

2.5A INTERPRETATION OF RADAR RETURNS FROM THE MESOSPHERE

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INTRODUCTION

Since the first study of VHF radar signals from the mesosphere by WOODMAN and GUILLEN (1974) it has been clear that neutral atmosphere turbulence play a central role in generating the refractive index irregularities that backscatter the radio waves. It is generally believed (HARDY, 1972; ROTTGER, 1980, LIU and YEH, 1980) that an increase in the turbulent energy dissipation rate will result in a decrease in signal correlation time and an increase in scattered signal power. Thus, in turbulence-generated radar echoes a negative correlation between echo power and signal correlation time is expected (LIU and YEH, 1980). We will in the following adopt the notation P/C correlation for this relationship.

It has become apparent that the P/C correlation changes as a function of altitude (RASTOGI and BOWHILL 1976b; HARPER and WOODMAN, 1977). The P/C correlation is negative in the upper part of the mesosphere as expected of turbulence-generated irregularities, whereas it is largely positive in the lower mesosphere. Similar changes in the P/C correlation have also been observed in the stratosphere/troposphere region (ROTTGER and LIU, 1978). RASTOGI and BOWHILL (1976a,b) attempted to explain the positive P/C correlation observed in the lower mesosphere in terms of stronger turbulence occurring in narrower layers. However, LIU and YEH (1980) have argued that the effect of a narrowing layer is not strong enough to cause the positive P/C correlation. Instead ROTTGER and LIU (1978) suggested that the positive P/C correlation is a manifestation of partial reflection from stratified layers of refractive index gradient.

Partial reflection would explain another observational fact, namely that the scattered signal is aspect sensitive in the lower part of the mesosphere with maximum echo power coming from the vertical direction (FUKAO et al., 1980). On the other hand, perfect stratification would result in partial reflection of the radar waves, and no signal should be observed if the radar probing direction was oblique to the vertical. Since radar echoes, although reduced from that received from the vertical direction, are observed at very oblique angles it has been suggested that stratified layers modified by turbulence is needed to explain the observations from the lower mesosphere.

We shall examine the turbulence theory and assumptions made in previous studies and show that the scattered signal from the mesosphere is compatible with this theory if reasonable assumptions are made. We will show that under situations of stable turbulence the P/C correlation is positive if the radar Bragg wavelength (λ_r) is within the inertial subrange. Finally we will compare the results of the calculations to VHF radar data.

THEORY OF TURBULENCE

Turbulent motions in a fluid like the atmosphere are described by the Navier-Stokes equations, an unclosed set of nonlinear equations with no known general solution. Considerable work has gone into finding special solutions based on assumptions about, and observations of, the physical state of turbu-

lence. One of the assumptions usually made is that the turbulence is isotropic at all scales substantially smaller than the largest (outer) scale. A general understanding of turbulence has emerged (TATARSKII, 1971; TENNEKES and LUMLEY, 1972); however, the resulting equations that describe the turbulent spectra are more a result of limited physical insight and assumptions than of strict mathematical analysis, and thus should be carefully compared to experimental results.

In the inertial subrange of turbulence the velocity and irregularity spectra, $E(k)$ and $E_{\theta}(k)$, respectively, are given by TATARSKII (1971), and TENNEKES and LUMLEY (1972) as follows

$$E(k) = \alpha \epsilon^{2/3} k^{-5/3} \quad (1)$$

and

$$E_{\theta}(k) = \beta N \epsilon^{-1/3} k^{-5/3} \quad (2)$$

Here $\alpha = 1.5$ and $\beta = 0.5$ are experimentally determined constants, N is a variable related to the source of the refractive index gradient and ϵ is the turbulent energy dissipation rate. Note that both spectra follow the $-5/3$ power law in wave number k . The inner scale of the two spectra are defined separately as $\eta = (\nu^3/\epsilon)^{1/4}$ and $\eta_{\theta} = (\gamma^3/\epsilon)^{1/4}$, respectively. Here ν is the kinematic viscosity and γ the thermal diffusivity. For simplicity we shall assume $\nu \sim \gamma$ so that the inner scale (Kolmogorov microscale) of the spectra are equal.

