4.2D ON THE SPECTRUM OF ATMOSPHERIC VELOCITY FLUCTUATIONS SEEN BY MST/ST RADAR AND THEIR INTERPRETATION

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ABSTRACT

We consider the observations of the spectrum of atmospheric motions over the range of periods from a few minutes to many hours that have been made with ST/MST radars in the past five years. This range of periods includes the periods associated with buoyancy waves and the scale of atmospheric motions often referred to by meteorologists as the mesoscale. We consider both the spectra of horizontal and vertical velocities and examine their interpretation in terms of buoyancy wave theory and turbulence theory. To help in interpreting these spectra we present some recently determined aircraft wave number spectra.

INTRODUCTION

In recent years it has become possible to observe continuously the atmospheric wind field above a particular location using clear-air Doppler radars. These radars are referred to here as ST/MST radars although they are increasingly being called "wind profilers" by meteorologists. Several studies have been made of wind variability in the troposphere and stratosphere using the Sunset and Platteville ST radars in Colorado, the Poker Flat MST radar in Alaska and the SOUSY MST radar in the Federal Republic of Germany, besides brief campaigns elsewhere.

Time-series analysis of wind records measured by clear-air Doppler radar have yielded spectra of the horizontal and vertical winds. The frequency spectra of the horizontal wind typically possess an $f^{-5/3}$ dependence while the frequency spectra of the vertical velocity is rather flat with a sharp fall-off at periods below the Brunt-Vaisala period. Although it is fairly clear that the vertical velocity spectra are due to waves, the horizontal velocity spectra have been attributed to quasi-two-dimensional turbulence by some authors and to a spectrum of internal (buoyancy) waves by others.

In order to explore the nature of the radar-observed spectrum of horizontal wind we make use of velocity spectra in the wave number domain obtained recently from spectral analysis of a large set of aircraft winds obtained during the Global Air Sampling Program (GASP). In addition to the GASP horizontal wind spectra we also make use of the temperature spectra obtained from the GASP data set. As a final diagnostic tool we compare the consistency of the horizontal velocity spectra with the radar-observed vertical velocity spectrum under the hypothesis that both spectra are due to a common spectrum of internal waves.

SPECTRA OBSERVED BY ST/MST RADARS

During the past five years several authors have examined the spectral properties of the atmospheric wind field using ST/MST radars. GAGE and CLARK (1978) studied the wind variability observed by the Sunset radar during a jet stream passage. ROTTGER (1981) presented a study of stratospheric wind variability using the spaced-antenna technique. BALSLEY and CARTER (1982) and LARSEN et al. (1982) presented spectra determined from wind measurements of the Poker Flat radar. The Balsley-Carter spectra for zonal velocity at 8 km and 86 km are reproduced in Figure 1. They show a clear $f^{-5/3}$ dependence down to the smallest scales measured after correction for contamination by vertical velocities near the Brunt-Vaisala period.

The spectrum of vertical velocity can similarly be determined from time-series analysis of radar-measured vertical velocities. RÖTTGER (1981) presented the first spectra of vertical velocity. Vertical velocity spectra were subsequently determined from an array of vertically looking ST radars used in France during ALPEX (BALSLEY et al., 1983; ECKLUND et al., 1983). Figure 2 contains the vertical velocity spectra determined from the ALPEX data together with a vertical velocity spectrum determined from Poker Flat data. As we shall show below, the spectrum of vertical velocity is useful in the quest to differentiate waves from turbulence.

INTERPRETATION

In this section we briefly summarize the theoretical arguments which form the basis of the two interpretations which have been offered to explain the spectra presented here.

(a) Quasi-Two-Dimensional Turbulence

The general theory for two-dimensional turbulence has been presented by KRAICHNAN (1967) and KRAICHNAN and MONTGOMERY (1979). In the atmosphere, of course, turbulence is not strictly two-dimensional as pointed out by CHARNEY (1971). According to Charney energy should be distributed equally between the

