2\sigma_{yy}(I,J) + \sigma_{yy}(I,J-1)] \qquad \text{(A-6a)}$$



Assumes $t > t_r$

(NO BUCKLING)

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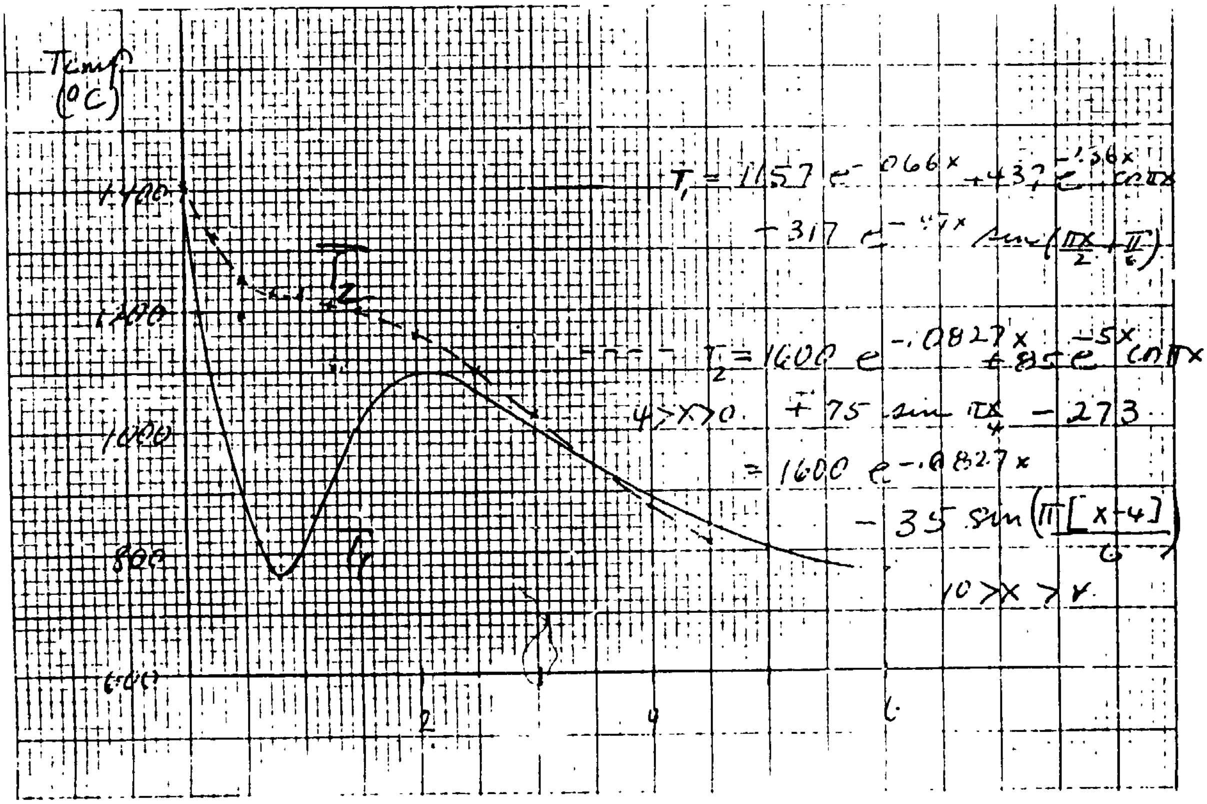
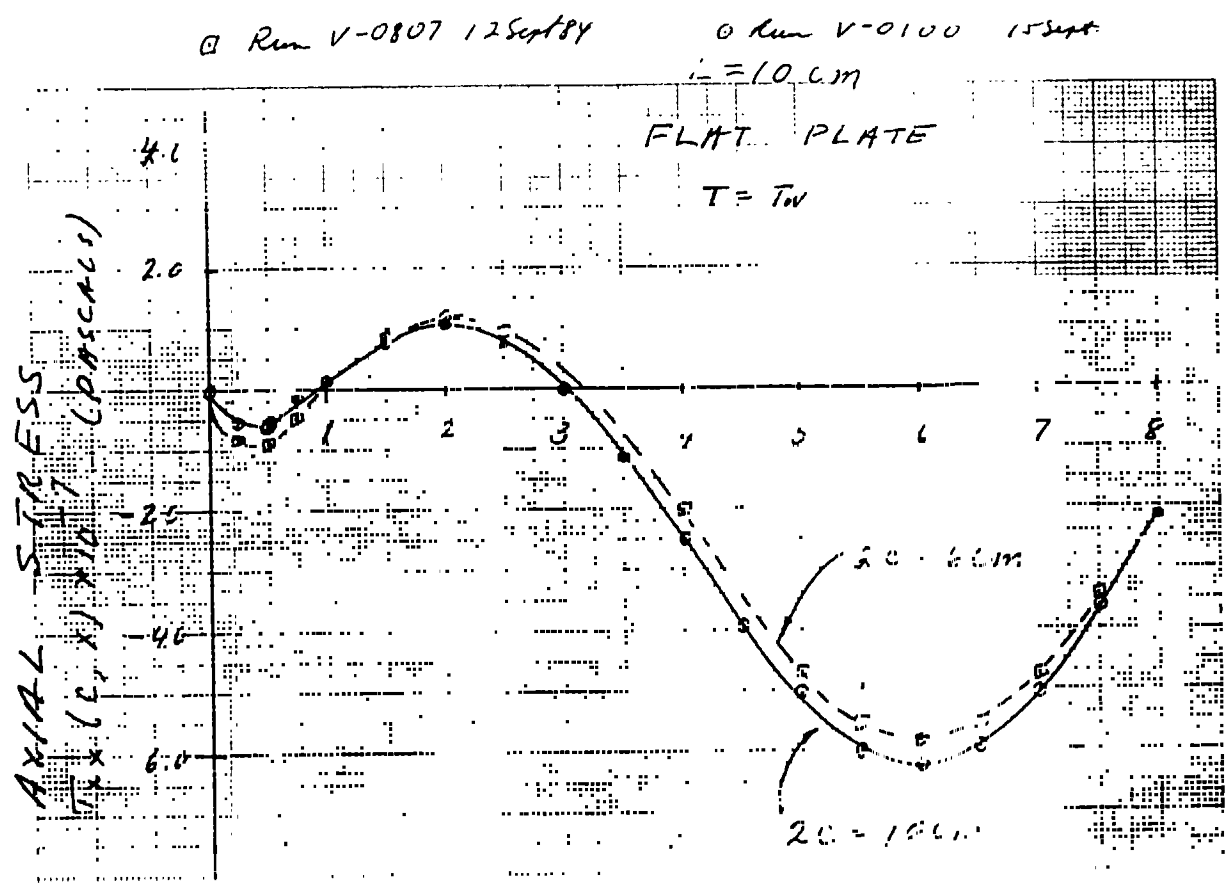