In the dissipation subrange of turbulence the spectral forms are different and given by TENNEKES and LUMLEY (1972) as

$$E(k) = \alpha \epsilon^{2/3} k^{-5/3} \exp[-3/2 \alpha (k\eta)^{4/3}] \quad (3)$$

and

$$E_{\theta}(k) = \beta N \epsilon^{-1/3} k^{-5/3} \exp[-3/2 \beta (k\eta_{\theta})^{4/3}] \quad (4)$$

for the velocity and irregularity spectra, respectively. For a discussion of the validity range of these spectra see TENNEKES and LUMLEY (1972). In what follows only dimensional analysis will be made.

A first inspection of equation (1) and (3) which determines the line-of-sight velocity variations at the Bragg wavelength indicate a continuously increasing velocity as a function of an increase in ϵ , both in the dissipation and the inertial subranges. However, the radar signal correlation time observed at one radar wavelength is related to the sum of all velocity fluctuations at wavelengths larger than the Bragg wavelength. As the scattering irregularities move around, their motions are affected by all turbulent cells of size larger than the Bragg wavelength. Thus we have the relationship between the radar rms velocity v and the turbulent velocity spectrum

$$\langle v^2 \rangle \propto \int_{k_L}^{k_T} E(k) dk \quad (5)$$

Here k_T is the radar Bragg wave number and k_L is the wave number corresponding to the outer scale of turbulence which is given by TENNEKES and LUMLEY (1971) as

$$L \propto \frac{v^3}{\epsilon} \quad (6)$$

V is the shear velocity-difference across the turbulent layer. Assuming that there is an inertial subrange and that $k_e \gg k_L$ we get by integrating equation (5) and substituting equation (6)

$$\langle v^2 \rangle \propto \epsilon^{2/3} k_L^{-2/3} \propto v^2 \quad (7)$$

Equation (7) shows that v is directly proportional to the velocity difference across the turbulent layer. Rearranging equation (6) we get

$$\epsilon \propto v^2 \frac{v}{L} \quad (8)$$

For further simplification we need to find a relationship between velocity difference and the layer thickness L .

The Richardson number is defined as

$$R_i = \omega_B^2 / (dV/dz)^2$$

where ω_B is the Brunt-Vaisala frequency, and the average R_i across the turbulent layer is $R_i = \omega_B^2 / (V/L)^2$. The critical Richardson number for onset of turbulence is known experimentally to be $R_i = 1/4$. The Brunt-Vaisala frequency is also a constant at a given altitude so that the ratio V/L is a constant and equation (8) reduces to

$$\epsilon \propto v^2$$

and thus from (7) and (8) we get

$$v \propto \epsilon^{1/2} \propto T^{-1} \propto v \quad (9)$$

here T is the signal correlation time. Since the line-of-sight rms velocity is proportional to the square root of the energy dissipation rate, and the signal correlation time is inverse proportional to the rms velocity, we conclude that the signal correlation time is proportional to $\epsilon^{-1/2}$ regardless of whether the Bragg wavelength is in the inertial or dissipation subrange.

To calculate the relationship between radar echo power and energy dissipation rate we first need to consider the factor N in equations (2) and (4). To do so we need physical insight in the meaning of N . N is a measure of the dissipation of refractive index fluctuations. Assuming for the time being that the refractive index fluctuations are due to temperature (θ) variations and following TATARSKII (1971) we have

$$N = \gamma \frac{\partial \theta}{\partial z} \frac{d\theta}{dz}$$

In a steady state N is also a measure of the combined heat sources of a turbulent region. In considering equation (2) both TATARSKII (1971) and HARDY (1972) assumed a constant overall temperature gradient through the turbulent layer. However, the effective thermal diffusion coefficient (γ) changes order of magnitude with the onset of turbulence, so to maintain a constant temperature gradient it is required that the source of the turbulent irregularity structures (N) change as a function of ϵ . This leads to the following relation between the radar scattering cross section σ and ϵ given by HARDY (1972)

$$\sigma \propto \epsilon^{2/3}$$

This indicates a positive correlation between radar echo power and turbulent energy dissipation and led ROTTGER (1980) to conclude that a negative P/C correlation existed in the inertial subrange. However, the requirement that the source of temperature fluctuations changes in order to keep up with the