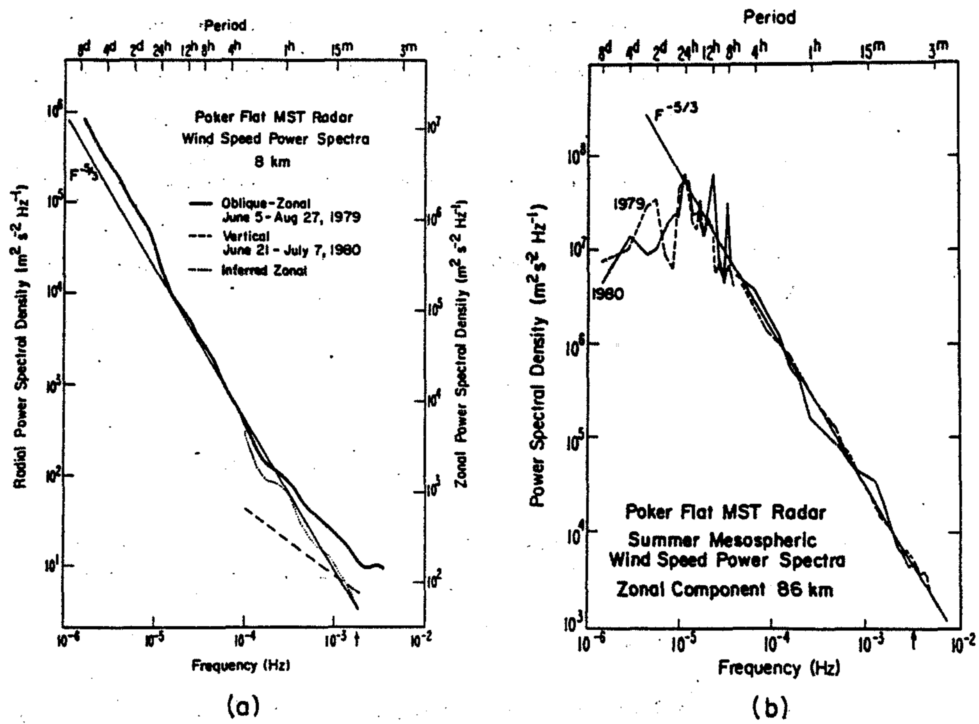

Figure 1. Power spectrum of the zonal wind observed at Poker Flat, Alaska, radar: a) 8 km and b) 86 km (BALSLEY and CARTER, 1982).

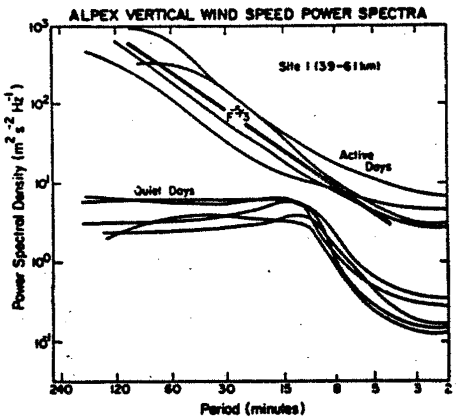

Figure 2. Power spectra of vertical velocity for quiet and active periods observed in southern France during ALPEX (BALSLEY et al., 1983).

two components of horizontal kinetic energy (zonal and meridional) and potential energy. Kraichnan showed that two inertial ranges are pertinent to turbulence in two dimensions: a k^{-3} enstrophy cascading range at scales smaller than the scale of energy insertion and a $k^{-5/3}$ reverse energy cascading inertial range at scales larger than the scale of energy insertion. GAGE (1979) suggested that this theory of turbulence might explain the observed mesoscale spectra if there were two sources of atmospheric turbulence kinetic energy: a large-scale one and a small-scale one as shown schematically in Figure 3. LILLY (1983) explored these ideas in more detail and concluded that only a few percent of small-scale energy would need to be reverse cascaded to larger scales in order to explain the observed mesoscale spectra.