We assume the Sumino model of viscoplastic behavior, i.e.

$$\dot{\varepsilon}^{PL}_{ij} = \frac{bB}{\tau_0^m} \frac{N_m(\sqrt{J_2} - D\sqrt{N_m})^m e^{-Q/kT}(\sigma_{ij} - \sigma_{KK}\,\delta_{ij}/3)}{\sqrt{J_2}}$$

$$\sqrt{J_2} > D\sqrt{N_m}$$

DISLOCATION DENSITY CHANGES!!

$$\dot{N}_m = K_1\, N_m(\tau - D\sqrt{N_m})^{\lambda+m} e^{-Q/k}$$

MOBILE

NEW: SOME BASIS FOR A "SHAPE" FACTOR

$$K_1 \rightarrow \frac{K_1}{10}$$

λ, b, B, τ_o, m, D, K_1, Q } MATERIAL "CONSTANTS"

$$\frac{\partial^2 \dot{\sigma}_{xx}}{\partial x^2} = \frac{\partial^2 \dot{\sigma}_{yy}}{\partial y^2} \qquad (1)$$

The compatibility equation is

$$\frac{\partial^2 \dot{\varepsilon}_{xx}}{\partial y^2} + \frac{\partial^2 \dot{\varepsilon}_{yy}}{\partial x^2} = 2\,\frac{\partial^2 \dot{\varepsilon}_{xy}}{\partial x \partial y} \qquad (2)$$

$$\dot{\varepsilon}_{ij} = \frac{1+\nu}{E}\,\dot{\sigma}_{ij} - \frac{\nu}{E}\,\dot{\sigma}_{KK}\,\delta_{ij} + \alpha\,\dot{T}\,\delta_{ij} + \dot{\varepsilon}_{ij}^{PL} \qquad (3)$$