changing turbulent energy dissipation rate seems physically unreasonable. It seems much more reasonable to assume that the source is independent of the turbulence. In this case N is independent of ϵ and onset of turbulence produces a decrease in temperature gradient across the turbulent layer to compensate for the increase in effective diffusion. Thus inside the inertial subrange we get from equation (2) the following relationship

$$E_{\theta}(k_r) \propto \epsilon^{-1/3}. \quad (10)$$

For a backscatter radar that operates at a fixed frequency with Bragg wave number k_r , it can be shown that the scattered power $P(k_r)$ is proportional to $E_{\theta}(k_r)$ (OTTERSTEN, 1969). Combining equations (9) and (10) gives the relationship $P \propto T^{2/3}$. Thus we find that there should be a positive correlation between radar echo power and signal correlation time if the radar Bragg wavelength is within the inertial subrange of the turbulent spectrum.

If, on the other hand, the radar operates in the dissipative subrange of the turbulence spectrum we have to examine equation (4) to obtain the relationship between the energy dissipation rate and the spectral power E_{θ} . Substituting for η_0 in equation (4) and again assuming that N is a constant we get

$$E_{\theta}(k_r) \propto \epsilon^{-1/3} \exp(-K \epsilon^{-1/3}) \quad (11)$$

where K is a constant. Considering equation (11) we conclude that if the radar Bragg wavelength is sufficiently inside the dissipative subrange, an increase in energy dissipation rate will result in an increase in radar echo power since the exponential term will overpower the power term. Thus considering equations (9) and (11) we conclude that there will be a negative P/C correlation if the radar Bragg wavelength is within the dissipative subrange of the turbulent spectrum.

DISCUSSION

In the previous section it was pointed out that turbulent theory as presented by TATARSKII (1971), and TENNEKES and LUMLEY (1972) can easily explain both positive and negative P/C correlation in VHF radar data provided the source of the refractive index variations (N) is a constant independent of the turbulent energy dissipation rate. The equations were developed on the assumption of isotropic and homogeneous shear layer turbulence.

RASTOGI and BOWHILL (1976b) have estimated that the Jicamarca VHF radar operates at a Bragg wavelength (3 m) that is within the dissipative subrange throughout the mesosphere, and recent comparison between Jicamarca radar data and rocket electron-density data (ROYRVIK and SMITH, 1984) have shown that this is true for a turbulent layer located at an altitude of 86 km. However, Royrvik and Smith also noted that the inertial subrange of the spectrum extended to higher wave numbers than estimated by RASTOGI and BOWHILL (1976b). Thus we would expect a negative P/C correlation in the altitude region around 85 km as observed by several investigators (HARPER and WOODMAN, 1977; COUNTRYMAN and BOWHILL, 1979; FUKAO et al., 1980, and ROYRVIK, 1983).

In order to determine if the positive P/C correlation observed in the lower mesosphere (RASTOGI and BOWHILL, 1976b; COUNTRYMAN and BOWHILL 1979, FUKAO, et al. 1980 and ROYRVIK, 1983) is caused by the radar Bragg wavelength being in the inertial subrange of the irregularity spectrum, we have recalculated the range of the inner scale of turbulence that can be expected in the mesosphere. In doing so we have calculated the turbulent energy dissipation rate (ϵ) from the signal correlation time; and the inner scale of turbulence η from ϵ and the kinematic viscosity ν taken from the U.S. Standard Atmosphere model. Outer limits on correlation time have been estimated from data available from the Jicamarca radar (RASTOGI and BOWHILL, 1976b; HARPER and WOODMAN, 1977;

COUNTRYMAN and BOWHILL, 1979; FUKAO et al., 1980 and ROYRVIK, 1983). The resulting limits on the inner and outer scales of turbulence have been plotted in Figure 1. It appears that for strong turbulence the Jicamarca radar Bragg wavelength will be in the inertial subrange of turbulence below about 70 km, and positive P/C correlation should be expected for this altitude range at least part of the time.