The turbulence theory invokes dimensional arguments to determine the spectral shape within the two inertial ranges. For the enstrophy cascading range the spectrum has the form

$$E(k) = \alpha_1 \eta^{2/3} k^{-3} \qquad (1)$$

where η is the enstrophy cascade rate and α_1 is a universal constant. For the reverse energy cascading range

$$E(k) = \alpha_2 \left(\frac{dE}{dt}\right)^{2/3} k^{-5/3} \qquad (2)$$

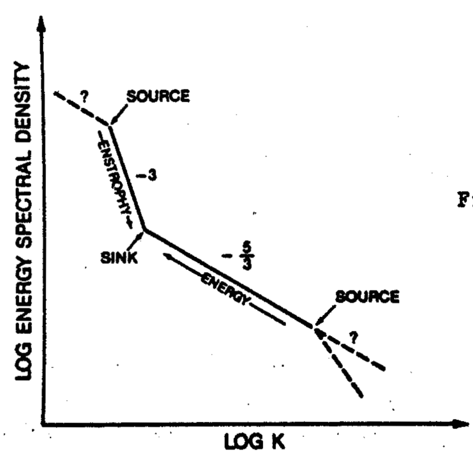

Figure 3. Schematic two-dimensional turbulence spectrum (LARSEN et al., 1983).

where dE/dt is the rate of energy insertion at the small-scale source and α_2 is also a universal constant. While the turbulence theory is formulated in the spatial domain, spectral shape of frequency and wave number spectra should be identical to the extent that the Taylor transformation is valid, as discussed below.

(b) Internal Wave Spectra

The development of a dynamical theory for a spectrum of internal waves has proceeded rapidly over the past decade. GARRETT and MUNK (1972, 1975) pioneered this effort by showing how diverse observations of ocean spectra could be given a unified treatment by hypothesizing that they were all due to different sampling strategies of a common spectrum of internal waves. A central element of the theory is that the waves obey the dispersion relation

$$\left(\frac{\omega}{N}\right)^2 = \frac{k_H^2}{k_H^2 + k_z^2} \qquad (3)$$

In Equation (3) k_H and k_z are horizontal and vertical wave numbers, ω is the angular frequency ($\omega \equiv 2\pi/\tau$; where τ is the period) and N is the Brunt-Vaisala or buoyancy frequency ($N^2 = g/\theta\ \partial\theta/\partial z$; where g is gravitational acceleration and θ is potential temperature and z is the vertical coordinate). This dispersion relation dictates the relative contribution of wave energy to vertical and horizontal velocity components as a function of frequency. Briefly, the u-, v-, and w-spectra are related through polarization relations and each is a universal function of frequency when scaled by the Brunt-Vaisala frequency N. Kinetic energy is supposed to be distributed equally between the two horizontal velocity components and the horizontal kinetic energy is supposed to equal the potential energy.

As the theory has been developed for the ocean, and ω^{-2} spectrum is pertinent to horizontal velocities and temperature or vertical displacement. An ω° spectrum for vertical velocity with sharp cutoffs at inertial and buoyancy frequencies is consistent with this picture. The k_x and k_z spectra are related to the above through the dispersion relation. In particular, k_x spectra have a k^{-2} power law dependence out to a transition wave number k_* and beyond that a $k^{-2.5}$ dependence.

The theory of Garrett and Munk depends upon the empirical fitting of observed spectra. While this was done successfully for the ocean and has been tested extensively, only recently has a serious attempt been made to extend the theory to the atmosphere. VANZANDT (1982) showed that it was possible to extend the internal wave description of velocity spectra to the atmosphere with only apparently minor modifications. The modifications required included a change in spectral slope from -2 for the ocean to -5/3 for the atmosphere.

WAVE NUMBER SPECTRA FROM GASP AIRCRAFT DATA

As mentioned above, a complete description of atmospheric fluctuation spectra requires analysis in the spatial domain as well as the temporal domain. In many ways the internal wave spectra are best described in the temporal domain while turbulence theory is predicted on a spatial description. Within the context of turbulence theory it is conventional to relate spatial and temporal properties through a Taylor transformation. This transformation simply assumes that all scales of turbulence fluctuations move with the same advection velocity. This aspect of the subject has been explored in some detail by BROWN and ROBINSON (1979). These authors were able to show the validity of the Taylor transformation on scales of order 500-1000 km from an examination of eastern European rawinsonde data.

The GASP data permit an examination of atmospheric spectra over scales ranging from a few km to nearly 10,000 km. Thus, there is an overlap of scale sizes in the GASP wave number spectra with earlier large-scale spectral analyses e.g., CHEN and WIIN-NIELSEN (1978), BOER and SHEPHERD (1983). The data collection phase of GASP was conducted during 1975-1979, with meteorological and trace constituent data automatically recorded with instruments placed aboard Boeing 747 airliners in routine commercial service. Wind data were taken from the onboard computer, which was linked to the inertial navigation system, and have a random error of five percent of the reported value. There are 6945 flights in the GASP data set, with over 0.6 million observations. All GASP data are archived at the National Climatic Center, Asheville, North Carolina.