Eqs (2) and (3), to yield

$$\frac{\partial^2(\dot{\sigma}_{xx} + \dot{\sigma}_{yy})}{\partial x^2} + \frac{\partial^2}{\partial y^2}(\dot{\sigma}_{xx} + \dot{\sigma}_{yy}) \qquad (4)$$

$$= -\,E\alpha\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\dot{T}$$

$$+ \left(\frac{\partial^2 \dot{\varepsilon}_{xx}^{PL}}{\partial y^2} + \frac{\partial^2 \dot{\varepsilon}_{yy}^{PL}}{\partial x^2} - 2\,\frac{\partial^2 \dot{\varepsilon}_{xy}^{PL}}{\partial x \partial y}\right) E$$

PLASTIC

$$\frac{\partial^2(\sigma_{xx}+\sigma_{yy})}{\partial x^2} + \frac{\partial^2(\sigma_{xx}+\sigma_{yy})}{\partial y^2} = -\alpha E \frac{\partial^2 T}{\partial x^2} -$$

$$+ \int_0^x \frac{E}{v}\left(\frac{\partial^2 \dot{\varepsilon}_{xx}^{PL}}{\partial y^2} + \frac{\partial^2 \dot{\varepsilon}_{yy}^{PL}}{\partial x^2} - 2\,\frac{\partial^2 \dot{\varepsilon}_{xy}^{PL}}{\partial x \partial y}\right) dx$$

PULL RATE

$$\frac{\partial^2 \sigma_{xx}}{\partial x^2} = \frac{\partial^2 \sigma_{yy}}{\partial y^2}$$

$\dot{\varepsilon}_{ij}^{PL}$ = function of stresses and N_m

$$\dot{N}_m = K_1 N_m (\tau - D\sqrt{N_m})^{\lambda+m} e^{-Q/RT}$$

∴ SOLVE VIA INTERATION!!!!

"0" = ELASTIC
⋮
"20"

OUTPUT STRESSES (x,y)
STRAIN RATES (x,y)
VERY NEW ⟶ DISLOCATION DENSITY (x,y)

$L = 1$ cm $\quad \dot{\varepsilon} \doteq 10^{-2}\ sec^{-1}$

$C = 2$ cm

$T = T_w \quad \dot{\varepsilon}^{PL} \doteq 10^{-7}\ sec^{-1}$

$\sigma^{el}_{xx_{max}} = .1147 \times 10^{8}$ pascals

$\sigma^{PL}_{xx_{max}} = .1086 \times 10^{8}$ pascals

(Typical)