This result is only partly satisfying since positive P/C correlation has been observed as high as 75 km in a region where $k_\eta < k_i$. It should be noted, however, that equations (3) and (4) are valid only for $k_\eta \ll k_i$ and the changeover from negative to positive P/C correlation may occur not at $k_\eta = k_i$ but at $Ak_\eta = k_i$ where A is a factor somewhat larger than 1. Another possible source for the discrepancy is that the kinematic viscosity given by the U.S. Standard Atmosphere is somewhat overestimated and thus has reduced the estimate of k_η in the lower mesosphere.

Additional support for this theory comes from HF partial-reflection data from the 2.6-MHz Urbana radar. Correlation between scattered power and signal correlation time is positive throughout the D region as can be seen in Figure 2. This is as expected of turbulent scattering since the Bragg wavelength for this radar (~56 m) is within the inertial subrange at least up to 90 km. The situation has been summarized in Figure 3 which shows the spectra of irregularities for two different energy dissipation rates.

So although there is a small discrepancy between observations and the calculations in this paper we feel justified in concluding that the turbulent model as described here correctly predicts positive and negative P/C correlation in the lower and upper mesosphere, respectively.

The model of turbulent scattering of radio waves suggested here is very similar to a model suggested by BOLGIANO (1968) in which very strong turbulence results in a layer of nearly uniformly mixed refractive index bordered by two sharp ledges. Very little is known about the shape of these ledges; however,

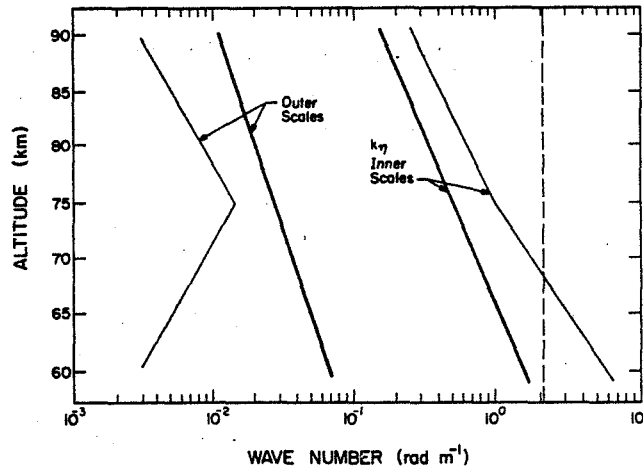


Figure 1. Profiles of inner and outer scales of turbulence in the mesosphere for two extremes of energy dissipation rates. The maximum and minimum energy dissipation rates were calculated from the maximum variation in signal correlation time at each altitude.