Figure 4 shows the spectra of zonal and meridional winds over the range of wavelengths from 150 to 4800 km. Tropospheric spectra and stratospheric spectra are contained in Figure 4a and 4b, respectively. These spectra were obtained by analysis of the longest flights at constant altitude in the GASP data set. GASP data were recorded at five minute intervals (75 km intervals at a nominal ground speed of 250 ms^{-1}) at all times during flights above 6 km; about 80 percent of the data fall in the altitude range 9-13 km.

Several features of the wave number spectra of Figure 4 are worth noting. At wavelengths larger than 500-700 km the spectral slope approaches k^{-3}. At wavelengths less than about 500 km the spectral slope is close to $k^{-5/3}$. The spectral amplitude is nearly the same for the zonal and meridional components of velocity and the magnitude of the spectra vary little between the troposphere and the stratosphere. A more detailed analysis of the GASP data, including a limited sample of high-resolution data, has recently been prepared (NASTROM and GAGE, 1984).

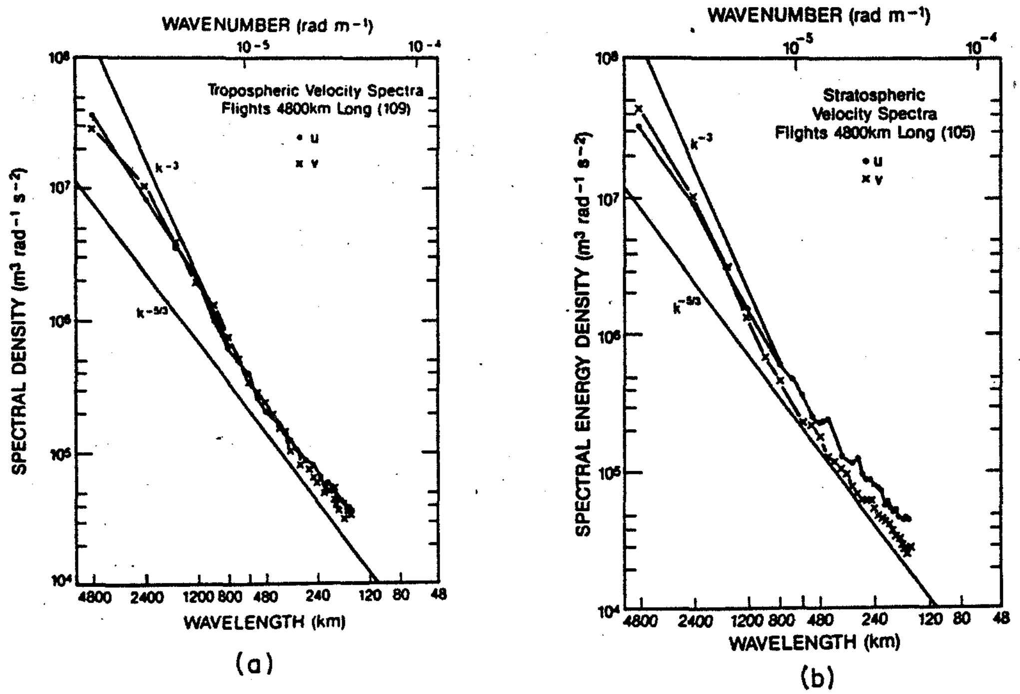

Figure 4. Horizontal velocity spectra determined from GASP flights at least 4800 km long: a) troposphere and b) stratosphere.

COMPARISON OF WAVE NUMBER SPECTRA DETERMINED BY AIRCRAFT WITH FREQUENCY SPECTRA DETERMINED BY RADAR

The radar-observed frequency spectra of horizontal velocity can be compared with aircraft-observed wave number spectra by applying the Taylor transformation to the frequency spectra. Such a comparison is shown in Figure 5 which has been adapted from LILLY and PETERSEN (1983). In this figure the Taylor-transformed Balsley-Carter spectrum is compared with the GASP wave number spectrum of Figure 4. Throughout the atmospheric mesoscale the two spectra agree very well, both exhibiting a -5/3 slope. While the agreement seen here is entirely consistent with expectations for turbulence it does not rule out the possibility the spectrum is comprised of internal waves or due to some other process. Indeed, the shape and magnitude of these spectra are in reasonable accord with the model internal wave spectra of VANZANDT (1982).