$N_{init} = 1.0/cm^2$

$N_{final} = 12/cm^2$ at outer edge

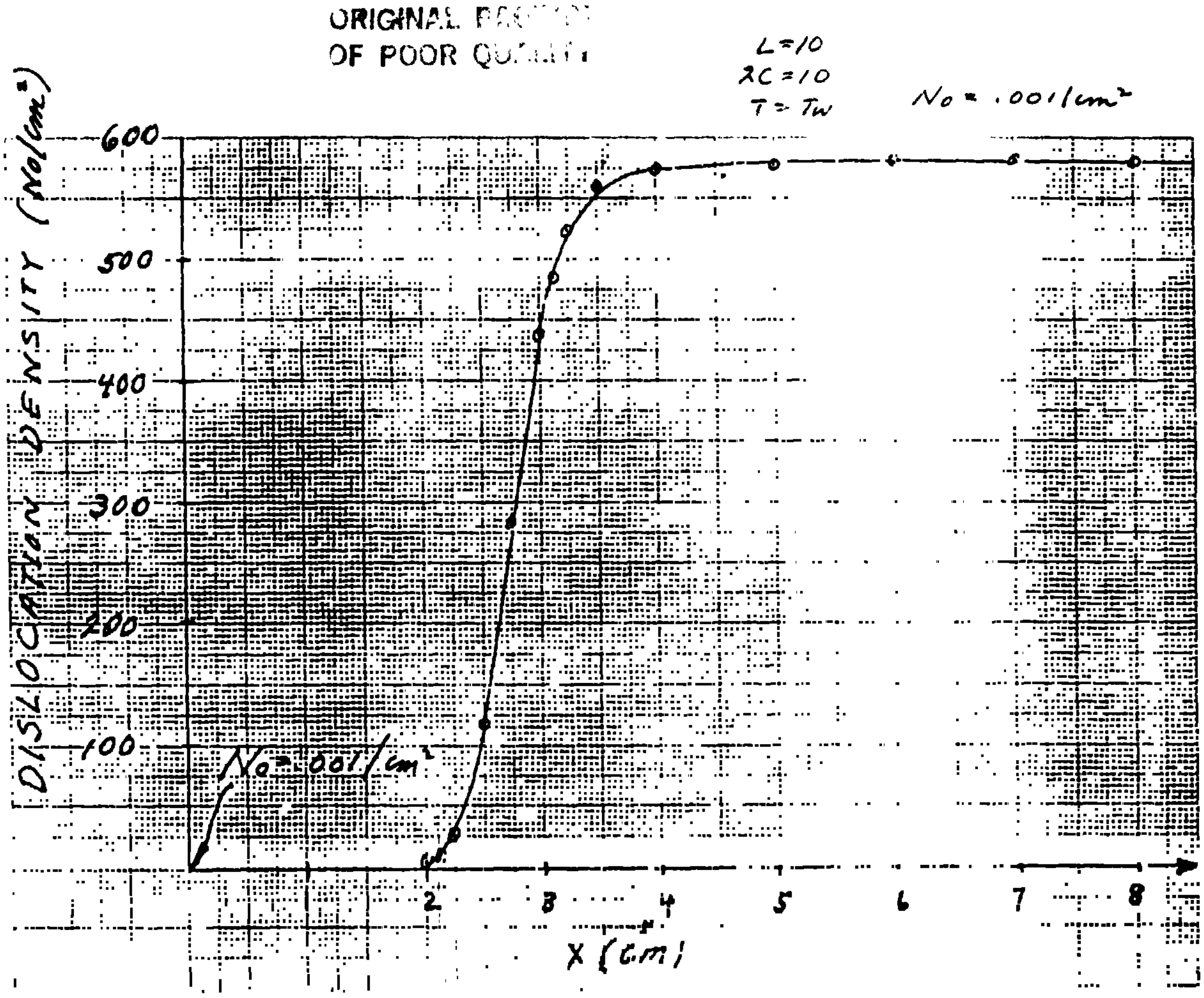
$6/cm^2$ at center

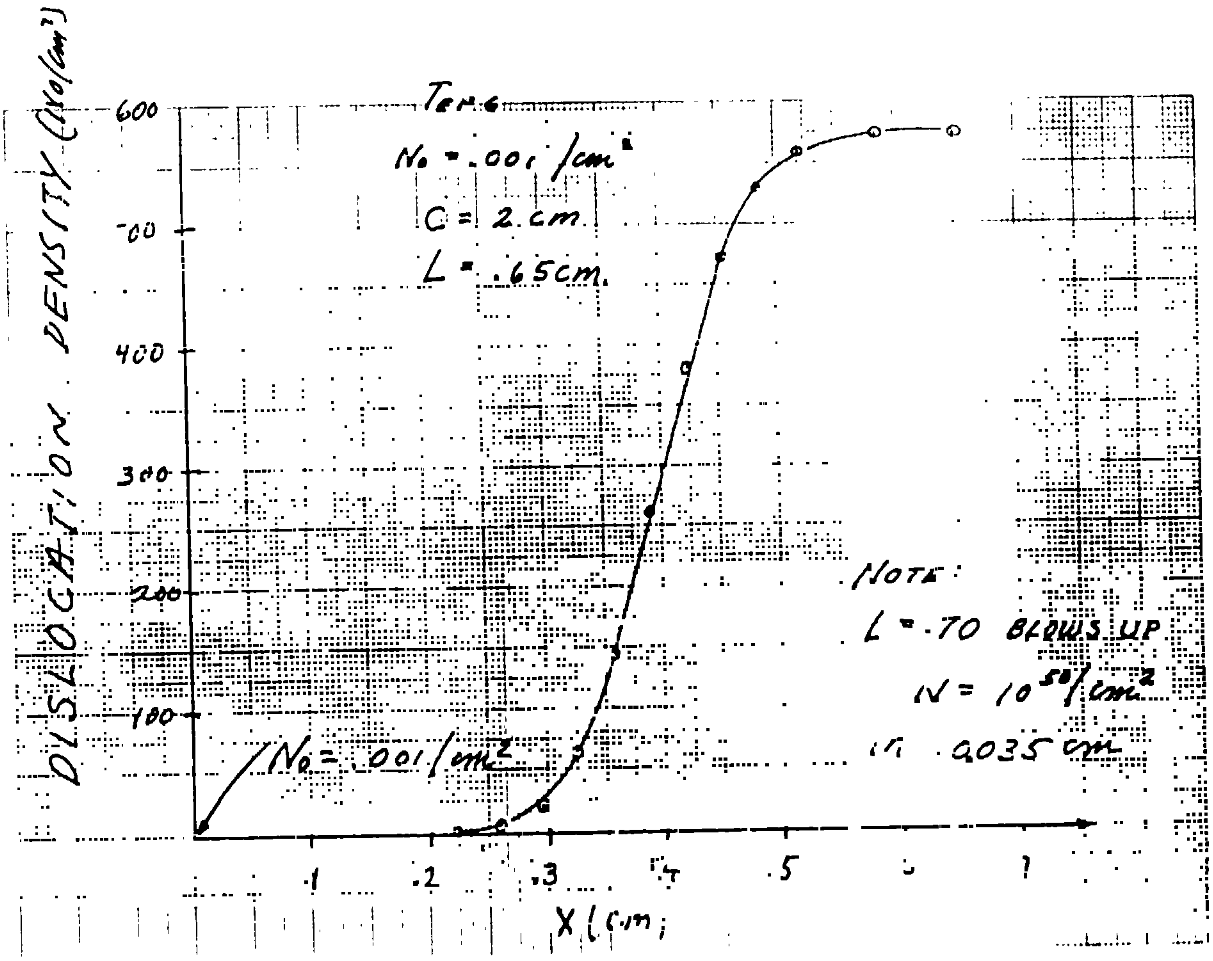
EVERY PROBLEM DIFFERENT

$\dot{\varepsilon} \doteq 10^{-3} \rightarrow 10^{-2}\ sec^{-1}$

$\dot{\varepsilon}^{PL} \doteq 10^{-9} \rightarrow 10^{-4}\ sec^{-1}$

$\rightarrow 10^{-3}\ sec^{-1}$ once in awhile


L=10
λC=10
T>Tw
No = .001/cm²
DISLOCATION DENSITY (No/cm²)
600
500
400
300
200
100
No=.001/cm²
2
3
4
5
6
7
8
X (cm)



DISLOCATION DENSITY (No/cm²)
No = .001 /cm²
C = 2. cm
L = .65cm.
NOTE:
L = .70 BLOWS UP
No = .001/cm²
600
400
300
200
100
.1
.2
.3
.5
X (cm)