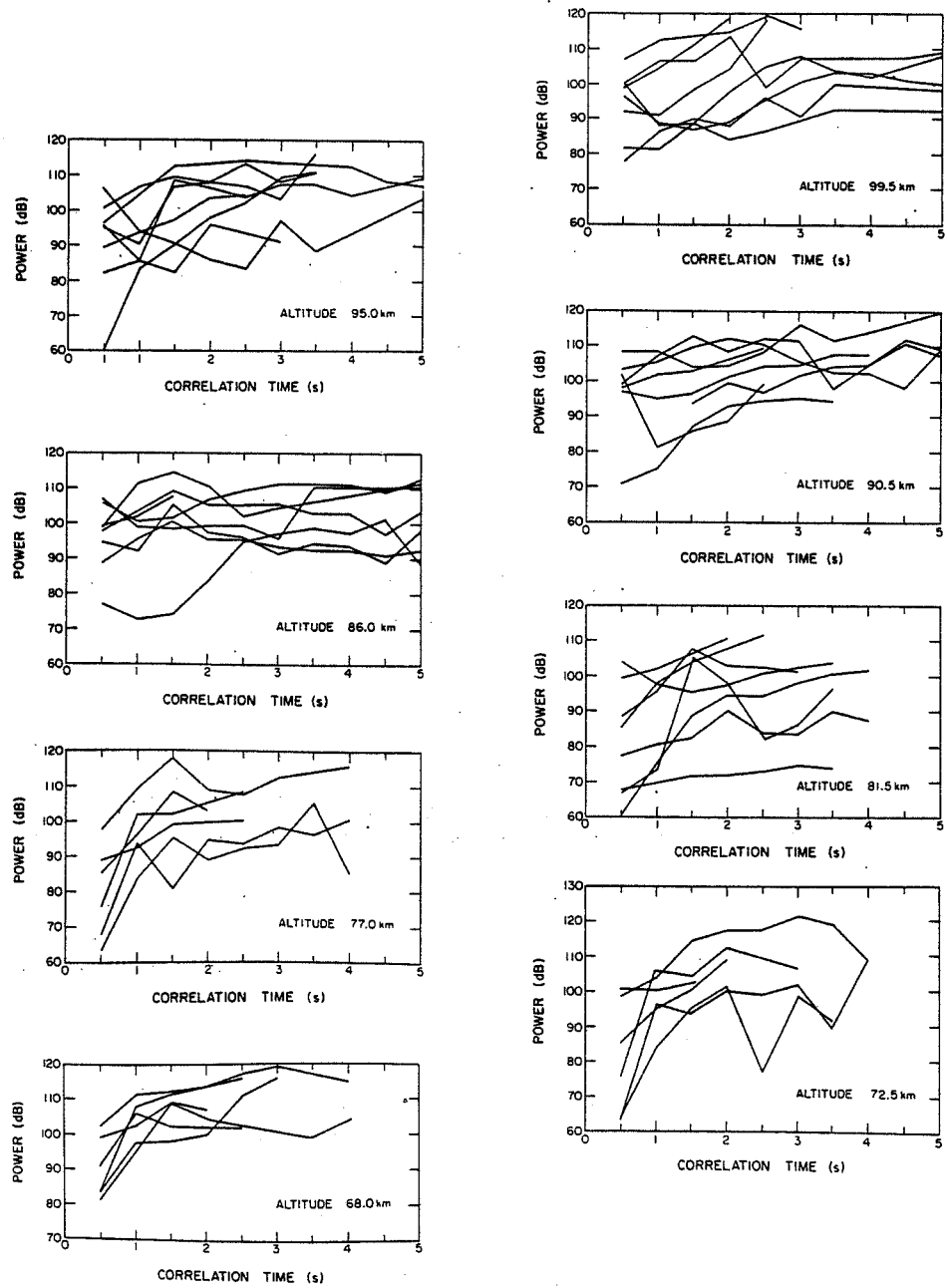


Figure 2. Correlogram of echo power versus signal correlation time for the Urbana 2.6-MHz partial-reflection radar for data obtained on 24-26 April 1982. Each curve represents from four to seven hours of data.

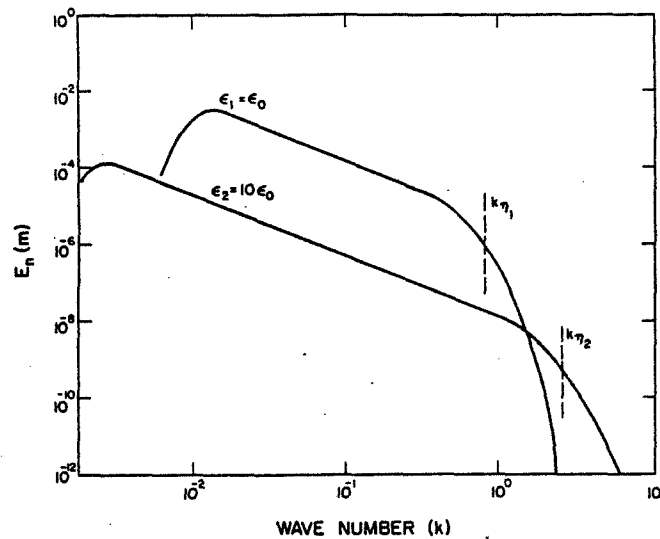


Figure 3. Schematic representation of refractive index irregularity spectra for two different energy dissipation rates.

it seems reasonable to assume that at Bragg wavelengths equal to, or smaller than, the inner scale of turbulence no additional radar echo will be received (ROTTGER et al., 1979). On the other hand if the Bragg wavelength is a substantial fraction of the outer scale of turbulence these ledges may cause partial reflection that come in addition to the scattering from the turbulent layer itself. This may be the reason for observations made at MF/HF frequencies by HOCKING and VINCENT (1982) and others.

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