COMPARISON OF GASP KINETIC AND POTENTIAL ENERGY SPECTRA

According to turbulence theory (CHARNEY, 1971), there should be an equipartitioning between each of the two components of horizontal velocity and potential energy. That is, for turbulence, kinetic energy should be about twice the potential energy. For a spectrum of waves there should be equipartitioning between the total kinetic energy and potential energy. An examination of the ratio of kinetic to potential energy therefore can be used to help to differentiate between waves and turbulence.

The potential energy spectrum $\Phi_{P.E.}$ is related to the temperature spectrum $\Phi_{\theta\theta}$ by

$$\Phi_{P.E.} = \frac{N^2}{\left(\frac{\partial\theta}{\partial z}\right)^2} \cdot \frac{1}{2}\,\Phi_{\theta\theta} = \frac{g^2}{2N^2\theta^2}\,\Phi_{\theta\theta} \qquad (4)$$

Wave number spectra of potential temperature from GASP data are presented in Figure 6. Note that the magnitude of the potential temperature spectrum is significantly larger in the stratosphere than in the troposphere. The reason for this is the larger hydrostatic stability in the stratosphere.

The potential temperature spectra contained in Figure 6 have been converted to potential energy spectra by use of Equation (4). The factor $g^2/2N^2\theta^2$ used to convert tropospheric and stratospheric spectra were 4 and 1.5, respectively. Potential energy spectra obtained in this way are compared with

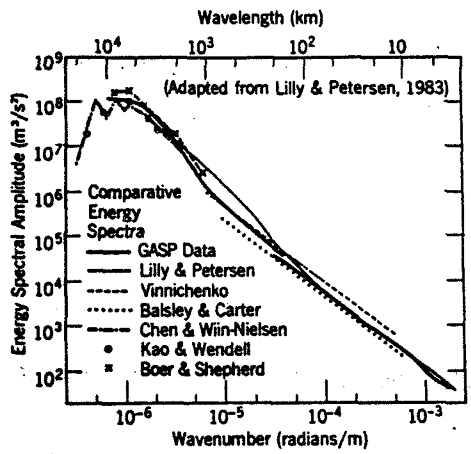

Figure 5. Composite horizontal energy spectra showing comparison of aircraft wave number spectra with Taylor-transformed radar frequency spectrum (adapted from LILLY and PETERSEN, 1983).

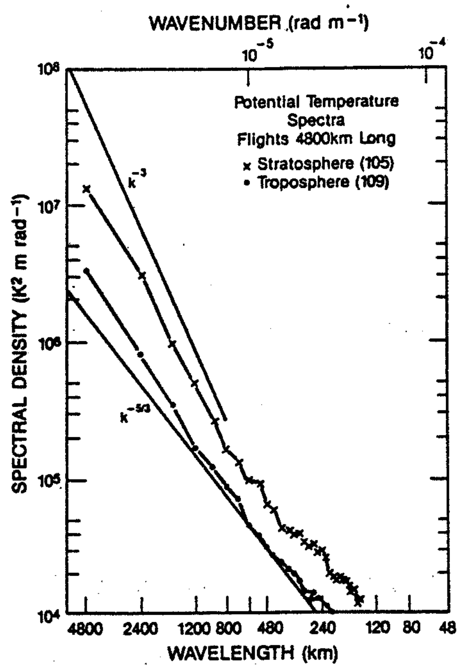

Figure 6. Potential temperature spectra in the troposphere and stratosphere from GASP aircraft flights at least 4800 km long.

the kinetic energy spectra obtained on the same flights in Figure 7. For both the troposphere and the stratosphere the comparisons in Figure 6 show a ratio of potential to kinetic energy closer to 2 than 1 over much of the spectrum. While this result favors the turbulence interpretation, it cannot be considered conclusive because of the uncertainty in the value of $g^2/2N^2\theta^2$ used to convert the potential temperature spectrum to a potential energy spectrum (GAGE and NASTROM, 1984).