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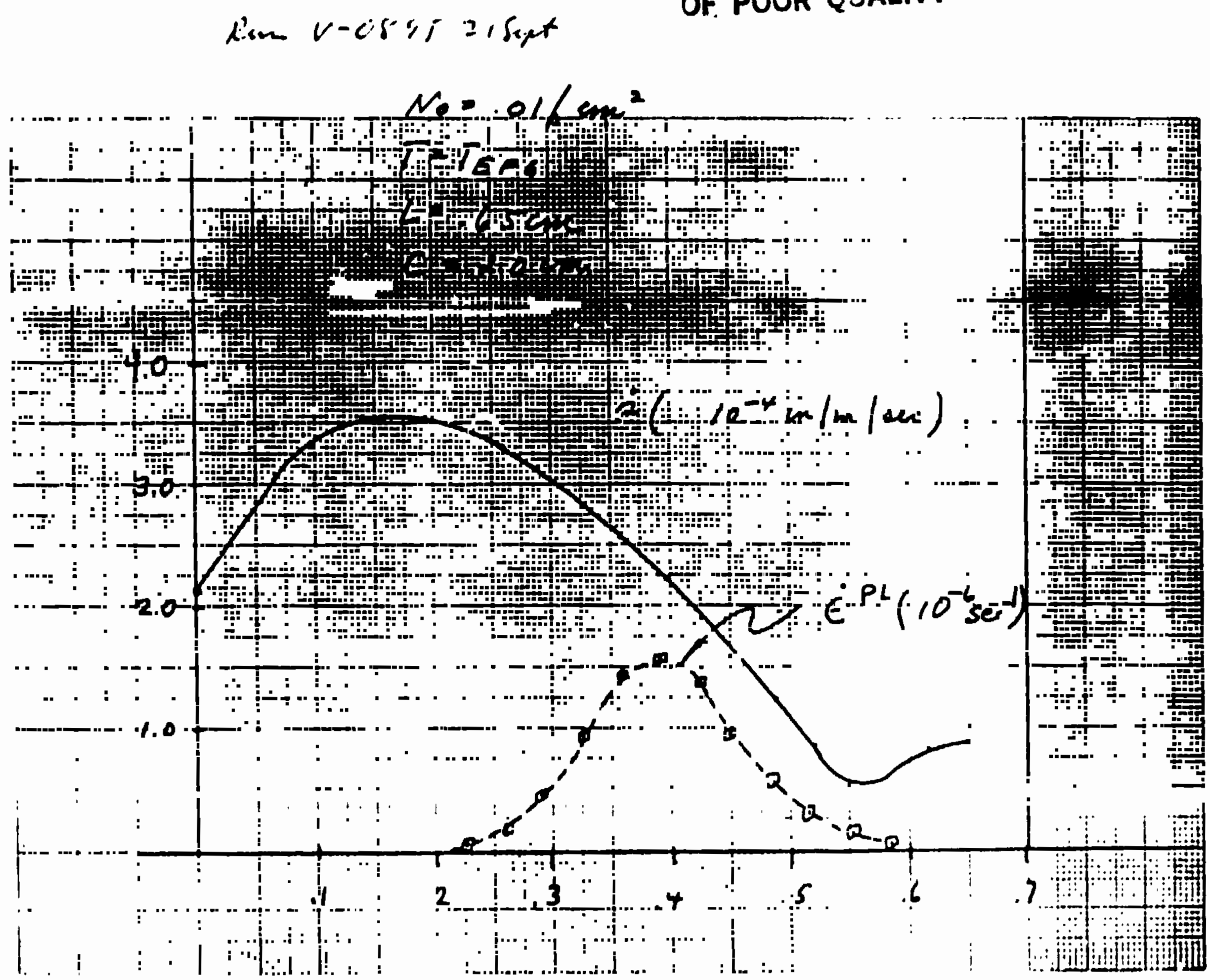

NUMERICAL PROCEDURE
- CONVERGES RAPIDLY: change in σ is $\doteq 10^{-4}\sigma$
- DOES NOT CONVERGE

NOTE 1: ELASTIC REGION NO PROBLEM

NOTE 2: DOES NOT CONVERGE
MEANS $\dot{\epsilon}^{PL} = 10^3\ sec^{-1}$
$N_m = 10^{50}/cm^2$

NOTE 3:
MAJOR OBSERVATION
- L = .65 cm CONVERGES
- L = .70 cm DOES NOT

L = 0.5 → 10. CONVERGES WITH [illegible] TEMP PROFILE

FOR STRESSES USE ELASTIC

FOR RESIDUAL STRESSES

PLASTICITY

$\dot{\epsilon}^{PL}_{ij}(x,y)$ ← THIS IS PROBLEM.

PLASTICITY IS SMALL BUT IS JUST AS BAD AS A LOT.

$\dot{\epsilon}$ is HIGH σ_{ij} ARE HIGH

$T \doteq 750°$ x ≈ .75 cm

MODEL INADEQUATE

Re: DEFORMATION MODE CHANGES (twins)

Problems and Concerns

1. N_O at MELT INTERFACE (x=0)
2. VALIDITY OF CONSTRANTS FOR SUMINO MODEL FOR RIBBON
3. SHOULD WE THINK "TWINS"?
4. SHOULD WE WORK OTHER PROFILES?
5. ROLE OF CHANGING "L"