COMPARISON OF RADAR OBSERVED HORIZONTAL AND VERTICAL VELOCITY SPECTRA

Most of this paper has been concerned with the interpretation of the horizontal velocity frequency spectrum observed by radar in the light of a recent study of wave number spectra obtained from aircraft observations. These comparisons have shown that the observed spectra are quite similar in the wave number and frequency domains. While these results support the turbulence interpretation they do not rule out the wave interpretation. In this section we compare the horizontal and vertical velocity spectra observed by radar to determine their consistency with the idea that these spectra are both due to internal waves. From the comparison of horizontal velocity spectra presented in Figure 5 it can be seen that these spectra are fairly universal. Not so much is known about the universality of the vertical velocity spectra although they would necessarily be fairly universal if both the horizontal and vertical spectra were due to waves.

Before examining the consistency of atmospheric spectra of horizontal and vertical velocities, it is useful to review the situation in the ocean where both horizontal and vertical spectra are thought to be due to internal waves. Figure 8 contains a comparison of vertical velocity spectra and horizontal velocity spectra from the ocean (ERIKSEN, 1978). These spectra are fairly universal in accord with the Garrett-Munk theory. The horizontal spectrum $E_H(\omega)$ can be expressed in terms of the vertical spectrum $E_V(\omega)$ using the model equation (ERIKSEN, 1978):

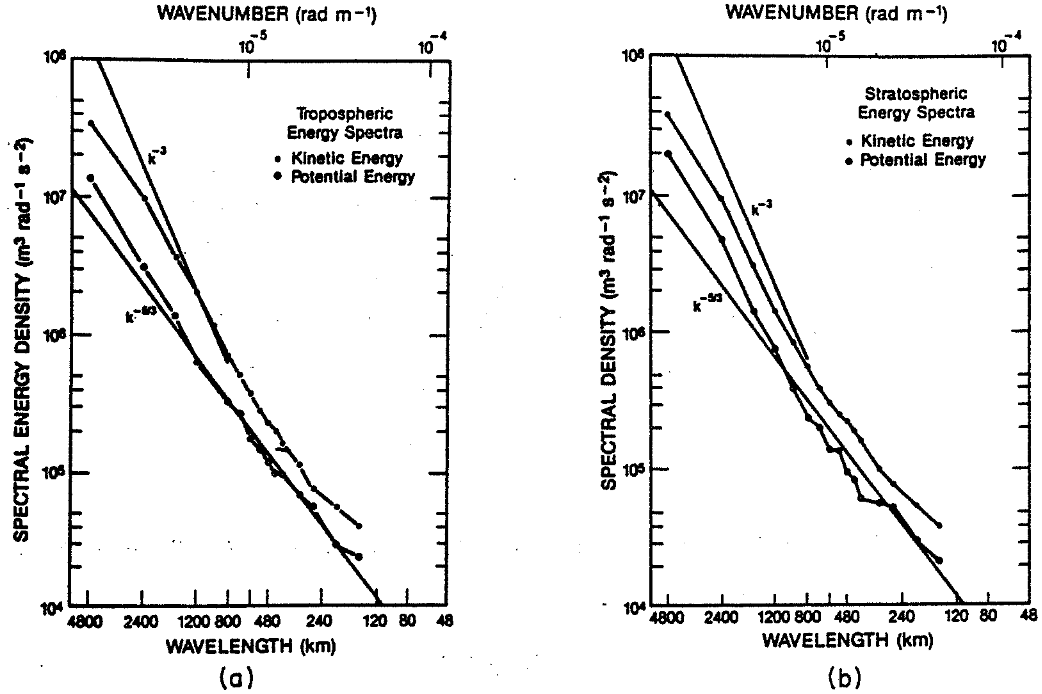

Figure 7. Kinetic and potential energy spectra from GASP aircraft flights at least 4800 km long: a) troposphere and b) stratosphere.

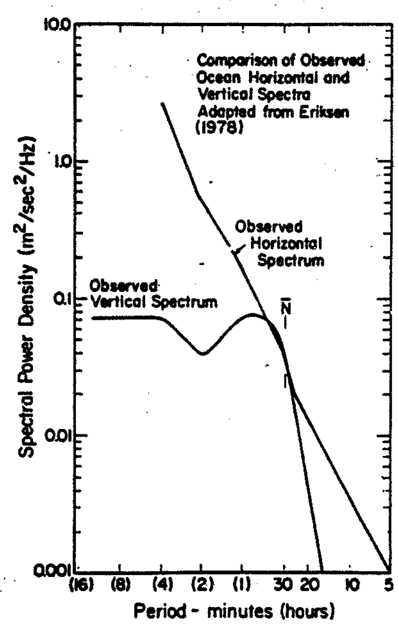

Figure 8. Internal wave spectra of horizontal and vertical velocity in the ocean (adapted from ERIKSEN, 1978).

$$E_H(\omega) = \left(\frac{N^2 - \omega^2}{\omega^2}\right) \left(\frac{\omega^2 + f^2}{\omega^2 - f^2}\right) E_V(\omega) \qquad (5)$$

where f is the coriolis frequency [$f \equiv 2\Omega \sin$ (latitude); Ω is the earth angular velocity].

Figure 9 contains a comparison of horizontal and vertical spectra observed by the radar. Starting with the vertical spectrum the dashed curve determines the location of the horizontal internal wave spectra that would be consistent with the polarization relations. The observed atmospheric spectra of horizontal velocity are shown for comparison. The fact that the magnitude of observed spectra exceeds the magnitude of the calculated wave spectra, and that the slope of the observed horizontal spectra differs significantly from the calculated wave, spectra strongly suggests that different processes are responsible for the horizontal and vertical spectra. Of course, this analysis has been made with horizontal and vertical spectra taken at different locations. To the extent that the spectra are universal this does not matter. However, the analysis should be repeated with simultaneous, co-located determinations of horizontal and vertical velocity spectra.

CONCLUDING REMARKS

In this paper we have examined the nature of the horizontal velocity spectra observed by MST/ST radar. By comparison with the aircraft-determined wave number spectra we conclude that the radar and the aircraft observe the same spectrum of atmospheric motions. Furthermore, the magnitude and shape of the Taylor-transformed radar spectra are comparable to the magnitude and shape of the aircraft wave number spectra. Finally, these spectra are in reasonable accord with expectations from quasi-two-dimensional turbulence. An examination of kinetic and potential energies deduced from the aircraft spectra shows spectral energy is partitioned equally between the two horizontal components of velocity and potential energy as expected for turbulence theory.

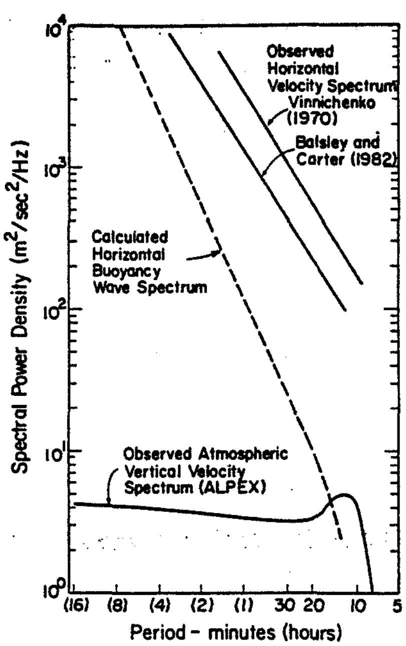

Figure 9. Comparison of observed horizontal velocity spectra in the atmosphere with horizontal spectra of internal waves (dashed line) determined from Equation (5) and observed vertical velocity spectrum.

A comparison of the energy levels and shapes of horizontal and vertical velocity spectra show that the magnitude of atmospheric spectra of horizontal velocity are too large to be consistent with the magnitude of the vertical velocity spectrum, assuming both spectra are due to waves. The spectral slope of the horizontal velocity spectrum is also inconsistent with the slope anticipated for a spectrum of internal waves given the observed shape of the vertical velocity spectrum.

Taken together these results strongly suggest that the radar spectra of vertical and horizontal velocities are indicative of different processes. They suggest that internal waves and quasi-two-dimensional turbulence are both important in determining the observed velocity spectra. The MST/ST radar observations should help in sorting out the complementary roles of waves and turbulence in middle atmospheric dynamics.

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